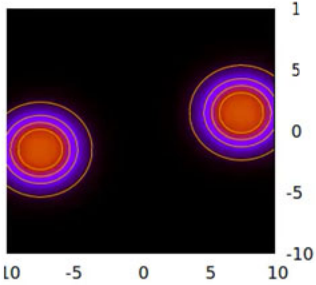
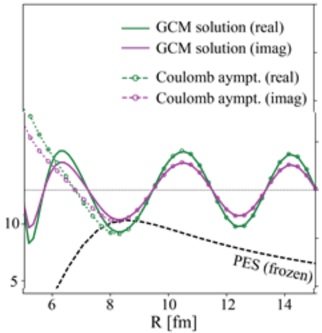
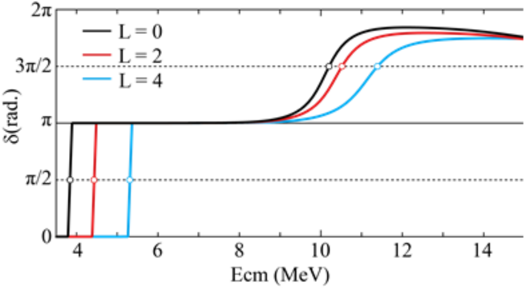
GCM + Time-Dependent Approach for sub-barrier reaction

- Toward microscopic description of sub-barrier fusion -

M. Kimura (RIKEN)

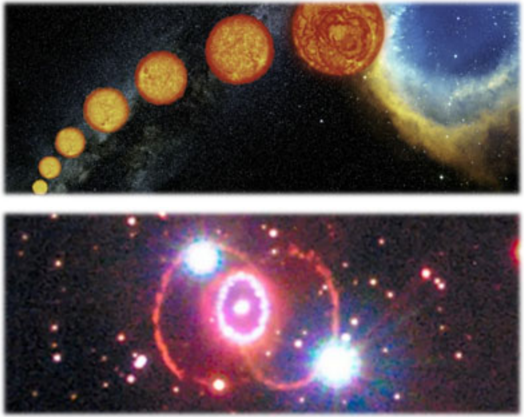






Astrophysical fusion reactions

Fusion of light elements such as $^{12}\mathrm{C}+^{12}\mathrm{C}$ and $^{16}\mathrm{O}+^{16}\mathrm{O}$ play essential roles in various astrophysical phenomena.



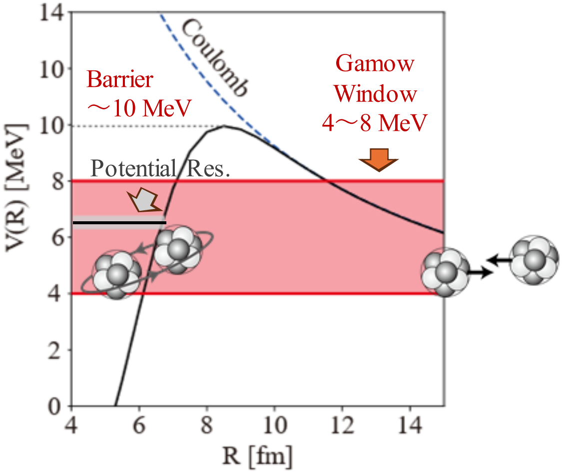
- Stellar evolution and nucleosynthesis
- Type Ia supernovae
- Core-collapse supernovae
- X-ray superbursts

Reaction	Astrophysical phenomena	T[GK]	Gamow Window
$^{12}\mathrm{C}+^{12}\mathrm{C}$	SN Ia, Superbursts	0.5 – 1.0	1.5 – 3.0 MeV
$^{12}\mathrm{C}+^{16}\mathrm{O}$	Massive-star evolution	1.0 – 2.0	3.0 – 5.0 MeV
$^{16}\mathrm{O}+^{16}\mathrm{O}$	Core-collapse SN	1.5 – 2.5	4.0 – 8.0 MeV

Astrophysical fusion reactions

All these reactions occur well below the Coulomb barrier

e.g. ^{16}O+^{16}O: Coulomb barrier 10MeV, GW 4 – 8 MeV.



