

Microscopic modeling of compound nucleus reactions  
: low-energy induced fission and astrophysical nuclear fusion reactions

Kouichi Hagino  
Kyoto University

George F. Bertsch (Seattle)  
Kotaro Uzawa (JAEA)  
Kosei Nagao (Kyoto)



1. Microscopic calculations for low-energy induced fission
2.  $^{12}\text{C}+^{12}\text{C}$  fusion reactions at astrophysical energies
3. Discussion: the validity of compound nucleus picture at low energies?
4. Summary

Microscopic modeling of compound nucleus reactions  
: low-energy induced fission and astrophysical nuclear fusion reactions



Chongqing, March 2011

Kouichi Hagino  
Kyoto University

George F. Bertsch (Seattle)  
Kotaro Uzawa (JAEA)  
Kosei Nagao (Kyoto)



1. Microscopic calculations for low-energy induced fission
2.  $^{12}\text{C}+^{12}\text{C}$  fusion reactions at astrophysical energies
3. Discussion: the validity of compound nucleus picture at low energies?
4. Summary

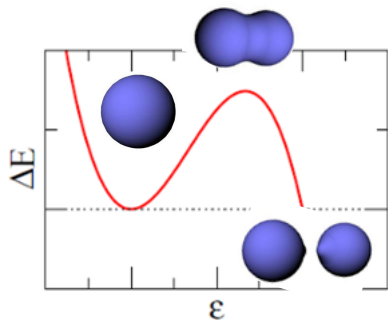
## Microscopic understanding of nuclear fission

How well can one describe nuclear fission microscopically?

### ➤ macroscopic understanding of fission:

a competition between the surface and the Coulomb energies

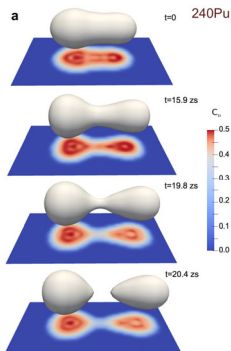
→ **fission barrier**



### ➤ a microscopic understanding of fission:

large change of nuclear shape

→ **far from complete**

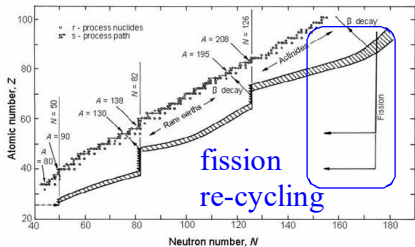


G. Scamps and C. Simenel,  
Nature 564 (2018) 382

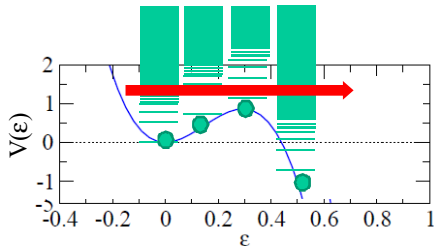
# Microscopic calculations for low energy induced fission

## Why is a microscopic theory for fission important?

### ➤ r-process nucleosynthesis



### ➤ barrier-top fission

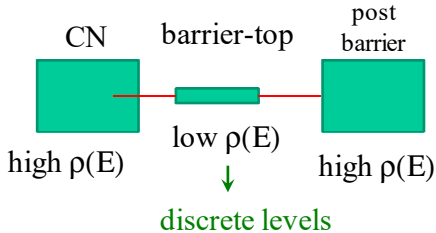


cf. talks by Kajino, Nishimura, and Giuliani tomorrow morning

fission of neutron-rich nuclei

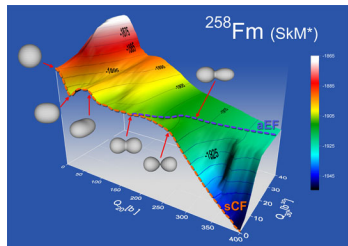
→ low  $E^*$  and low  $\rho(E^*)$

- ✓ Validity of statistical models?
- ✓ Validity of the Langevin approach?



## Microscopic approaches to fission

### ➤ spontaneous fission



A. Staszczak et al., PRC80 ('09) 014309

### constrained Hartree-Fock (+B) method:

$$\delta\langle\Phi|H - \lambda Q_{20}|\Phi\rangle = 0$$

$$\rightarrow \Phi(Q_{20}), E(Q_{20})$$

$$\rightarrow P = \exp\left[-2 \int dq \sqrt{\frac{2B(q)}{\hbar^2}(V(q) - E)}\right]$$

cf. the next talk by K. Washiyama

### ➤ induced fission: developments → only recently

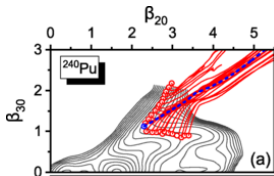
#### ✓ Time-dependent GCM

$$|\Psi(t)\rangle = \int dq f(q, t) |\Phi_q\rangle$$

D. Regnier et al., PRC93, 054611 (2016)

J. Zhao et al., PRC99, 054613 (2019)

B. Li et al., PRC111, L051302 (2025)



#### ✓ TD-SLDA

A. Bulgac et al., PRL135, 062501 (2025)

#### ✓ Finite $T$ -TDHF

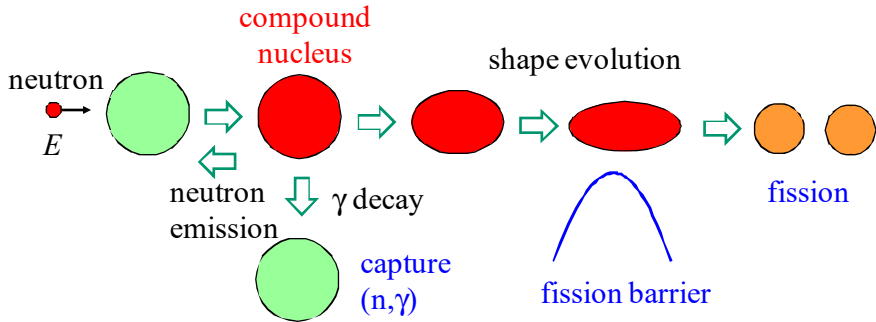
B. Li et al., PRC110, 034302 (2024)

#### ✓ Our approach

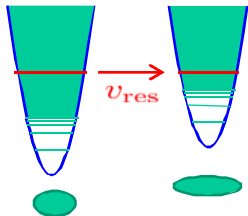
in the same philosophy as TDGCM,  
but with a time-independent approach using  
a Green's function

G.F. Bertsch and K.H., Phys. Rev. C107, 044615 (2023).

