Fragments spin generation in low-energy fission: Beyond thermalization

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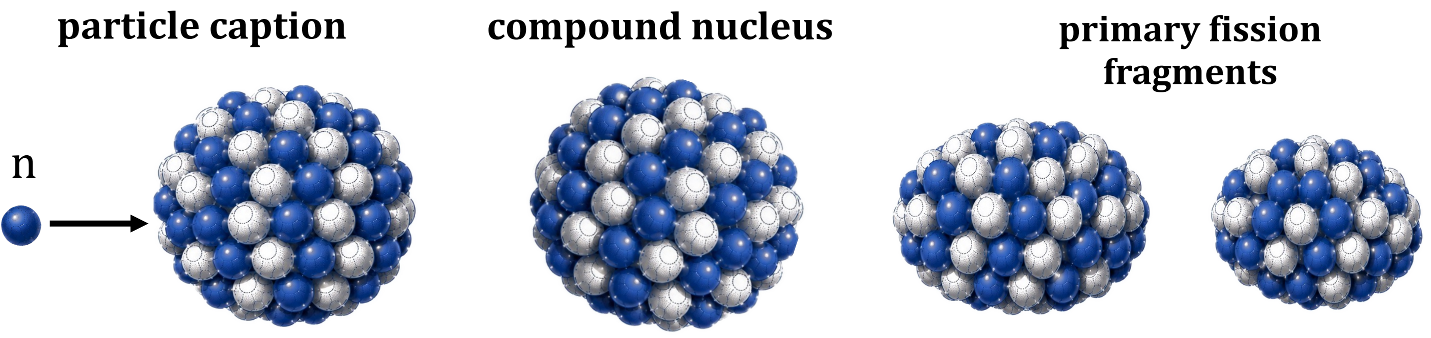
Russian Science Foundation

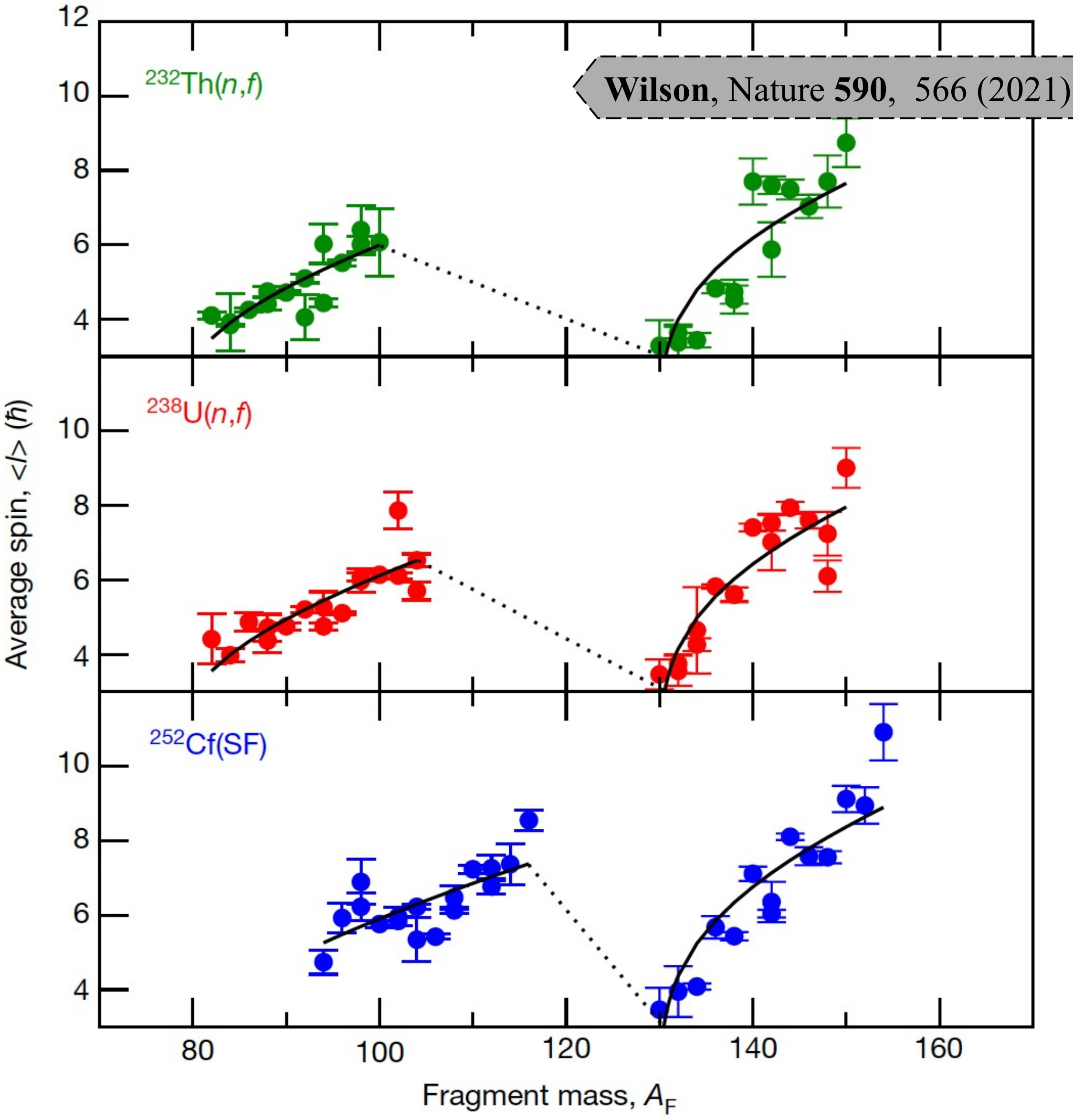
Outline

Problem: angular momentum generation of FF

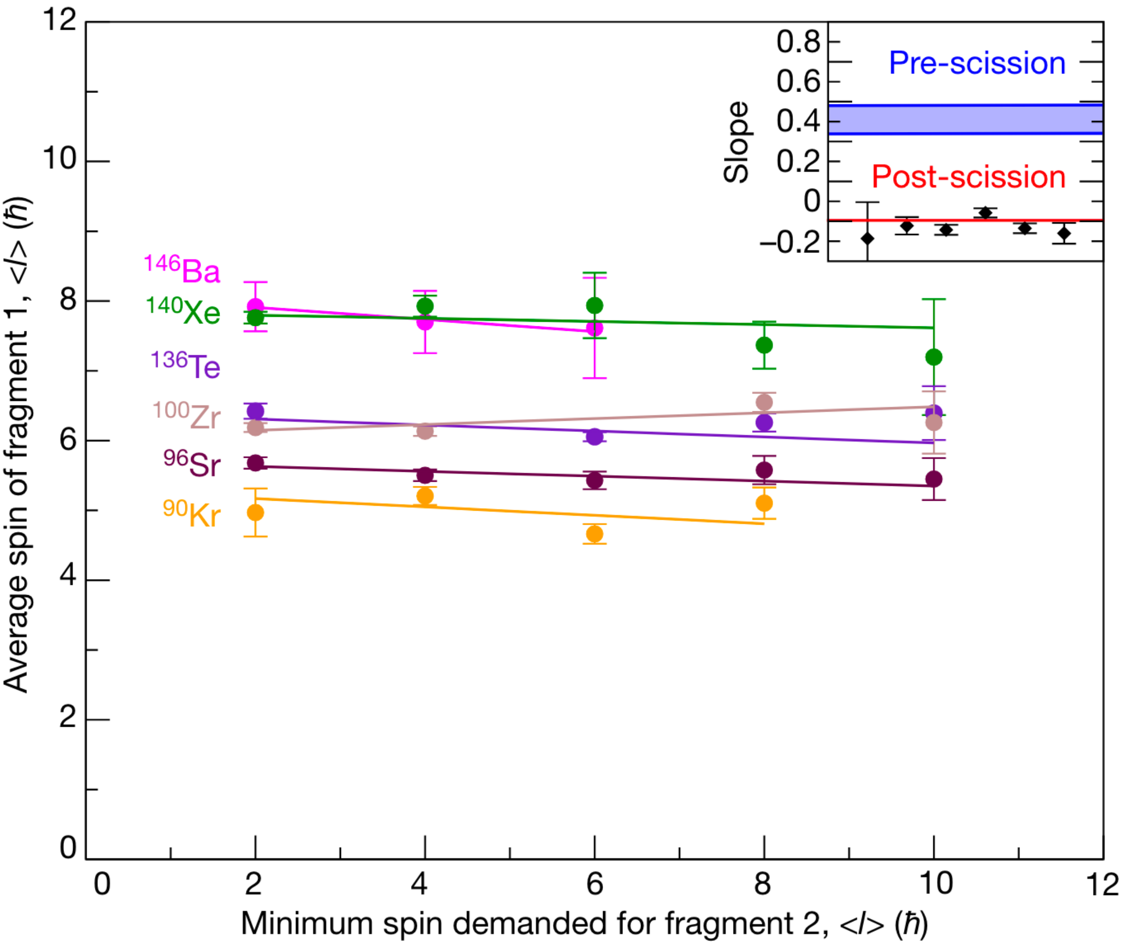


$(A, Z, \mathcal{J} \sim \frac{\hbar}{2})$ $(A+1, Z, \mathcal{J} \sim \hbar)$ $A_1, Z_1, \mathcal{J}_1$ $A_2, Z_2, \mathcal{J}_2$

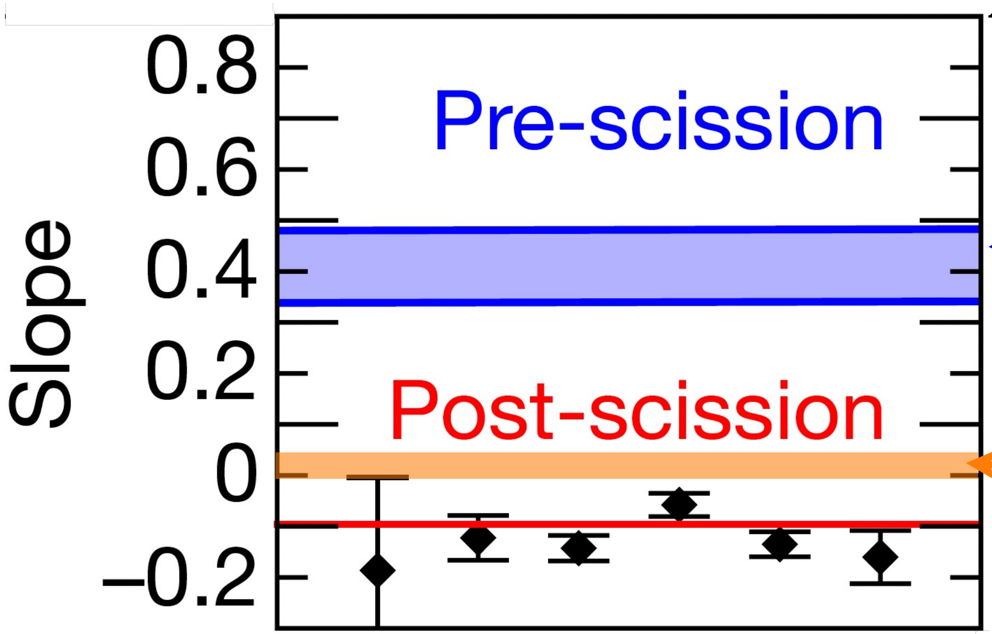
Reaction	Average total angular momentum $\bar{\mathcal{J}}, \hbar$	Paper
^{232}Th(n,f)	10	**Naik**, Nucl. Phys. A **587**, 273(1995)
^{233}U(n,f)	7	**Hoffman**, PRC **113B**, B714(1964)
^{235}U(n,f)	8	
^{239}Pu(n,f)	7	
^{252}Cf(sf)	7	**Wilhelmy**, PRC **5**, 2041(1972)



