

Microscopic pairing in fission description

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Workshop on Fission Dynamics 2026, 11-15 May 2026



1. Motivation: Relevance of spontaneous fission

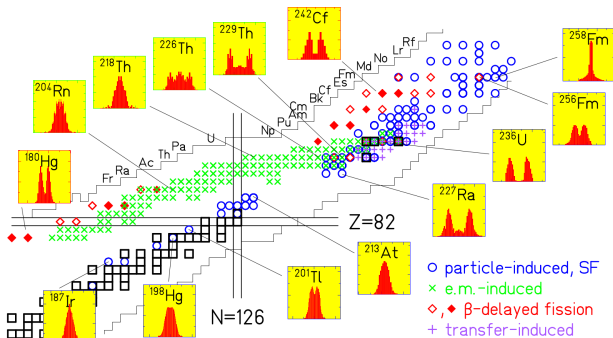
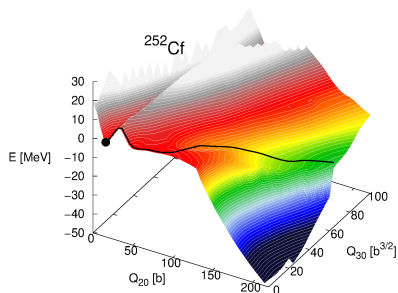


Fig. 1. K.H. Schmidt, B. Jurado *Rep. Prog. Phys.* **81** 106301 (2018).

Fission observables as a benchmark:

- **Half-lives:** SHE experiments' preparation & measurements analysis
- Mass/Charge yields
- TKE distributions
- Neutron multiplicities

1. Motivation: Spontaneous fission half-lives. Least-energy approximation.



- Least-energy integral:

$$S = 2 \int_{s_1}^{s_2} \sqrt{\frac{2}{\hbar} (V(s) - E) B_s(s)} ds$$

- Tunneling probability: $P = [1 + e^S]^{-1}$
- SF half-life: $T_{1/2}^{\text{SF}} = \frac{\ln 2}{P\nu}$

RESULTS: Typically overestimated by 2 orders of magnitude.

Strategies for refinement:

- Triaxial calculations,
- Dynamical treatment,
- The incorporation of non-"geometrical" degrees of freedom

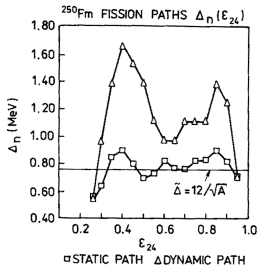
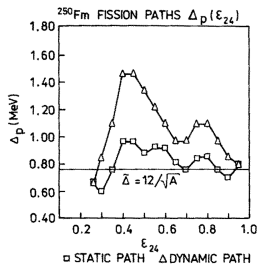
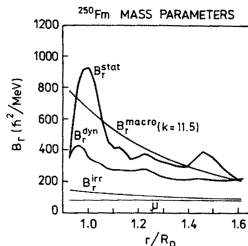
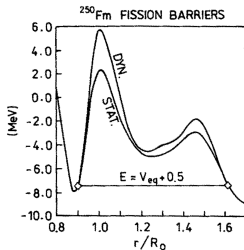
1. Motivation: Role of pairing

Fission half-lives and mass distributions calculations - crucial ingredients:

- Determination of the **least action** path. $S(L) = 2 \int_{s_1}^{s_2} \sqrt{\frac{2}{\hbar} (V(s) - E) B_s(s)} ds$
- An increase in the **pairing** gap parameter leads to an increase in the penetrability of the fission barrier \implies shorter half-lives.*
- Strong interplay between the **potential energy/collective inertia** - increasing/decreasing as an inverse of the pairing gap squared.

*L. Moretto and R. Babinet, Phys. Lett. B 49, 147 (1974).