# Microscopic calculations for low energy induced fission



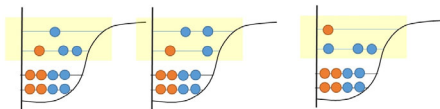
the basic idea:



- Many-body configurations in a MF pot. at each shape
- hopping due to residual interactions  
→ **shape evolution**

## Microscopic calculations for low energy induced fission

Shell model approach for nuclear structure



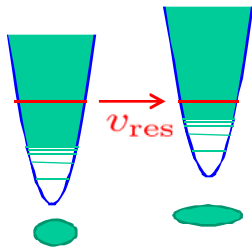
$$|\Psi\rangle = v_1|m_1\rangle + v_2|m_2\rangle + v_3|m_3\rangle + \dots$$

Figure: Noritaka Shimizu (Tsukuba)

many-particle many-hole configurations  
in a given mean-field potential

→ mixing by residual interactions

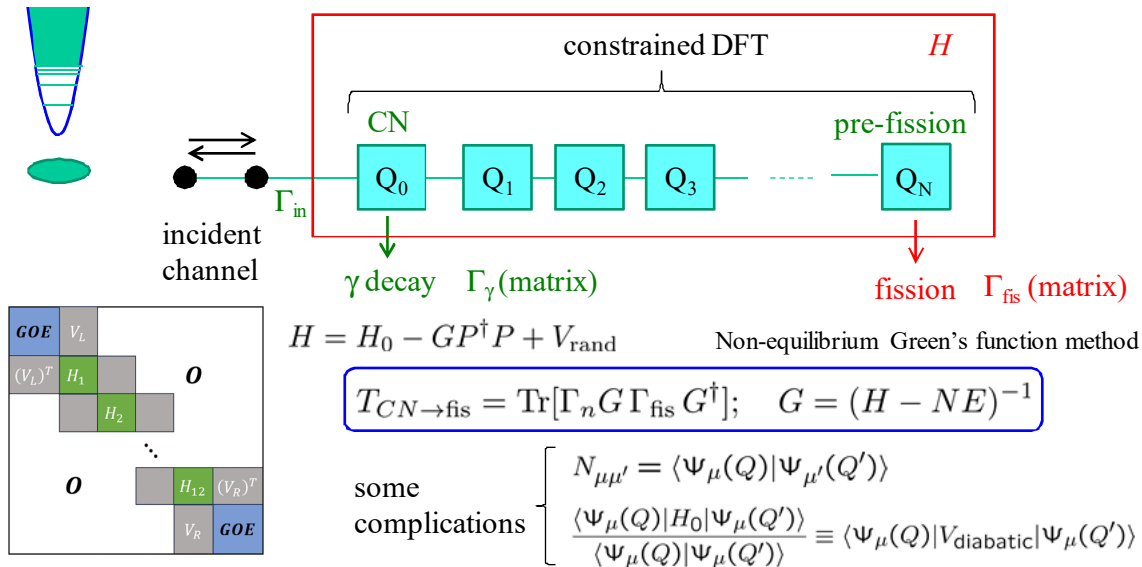
for induced fission



shape dependent mean-field potentials

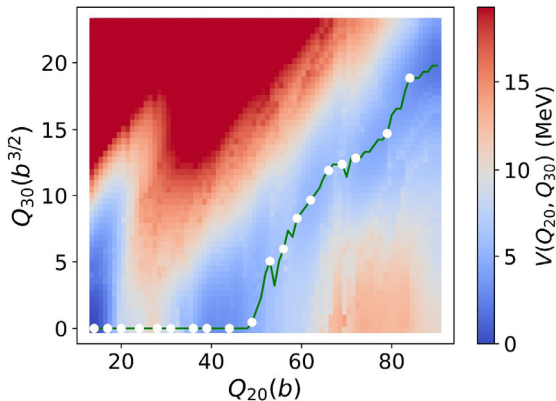
G.F. Bertsch and K.Hagino,  
Phys. Rev. C107, 044615 (2023).  
K. Uzawa and K. Hagino,  
Phys. Rev. C110, 014321 (2024).

# Microscopic calculations for low energy induced fission



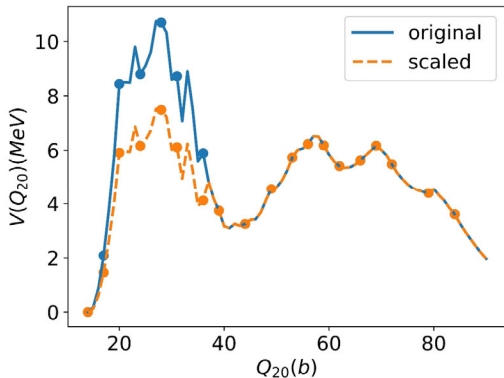
K. Uzawa and K. Hagino, Phys. Rev. C112, 014326 (2025).

