Microscopic study of the partition of particle number, energy, and angular momentum in nuclear fission

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2026年5月18日

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- Nuclear Fission dynamics
- Combined the projection in partial space
- Extracted fission observables
- Fission of rapidly rotating compound nuclei
- Summary & Outlooks

Nuclear Fission Dynamics

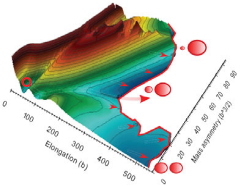
Microscopic Fission Theory

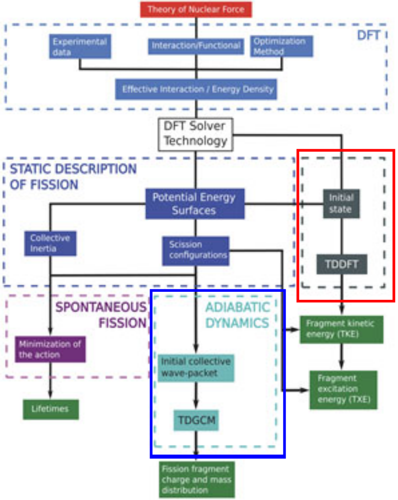
Collective Fluctuation √

Distribution Width

Fragment Mass &

charge Distribution





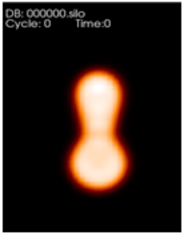
Non-adiabatic effect √

Intrinsic excitation

Scission configuration

Fragment kinetic

& excitation energy



Jie Zhao, T. Nikšić, D. Vretenar, and Shan-Gui Zhou, Phys. Rev. C **99** 014618 (2019)

N. Schunck and L. M. Robledo, 2016 *Rep. Prog. Phys.* **79** 116301

Fission dynamics

$$\mathcal{S}[\Psi] = \int_{t_0}^{t_1} \langle\Psi(t)|\hat{H} - i\hbar\frac{\partial}{\partial t}|\Psi(t)\rangle dt$$

Variational wavefunction(w.f.) space :

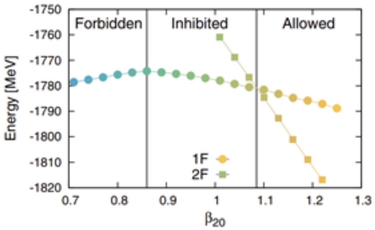
HF w.f. (Slater determinant): $|\Psi\rangle = \prod_{k=1}^{N} \hat{a}_k^\dagger|0\rangle$

$$i\frac{\partial}{\partial t}\rho(t) = [h(t), \rho(t)]$$

$$\rho(t) = \sum_{k=1}^{N} |\phi_k(t)\rangle\langle\phi_k(t)|$$

TDHF

Pairing as a fission lubricant!



P. Goddard, P. Stevenson, A. Rios Phys. Rev. C, 92 (2015), 054610

HFB w.f. (quasi-particle Slater determinant) : $|\Psi\rangle = \prod_k \beta_k|0\rangle, \beta_k = \sum_l U_{lk}^* c_l + V_{lk}^* c_l^\dagger$

$$i\frac{\partial}{\partial t}R = [\mathcal{H}, R] \quad R \equiv \begin{pmatrix} \rho & \kappa \\ -\kappa^* & 1-\rho^* \end{pmatrix} \quad \mathcal{H} \equiv \begin{pmatrix} h & \Delta \\ -\Delta^* & -h^* \end{pmatrix} \quad \begin{matrix} \rho = V^* V^T \\ \kappa = V^* U^T \end{matrix}$$

TDHFB

Fission dynamics

BCS w.f. : $$|\Psi\rangle = \prod_{k>0}(u_k + v_k \hat{a}_k^\dagger \hat{a}_{\bar{k}}^\dagger)|0\rangle,\ \rho_k = v_k^2,\ \kappa_k = u_k v_k$$

$$i\frac{\partial}{\partial t}|\phi_k(t)\rangle = [h(t) - \eta_k(t)]|\phi_k(t)\rangle$$

$$i\frac{d}{dt}\rho_k(t) = \kappa_k(t)\Delta_k^*(t) - \kappa_k^*(t)\Delta_k(t)$$

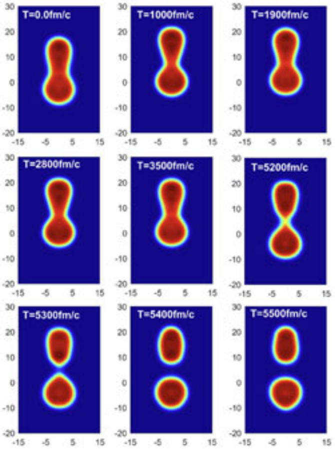
TDBCS

$$i\frac{d}{dt}\kappa_k(t) = [\eta_k(t) + \eta_{\bar{k}}(t)]\kappa_k(t) + \Delta_k(t)[2\rho_k(t) - 1]$$