1. Motivation: Some background



- GCM+GOA collective hamiltonian
- shape + pairing degrees of freedom

1. Motivation: Some background

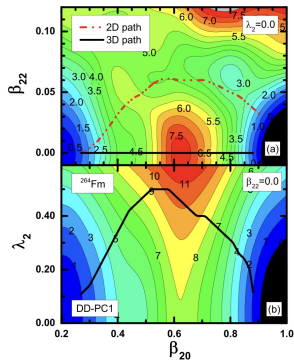
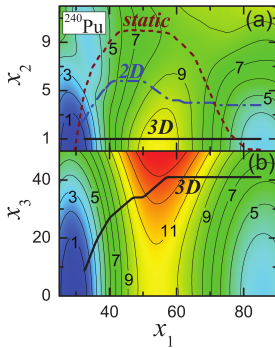
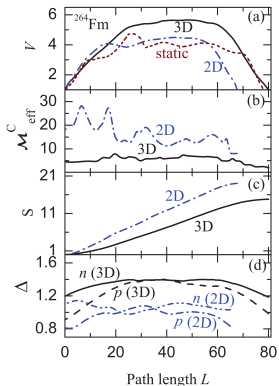
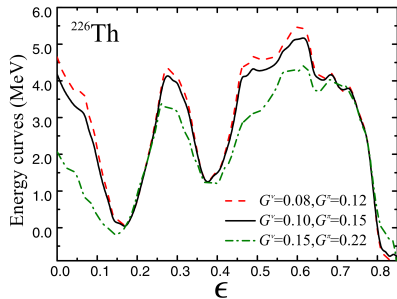


FIG. 3. (Color online) Projections of the three-dimensional (3D) dynamic path (solid curves) for the spontaneous fission of ^{264}Fm on the (β_{20}, β_{22}) plane for $\lambda_2 = 0$ (a), and the (β_{20}, λ_2) plane for $\beta_{22} = 0$ (b), calculated using the dynamic programming method. The dash-dot-dot curve denotes the two-dimensional (2D) path computed without the inclusion of dynamic pairing correlations.

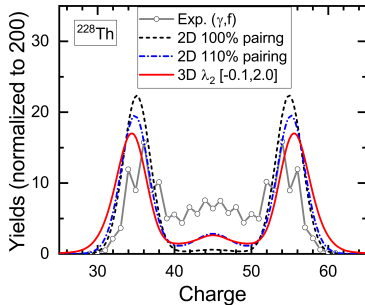
J. Sadhukhan, J. Dobaczewski, W. Nazarewicz, J.A. Sheikh, A. Baran, PRC 90(2014).

J. Zhao, B-N. Lu, T. Niksić, D. Vretenar, S-G. Zhou., PRC 93, 044315 (2016)

1. Motivation: Some background



X. Guan, Y. Xin, Y. Chen, X. Wu, Z. Li, PRC 104 (2021).



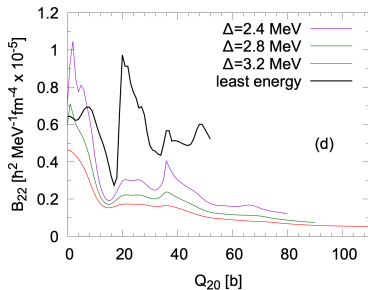
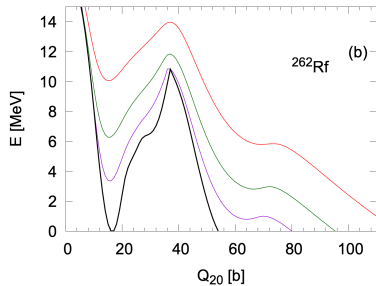
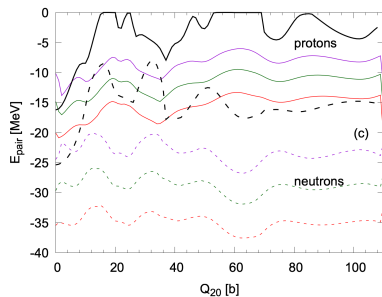
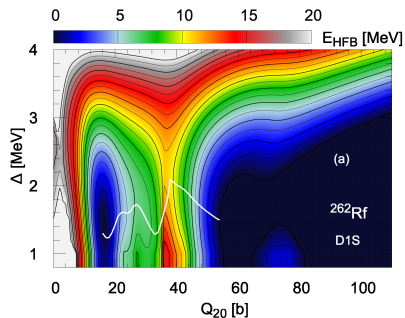
J. Zhao, T. Niksic, D. Vretenar, PRC 104 (2021).