Wilson, Nature **590**, 566 (2021)



Wilson, Nature **590** 566 (2021)

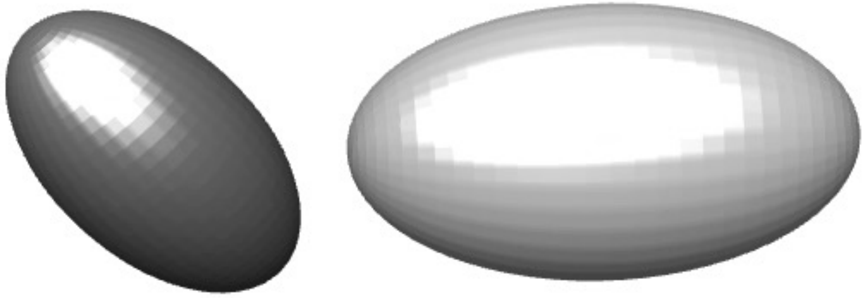


At fixed orbital momentum *L*

However, if *L* absorbs the fluctuations pre-scission predicts

Pre-scission spin generation

At the scission point the pre-fragments have a various vibration modes



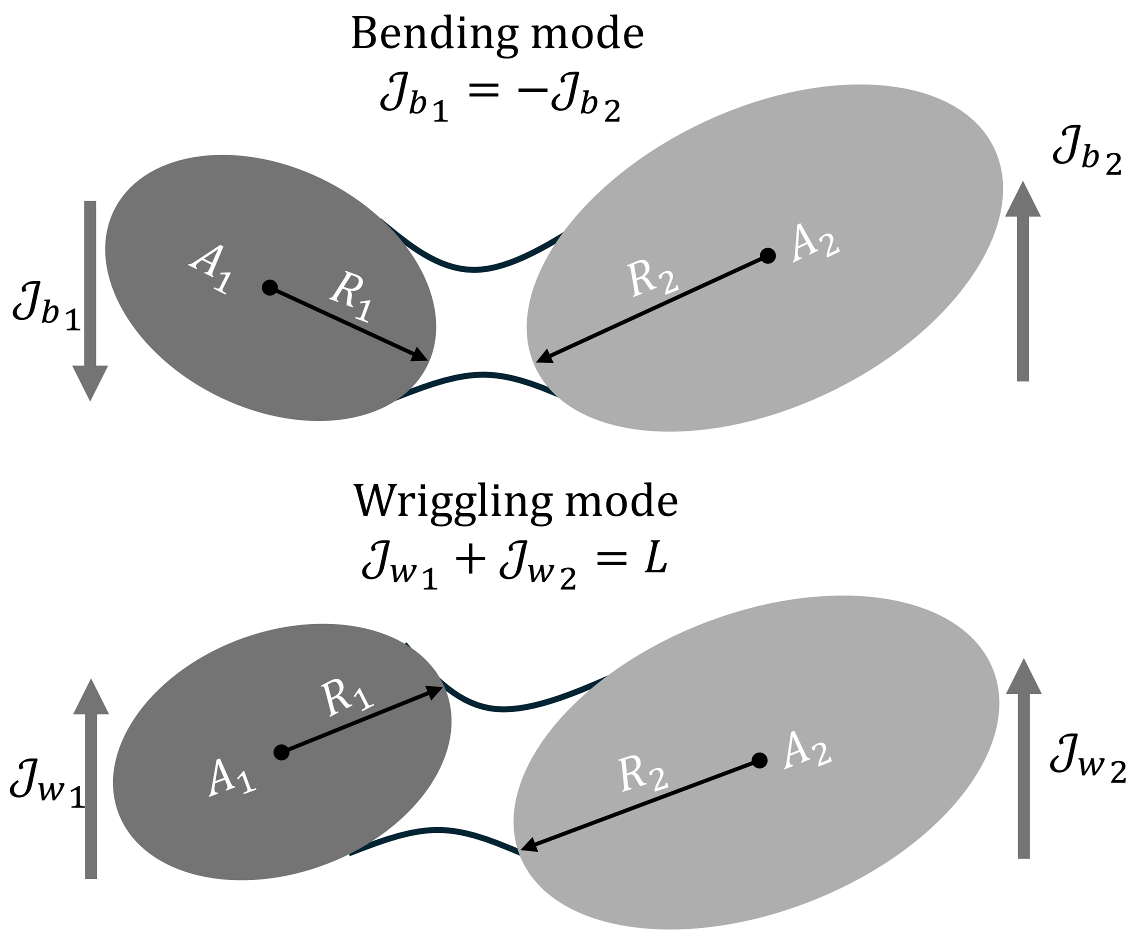
In this case the distribution of any of type such could be describe by function

$$P(\mathcal{J}_{t_i}) = \frac{1}{\sqrt{\pi C_t}} \exp\left[-\frac{\mathcal{J}_{t_i}^2}{C_t}\right]$$

t is type of mode

Kadmensky et al, Nucl At Phys 87, 359 (2024)

But the most important for $\mathcal{J}$ two of them



Nix & Swiatecki, Nucl Phys **71**, 1 (1965)

Constructing the distribution of angular moments

$$P(\mathcal{J}_1, \mathcal{J}_2, \varphi) = \frac{2\mathcal{J}_1\mathcal{J}_2}{\pi C_w C_b} \exp[-\mathcal{J}_1(aI_2^2 + b) - \mathcal{J}_2(aI_1^2 + b) + 2\mathcal{J}_1\mathcal{J}_2 \cos\varphi\,(aI_1I_2 - b)]$$

Kadmensky et al, Nucl At Phys 8**7**, 359 (2024)

where $a = [C_b(I_1 + I_2)^2]^{-1};\ b = C_w^{-1}$

$\Downarrow$

$$P(\mathcal{J}_i) = \iint P(\mathcal{J}_i, \mathcal{J}_j, \varphi)\, d\mathcal{J}_j\, d\varphi \Longrightarrow P(\mathcal{J}_i) = \frac{2\mathcal{J}_i}{\xi_i} \exp\left[-\frac{\mathcal{J}_i^2}{\xi_i}\right]$$

$\Leftarrow$

$$\bar{\mathcal{J}}_i = \int_0^\infty P(\mathcal{J}_i)\mathcal{J}_i\, d\mathcal{J}_i = \frac{1}{2}\sqrt{\frac{\pi}{\xi_i}}$$

with $\xi_i = \frac{I_i^2 C_w}{(I_1+I_2)^2} + C_b$

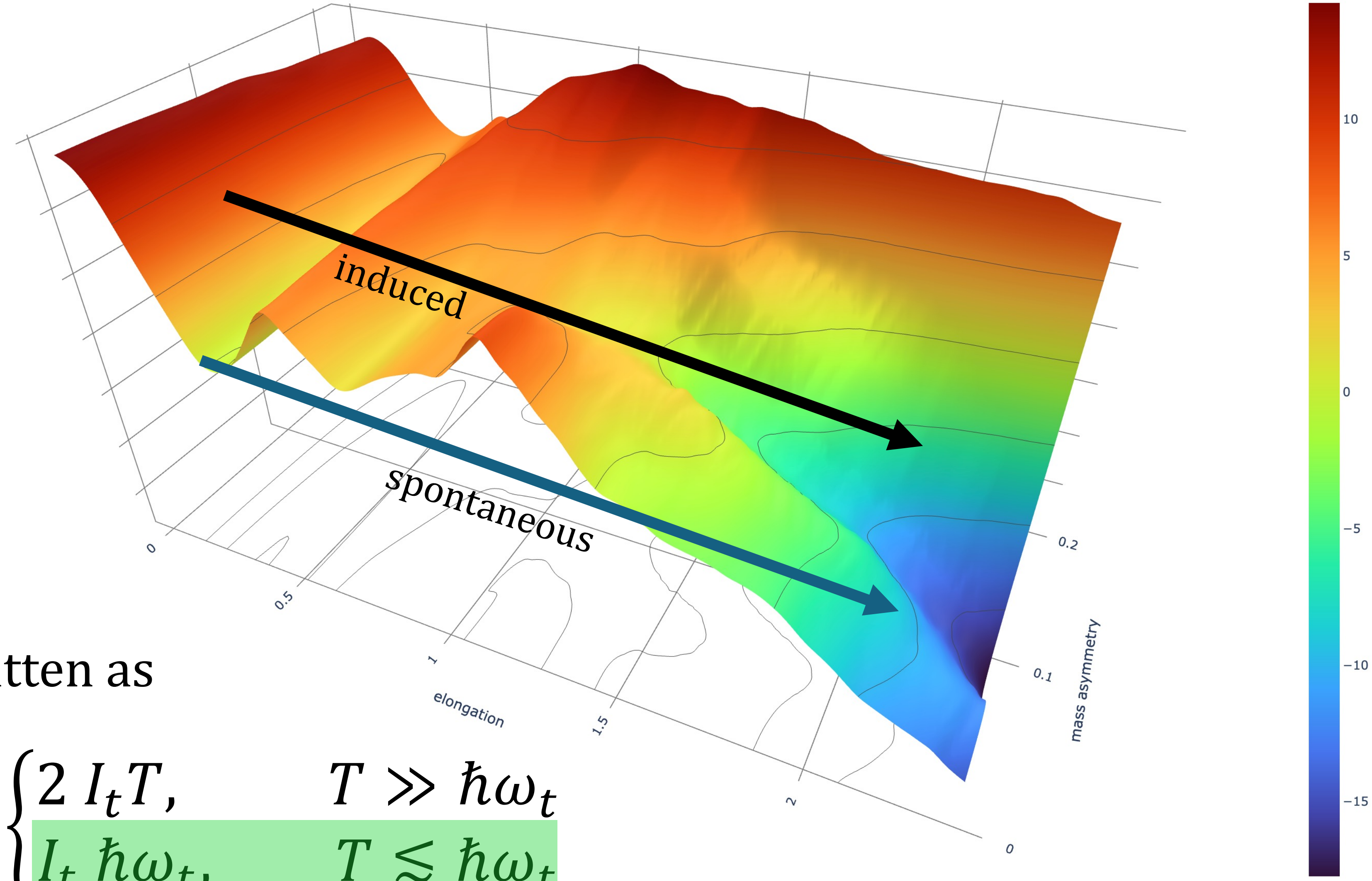
Collective motion of low-energy fission

“Cold” fission approach

$$TXE_{A_c} \ll \sum \Delta V\left(def_{A_i}\right)$$

$$T_{A_c} < 1\ \mathrm{MeV}$$

$$P(\mathcal{J}_{t_i}) = \frac{1}{\sqrt{\pi C_t}} \exp\left[-\frac{\mathcal{J}_{t_i}^2}{C_t}\right]$$



In this case the C_t could be written as

$$C_t = I_t \hbar\omega_t \coth\left[\frac{\hbar\omega_t}{2T}\right] \longrightarrow \begin{cases} 2\, I_t T, & T \gg \hbar\omega_t \\ I_t\, \hbar\omega_t, & T \lesssim \hbar\omega_t \end{cases}$$

Nix & Swiatecki, Nucl Phys **71**, 1 (1965)