fission path based on Skyrme DFT with UNEDF1



$$\langle \Psi_{\mu}(Q) | \Psi_{\mu}(Q') \rangle \sim 0.52$$

fission barrier without pairing

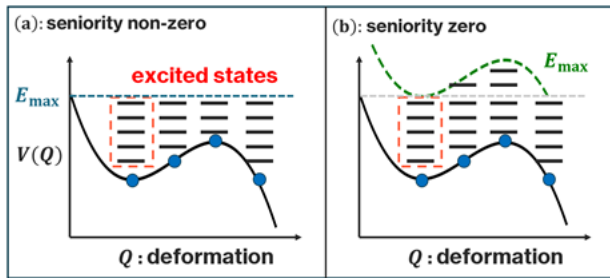


scale the first barrier

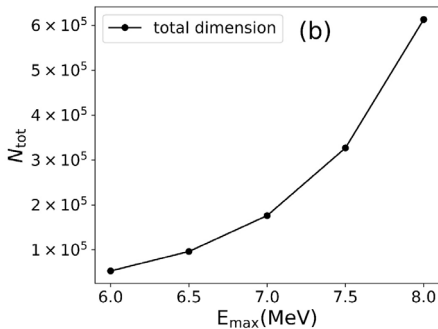
$\rightarrow B \sim B_{\text{empirical}}$  if with pairing

K. Uzawa and K. Hagino, Phys. Rev. C112, 014326 (2025).

$np$ - $nh$  configurations up to  $E_{\text{max}}$



dimension of  $H \sim O(10^5)$



$$H = H_0 - GP^\dagger P + V_{\text{rand}}$$

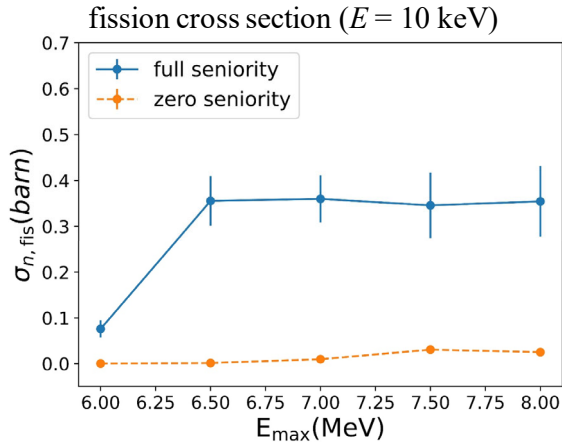
- $G \leftarrow 0^+_{2-}$  in  $^{236}\text{U}$  for a given model space
- $V_{\text{rand}} \leftarrow$  systematics
- $\Gamma_{\text{cap}}, \Gamma_{\text{n}} \leftarrow$  empirical
- $\Gamma_{\text{fis}}$  for a pre-fission config.  $\leftarrow$  insensitivity

inverse of a huge matrix

- LSMR method
- shift-invert Lanczos method

K. Uzawa and K. Hagino  
PRE110, 055302 (2024).

K. Uzawa and K. Hagino, Phys. Rev. C112, 014326 (2025).



the empirical value:  $\sigma_{n,\text{fis}} = 3.116$  b

→ reproduced within a factor of 10

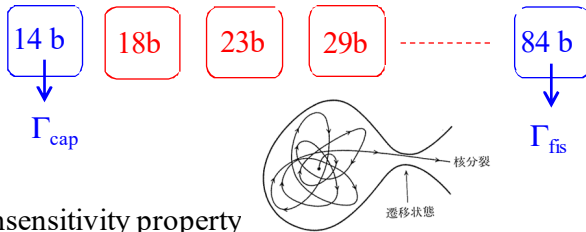
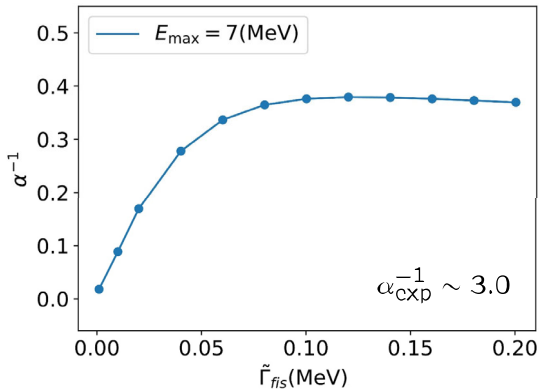
possible origins of the discrepancy:

- axial symmetry → triaxiality
- $G = \text{const.} \rightarrow G(\beta)$
- Skyrme UNEDF1 → other functionals?
- collective coordinates:  $Q_2$  and  $Q_3$ ?  
e.g., the momentum dependence?

convergence at  $E_{\max} \sim 6.5$  MeV

K. Uzawa and K. Hagino, Phys. Rev. C112, 014326 (2025).

fission-to-capture branching ratio



insensitive to  $\Gamma_{fis}$  (post-barrier dynamics)  
→ the main assumption of the transition state theory (TST)

cf. K.Hagino and G.F. Bertsch, JPSJ 93, 064003 (2024).

cf. strong absorption in nuclear reactions

## Application to low-energy fission of $^{236}\text{U}$

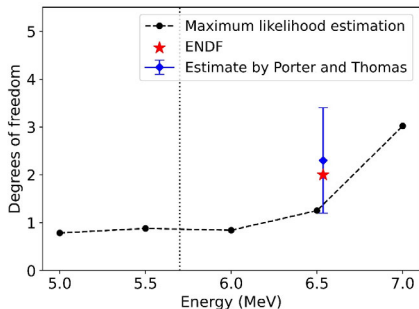
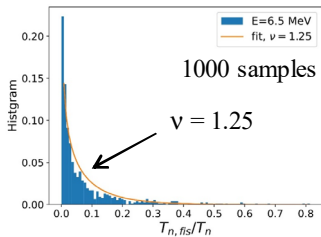
the effective number of freedom for fission

the distribution of  $T_{n,\text{fis}}$  → fit with the chi-square function

$$P_\nu(x) = \frac{\nu}{2\Gamma(\nu/2)} \left(\frac{\nu x}{2}\right)^{\nu/2-1} e^{-\nu x/2} \quad \nu: \# \text{ of d.o.f.} \quad (\text{cf. } \nu=1: \text{ the PT distribution})$$