2. Fission description: Microscopic pairing

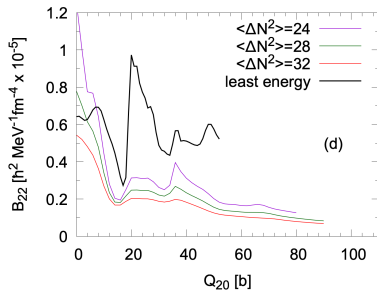
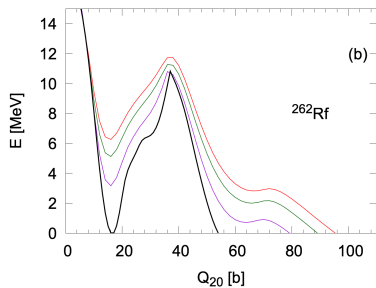
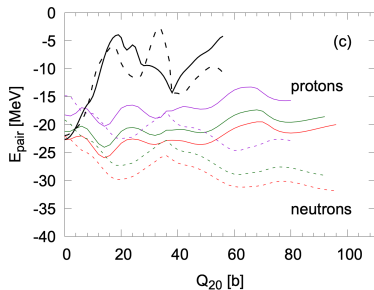
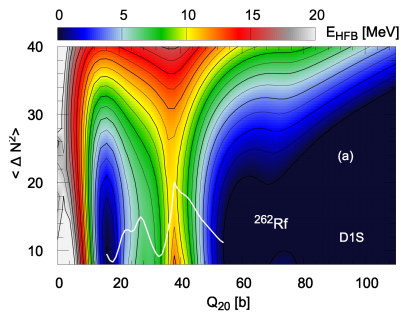
There are 3 ways of studying the effect of pairing correlations in the microscopic fission description by considering:

- the pairing gap operator: $\Delta = G\langle\hat{P}^+\rangle$,
- the particle number fluctuations: $\Delta\hat{N}^2 = \hat{N}^2 - \langle\hat{N}\rangle^2$,
- the quenching factor: QF (renormalization of the HFB pairing field).