Ingredients: Moments of inertia

$$I_t \to I_w = \frac{(I_1 + I_2)I_0}{I},$$

$$I_t \to I_b = I_1 + I_2\left(\frac{R_1}{R_2}\right)^2$$

Randrup, PRC 106, 014609 (2022)

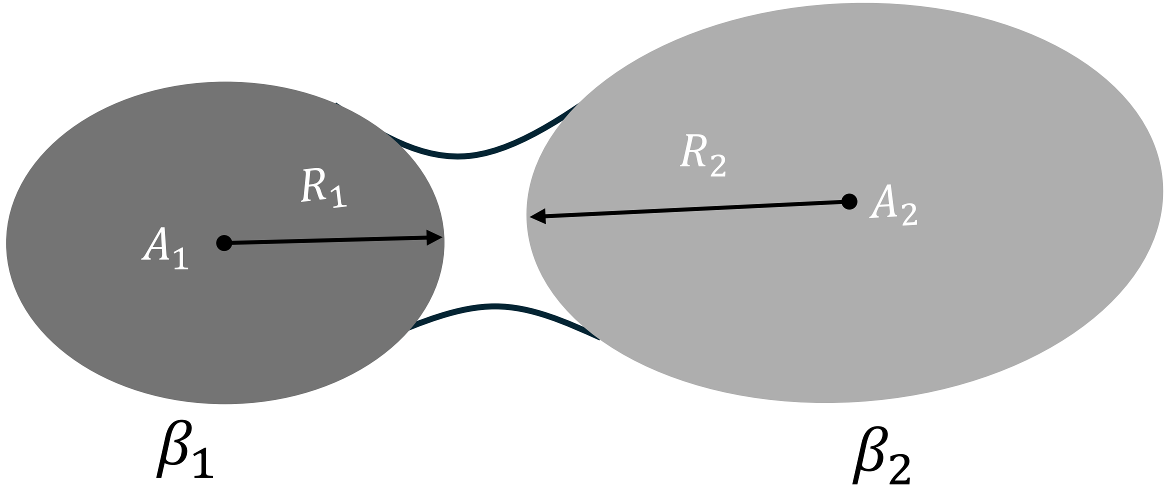
$$I = I_0 + I_1 + I_2$$

$$I_0 = \mathcal{M}(R_1 - R_2)^2$$

$$R_i(\beta_i) = r_0 A_i^{1/3}\left(1 + \sqrt{\frac{5}{4\pi}}\beta_i - \frac{\beta_i^2}{4\pi}\right)$$

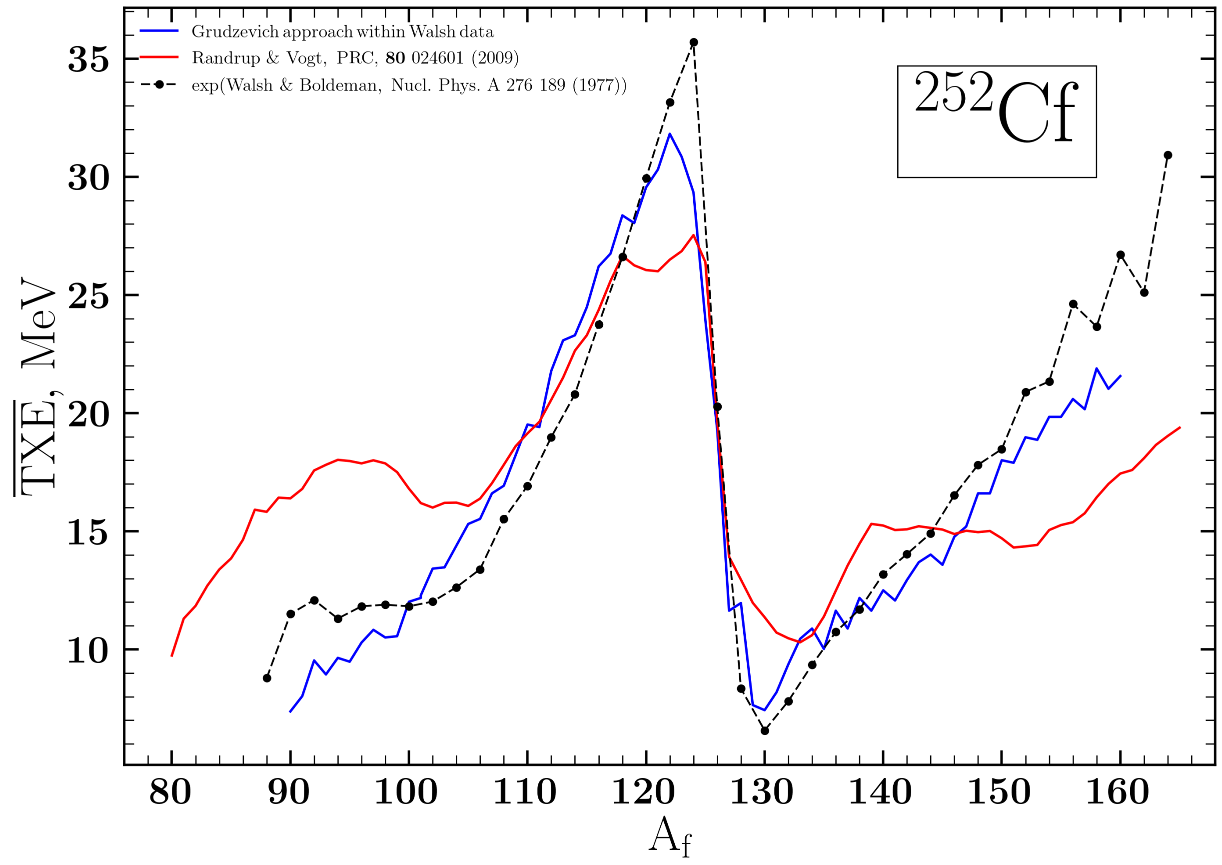
Model
Rigid body
Hydrodynamical
Superfluid: rectangular well, harmonic oscillator

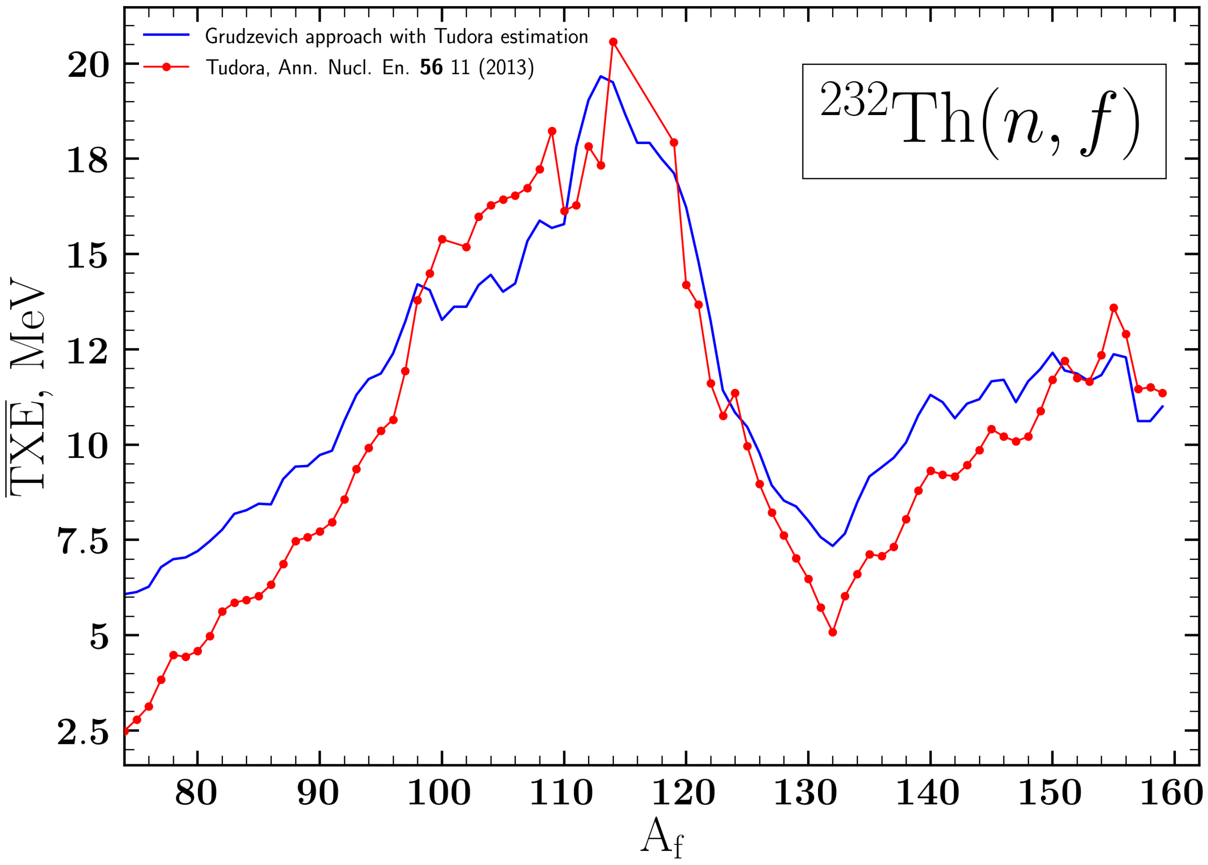
Lyubashevsky, CPC 49, 034104 (2025)



So what about FF deformations β_i and vibration energies $h\omega_t$?

Ingredients: Deformation parameter estimates





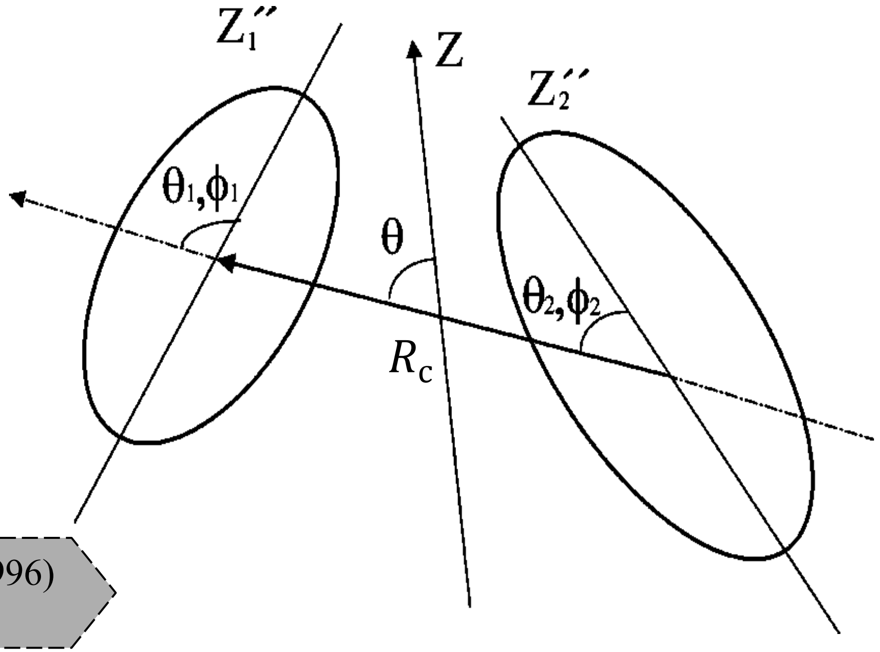
$$\boxed{\nu^2 + 4\nu + 5 = \overline{\mathrm{TXE}}} = \sigma A^{\frac{2}{3}}\left(\frac{2}{5}(1-x)\beta^2 - \frac{4}{105}(1-2x)\beta^3\right)$$