- Classically forbidden,
  purely quantum phenomena
- S-wave ($J = 0$) dominance
  (requiring J-projected description)
- Potential resonance & compound state

Quite challenging issue in
quantum many-body problems

Coupled Channel approach to sub-barrier fusion

Schematic two-channel model for $^{16}O+^{16}O$ (coupling to the 3^- octupole excitation)

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dR^2}+\begin{pmatrix} U_L(R) & V_{cc}(R)\\ V_{cc}(R) & U_{L'}(R)+E^*_{3^-}\end{pmatrix}\right]\begin{pmatrix}\chi_{g.s.}(R)\\ \chi_{3^-}(R)\end{pmatrix} = E_{cm}\begin{pmatrix}\chi_{g.s.}(R)\\ \chi_{3^-}(R)\end{pmatrix}$$

K. Hagino et al., CPC 123 (1999), PTP 128, 1061 (2012)

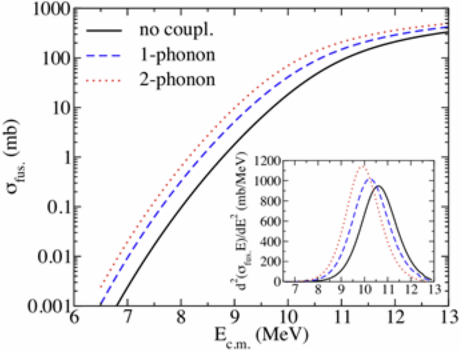
In the adiabatic/sudden limit, channel coupling generates eigen-barriers, effectively lowering the fusion barrier.

Direct and coupling potentials are usually introduced phenomenologically. (Use of TDHF/DC-TDHF have also been explored)

A. S. Umar and V. E. Oberacker, PRC 74, 021601(R) (2006)

Full-microscopic description is challenging and interesting

C. Simenel, et al, PRC88 (2013)



Motivation and theoretical recipe

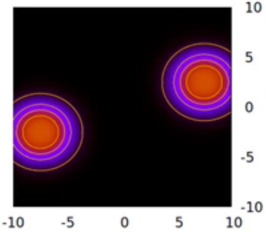
Ultimate goal: Microscopic description of quantum tunneling in heavy-ion reactions

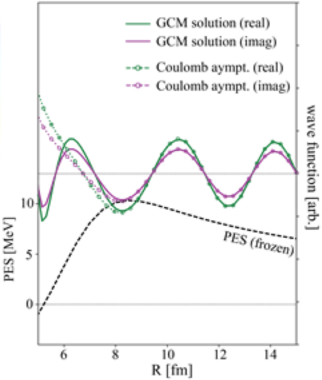
Key question: How do effective channel couplings emerge from many-body dynamics?

First step: We employ a computationally tractable **time-dependent cluster model** and focus on **subbarrier scattering** to investigate essential collective dynamics

Theoretical Recipe

Step 1: Time-dependent cluster dynamics
Generate dynamical basis states from above-barrier reactions





Step 2: GCM + Kohn variational method for scattering
Extract phase shifts and resonance structures

Microscopic α-cluster model

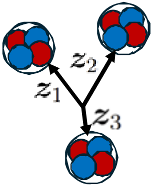
The α-cluster model simplifies a nucleus as composed of α-clusters

$$\Phi_{N\alpha}(\boldsymbol{z}_1, ..., \boldsymbol{z}_N) := \mathcal{A}' \{ \Phi_\alpha(\boldsymbol{z}_1) \cdots \Phi_\alpha(\boldsymbol{z}_N) \},$$

Each α cluster has a $(0s)^4$ configuration
localized at a phase-space point $\boldsymbol{z}$ ($\langle \boldsymbol{r} \rangle \propto \Re \boldsymbol{z}, \quad \langle \boldsymbol{p} \rangle \propto \Im \boldsymbol{z}$)

$$\Phi_\alpha(\boldsymbol{z}) := \mathcal{A} \{ \phi(\boldsymbol{r}_1, \boldsymbol{z}) \chi_{p\uparrow} \cdots \phi(\boldsymbol{r}_4, \boldsymbol{z}) \chi_{n\downarrow} \},$$

$$\phi(\boldsymbol{r}, \boldsymbol{z}) := \left(\frac{2\nu}{\pi} \right)^{3/4} \exp \{ -\nu (\boldsymbol{r} - \boldsymbol{z})^2 \},$$



e.g., 3 α system (^{12}C)