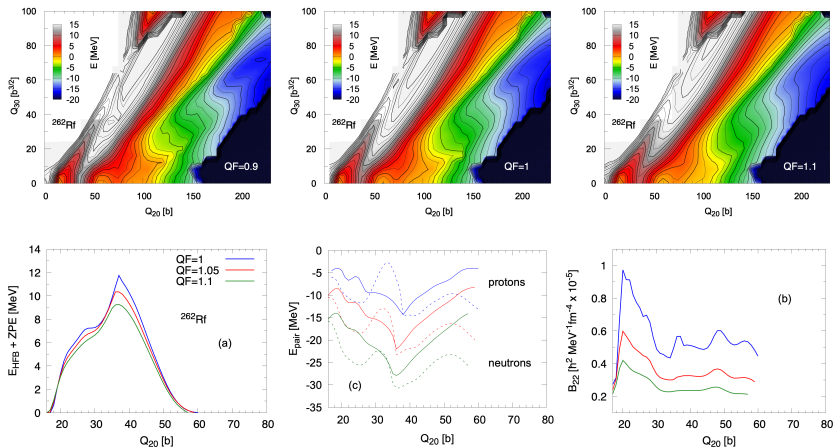
2. Fission description: Pairing gap



2. Fission description: Particle number fluctuations

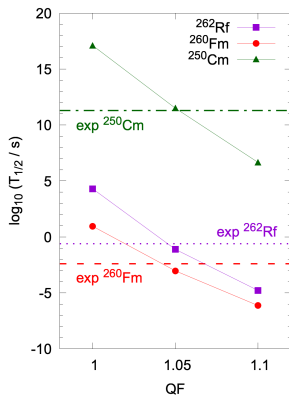
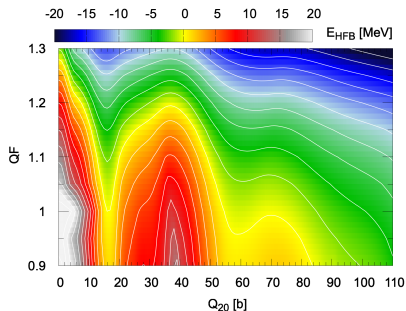


2. Fission description: Pairing strength



A. Zdeb, *et. al.* PRC 111, 044607 (2025).

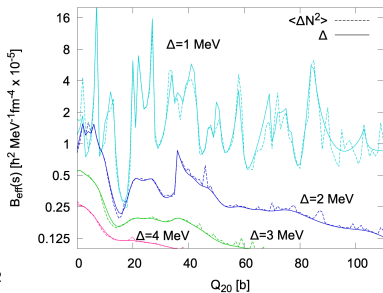
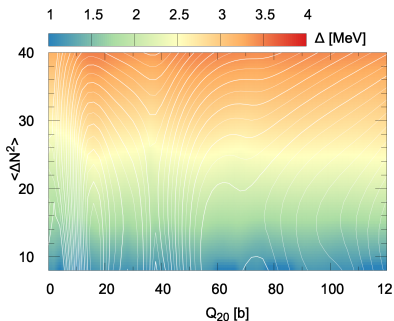
3. Results: Spontaneous fission half-lives - SHE



- ▶ monotonic decrease in the potential energy of the nucleus and a flattening of the fission barrier

A. Zdeb, et. al. PRC 111, 044607 (2025).

3. Results: Collective inertias with constraints on Δ and $\Delta\hat{N}^2$



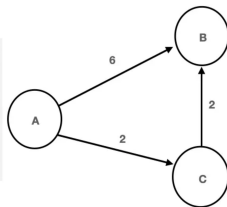
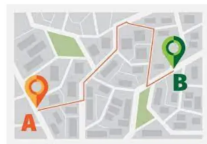
$$B_{\text{eff}}(s) = \sum_{i,j} B_{ij} \frac{dq_i}{ds} \frac{dq_j}{ds}$$

- ▶ Only the quasiparticle one-body part of $\Delta\hat{N}^2$ is considered.
- ▶ Both inertias become more consistent as the value of Δ and $\Delta\hat{N}^2$ increases.

4. Dijkstra's algorithm

The shortest path algorithm used in many real-world applications where we need to find the most efficient route through a network:

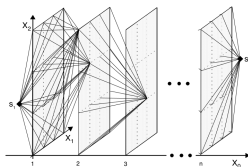
- Computer networks (routing protocols);
- GPS and navigation systems;
- Logistics and transport optimization;
- Telecommunications;
- ...



- Graph representation of the system - nodes with certain weights (positive values);
- The algorithm keeps track of all node's distances from the start node;
- Expands on the node with the current shortest distance from the start;
- Each node is visited only once.

4. Dijkstra's Algorithm vs. Dynamic Programming Method

DPM

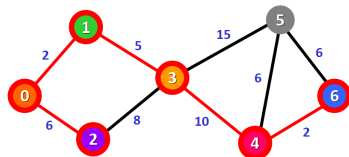


$$\{X_n\} = X_n \oplus \{X_{n-1}\}$$

A. Staszczak, arXiv:nucl-th/9603033v1

- A sequential structure of the problem is assumed: the optimal value for a given point depends only on the preceding points.
- Requires a single “preferred direction”.
- If the PES is complex, with local minima and multiple possible paths in different directions, DP may miss the globally minimal path.