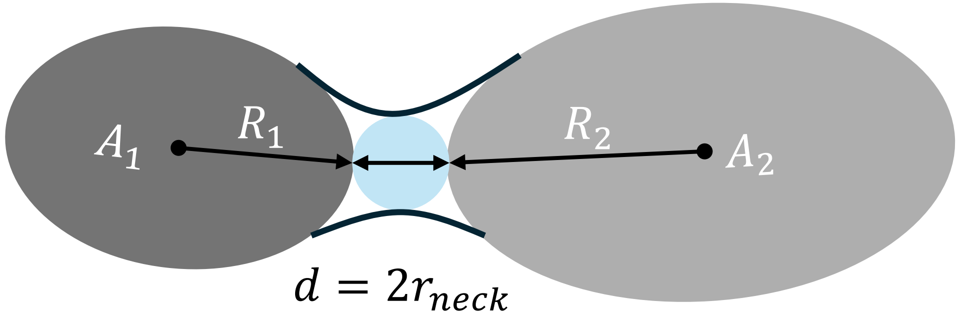
Grudzevich, Probl. At. Sci. Technol. Ser: Nucl. Const. **1** 39 (2000)

Strutinsky, Nucl. Phys. A **95** 420 (1967)

Ingredients: Estimation of the vibration's energy

$$\omega_t = \sqrt{\frac{K_t}{I_t}}$$





Adamian., Int. J. Mod. Phys. E **5** 191 (1996)
Shneidman, PRC **65** 064302 (2002)

$$R_c = R_1(\beta_1) + R_2(\beta_2) + d$$

$$\mathrm{U}(\beta_1, \beta_2, d, \Omega_i) = \mathrm{U}_{nucl}(\beta_1, \beta_2, d, \Omega_i) + \mathrm{U}_{Coul}(\beta_1, \beta_2, d, \Omega_i)$$

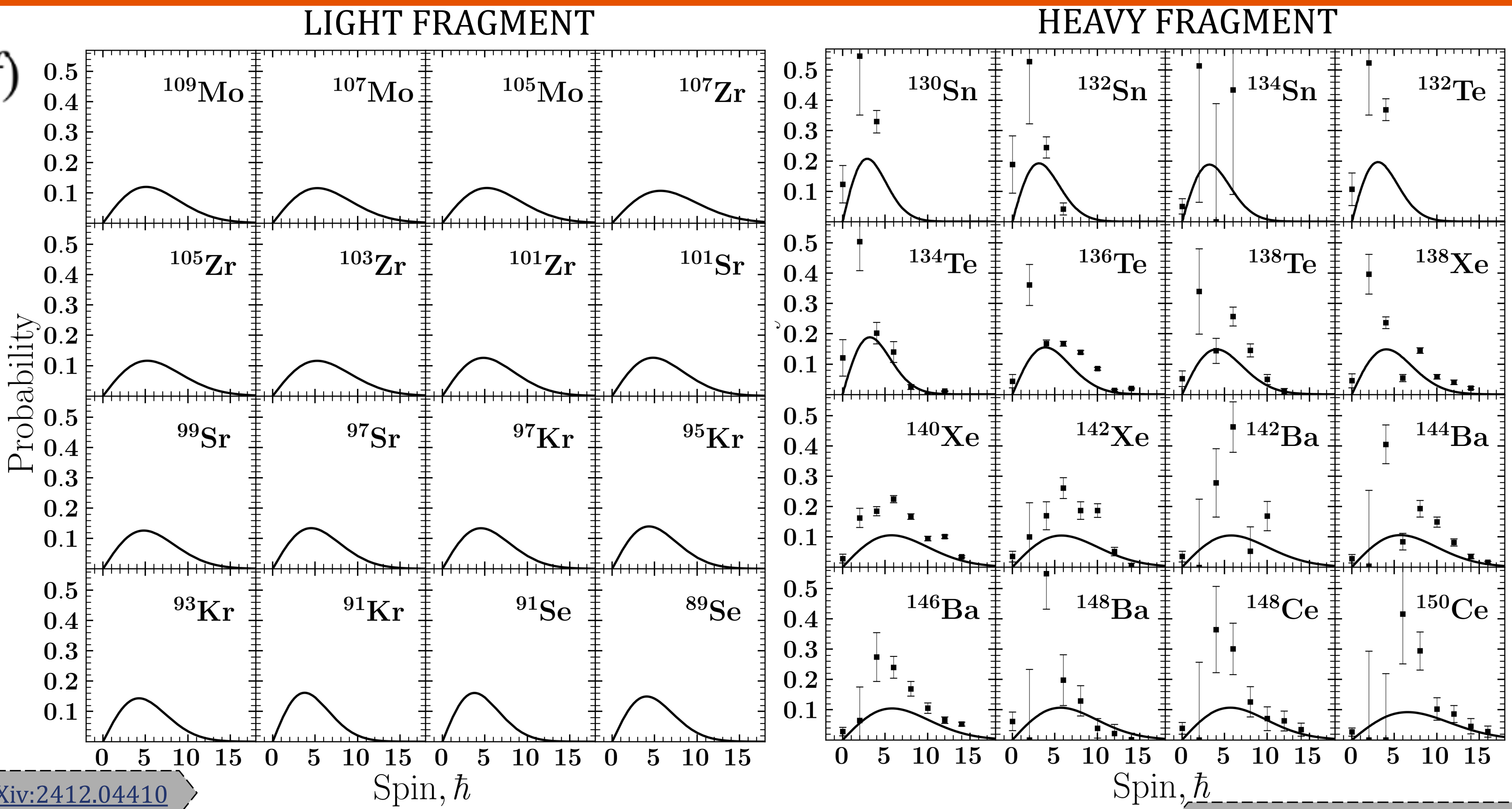
By differentiating U for each type of vibration one yields

$$K_b = K_{11} - 2K_{12}\frac{R_1}{R_2} + K_{22}\left(\frac{R_1}{R_2}\right)^2 \qquad K_w = K_{11} + 2K_{12}\frac{R_1}{R_2} + K_{22}\left(\frac{R_1}{R_2}\right)^2$$

Adamian, Phys At Nucl, **70** 1350 (2007)

Results: Spin distributions

$^{238}\mathbf{U}(n,f)$

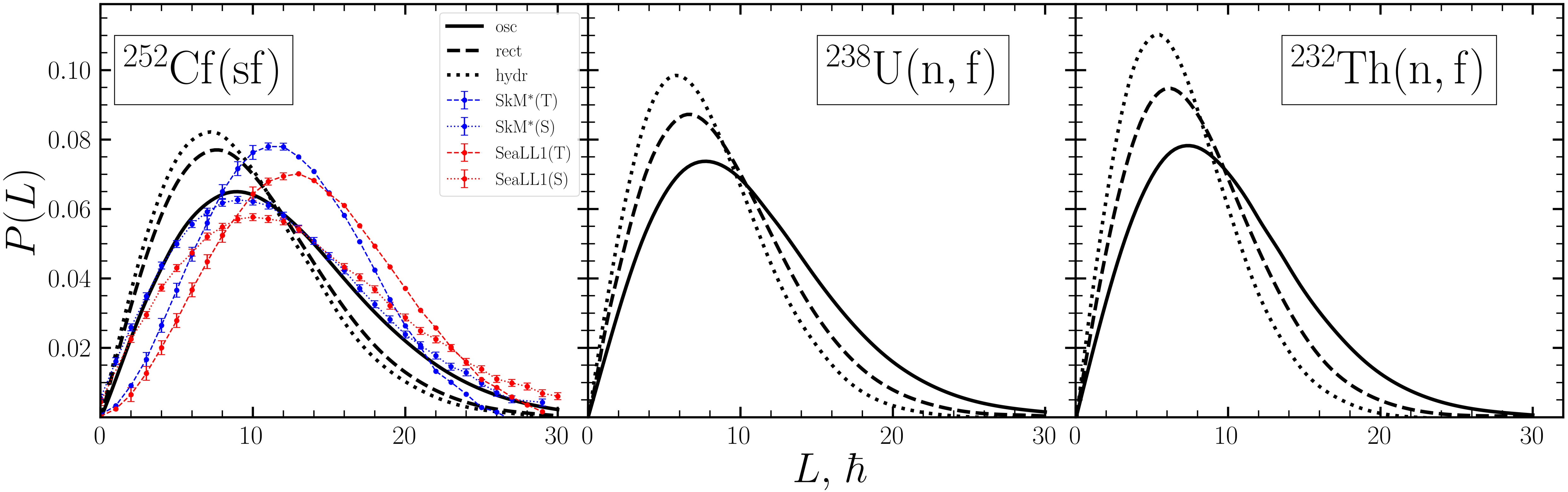


Lyubashevsky, arXiv:2412.04410

Wilson, Nature **590** 566 (2021)

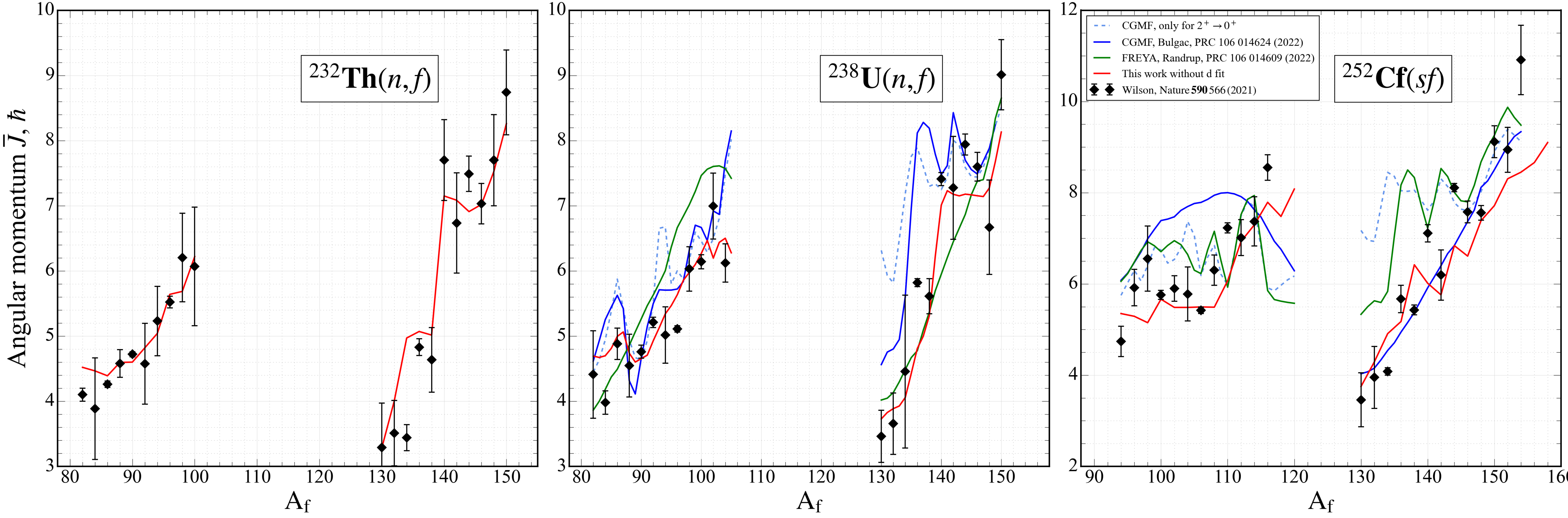
Results: Orbital momentum distribution

$$P(L) = \int_0^{\pi} \frac{2L}{\pi C_w} \exp\left[-\frac{L^2}{C_w}\right] d\varphi_L = \frac{2L}{C_w} \exp\left[-\frac{L^2}{C_w}\right] \Rightarrow \boxed{\bar{L} = \int_0^{\infty} L \frac{2L}{C_w} \exp\left[-\frac{L^2}{C_w}\right] dL = \frac{1}{2}\sqrt{\pi C_w}}$$