Microscopic Hamiltonian with effective NN and NNN interactions

- Reproduces α - α phase shifts
- Reasonable description of excitation spectra in ^{12}C and ^{16}O

$$H = \sum_i t_i - t_{cm} + \sum_{i<j} v_{ij} + \sum_{i<j<k} v_{ijk}$$

A. Tohsaki , PRC 49, (1994)

GCM with microscopic α - cluster model

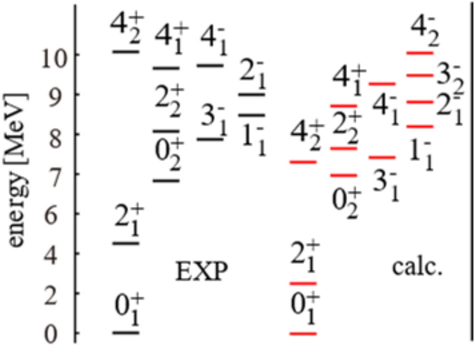
- Low-lying spectra of ^{12}C and ^{16}O are reasonably described within the GCM framework.

$$\Phi_{GCM} = \sum_{p} c_p P^J \Phi_{N\alpha}(z_1^{(p)}, \ldots, z_N^{(p)})$$

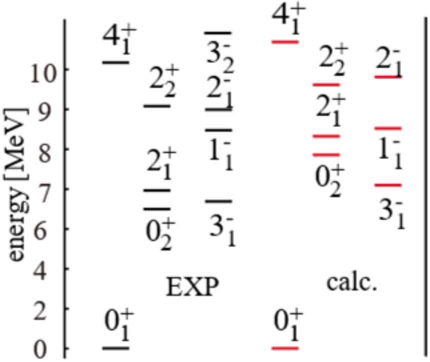
Imai et al., PRC99 (2019)

- The 3^- state of ^{16}O, which is important for subbarrier reactions, is reasonably reproduced.

3α system (^{12}C)



4α system (^{16}O)



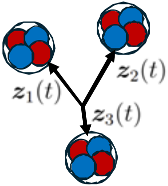
Time-dependent α - cluster model

The phase space positions of α particles are time dependent.

$$\boldsymbol{z} \to \boldsymbol{z}(t)$$

E. Caurier, et al., PLB109, (1982)

$$\Phi_{N\alpha}(t) := \Phi_{N\alpha}(\boldsymbol{z}_1(t), \dots, \boldsymbol{z}_N(t)) = \mathcal{A}'\left\{\Phi_\alpha(\boldsymbol{z}_1(t))\cdots\Phi_\alpha(\boldsymbol{z}_N(t))\right\},$$



TDVP

$$\delta \int dt\, \frac{\langle \Phi_{N\alpha}(t)| i\hbar\partial_t - H |\Phi_{N\alpha}(t)\rangle}{\langle \Phi_{N\alpha}(t)|\Phi_{N\alpha}(t)\rangle} = 0$$

EOM

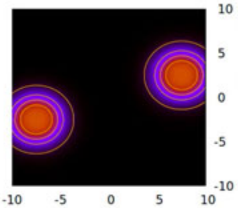
$$i\hbar \sum_j C_{ij} \frac{d}{dt}\boldsymbol{z}_j(t) = \frac{\partial}{\partial \boldsymbol{z}_i^*} \frac{\langle \Phi_{N\alpha}(t)|H|\Phi_{N\alpha}(t)\rangle}{\langle \Phi_{N\alpha}(t)|\Phi_{N\alpha}(t)\rangle},$$

PROS

- Free from spurious center-of-mass excitation
- Small comp. cost ⇨ Projections, GCM, etc.

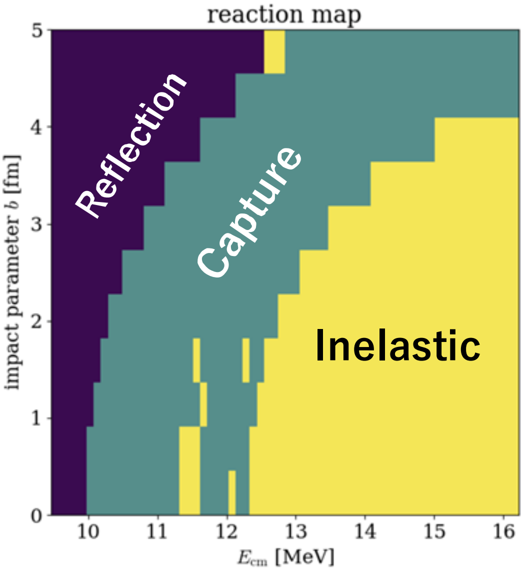
CONS

- Too small DOF, oversimplified
- Weak one-body dissipation, No two-body dissipation



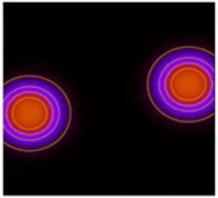
Reaction map of $^{16}O+^{16}O$ by TDCM

- Reaction pattern can be classified to “Reflection”, “Capture”, “Inelastic”
- Narrow window for “Capture”
  due to weak dissipation (small DOF)
- In the “Inelastic” process,
  ^{16}O is octupole excited.