Dijkstra's Algorithm

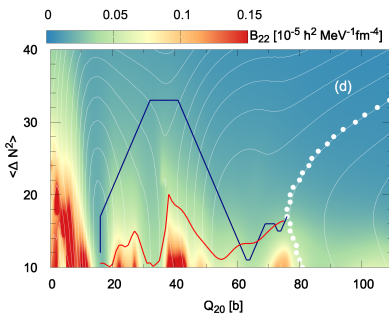
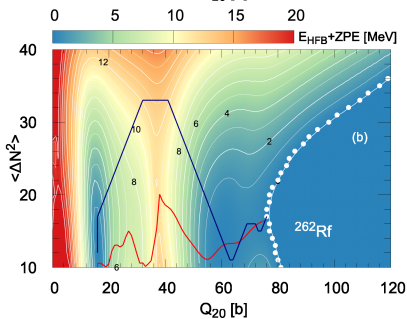
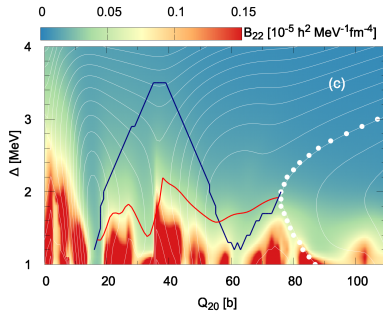
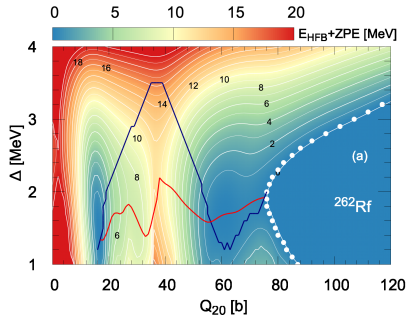


<https://www.freecodecamp.org/news/dijkstras-shortest-path-algorithm-visual-introduction/>

- Finds the shortest path from one node to all other nodes, guaranteeing the global minimum.
- Does not require a single chosen direction.
- The globally minimal path is guaranteed for positive weights (this condition is fulfilled in nuclear physics).

Dynamic programming assumes a sequential progression along a single direction and may fail to detect optimal points if the valley spreads vertically.

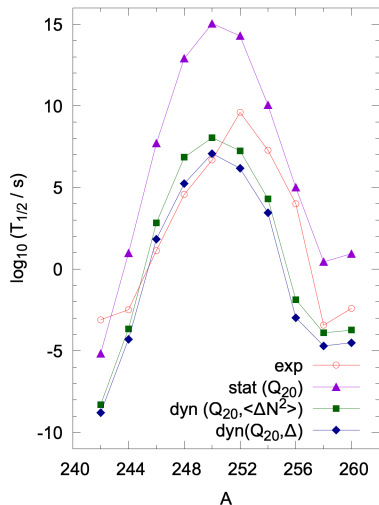
5. Results: Dynamical pairing



Zdeb, et. al. PRC 112, 044602 (2025).



3. Results: Dynamical pairing - Fermium isotopic chain



A. Zdeb, *et. al.* PRC 112, 044602 (2025).

- ▶ Reduction of the spontaneous fission half-lives by 3-8 orders of magnitude when pairing treated dynamically.
- ▶ Possible sources of the underestimations in light isotopes:
 - collective inertias,
 - fixed E_0 parameter value,
 - neglecting higher order multiple moments
 - ...

4. Summary

- The quenching factor is not an appropriate tool to handle pairing correlations.
- The Δ and $\Delta\hat{N}^2$ are proportional to each other and lead to the similar results.
- The one-body approximation of the collective inertias associated with the $\Delta\hat{N}^2$ constraint works well along fission paths of the studied isotopes.
- Considerable improvement in reproducing the spontaneous fission half-lives data is achieved in most cases compared to the static approach.
- Dynamical pairing + higher-order multipole moments seem to be a promising direction for the future research



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