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Ternary quasi-fission and intrinsic angular momentum of fragments in heavy-ion reactions

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May 15, 2026

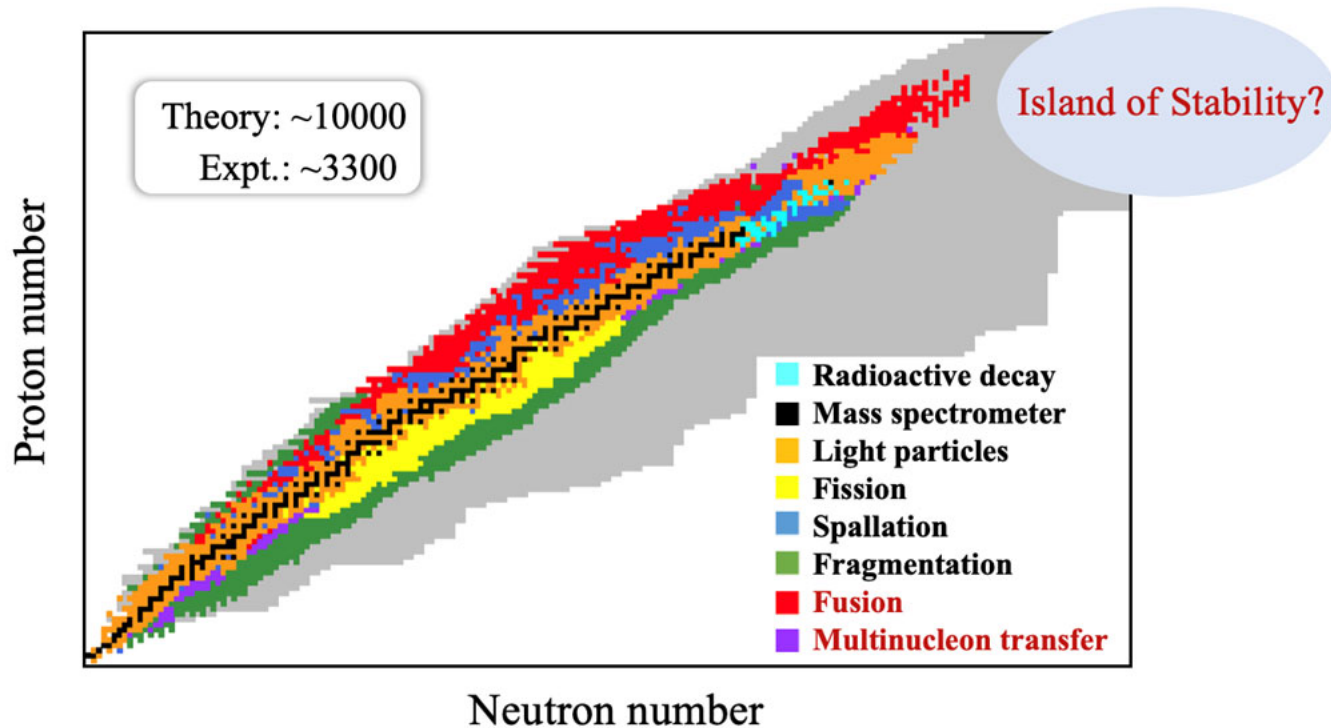
Collaborators: B. Li, J. Meng, T. Nikšić, Z. X. Ren, D. Vretenar, P. W. Zhao

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- ◆ Introduction
- ◆ Relativistic time-dependent density functional theory
- ◆ Ternary quasi-fission
- ◆ Intrinsic angular momentum of fragments in reactions
- ◆ Summary and outlook

Heavy-ion reactions

- Heavy-ion reactions, induced by ions heavier than α particles, are one of the important methods for synthesizing new nuclides.



Thoennesen, *The Discovery of Isotopes* (2016)
Xia et al., *ADNDT* 121, 1 (2018)

- **Fusion**: synthesizing superheavy nuclei (SHN).
- **Multinucleon transfer (MNT) reactions**: synthesizing neutron-rich nuclei.

Jiang, **DDZ**, Zhou. *Physics* 54, 599 (2025)

Fusion: Neutron-deficient superheavy nuclei

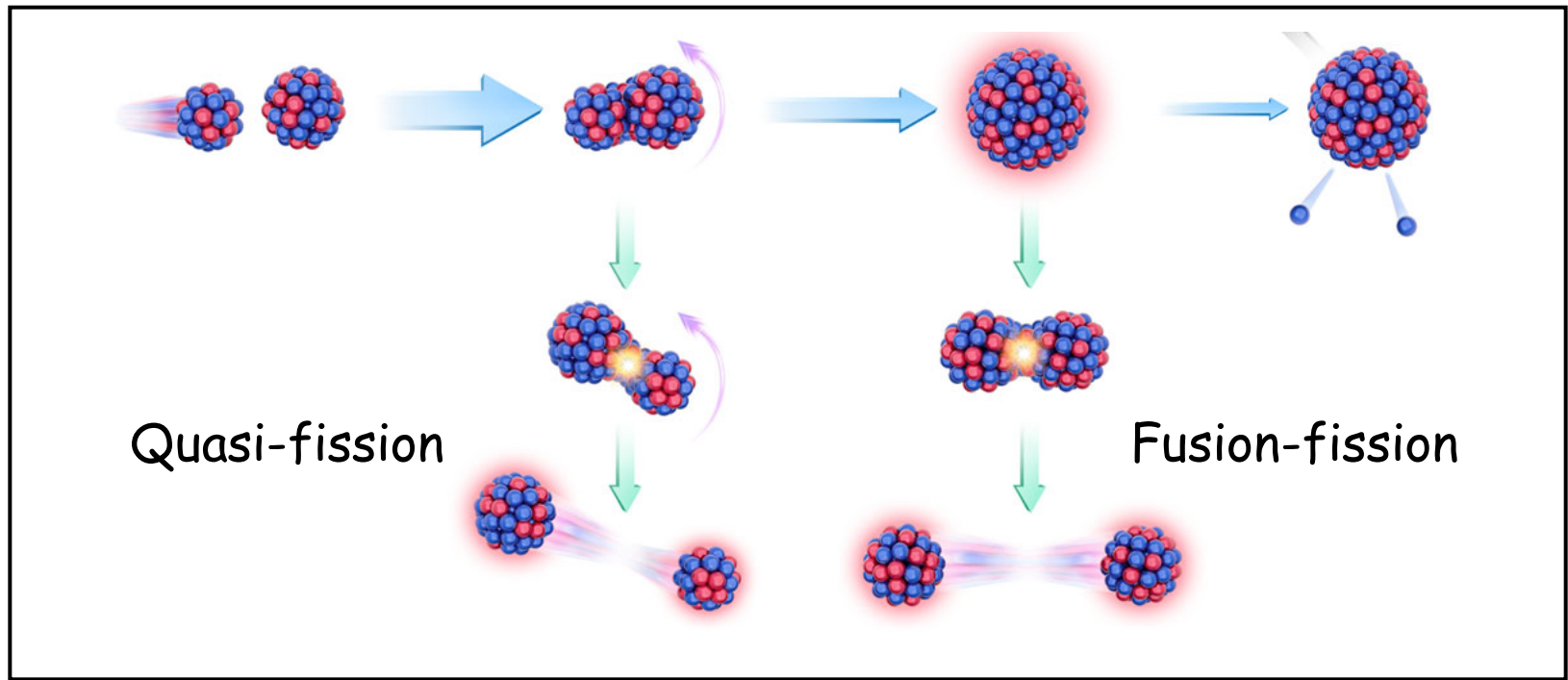
- Fusion is the only successful method for synthesizing SHN to date.
- However, the cross sections drop rapidly with increasing Z , hindering the synthesis of new elements.