K. Uzawa and K. Hagino,  
Phys. Rev. C110, 014321 (2024).

# calc. with a smaller  
model space (seniority 0 only)



$$G(E_n) = \frac{1}{H - i\Gamma/2 - E_n} = \sum_{\alpha} \frac{|\phi_{\alpha}\rangle\langle\tilde{\phi}_{\alpha}|}{E_{\alpha} - E_n}$$

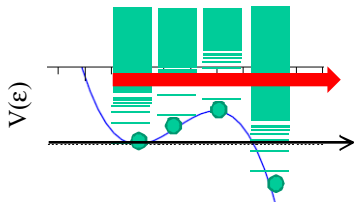
only a few eigenstates with  $\text{Re}(E_{\alpha}) \sim E_n$  contribute

“transition states”

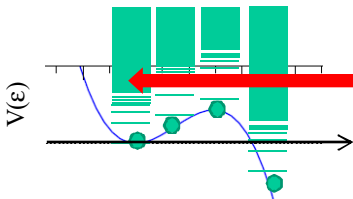
a small number of d.o.f. for induced fission ← transition state theory

# $^{12}\text{C}+^{12}\text{C}$ fusion reactions at astrophysical energies

fission



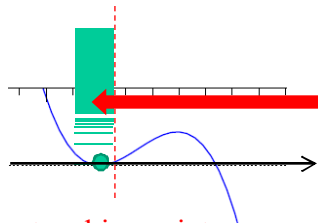
fusion



fusion for  
superheavy elements



a microscopic description  
for shape evolution



touching point

fusion of light nuclei  
(nuclear astrophysical  
reactions)

relative motion  
+ compound nucleus

## $^{12}\text{C}+^{12}\text{C}$ fusion reactions at astrophysical energies

$^{12}\text{C}+^{12}\text{C}$  fusion

: one of the key reactions in nuclear astrophysics



- ✓ Carbon burning in massive stars
- ✓ Type Ia supernovae
- ✓ X-ray superburst

C.L. Jiang et al., PRL110, 072701 (2013).

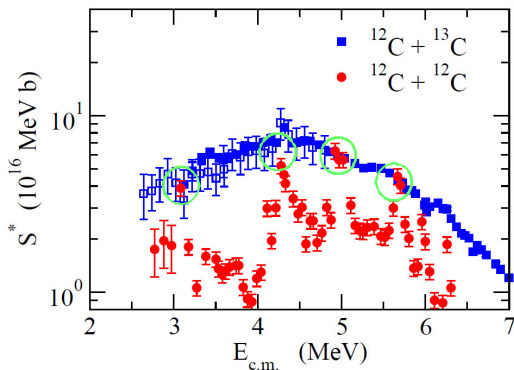
difference between  $^{24}\text{Mg}$  and  $^{25}\text{Mg}$

- $^{24}\text{Mg}$ : isolated CN resonances
- $^{25}\text{Mg}$ : overlapping CN resonances

need a new reaction model



$$S^*(E) = E\sigma(E)e^{2\pi(\eta-\eta_0)}$$

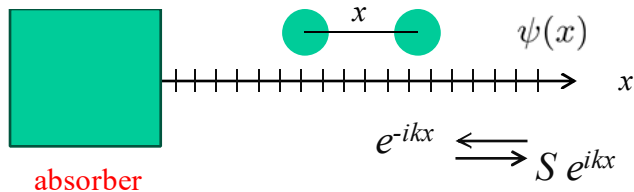


prominent resonance structure  
in  $^{12}\text{C}+^{12}\text{C}$  fusion

- ✓ off-resonance: fusion inhibition
- ✓ on-resonance: match with  $^{12}\text{C}+^{13}\text{C}$

a schematic model for the imaginary part : a microscopic understanding of the optical potential

□ scattering problem with absorption:



absorption  $\rightarrow |S| < 1$

cf. for strong absorption  
 $S \sim 0$

cf. optical potential

$$V_{\text{opt}}(r) = V(r) - \underbrace{iW(r)}$$

- elastic scattering
- absorption process

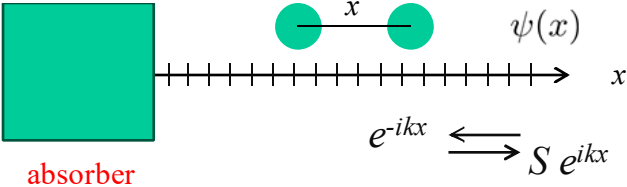


a loss of incident flux

- ✓ inelastic scattering
- ✓ transfer reaction
- ✓ fusion etc.

a schematic model for the imaginary part : a microscopic understanding of the optical potential

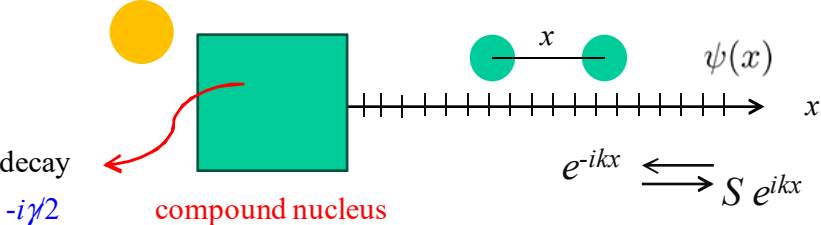
□ scattering problem with absorption:



absorption  $\rightarrow |S| < 1$

cf. for strong absorption  
 $S \sim 0$

□ a microscopic origin for absorption: compound nucleus formation (fusion)