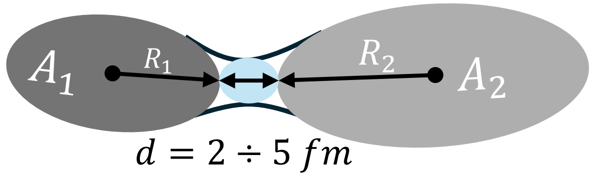


Best agreement: superfluid model with HO potential

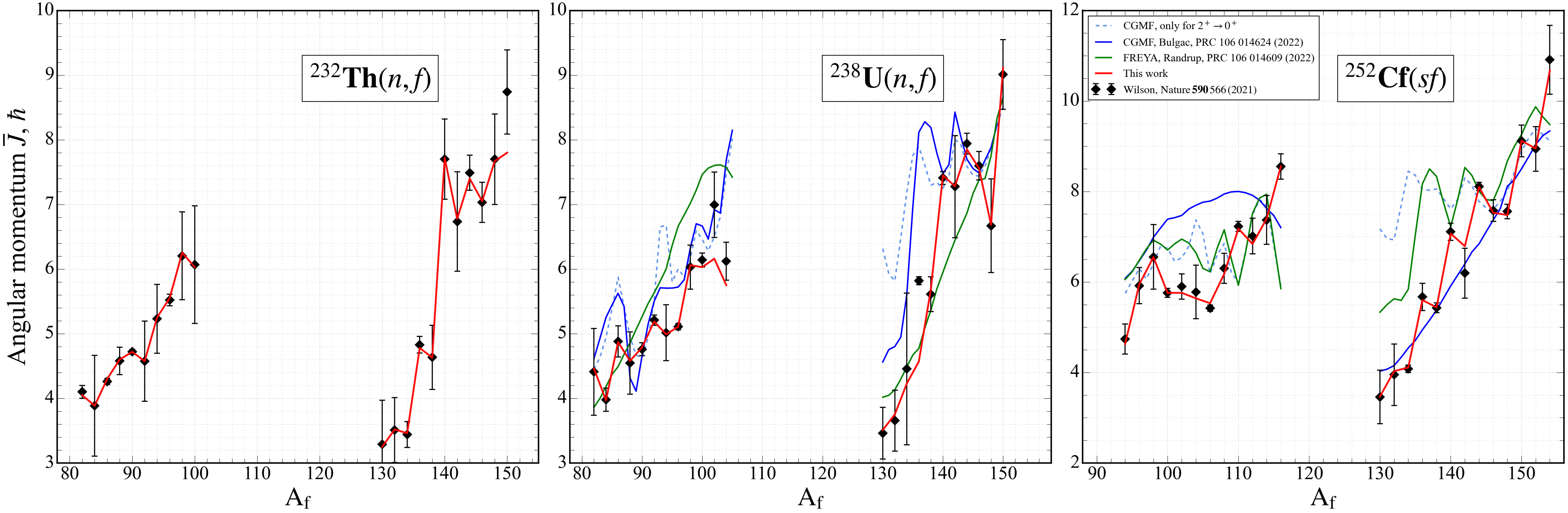
Results: Angular momentum distribution



Results: Angular momentum distribution



$$R_c = R_1(\beta_1) + R_2(\beta_2) + d(fit)$$



Results: Correlations

$$c_{\mathcal{J}_1\mathcal{J}_2}(A_1, A_2) = \frac{\mu_{\mathcal{J}_1\mathcal{J}_2}}{\sigma_1\sigma_2}$$

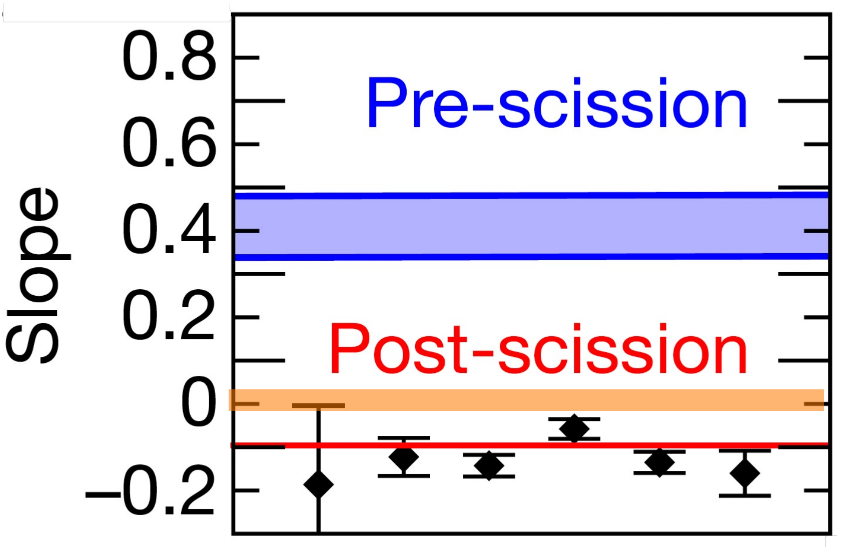
$$\tilde{c}_{\mathcal{J}_1\mathcal{J}_2}(A_1, A_2) = \frac{\sum c_{\mathcal{J}_1\mathcal{J}_2}(A_1, A_2) Y(A_1, A_2)}{\sum Y(A_1, A_2)}$$

Fragment	^{232}Th(n,f)		^{238}U(n,f)		^{252}Cf(sf)	
	$c_{\mathcal{J}_1\mathcal{J}_2}$	$Y(A_1, A_2)$	$c_{\mathcal{J}_1\mathcal{J}_2}$	$Y(A_1, A_2)$	$c_{\mathcal{J}_1\mathcal{J}_2}$	$Y(A_1, A_2)$
^{94}Sr	0,007	2,04	0,006	1,51	0,055	0,55
^{96}Sr	0,02	3,54	0,002	4,13	0,018	0,89
^{98}Sr	0,019	1,32	0	2,27	0,015	0,37
^{98}Zr	0,02	0,37	0	0,49	0,014	0,59
^{100}Zr	0,047	0,88	0,016	3,3	0,001	2,06
—	—	—	—	—	—	—
^{140}Xe	0,011	5,73	0,004	4,04	0,026	2,55
^{142}Xe	0,014	2,25	0,009	1,53	0,023	0,37
^{142}Ba	0,014	0,64	0,011	0,69	0,019	2,7
^{144}Ba	0,035	4,49	0,012	2,46	0,001	3,37
^{146}Ba	0,04	2,76	0,014	1,98	0	0,98
—	—	—	—	—	—	—
$\tilde{c}_{\mathcal{J}_1\mathcal{J}_2}$	0.034		0.02		0.017	
$c^{\text{FREYA}}_{S_1S_2}$	—		0.002		0.001	

Results: Correlations

$$c_{\mathcal{J}_1\mathcal{J}_2}(A_1, A_2) = \frac{\mu_{\mathcal{J}_1\mathcal{J}_2}}{\sigma_1\sigma_2}$$

$$\tilde{c}_{\mathcal{J}_1\mathcal{J}_2}(A_1, A_2) = \frac{\sum c_{\mathcal{J}_1\mathcal{J}_2}(A_1, A_2) Y(A_1, A_2)}{\sum Y(A_1, A_2)}$$

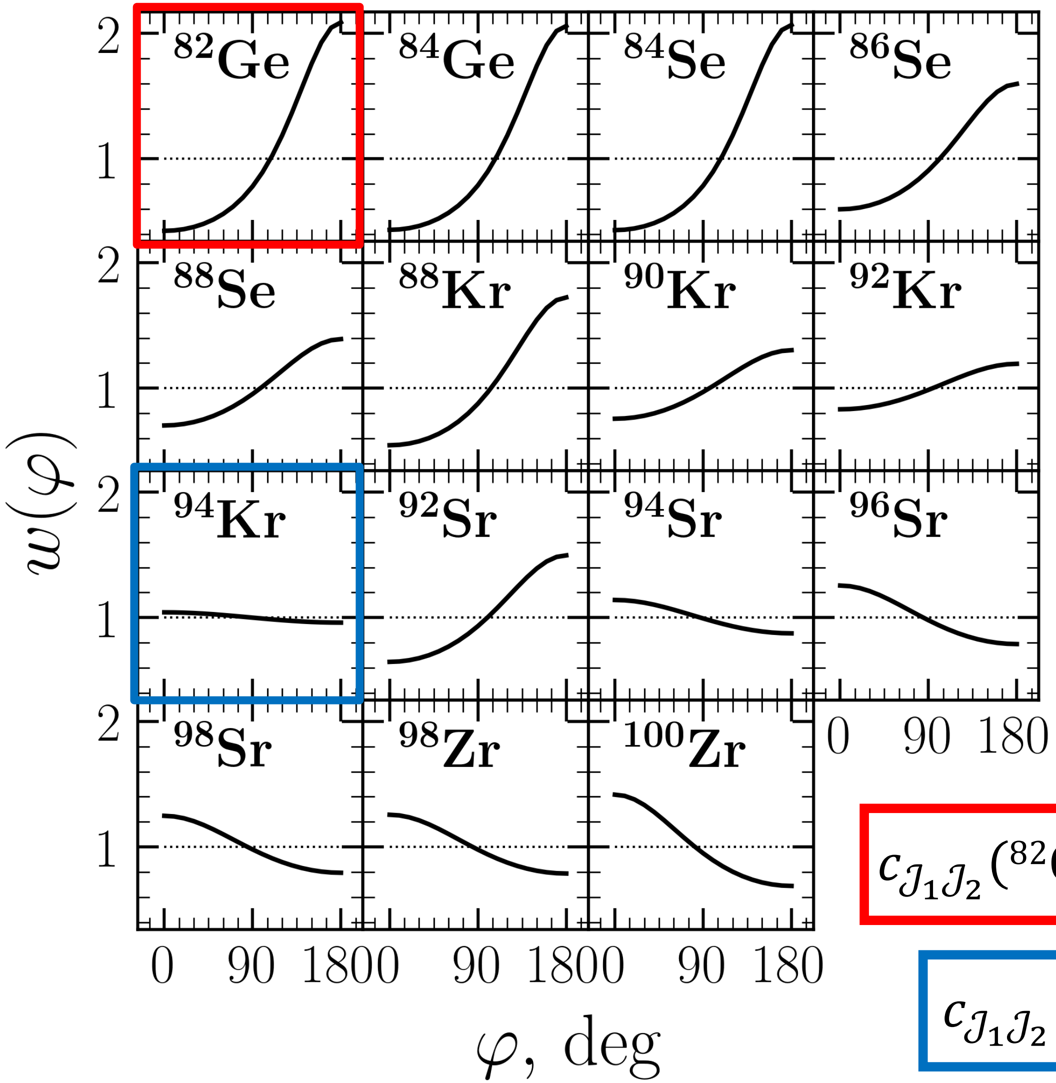


Fragment	^{232}Th(n,f)		^{238}U(n,f)		^{252}Cf(sf)	
	$c_{\mathcal{J}_1\mathcal{J}_2}$	$Y(A_1, A_2)$	$c_{\mathcal{J}_1\mathcal{J}_2}$	$Y(A_1, A_2)$	$c_{\mathcal{J}_1\mathcal{J}_2}$	$Y(A_1, A_2)$
^{94}Sr	0,007	2,04	0,006	1,51	0,055	0,55
^{96}Sr	0,02	3,54	0,002	4,13	0,018	0,89
^{98}Sr	0,019	1,32	0	2,27	0,015	0,37
^{98}Zr	0,02	0,37	0	0,49	0,014	0,59
^{100}Zr	0,047	0,88	0,016	3,3	0,001	2,06
—	—	—	—	—	—	—
^{140}Xe	0,011	5,73	0,004	4,04	0,026	2,55
^{142}Xe	0,014	2,25	0,009	1,53	0,023	0,37
^{142}Ba	0,014	0,64	0,011	0,69	0,019	2,7
^{144}Ba	0,035	4,49	0,012	2,46	0,001	3,37
^{146}Ba	0,04	2,76	0,014	1,98	0	0,98
—	—	—	—	—	—	—
$\tilde{c}_{\mathcal{J}_1\mathcal{J}_2}$	0.034		0.02		0.017	
$c^{\text{FREYA}}_{S_1S_2}$	—		0.002		0.001	