Münzenberg et al., EPJA 59, 21 (2023)

Oganessian, EPJA 60, 227 (2024)

Zhang et al., NST 36, 204 (2025)

- ✓ **Quasi-fission** and **fusion-fission** are two main competitive processes.



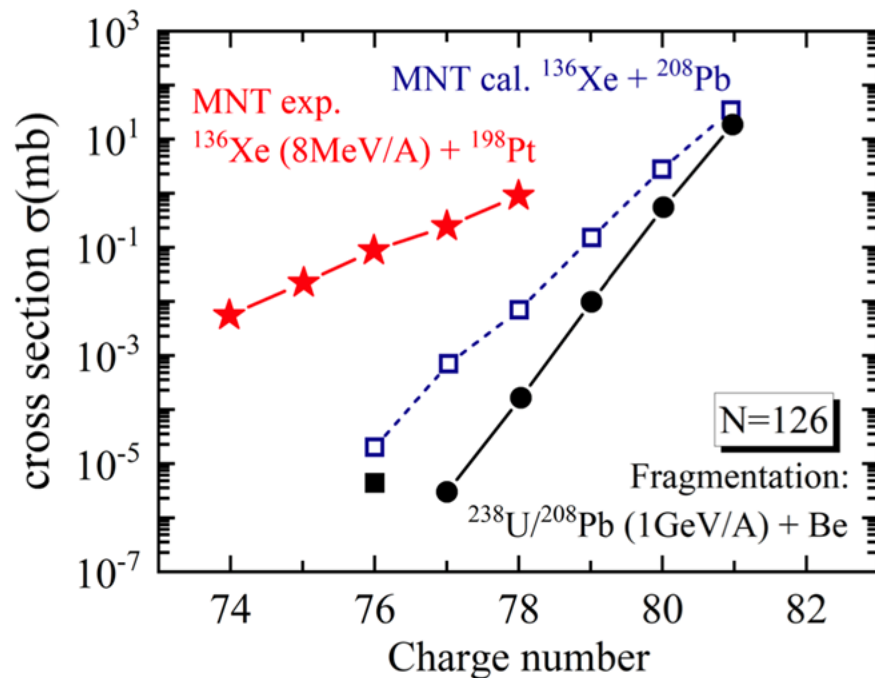
Jiang, DDZ, Zhou. Physics 54, 599 (2025)

MNT: Neutron-rich nuclei

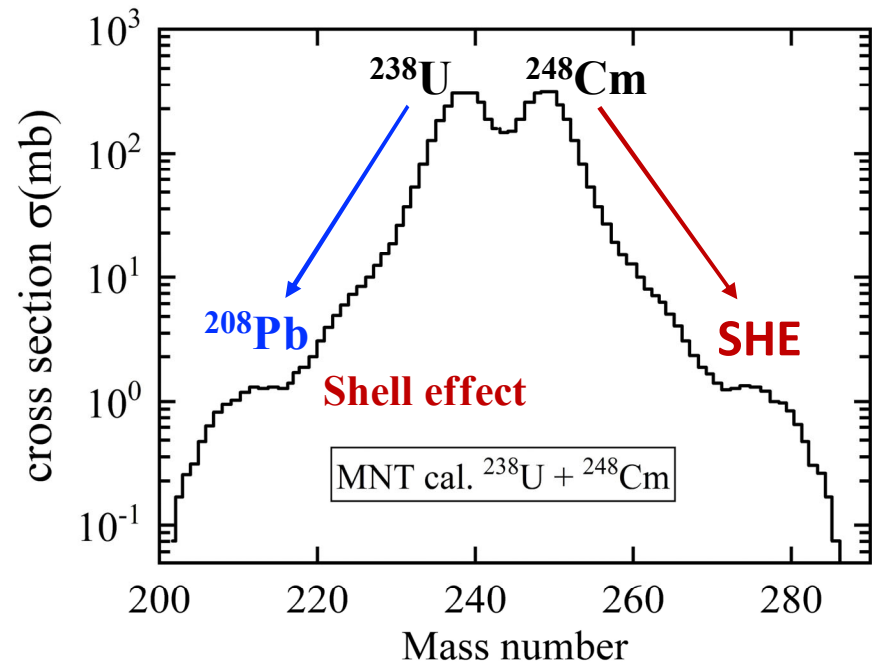
- Recently, the **MNT reaction** has been suggested as a possible pathway for the synthesis of neutron-rich heavy and superheavy nuclei

- ✓ Neutron-rich region around $N = 126$
- ✓ Possible reaction mechanism

Complex mass and angular distributions!



Zagrebaev and Greiner, PRL 101,122701 (2008)
Watanabe et al., PRL 115,172503 (2015)



Zagrebaev and Greiner, JPG 34, 2265 (2007)
Zagrebaev and Greiner, PRC 87, 034608 (2013)

Theoretical models for MNT reactions

■ The multinucleon transfer process is a **complex quantum many-body dynamical problem**. A broad range of theoretical approaches has been developed:

- ✓ Coupled channels Thompson, *Comput. Phys. Rep.* 7, 167 (1988); Scamps & Hagino, *PRC* 92, 054614 (2015) ...
- ✓ Dinuclear System Model
Adamian, Antonenko, and Scheid, *NPA* 618, 176 (1997); Adamian et al., *NPA* 633, 409 (1998)
Adamian et al., *PRC* 81, 024604 (2010); Feng, *PRC* 95, 024615 (2017); Zhu et al., *JPG* 42, 085102 (2015)
- ✓ GRAZING Model ...
Winther, *NPA* 572, 191 (1994); Yanez and Loveland, *PRC* 91, 044608 (2015); Wen et al., *PRC* 99, 034606 (2019)
- ✓ Langevin Model ...
Zagrebaev and Greiner, *PRL* 101, 122701 (2008); Zagrebaev and Greiner, *PRC* 78, 034610 (2008)
Zhu, *PLB* 849, 138423 (2024); Dai et al., *PRC* 109, 024617 (2024)
- ✓ (Improved) Quantum Molecular Dynamics ...
Wang et al., *PRC* 65, 064608 (2002); Li et al., *PRC* 93, 014618 (2016); Zhao et al., *PRC* 92, 024613 (2015)
- ✓ Time-Dependent Density Functional Theory (TDDFT) ...
Simenel, *PRL* 105, 192701 (2010); Sekizawa and Yabana, *PRC* 88, 014614 (2013)
Zhang et al., *PRC* 109, 024614 (2024); Li et al., *PRC* 110, 034611 (2024)
- ✓ Stochastic Mean-Field / TDRPA ...
Zhang et al., *PLB* 869, 139828 (2025)
- ✓ Other approaches ...
Simenel, *PRL* 106, 112502 (2011); Ayik et al., *PRC* 97, 054618 (2018);
Yilmaz et al., *PRC* 98, 034604 (2018); Sekizawa and Ayik, *PRC* 102, 014620 (2020);
Gao, Sekizawa, and Zhu, *PRC* 112, 014602 (2025);...