✓ couplings to CN

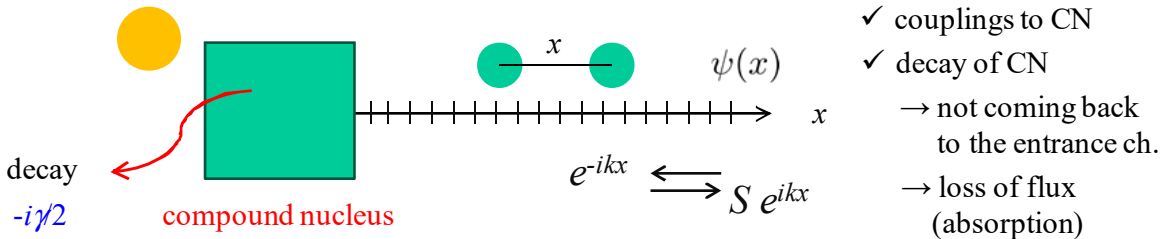
✓ decay of CN

$\rightarrow$  not coming back to the entrance ch.

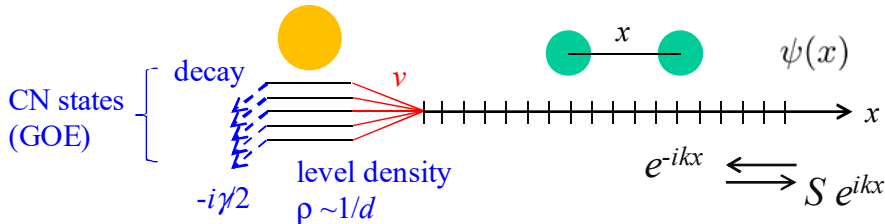
$\rightarrow$  loss of flux (absorption)

a schematic model for the imaginary part : a microscopic understanding of the optical potential

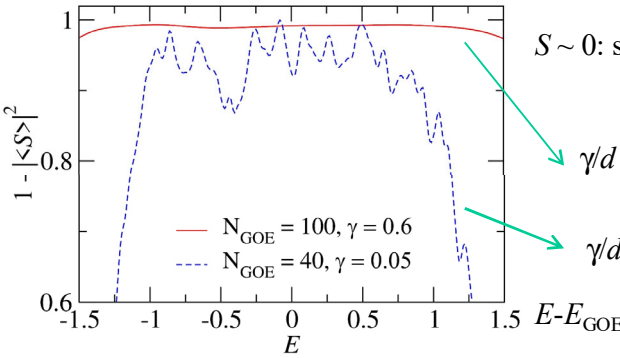
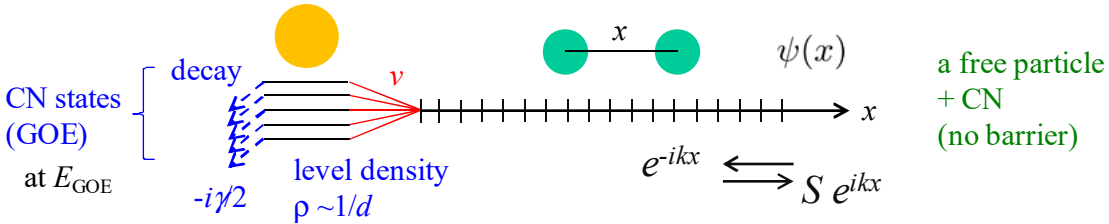
□ a microscopic origin for absorption: compound nucleus formation (fusion)



□ a modelling:



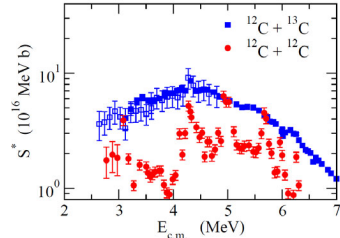
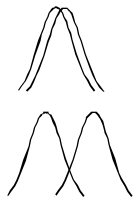
a schematic model with a random matrix



$S \sim 0$ : strong absorption

$\gamma/d = 20$

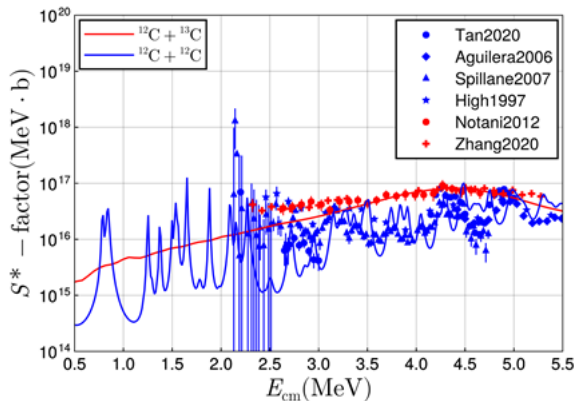
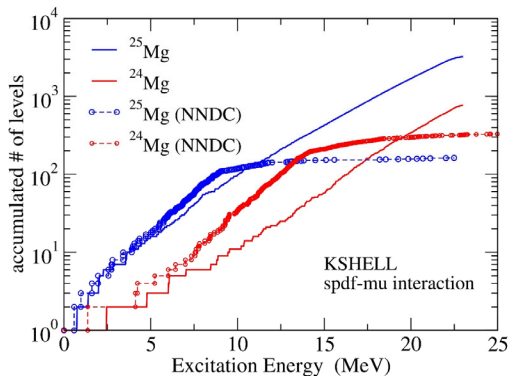
$\gamma/d = 1$



## a more realistic model with shell model

K. Nagao, K. Hagino, and K. Uzawa, in preparation.

- ✓ the spectrum of CN states: RMT  $\rightarrow$  shell model (KSHELL)
- ✓ decay widths  $\rightarrow$  statistical model
- ✓ 3D calculations with the Coulomb barrier



## Discussion: the validity of compound nucleus picture at low energies?

A question: at what  $E^*$  does the compound nucleus picture begin to be valid?