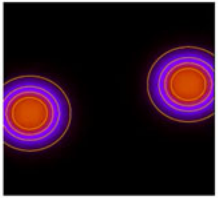


Analyze ^{16}O after scattering

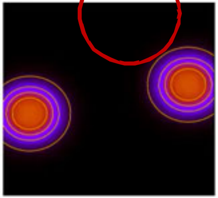
Reflection



Capture



Inelastic



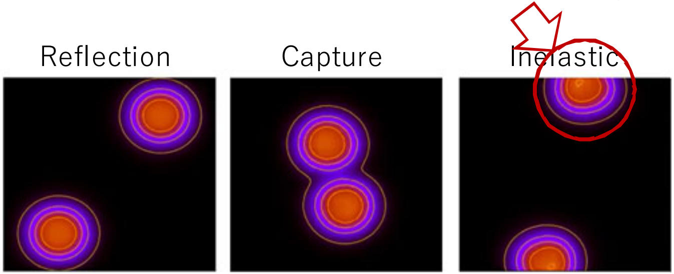
Anatomy of ^{16}O after the scattering

Excitation energy of ^{16}O

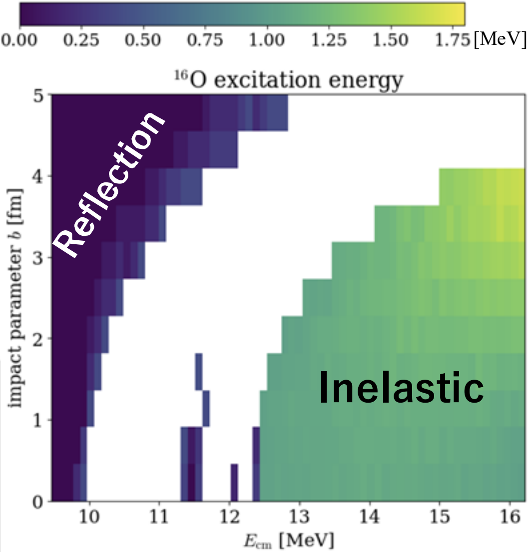
$$E^*(^{16}\mathrm{O}) = \langle \Phi_{4\alpha}(t) | H | \Phi_{4\alpha}(t) \rangle - E_{g.s.}(^{16}\mathrm{O})$$

- 1.0 ～ 1.7 MeV excitation after "Inelastic" event
- A few hundreds keV excitation after "Reflection"

Analyze ^{16}O after scattering



Reflection   Capture   Inelastic



Anatomy of ^{16}O after the scattering

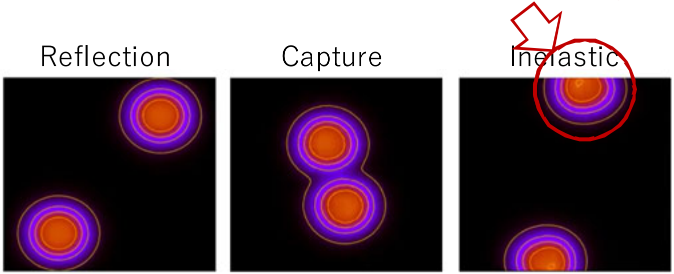
Angular momentum decomposition of ^{16}O

Probability to find the state with angular momentum J

$$P(J) = \sum_K \frac{\langle \Phi_{4\alpha}(t) | P^J_{KK} | \Phi_{4\alpha}(t) \rangle}{\langle \Phi_{4\alpha}(t) | \Phi_{4\alpha}(t) \rangle}$$

- 8〜12% are excited to $J \neq 0$ states after “Inelastic”
- A few % excitation after “Reflection”
- Most of them (80%) are $J = 3^-$