Relative TDDFT

- Relative TDDFT has been developed and applied to a series of nuclear dynamical phenomena.

Ren, Zhao, and Meng, PRC 102, 044603 (2020)

Ren, Zhao, and Meng, PLB 801, 135194 (2020)

Ren, Zhao, and Meng, PRC 105, L011301 (2022)

Ren, Zhao, Vretenar, Nikšić, Zhao, Meng, PRC 105, 044313 (2022)

Ren, Vretenar, Nikšić, Zhao, Zhao, Meng, PRL 128, 172501 (2022)

Li, Vretenar, Ren, Nikšić, Zhao, Zhao, Meng, PRC 107, 014303 (2023)

Li, Vretenar, Nikšić, Zhao, Meng, PRC 110, 034302 (2024)

Li, Zhao, Meng, PLB 856, 138877 (2024)

DDZ, Vretenar, Nikšić, Zhao, Meng, PRC 109, 024614 (2024)

DDZ, Li, Vretenar, Nikšić, Ren, Zhao, Meng, PRC 109, 024316 (2024)

DDZ, Vretenar, Nikšić, Zhao, Meng, PLB 869, 139828 (2025)

- Idea:** Transforming the time-dependent many-body problem to a time-dependent one-body problem.

Runge and Gross, PRL 52, 997(1984)

Leeuwen, PRL 82, 3863(1999)

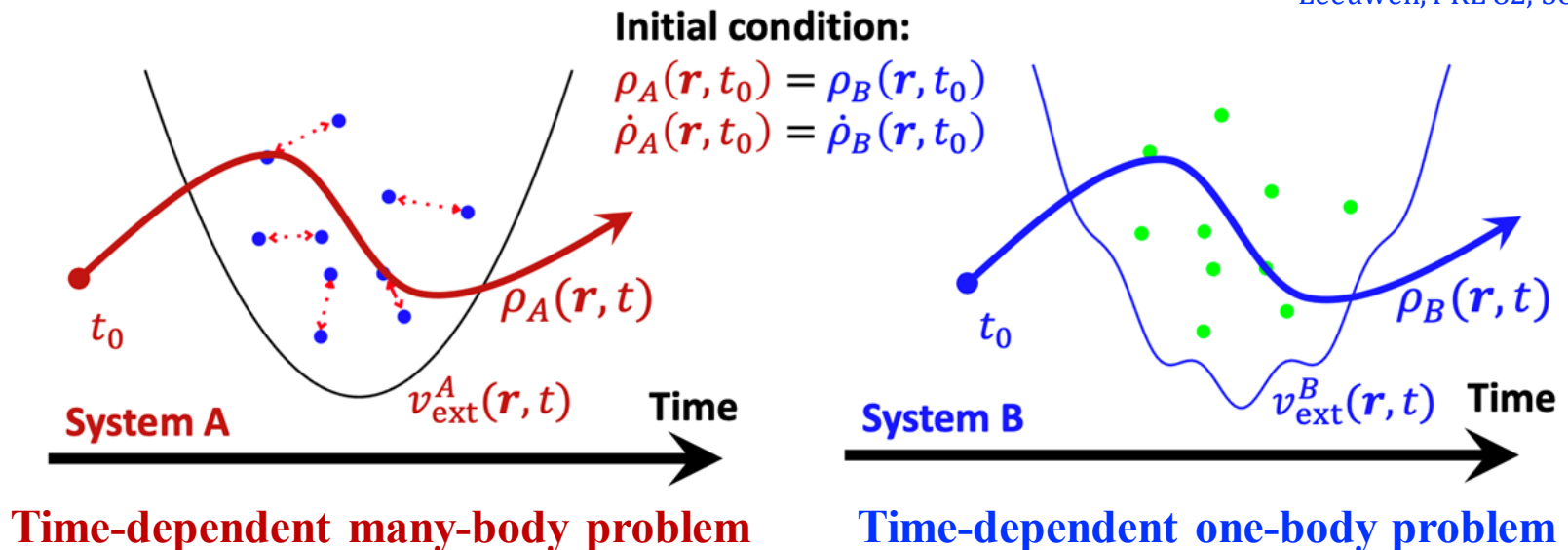


Figure is taken from Z. X. Ren

Relative TDDFT: Dirac equation

- In the relative TDDFT, nucleons satisfy the **Dirac equation**:

$$i\hbar \frac{\partial}{\partial t} \psi_k(\mathbf{r}, t) = \hat{h}(\mathbf{r}, t) \psi_k(\mathbf{r}, t).$$

where $\psi_k(\mathbf{r}, t)$ is the single-particle wavefunction, $\hat{h}(\mathbf{r}, t)$ **time-dependent** single-particle Hamiltonian.

Ren, Zhao, and Meng, PRC 102,044603(2020)

- Without memory effects, the form of $\hat{h}(\mathbf{r}, t)$ can be taken as

$$h(\mathbf{r}, t) = \alpha \cdot [p - V(\mathbf{r}, t)] + V^0(\mathbf{r}, t) + \beta[m + S(\mathbf{r}, t)],$$

where time-dependent potentials $S(\mathbf{r}, t)$ and $V^\mu(\mathbf{r}, t)$ reads

$$S(\mathbf{r}, t) = \alpha_S \rho_S + \beta_S \rho_S^2 + \gamma_S \rho_S^3 + \delta_S \Delta \rho_S,$$
$$V(\mathbf{r}, t) = \alpha_V j^\mu + \gamma_V (j_\nu j^\nu) j^\mu + \delta_V \Delta j^\mu + \tau_3 \alpha_{TV} j_{TV}^\mu + \tau_3 \delta_{TV} \Delta j_{TV}^\mu + e \frac{1 - \tau_3}{2} A^\mu,$$

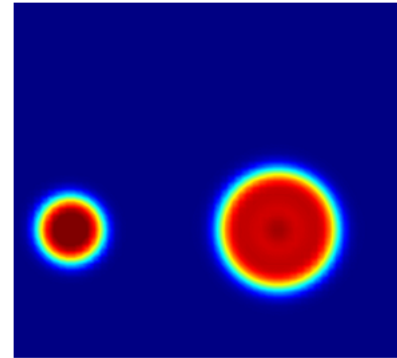
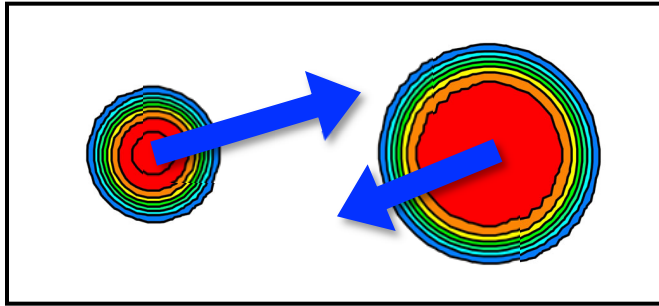
determined by the time-dependent density and currents,

$$\rho_S(\mathbf{r}, t) = \sum_{k=1}^A \bar{\psi}_k(\mathbf{r}, t) \psi_k(\mathbf{r}, t),$$

$$j^\mu(\mathbf{r}, t) = \sum_{k=1}^A \bar{\psi}_k(\mathbf{r}, t) \gamma^\mu \psi_k(\mathbf{r}, t), \quad j_{TV}^\mu(\mathbf{r}, t) = \sum_{k=1}^A \bar{\psi}_k(\mathbf{r}, t) \gamma^\mu \vec{\tau} \psi_k(\mathbf{r}, t).$$

Particle number and angular momentum projection

- TDDFT gives the most probable reaction path, and the wavefunction of reaction products typically lacks well-defined quantum numbers (particle number, spin)



- We extended relative TDDFT to include particle number projection and angular momentum projection.