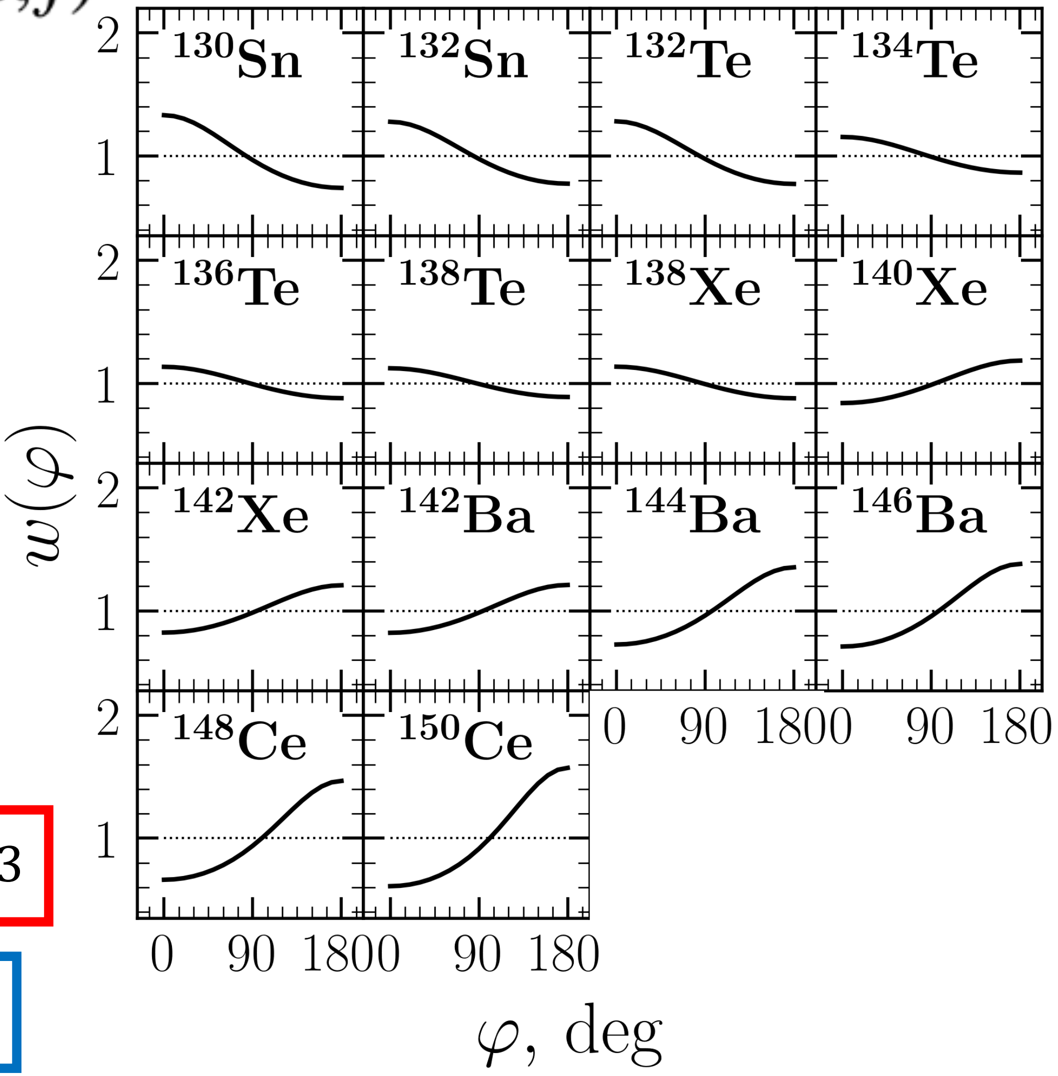
Results: Angular distribution of spins directions

$^{232}\mathbf{Th}(n,f)$

LIGHT FRAGMENT



HEAVY FRAGMENT



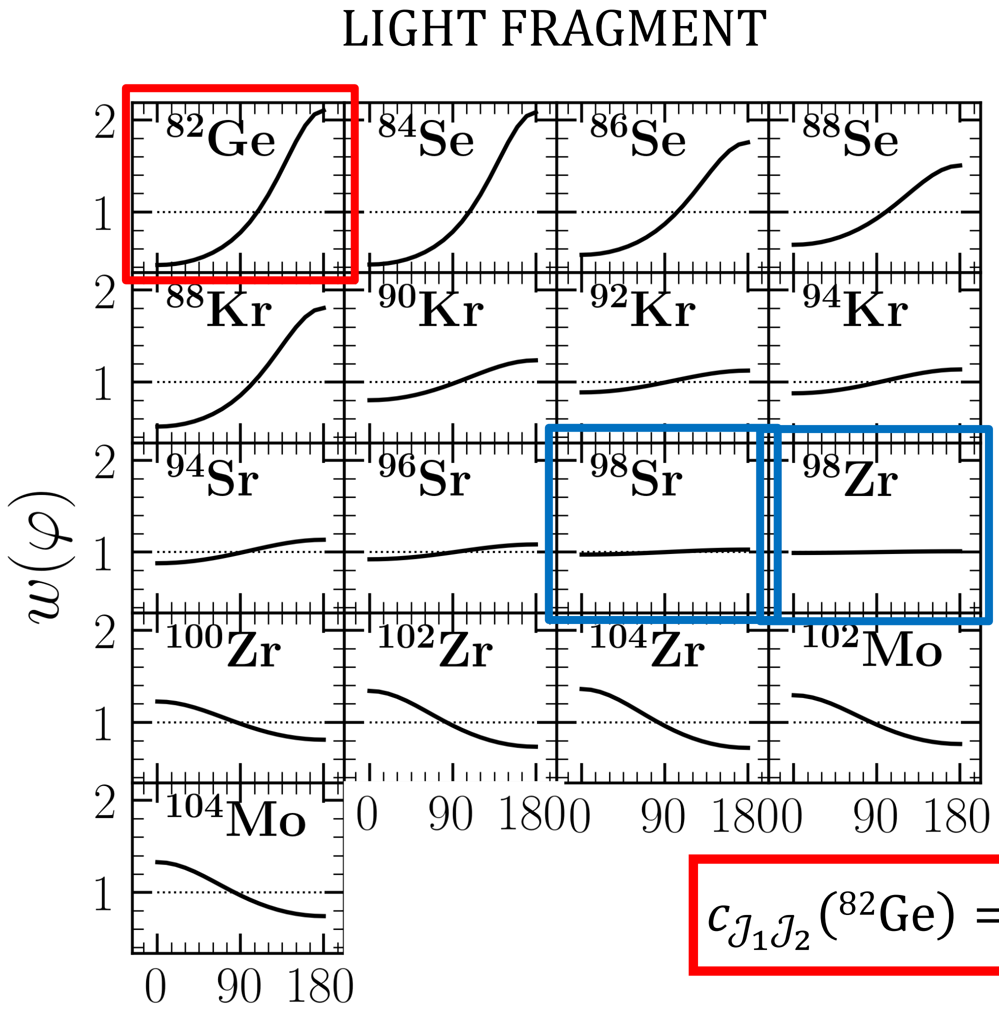
$c_{\mathcal{J}_1\mathcal{J}_2}(^{82}\mathrm{Ge}) = 0.203$

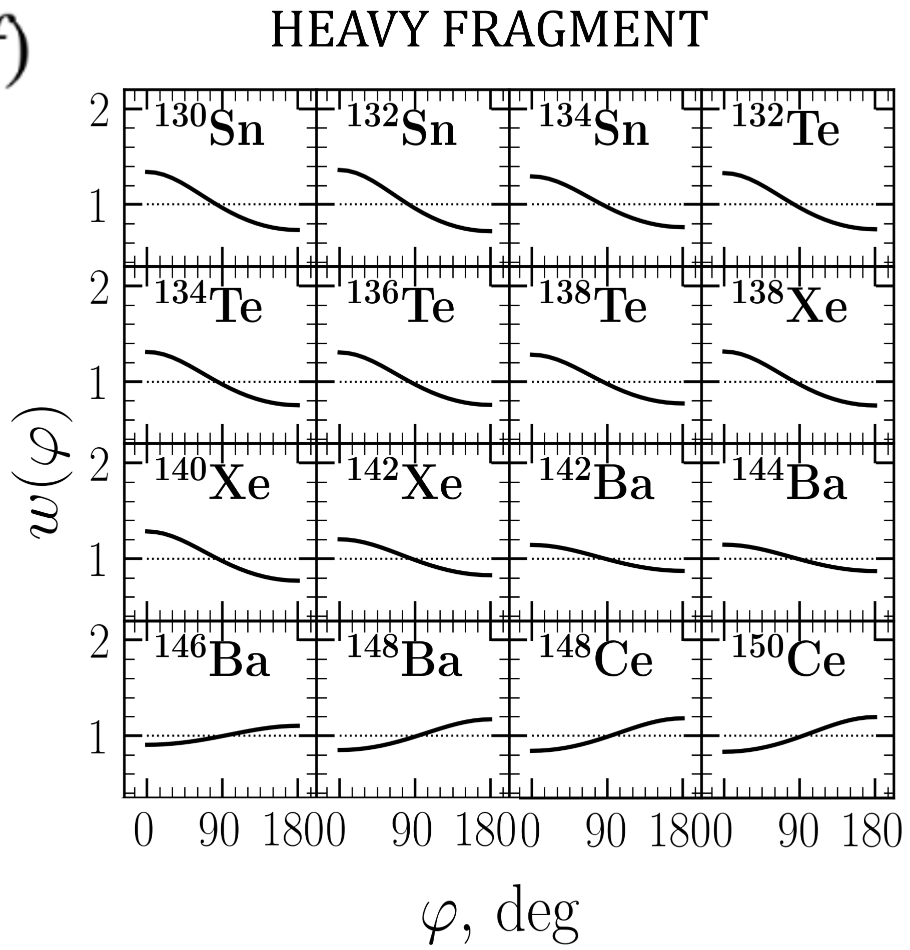
$c_{\mathcal{J}_1\mathcal{J}_2}(^{94}\mathrm{Kr}) = 0$

Lyubashevsky, PRC **111** 054601 (2025)