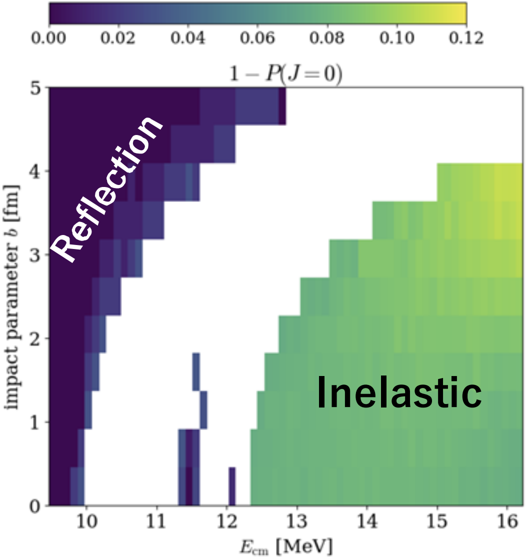
Analyze ^{16}O after scattering



Reflection

Capture

Inelastic



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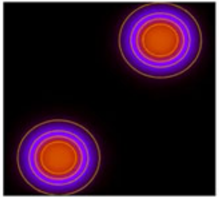
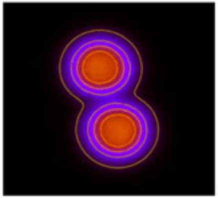
Analyze ^{16}O after scattering

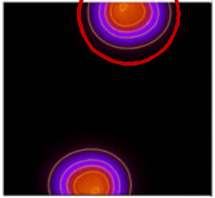
Reflection

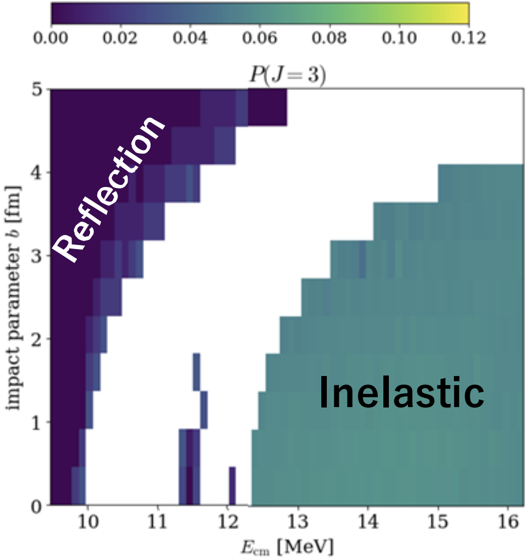


Capture



Inelastic





GCM for nucleus-nucleus scattering

○ Linear operator for scattering of ^{16}O+^{16}O

$$\mathcal{L} = H_{^{16}\mathrm{O}+^{16}\mathrm{O}} - 2H_{^{16}\mathrm{O}} - E_{cm}$$

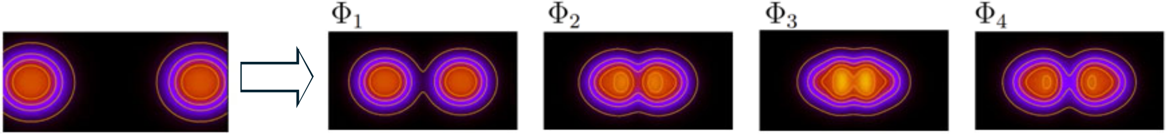
○ GCM ansatz for scattering of ^{16}O+^{16}O

$$\Phi_{GCM} = \boxed{\sum_i c_i \Phi_i} + \Phi^{(-)} + S_l \Phi^{(+)}$$

with boundary cond. (incoming/outgoing Coulomb wave)

$$\Phi^{(\pm)} \to H_L^{(\pm)}(k_{cm}R)\Phi_{^{16}\mathrm{O}}\Phi_{^{16}\mathrm{O}} \quad \text{as} \quad R \to \infty$$

Superpose
the TDCM snapshot



○ Kohn variational principle for scattering problems W. Kohn, Phys. Rev. 74 (1948)

$$\delta\left(S_l + i\frac{\mu}{k_{cm}}\langle\Phi_{GCM}|\mathcal{L}|\Phi_{GCM}\rangle\right) = 0$$

⇨ GCM equations for scattering.