DDZ, Vretenar, Nikšić, Zhao, Meng, PRC 109, 024614 (2024)

DDZ, Vretenar, Nikšić, Zhao, Meng, PLB 869, 139828 (2025)

Particle number projection

Particle number projection operator:

$$\hat{P}_n^V = \frac{1}{2\pi} \int d\theta e^{i\theta(n - \hat{N}_V)}.$$

$$P(n) = \langle \Psi | \hat{P}_n^V | \Psi \rangle, \quad P(n) = \frac{\langle \Psi | \hat{P}_n^V \hat{P}_N | \Psi \rangle}{\langle \Psi | \hat{P}_N | \Psi \rangle}.$$

Angular momentum projection

Angular momentum projection operator:

$$\hat{P}_{MK}^S = \frac{2S+1}{16\pi^2} \int d\Omega (D_{MK}^S(\Omega))^* \hat{R}(\Omega).$$

$$P(S, K) = \langle \Psi | (\hat{P}_{MK}^S)^+ \hat{P}_{MK}^S | \Psi \rangle = \langle \Psi | \hat{P}_{KK}^S | \Psi \rangle.$$

Simenel, PRL 105, 192701 (2010); Sekizawa and Yabana, PRC 88,014614 (2013)

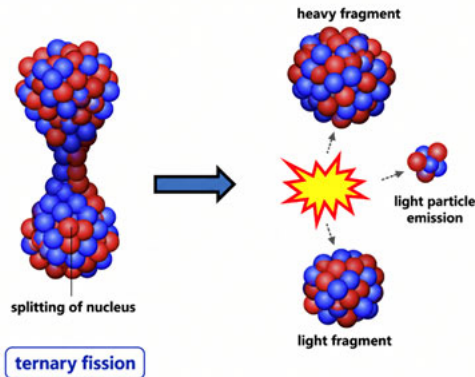
Scamps et al., PRC 108, L061602 (2023)

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Ternary fission

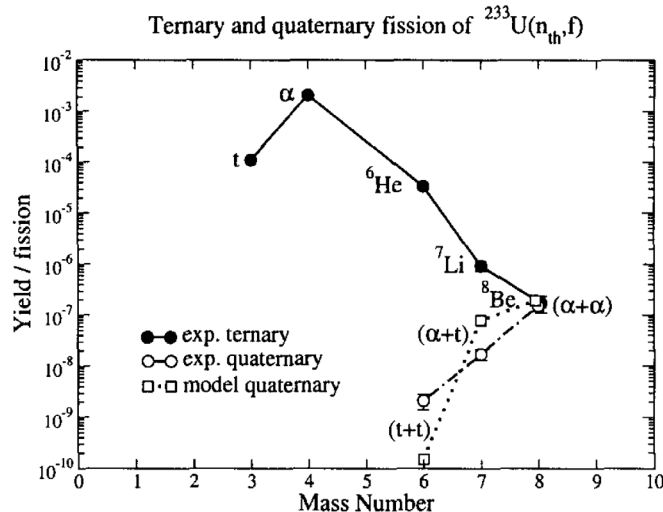
- Ternary (quaternary) fission was discovered by Tsien San-Tsiang, Ho Zah-Wei, and their French collaborators R. Chastel and L. Vigneron in 1947.



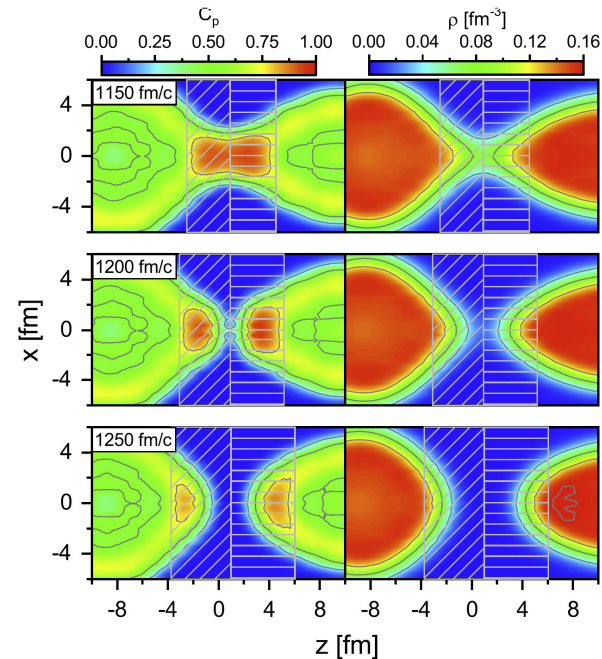
- ✓ TDDFT studies reveal proton localization values near 1 in the neck, indicating α -like clustering.

$$C_{q\sigma}(\vec{r}) = \left[1 + \left(\frac{\tau_{q\sigma} \rho_{q\sigma} - \frac{1}{4} |\vec{\nabla} \rho_{q\sigma}|^2 - \vec{j}_{q\sigma}^2}{\rho_{q\sigma} \tau_{q\sigma}^{\text{TF}}} \right)^2 \right]^{-1},$$

rarer than ordinary binary fission



Gönnenwein, NPA 734, 213 (2004)



Ren, Vretenar, Nikšić, Zhao, Meng, PRL 128, 172501 (2022)

Ternary quasi-fission in $^{238}\text{U} + ^{238}\text{U}$ collision

PRL **103**, 042701 (2009)

PHYSICAL REVIEW LETTERS

week ending
24 JULY 2009

Collision Dynamics of Two ^{238}U Atomic Nuclei

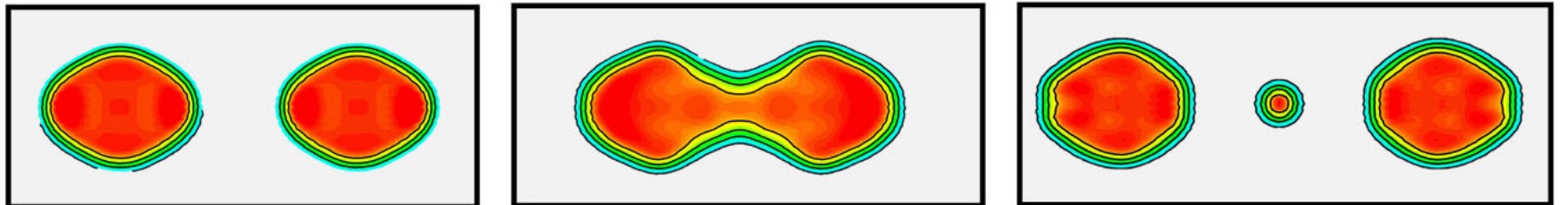
Cédric Golabek

GANIL (IN2P3/CNRS - DSM/CEA), BP 55027, F-14076 Caen Cedex 5, France

Cédric Simenel

CEA, Centre de Saclay, IRFU/Service de Physique Nucléaire, F-91191 Gif-sur-Yvette, France
(Received 20 April 2009; revised manuscript received 19 May 2009; published 24 July 2009)

“...Surprisingly, we also observe **ternary fission** due to purely dynamical effects.”