$E^*$



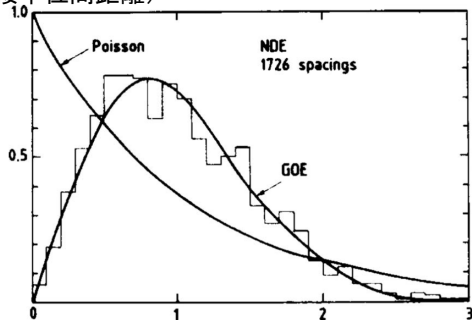
quasi-continuum  
spectrum  
→ statistical treatment

$$\rho(E) \sim e^{2\sqrt{aE^*}}$$



regular motion

random matrix: provides a good description of CN  
the nearest neighboring level spacing distribution  
(最近接準位間距離)

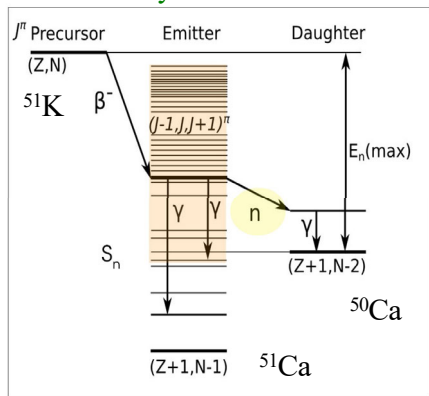


O. Bohigas et al., "Nuclear Data for Science and Technology", p. 809.

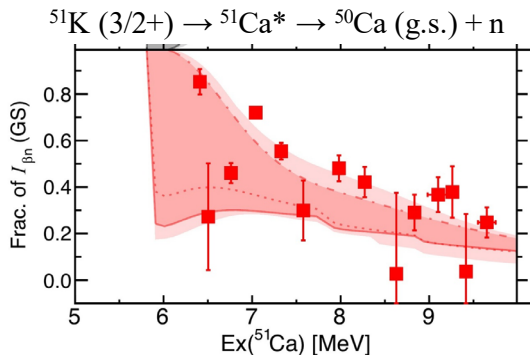
## Discussion: the validity of compound nucleus picture at low energies?

A question: at what  $E^*$  does the compound nucleus picture begin to be valid?

### beta-delayed neutron emission



a figure by R. Grzywacz (Tennessee)



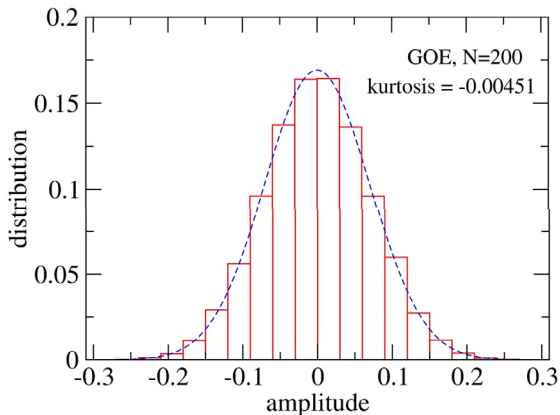
Z.Y. Xu, R. Grzywacz et al., PRL133, 042501 (2024)

Statistical model calculations seem to work well despite that the level density is low (10-20 states/MeV).

## Discussion: the validity of compound nucleus picture at low energies?

A question: at what  $E^*$  does the compound nucleus picture begin to be valid?

We address this question by investigating the kurtosis (尖度) of a distribution.



$$\psi_k = \sum_{\mu} C_{\mu}^{(k)} |\mu\rangle \quad \rightarrow \text{decay width} \sim |C|^2$$

Random Matrix (GOE):

$$P(x) = \sqrt{\frac{N}{2\pi}} e^{-Nx^2/2}; \quad (x = \{C_{\mu}^{(k)}\})$$

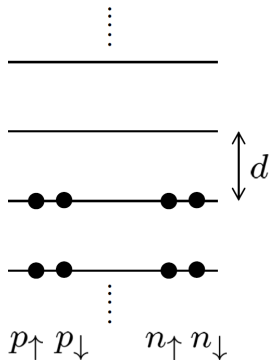
$$\text{kurtosis} \equiv \frac{\langle x^4 \rangle}{\langle x^2 \rangle^2} - 3$$

= 0 for a Gaussian distribution

## Discussion: the validity of compound nucleus picture at low energies?

uniform spacing single-particle levels

K. Uzawa and K. Hagino,  
Phys. Rev. C108, 024319 (2023).



$$H = H_0 - G \sum_{k,k'} a_k^\dagger a_{\bar{k}}^\dagger a_{\bar{k}'} a_{k'} - v \sum r a^\dagger a^\dagger a a$$