GCM for nucleus-nucleus scattering

○ GCM equation for scatt. problem

Y. Mito et al., Prog. Theor. Phys. 56 (1976)

Linear equations with coefficient c_i and S-matrix S_l as output
Computational cost is almost same with the ordinary GCM for the bound states

$$\begin{pmatrix} \langle P^L\Phi_1|\mathcal{L}|P^L\Phi_1\rangle & \cdots & \langle P^L\Phi_1|\mathcal{L}|P^L\Phi_N\rangle & \langle P^L\Phi_1|\mathcal{L}|\Phi^{(+)}\rangle \\ \vdots & & \vdots & \vdots \\ \langle P^L\Phi_N|\mathcal{L}|P^L\Phi_1\rangle & \cdots & \langle P^L\Phi_N|\mathcal{L}|P^L\Phi_N\rangle & \langle P^L\Phi_N|\mathcal{L}|\Phi^{(+)}\rangle \\ \langle \Phi^{(+)}|\mathcal{L}|P^L\Phi_1\rangle & \cdots & \langle \Phi^{(+)}|\mathcal{L}|P^L\Phi_N\rangle & \langle \Phi^{(+)}|\mathcal{L}|\Phi^{(+)}\rangle \end{pmatrix} \begin{pmatrix} c_1 \\ \vdots \\ c_N \\ -S_l \end{pmatrix} = - \begin{pmatrix} \langle P^L\Phi_1|\mathcal{L}|\Phi^{(-)}\rangle \\ \vdots \\ \langle P^L\Phi_N|\mathcal{L}|\Phi^{(-)}\rangle \\ \langle \Phi^{(+)}|\mathcal{L}|\Phi^{(-)}\rangle \end{pmatrix}$$

○ GCM equation for structure problems

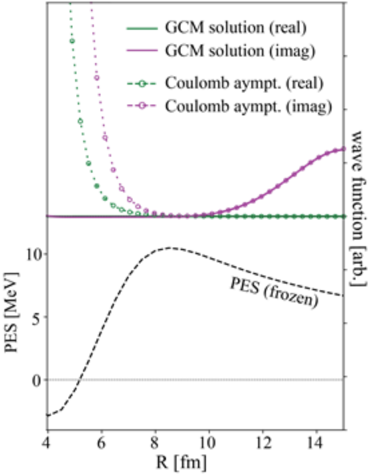
Eigenvalue problem with eigenvector c_i and eigen-energy E

$$\begin{pmatrix} \langle \Phi_1|H-E|\Phi_1\rangle & \cdots & \langle \Phi_1|H-E|\Phi_N\rangle \\ \vdots & & \vdots \\ \langle \Phi_N|H-E|\Phi_1\rangle & \cdots & \langle \Phi_N|H-E|\Phi_N\rangle \end{pmatrix} \begin{pmatrix} c_1 \\ \vdots \\ c_N \end{pmatrix} = 0$$

Solving GCM for the scattering

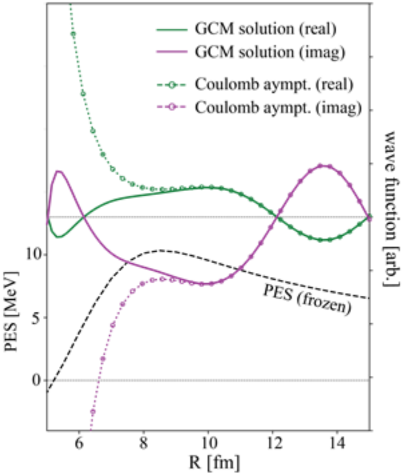
Below barrier

$E_{cm} = 8$ MeV, $\delta = 0$ deg.



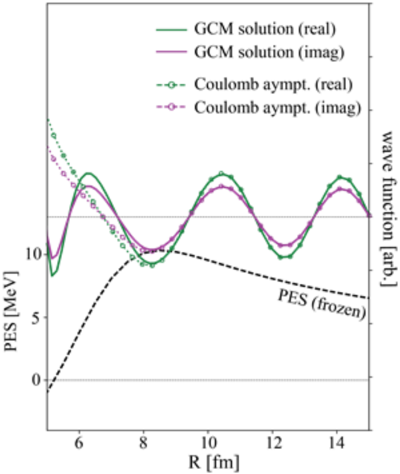
Just below barrier

$E_{cm} = 9.5$ MeV, $\delta = 24$ deg.