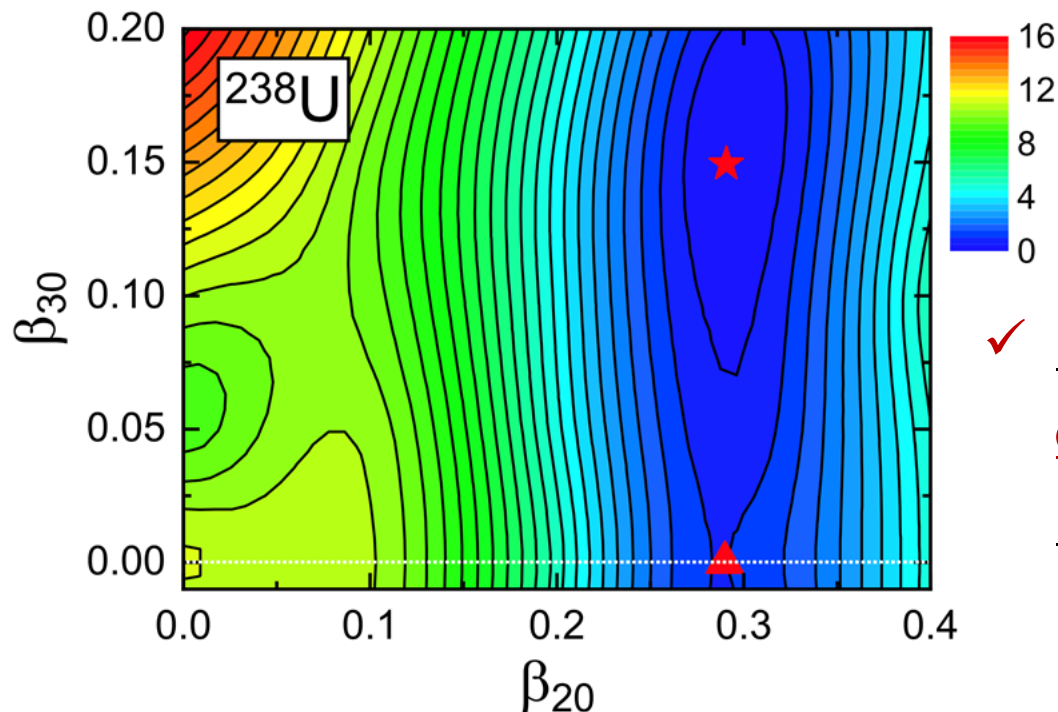
Time evolution

Ternary quasi-fission in collisions of actinide nuclei

Editors' Suggestion

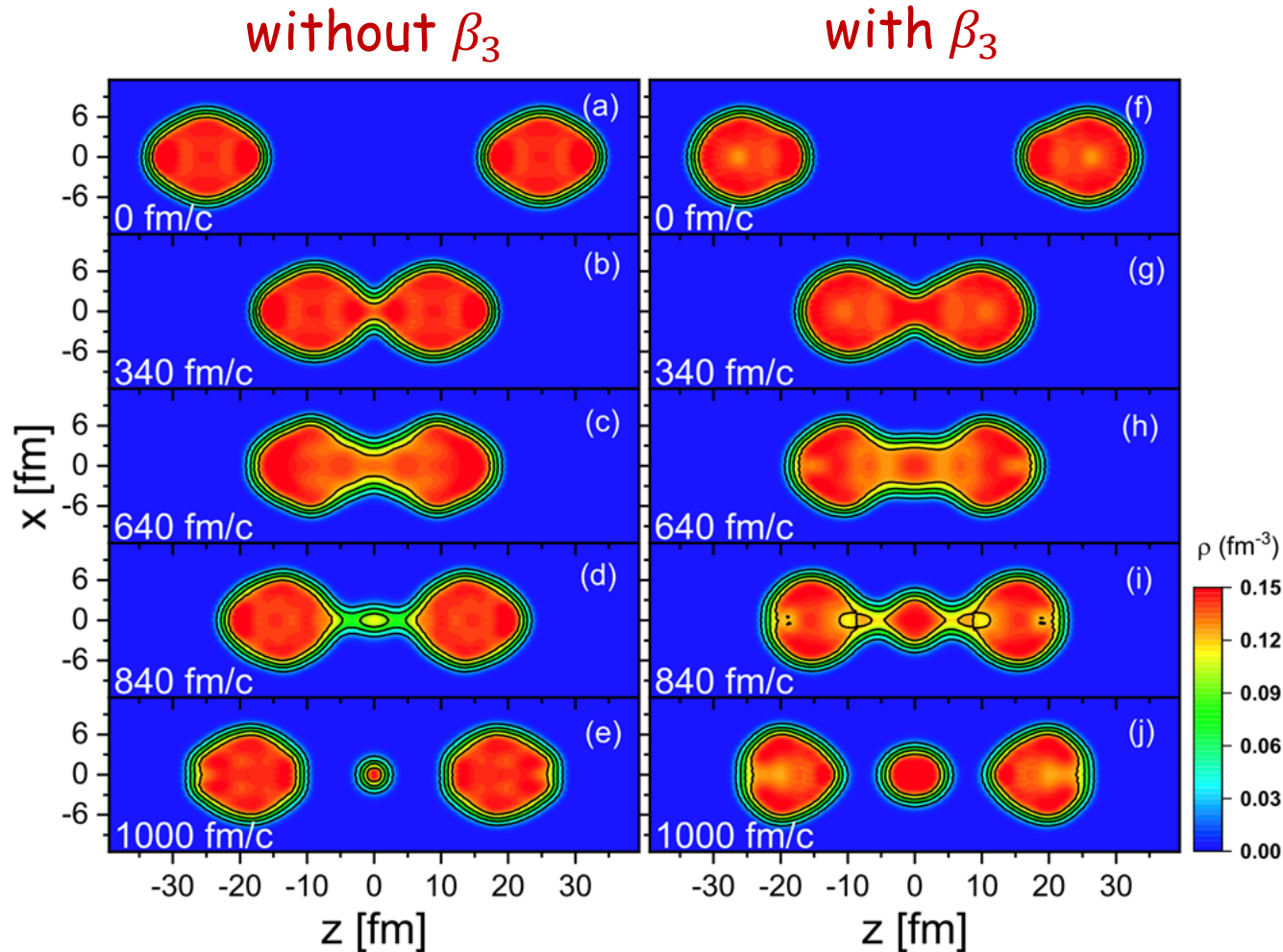
Ternary quasifission in collisions of actinide nuclei

D. D. Zhang, B. Li, D. Vretenar, T. Nikšić, Z. X. Ren, P. W. Zhao, and J. Meng
Phys. Rev. C **109**, 024316 – Published 13 February 2024



- ✓ The ground state of ^{238}U is **octupole-deformed** in the mean-file level.