Results: Angular distribution of spins

$^{238}\mathbf{U}(n,f)$

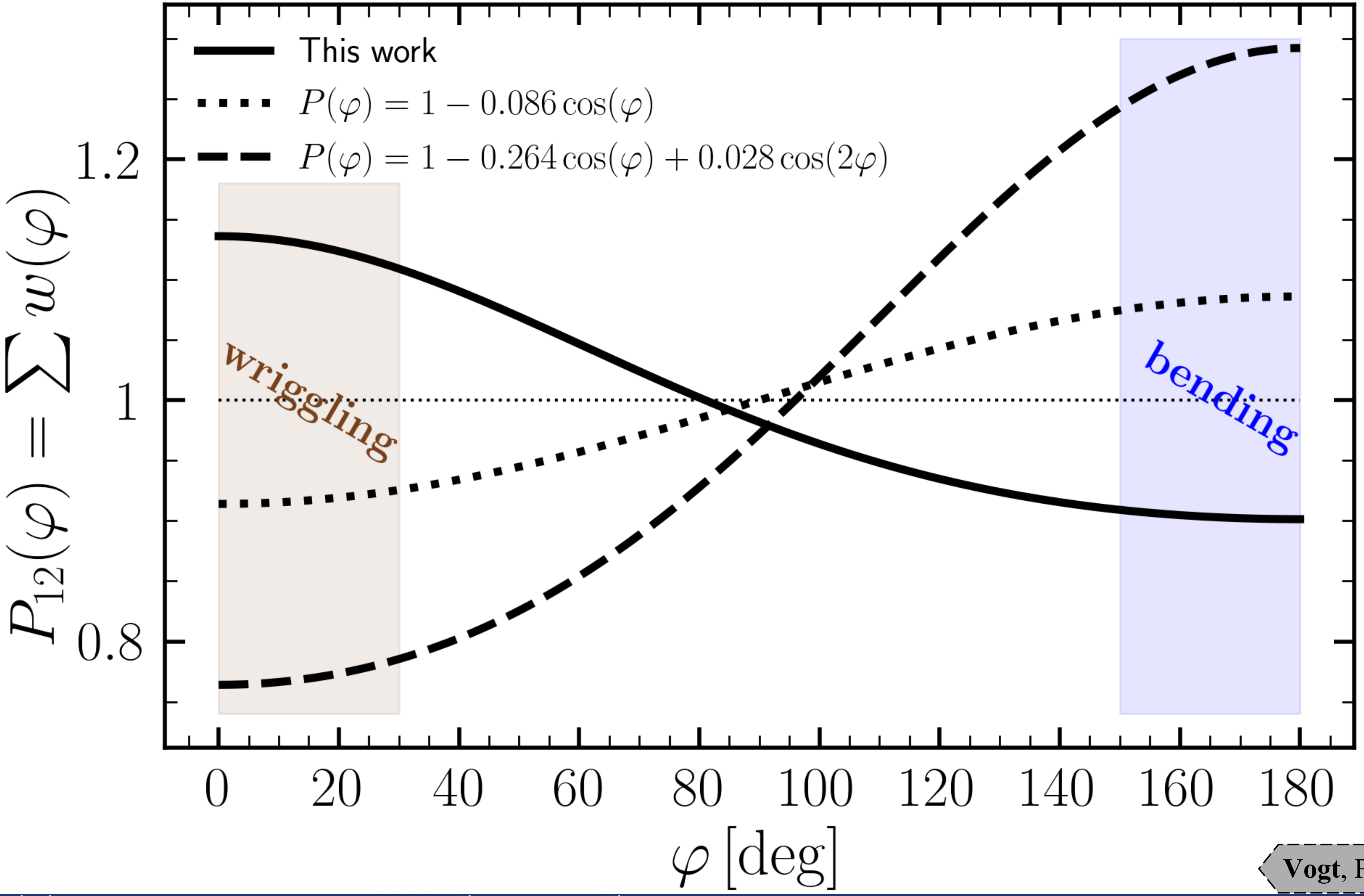




$c_{\mathcal{J}_1\mathcal{J}_2}(^{82}\mathrm{Ge}) = 0.207$

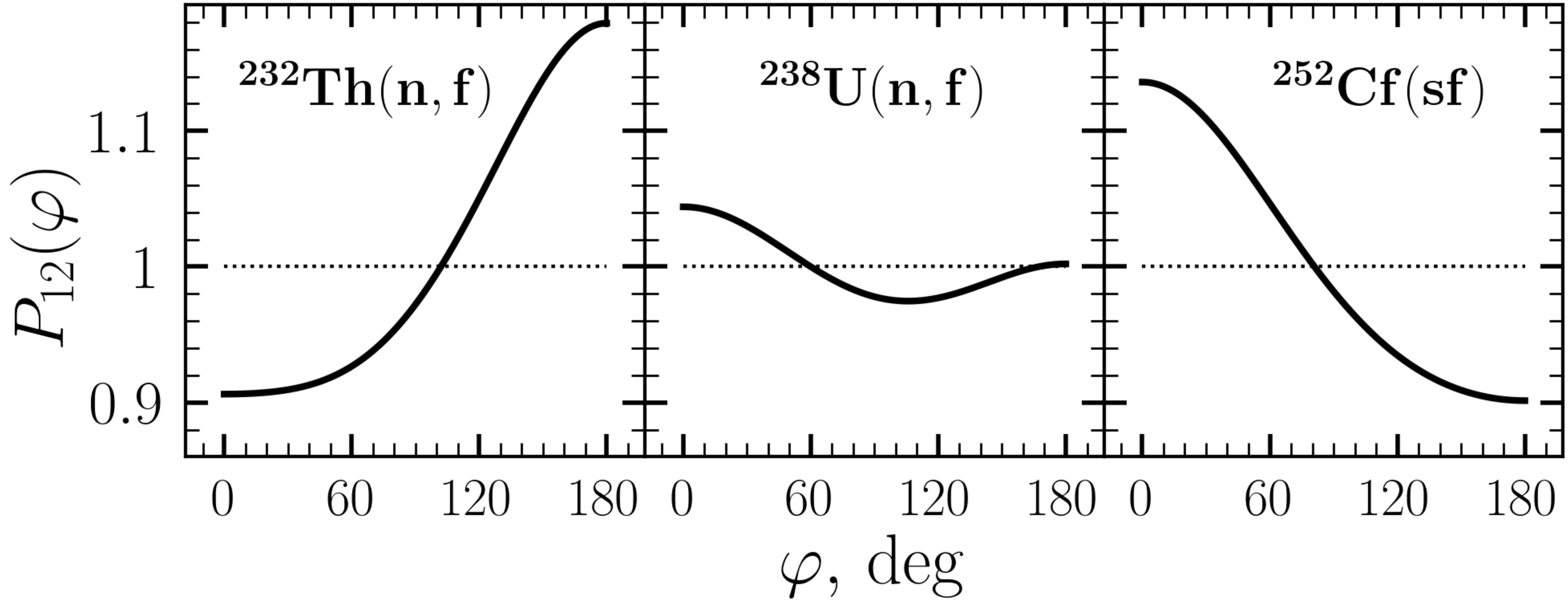
$c_{\mathcal{J}_1\mathcal{J}_2}(^{98}\mathrm{Sr}) = c_{\mathcal{J}_1\mathcal{J}_2}(^{98}\mathrm{Zr}) = 0$

Results: Angular distribution of spins

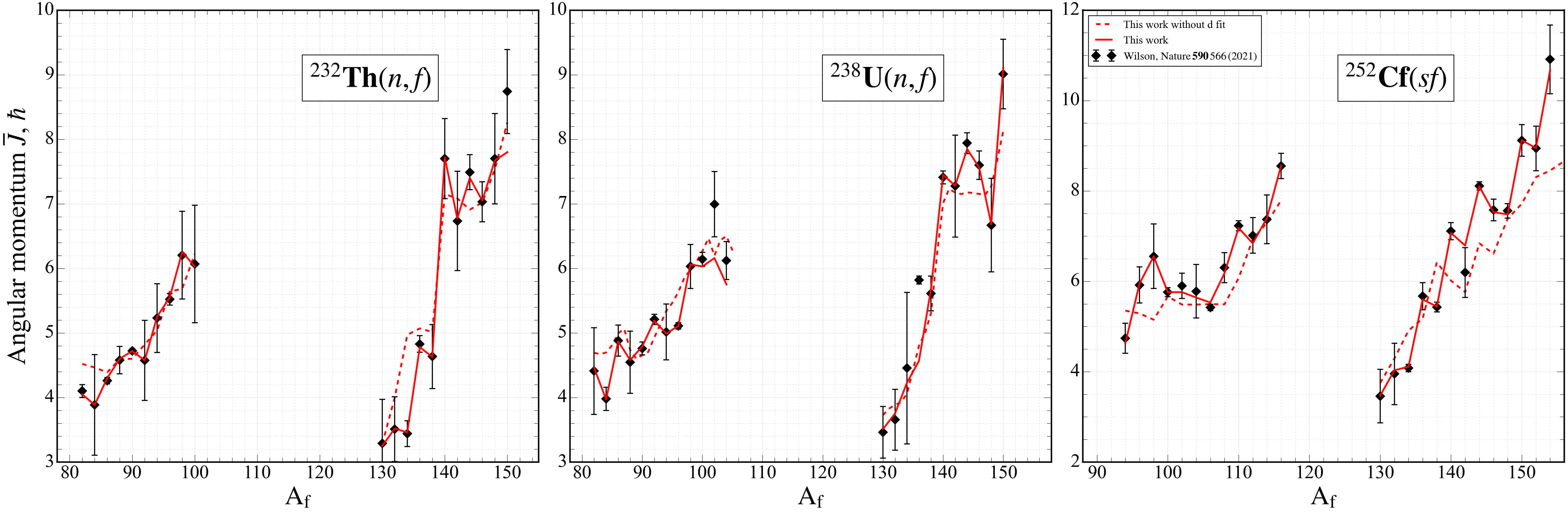


Vogt, PRC **103**. 014610 (2021)