$$G = 0.3d, \quad v_{np} = 0.025d, \quad v_{nn} = v_{pp} = 0.02d$$

(note) kurtosis = model space dependent, and basis dependent

$$|\psi_{N,k}\rangle = \sum_{\mu} C_{\mu}^{(k)} |\phi_{N,\mu}\rangle$$

$$[H_0 + H_{\text{pair}}] |\phi_{N,\mu}\rangle = E_{\mu} |\phi_{N,\mu}\rangle$$

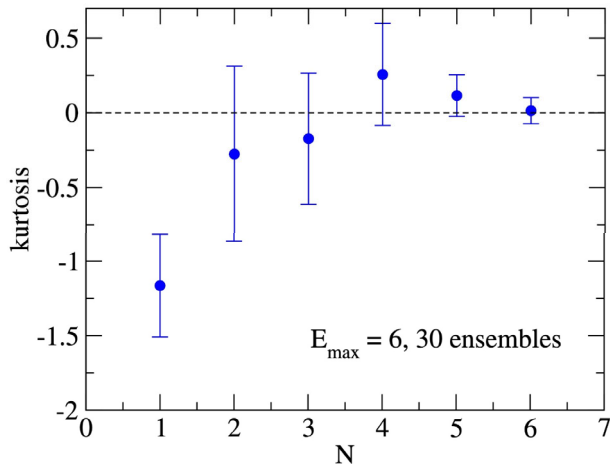
with  $E_{\mu} \sim Nd$

mixing due to the random interaction  $\rightarrow$  kurtosis

## Discussion: the validity of compound nucleus picture at low energies?

uniform spacing model

K. Uzawa and K. Hagino, Phys. Rev. C108, 024319 (2023).



# of levels for  $K_{\text{tot}} = 0$ :

4 ( $N=1$ )

16 ( $N=2$ )

48 ( $N=3$ )

133 ( $N=4$ )

332 ( $N=5$ )

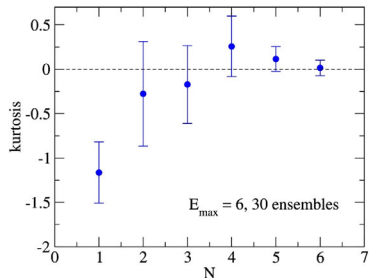
784 ( $N=6$ )

the kurtosis is consistent with 0  
even for  $N=2$ .

## Discussion: the validity of compound nucleus picture at low energies?

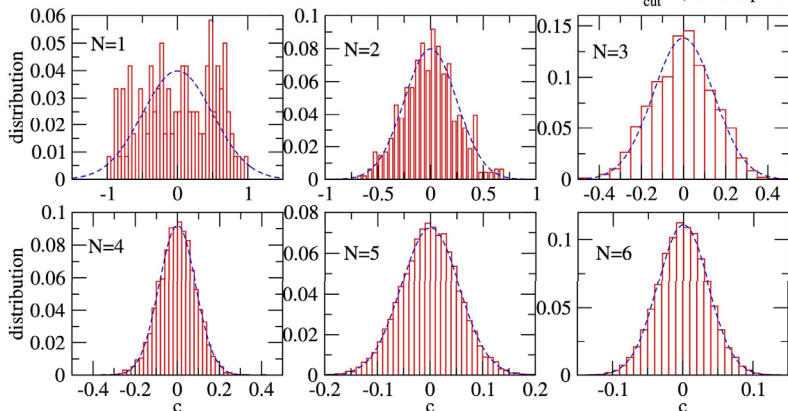
the distribution of wave function amplitudes

$E_{\text{cut}} = 6$ , 30 samples



$$|\psi_{N,k}\rangle = \sum_{\mu} C_{\mu}^{(k)} |\phi_{N,\mu}\rangle$$

$$[H_0 + H_{\text{pair}}] |\phi_{N,\mu}\rangle = E_{\mu} |\phi_{N,\mu}\rangle$$



K. Hagino and K. Uzawa, in preparation.

$N=2$ : approximately Gaussian  
 $N=6$ : almost complete Gaussian

## A short comment on Spectral Form Factor (SFF)

$$g(t, \beta) \equiv \frac{|Z(t, \beta)|^2}{Z(0, \beta)^2}$$
$$Z(t, \beta) = \sum_i e^{-(\beta+it)\epsilon_i}$$

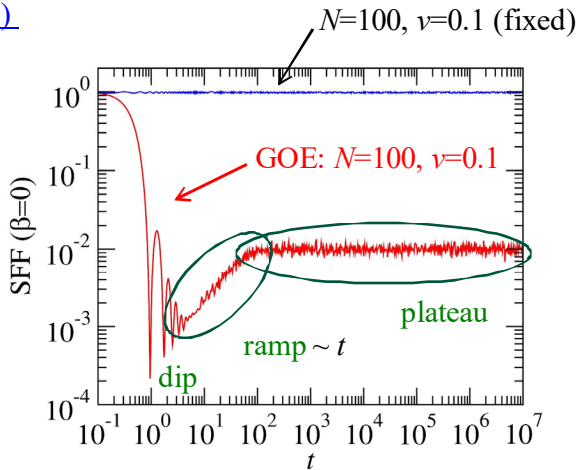
GOE:

$$g(t, \beta = 0) = (N + N^2 r_1(t)^2 - N r_2(t))/N^2$$

$$r_1(t) = J_1(2t)/t$$

$$r_2(t) = \begin{cases} 1 - \frac{t}{N} + \frac{t}{2N} \ln(1 + t/N) & (t < 2N) \\ -1 + \frac{t}{2N} \ln\left(\frac{t+N}{t-N}\right) & (t \geq 2N) \end{cases}$$

J. Liu, PRD98, 086026 (2018).

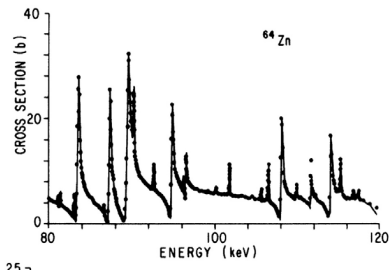


This quantity has been popular in the context of quantum chaos, but not in nuclear physics.

→ a new measure for compound nuclei?