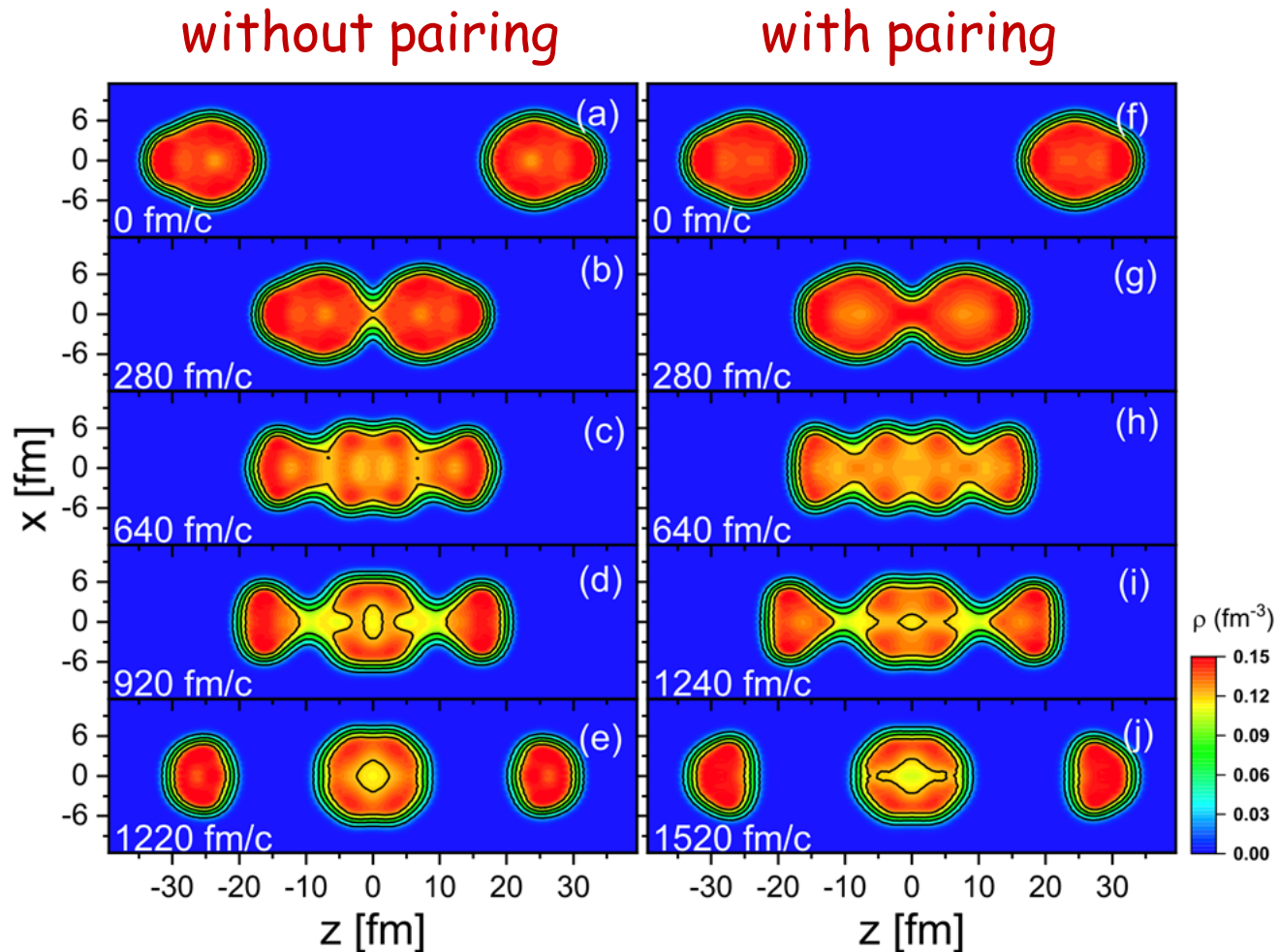
Octupole deformation



✓ The octupole deformation affects the ternary quasi-fission.

DDZ, Vretenar, Nikšić, Zhao, Meng, PRC 109, 024614 (2024)

The synthesis of neutron-rich heavy nuclei



✓ The neutron-rich heavy nuclei around uranium are produced in the middle.

DDZ, Vretenar, Nikšić, Zhao, Meng, PRC 109, 024614 (2024)

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Fragment intrinsic spins and their correlations

- In a system that consists of two fragments, the conservation of angular momentum requires

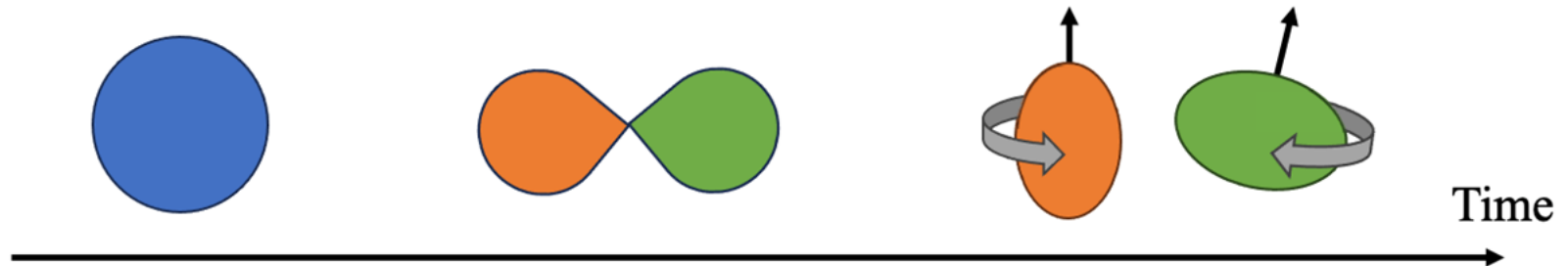
$$J = S^L + S^H + \Lambda.$$

S^L, S^H : spins of two fragments.

Λ : orbital angular momentum.

- The spontaneous fission of even-even nuclei ($J = 0$):

The generation of spins and their correlations.