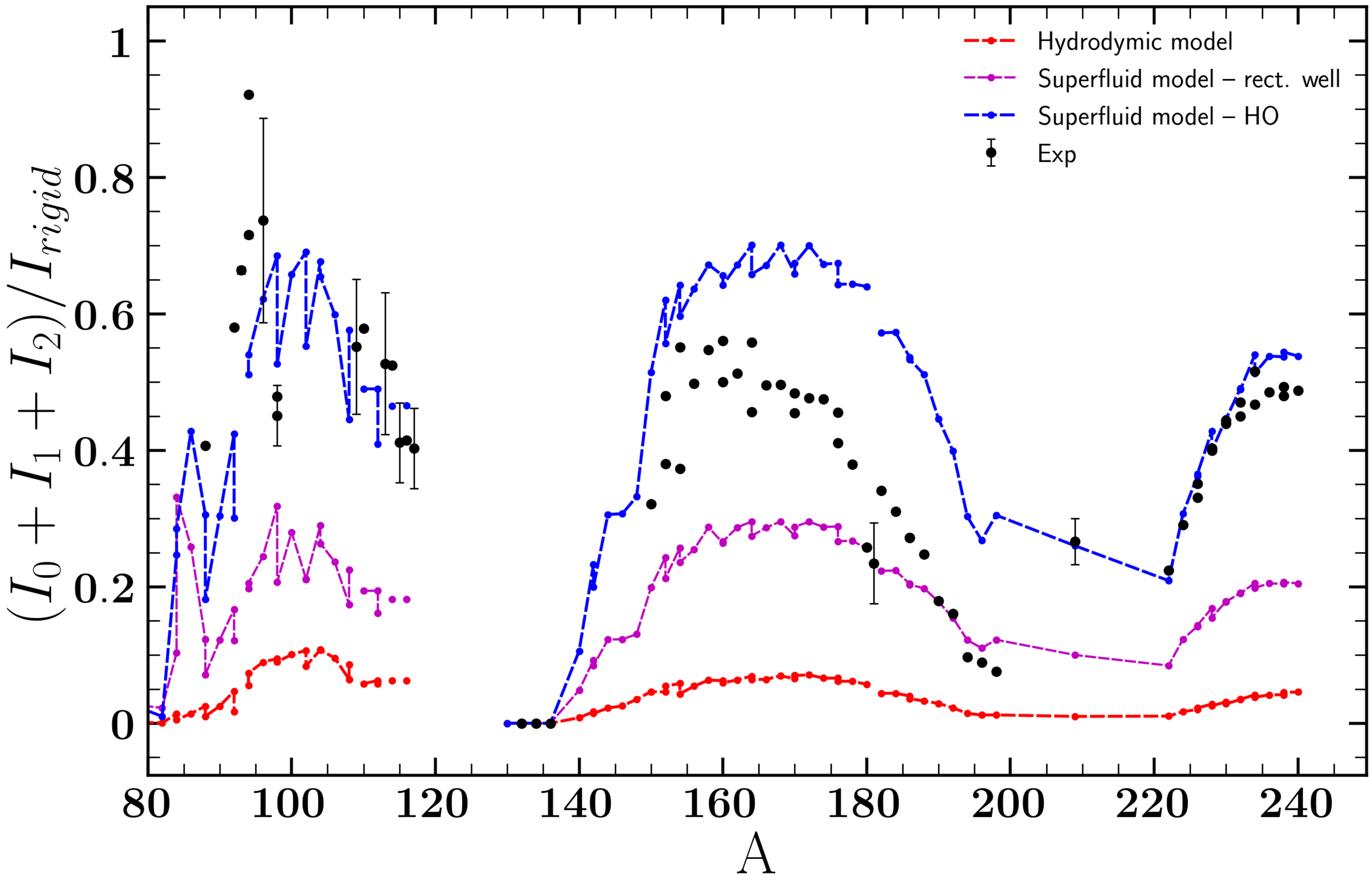
Results: Angular distribution of spins



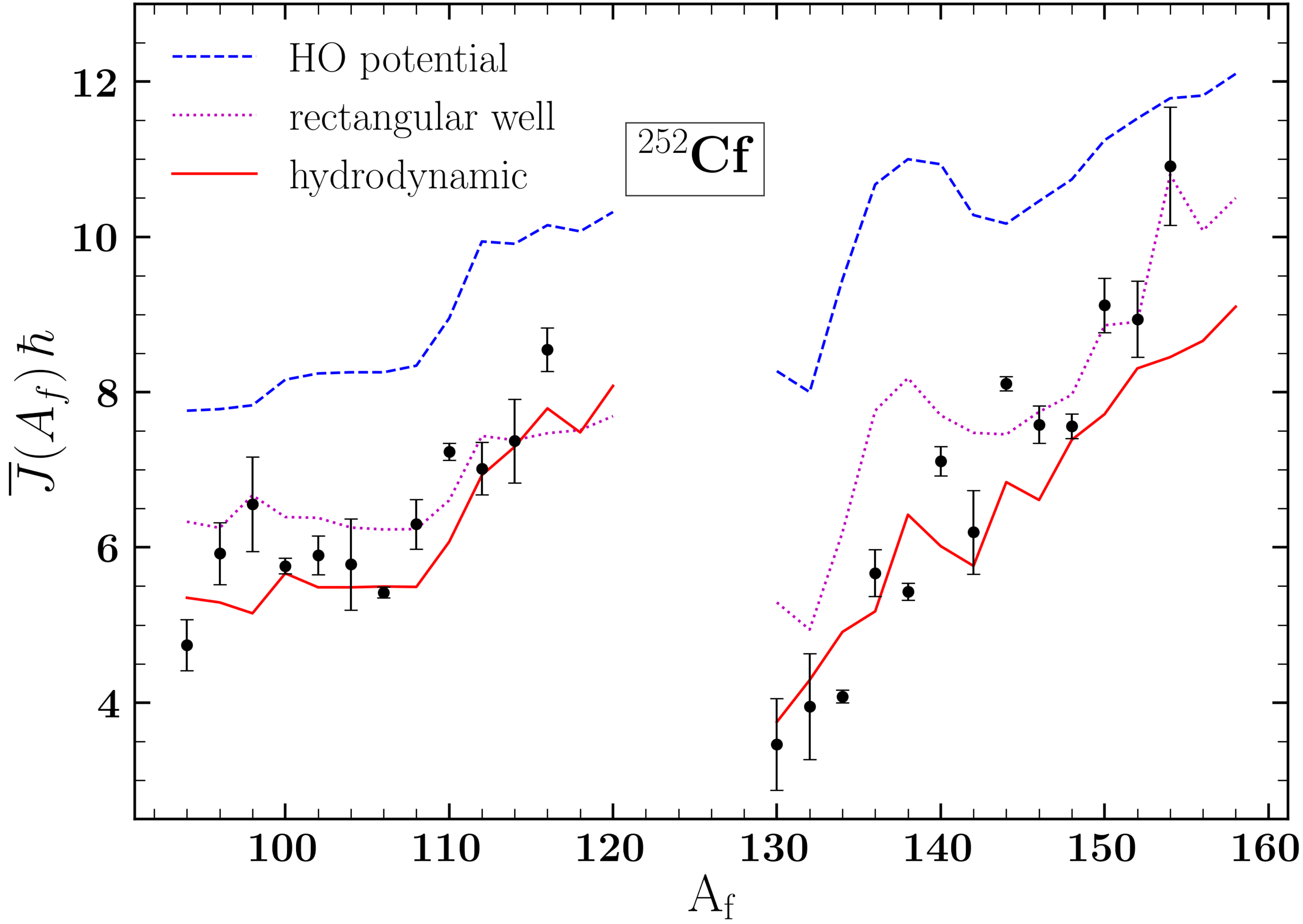
Beyond thermalization



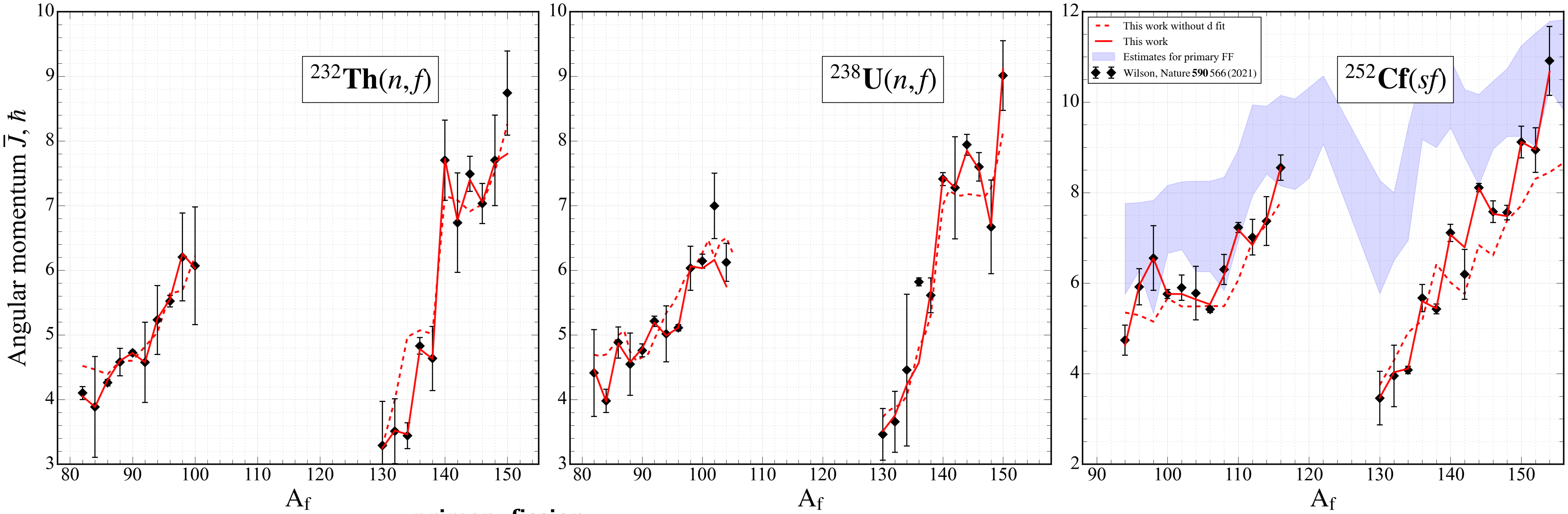
Beyond thermalization

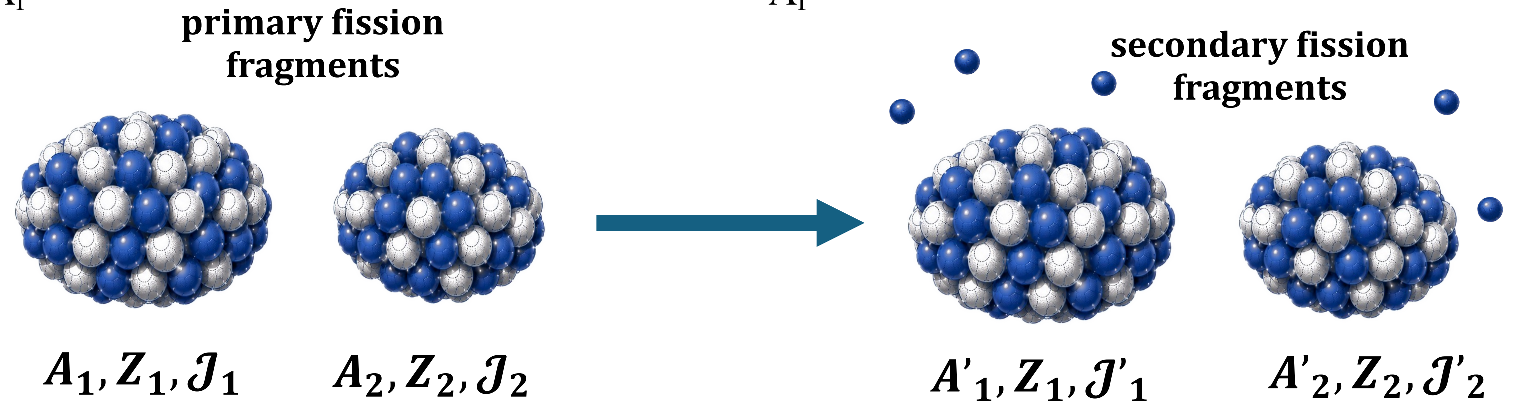


Beyond thermalization



Beyond thermalization





Summary

1. Developed simple low-energy pre-scission model reproduces the main features of fragment spin distributions across $^{232}Th(n,f)$, $^{238}U(n,f)$, and $^{252}Cf(sf)$.

2. Near-zero spin–spin correlations do not contradict the pre-scission mechanism; they follow naturally once the role of the orbital moment is accounted for.

3. The orbital angular momentum can serve as an intermediate reservoir absorbing angular momentum fluctuations between the two fragments.

4. Post-scission neutron evaporation is essential for the correct interpretation of experimental spin data.

THANK YOU FOR ATTENTION

Questions?Comments?