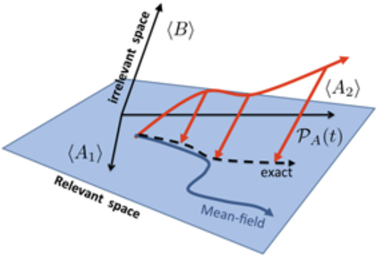
Extended to finite temperature

self-consistent description :

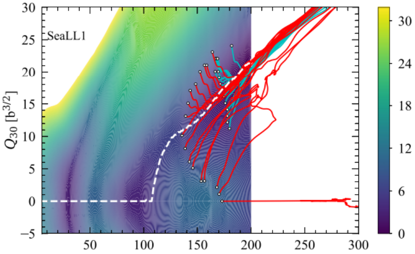
- partition of nucleon numbers and energy
- TKE release
- observables' energy dependent
- primary fission fragment deformation
- scission configuration



Fission dynamics



Lacroix, D., Ayik, S. *Eur. Phys. J. A* **50**, 95 (2014)



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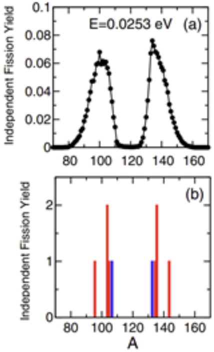


P. Goddard, et. al., Phys. Rev. C, 92 (2015), 054610

Conventional TD-DFT :

expectation values of operators ✓

their variances. ✗

The widths of fission observables: too small.

Higher-order many-body correlations ,quantum fluctuations beyond mean field important!

Fission dynamics

$$i\frac{\partial}{\partial t}|\phi_k(t)\rangle = [h(t) - \eta_k(t)]|\phi_k(t)\rangle$$

s.p. level random transition term:

$$i\frac{d}{dt}\rho_k(t) = \kappa_k(t)\Delta_k^*(t) - \kappa_k^*(t)\Delta_k(t) + q_r * C_{ik}\exp(-|\Delta E|/T_{eff})$$

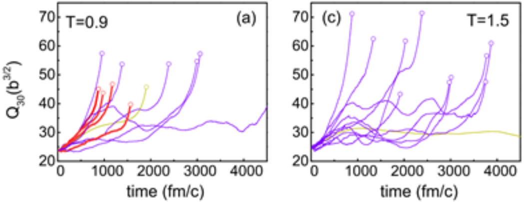
$$i\frac{d}{dt}\kappa_k(t) = [\eta_k(t) + \eta_{\bar{k}}(t)]\kappa_k(t) + \Delta_k(t)[2\rho_k(t) - 1]$$

Random transition: above and below fermi surface; random time

Simulating dynamical fluctuations during the evolution

Fluctuation by initial configuration

Dynamical fluctuation

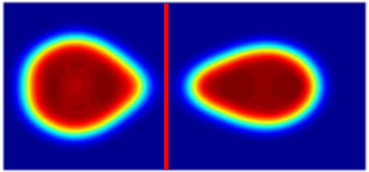


Yu Qiang, J.C. Pei, P. Stevenson, PRC 103, L031304 (2021)

Combined the projection in partial space

Particle number projection

Extract the specific fission channel (N_n, N_p conserved) from final-state of TD-DFT evolution



$$\hat{N}_\tau^P = \int dr \sum_k \hat{a}_k^\dagger(r)\hat{a}_k(r)\Theta(r)$$

Particle number operator in partial space

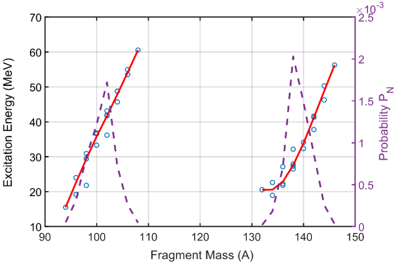
$$\hat{P}^q(N_T^q, N_P^q) = \frac{1}{4\pi^2} \iint d\theta_T d\theta_P e^{i\theta_T(\hat{N}_T^q - N_T^q)} e^{i\theta_P(\hat{N}_P^q - N_P^q)}$$