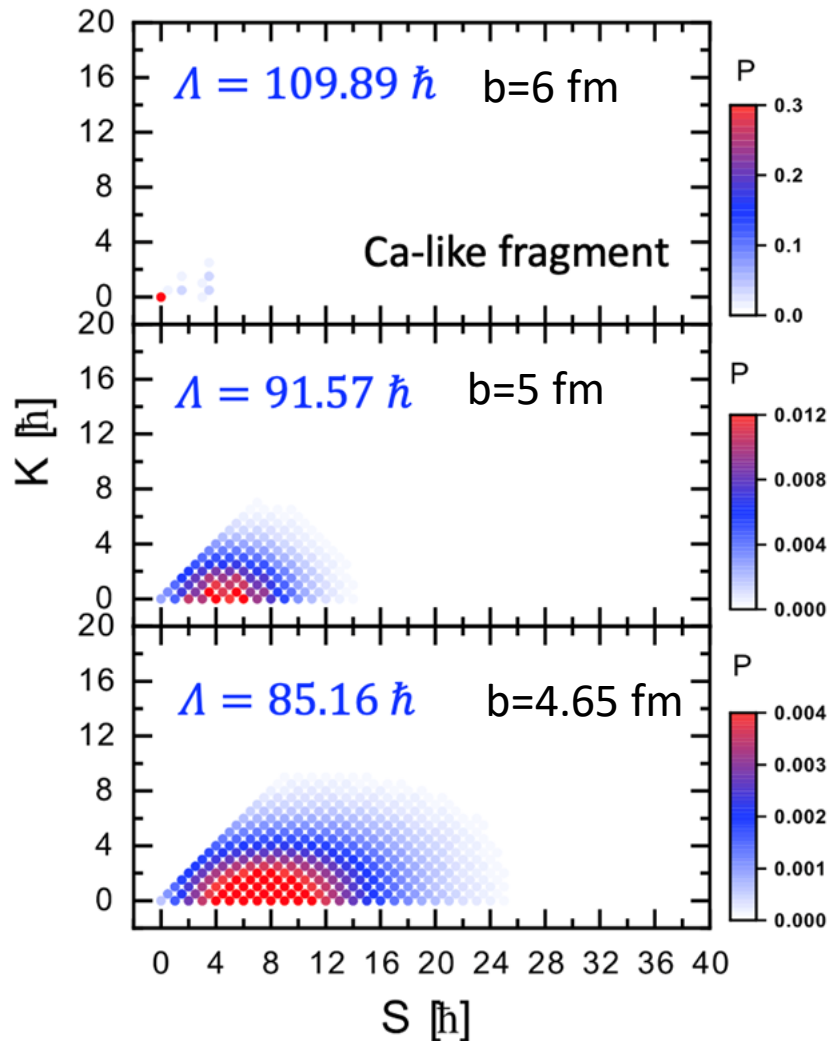


The fragment intrinsic spins and their correlations in fission have attracted broad interest.

As seen in the talks at the Workshop on Fission Dynamics 2026.

- What happens in reactions?

Spin distributions in the MNT reactions $^{40}\text{Ca} + ^{208}\text{Pb}$



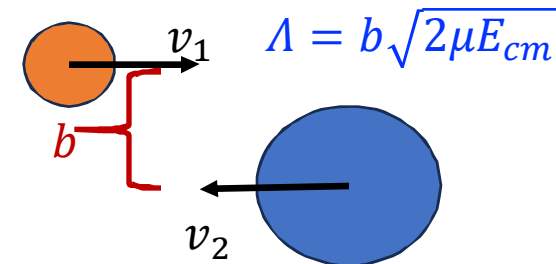
$$J = S^L + S^H + \Lambda.$$

At the beginning, $J = 0 + 0 + \Lambda$

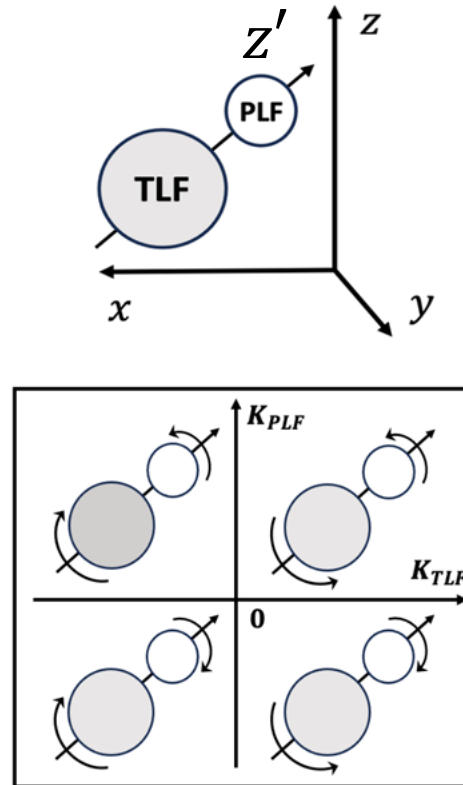
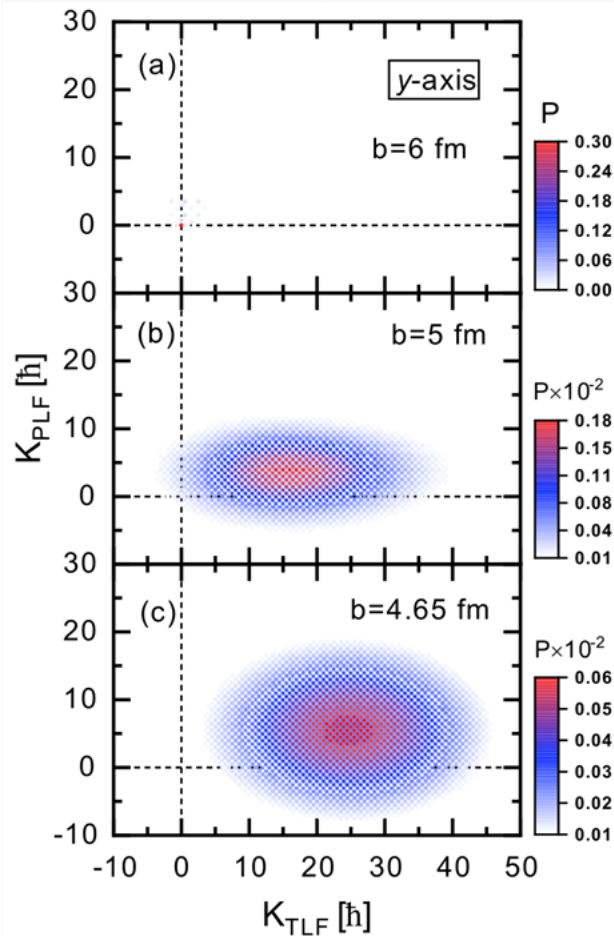
Almost no nucleon transfer

- ✓ The spins are generated.
- ✓ The orbital angular momentum is converted into intrinsic spin.