# Spectral Form Factor (SFF) with the experimental nuclear levels

## neutron resonance data



$^{64}\text{Zn}+n$  data

J.B. Garg et al., PRC23, 671 (1981).

see also: <https://www-nds.iaea.org/exfor/>

## energy levels

$E_n(\text{eV})$	$r_n(\text{eV})$
2627 ± 2	68 ± 3
4170 ± 2	19.4 ± 1.0
10775 ± 5	96 ± 5
13019 ± 5	190 ± 10
15212 ± 5	110 ± 5
21316 ± 10	67 ± 5
22676 ± 10	78 ± 5
29100 ± 10	56 ± 4
29640 ± 10	79 ± 5
30708 ± 10	201 ± 10
32450 ± 10	490 ± 25
39278 ± 10	23 ± 2
41010 ± 15	52 ± 5
47955 ± 15	219 ± 10
50700 ± 15	125 ± 6
52120 ± 15	128 ± 6
54300 ± 15	64 ± 6
60882 ± 15	6 ± 2
67440 ± 15	169 ± 15
68932 ± 15	39 ± 4
73135 ± 20*	1.5 ± 0.5
78345 ± 20	30 ± 3
78738 ± 20	50 ± 4
83570 ± 20	220 ± 10
87350 ± 20	178 ± 10
89520 ± 20	396 ± 20
94832 ± 20	139 ± 10
96555 ± 20	30 ± 3
100220 ± 25*	1.5 ± 0.5
108080 ± 20	128 ± 7
109970 ± 20	10 ± 2
.....	..
..	..

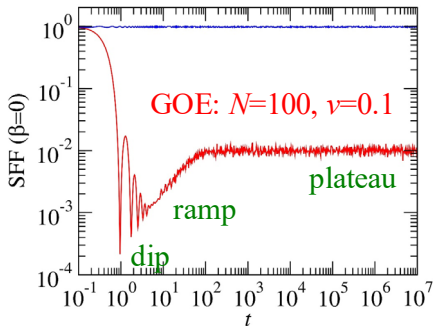
## SFF

$$g(t, \beta) \equiv \frac{|Z(t, \beta)|^2}{Z(0, \beta)^2}$$

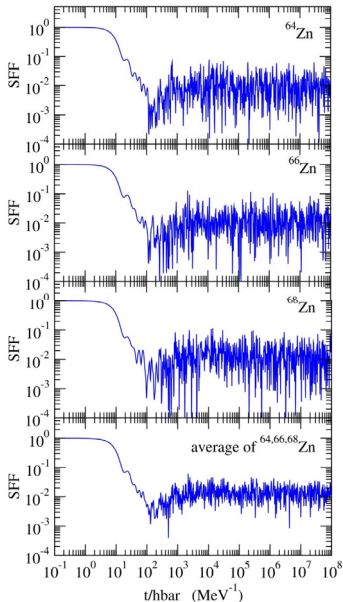
$$Z(t, \beta) = \sum_i e^{-(\beta+it)\epsilon_i}$$

## Spectral Form Factor (SFF)

$$g(t, \beta) \equiv \frac{|Z(t, \beta)|^2}{Z(0, \beta)^2}$$
$$Z(t, \beta) = \sum_i e^{-(\beta+it)\epsilon_i}$$



## SFF with experimental nuclear levels

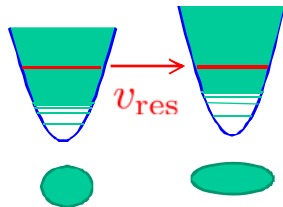


a clear dip-ramp-plateau structure!

K. Hagino, K. Uzawa,  
H. Watanabe, Y. Tanizaki,  
M. Tezuka, in preparation.

## Summary

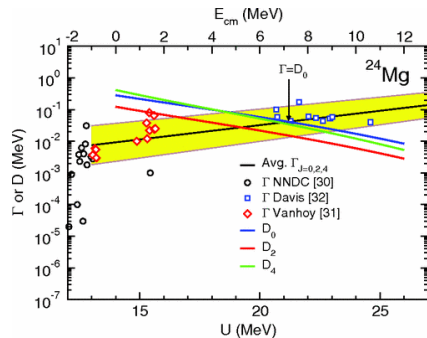
- r-process nucleosynthesis: fission of neutron-rich nuclei  
requires a microscopic approach applicable to low  $E^*$  and  $\rho(E^*)$   
→ a new approach: shell model + GCM  
an application to induced fission of  $^{236}\text{U}$  based on Skyrme EDF  
→
  - the insensitive property
  - a small value of d.o.f.← the transition state theory
- energy dependence of fusion cross sections for  $^{12}\text{C}+^{12,13}\text{C}$   
 $^{12}\text{C}+^{12}\text{C}$  : prominent resonance peaks  
 $^{12}\text{C}+^{13}\text{C}$  : a much smoother energy dependence  
a new model with shell model calculations for the CN
- the validity of the compound nucleus picture at low excitation energies
  - ✓ recent measurements of beta-delayed neutron emissions
  - ✓ the kurtosis from an uniform spacing model → consistent with kurtosis  $\sim 0$  even for  $N=2$



future direction: 1-channel  
→ coupled-channels



## Effects of isolated CN resonances

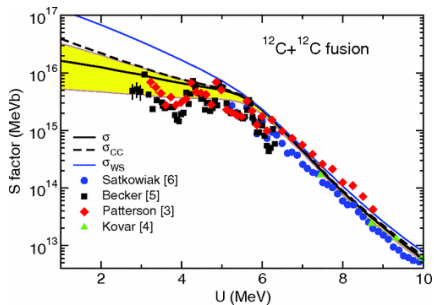


Moldauer factor

$$P_J = 1 - \exp(-2\pi\Gamma_J/D_J)$$

$$\rightarrow \sigma = \sum_J \sigma_{CC}^J P_J$$

C.L. Jiang et al., PRL110, 072701 (2013).



$E < 7 \text{ MeV}$

$\Gamma < D \rightarrow$  fusion inhibition

$E > 7 \text{ MeV}$

$\Gamma > D \rightarrow P_J \sim 1$