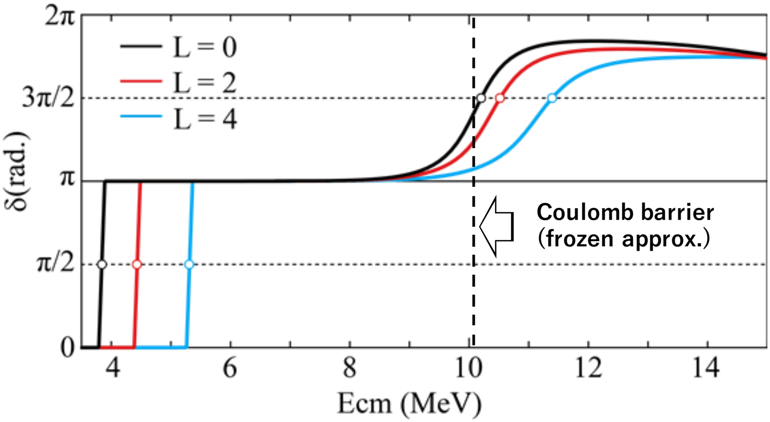
Above barrier

$E_{cm} = 14$ MeV, $\delta = 141$ deg.



^{16}O-^{16}O phase shift from GCM

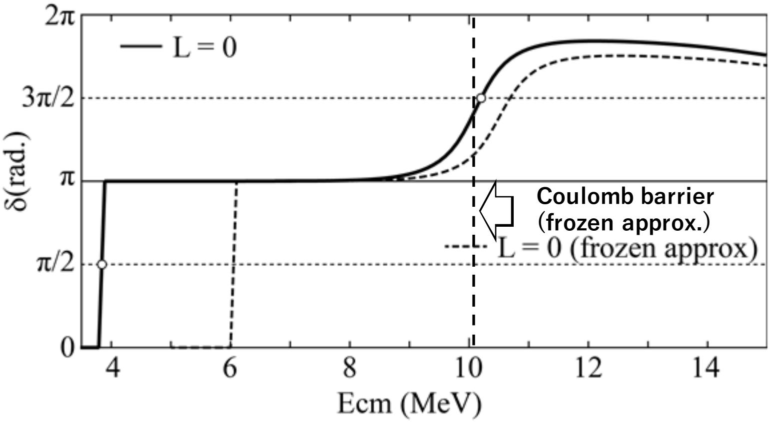
- Barrier top resonances with L=0, 2 and 4
- Below the barrier, the phase shift is already positive (tunneling)
- Deep sub-barrier resonances around E_{cm} = 4 to 6 MeV



^{16}O-^{16}O phase shift from GCM

Effect of the inelastic channel (3^-)

- Barrier top and deep sub-barrier resonances are shifted down
- Tunneling occurs at smaller energies and interaction becomes more attractive

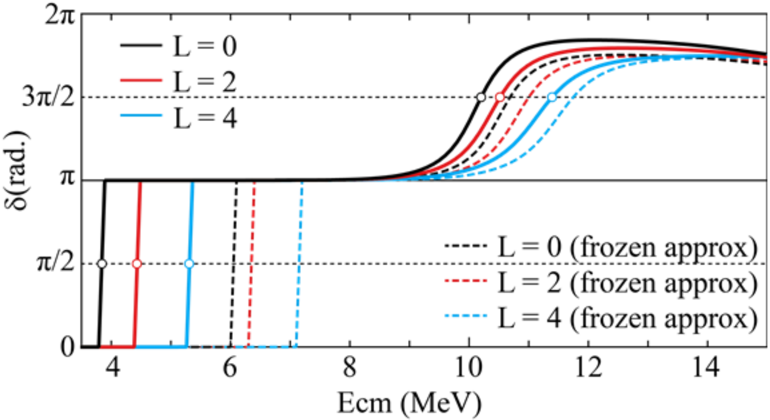


Coulomb barrier (frozen approx.)

^{16}O-^{16}O phase shift from GCM

Effect of the inelastic channel (3^-)

- Barrier top and deep sub-barrier resonances are shifted down
- Tunneling occurs at smaller energies and interaction becomes more attractive
- The trend is common to other partial waves



Summary and outlook

Summary

- Microscopic understanding of sub-barrier fusion is a challenging problem.
- We introduced a method to describe nucleus–nucleus scattering using basis states generated by TDCM.
- By solving the GCM for scattering problem, sub-barrier scattering and deep sub-barrier resonances are described microscopically.

Outlook

- Upgrade basis wave functions to more realistic frameworks such as TDAMD and TDHF.
- Extend the formalism to coupled-channel dynamics.
- Coupling to CN and their decay channels must be incorporated.