Nucleon transfer



The joint spin distribution of two fragments (y-axis)



Joint spin distribution:

$$P(K_{PLF}, K_{TLF}) = \langle \Psi | \hat{P}_{K_P} \hat{P}_{K_T} | \Psi \rangle.$$

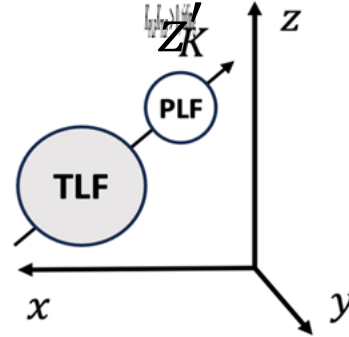
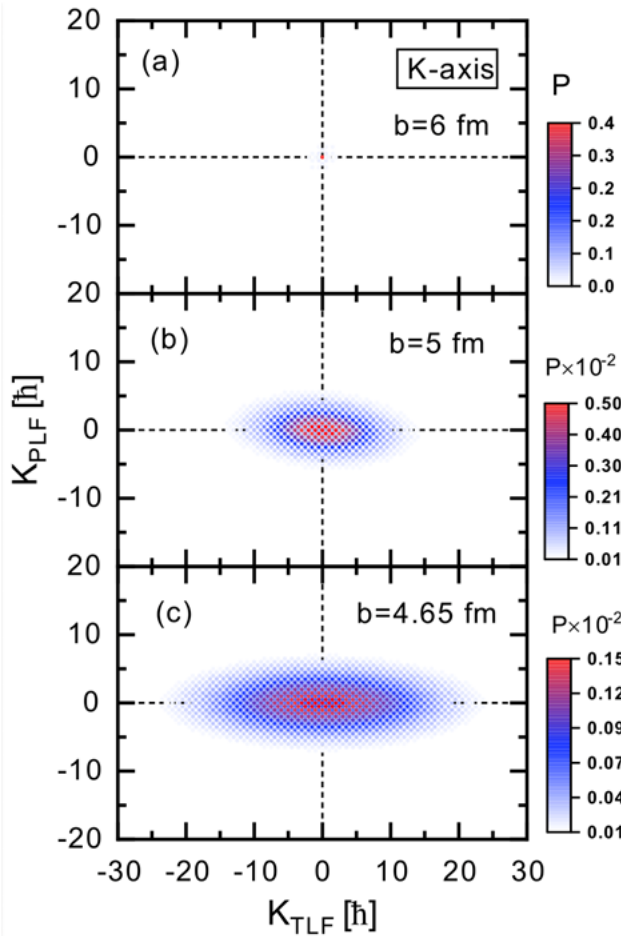
$K_{PLF}K_{TLF} > 0$, wriggling

$K_{PLF}K_{TLF} < 0$, bending

Various collective rotational modes are revealed.

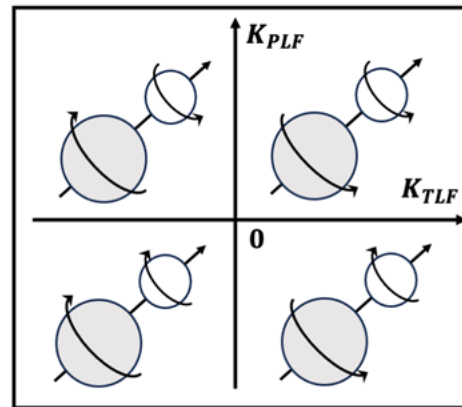
DDZ, Vretenar, Nikšić, Zhao, Meng, PLB 869, 139828 (2025)

The joint spin distribution of two fragments (z' -axis)



Joint spin distribution:

$$P(K_{\text{PLF}}, K_{\text{TLF}}) = \langle \Psi | \hat{P}_{K_P} \hat{P}_{K_T} | \Psi \rangle.$$



$K_{\text{PLF}}K_{\text{TLF}} > 0$, titing

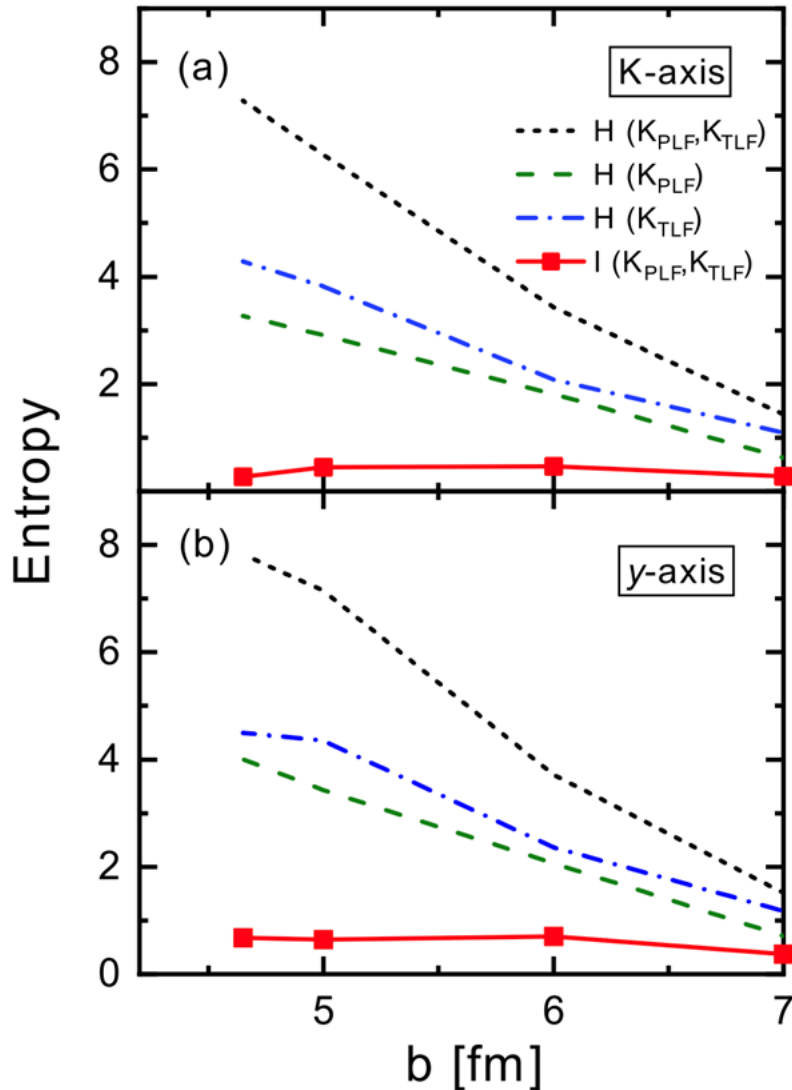
$K_{\text{PLF}}K_{\text{TLF}} < 0$, twisting

Various collective rotational modes are revealed.

DDZ, Vretenar, Nikšić, Zhao, Meng, PLB 869, 139828 (2025)

How to quantify the correlations between the two fragments' spins?

The correlations between two fragments



Entanglement Shannon entropy $I(A, B)$

$$I(A, B) = H(A) + H(B) - H(A, B).$$

$$H(A) = - \sum_A P(A) \ln(P(A)).$$

$$H(A, B) = - \sum_{A,B} P(A, B) \ln(P(A, B)).$$

The smaller entanglement Shannon entropy indicates fewer correlations.

The spins of the two fragments are correlated, but the correlations are rather weak.

DDZ, Vretenar, Nikšić, Zhao, Meng, PLB 869, 139828 (2025)

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Summary and outlook

- Relative TDDFT is extended to incorporate particle number projection and angular momentum projection, and then applied to
 - ✓ Ternary quasi-fission in collisions of actinide nuclei:
Octupole deformation plays an important role in ternary quasi-fission.
 - ✓ Intrinsic angular momentum of fragments in reactions:
The intrinsic angular momentum of fragments is generated with nucleon transfer, and their correlations are weak.
- Next step: How is the angular momentum of fragments distributed in the ternary quasi-fission?

Thanks for your attention!