$\langle\Psi|\hat{P}^q(N_T^q, N_P^q)|\Psi\rangle$ Probability of producing a fragment with specific N_n, N_p

$$E_{proj} = \frac{\langle\Psi|\hat{H}\hat{P}^n(N_T, N_P)\hat{P}^p(Z_T, Z_P)|\Psi\rangle}{\langle\Psi|\hat{P}^n(N_T, N_P)\hat{P}^p(Z_T, Z_P|\Psi\rangle}$$

Projected energy-excitation energy

for specific fission channel .



Angular momentum projection

For axially symmetric state(J_z=0):

$$\hat{P}_J^F = \frac{2\pi+1}{2}\int_0^{\pi} d\beta sin(\beta) P_J(cos(\beta))\hat{R}_x^F(\beta)$$

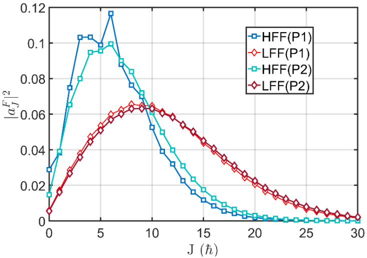
$$\hat{R}_x^F(\beta) = exp(-\hat{J}_x^F \beta/\hbar)$$

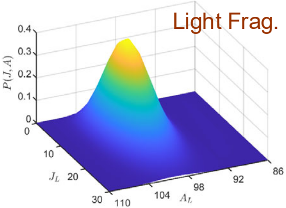
$$\vec{J}_F = [(\vec{r}-\vec{R}^F)\times(\vec{p}-m\vec{v}_F)+\vec{s}]\Theta(\vec{r})$$

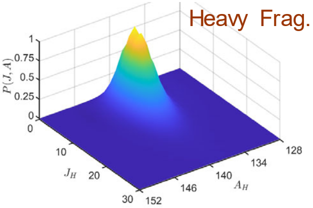
AMP for H/L fragments: $|a_J^F|^2 = \langle\Psi|\hat{P}_J^F|\Psi\rangle$

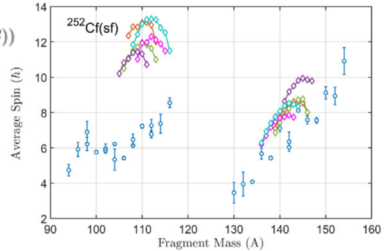
Simultaneously AMP and PNP:

$$P(N_F, J_F) = \langle\Psi|\hat{P}_N^F\hat{P}_J^F|\Psi\rangle = \frac{2J+1}{2}\int_0^{\pi} d\beta sin(\beta)\int_{-\pi}^{\pi}\frac{d\theta}{2\pi}\langle\Psi|e^{-\theta(\hat{N}_F-N_F)}e^{-\hat{J}_x^F\beta/\hbar}|\Psi\rangle P_{J_F}(cos(\beta))$$





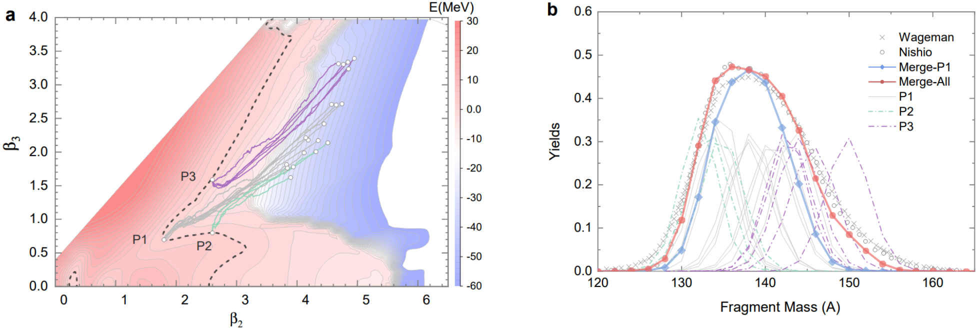




Extracted fission observables

Fission fragment mass yields

TD-BCS+ random s.p. level transition + particle number projection for fragments

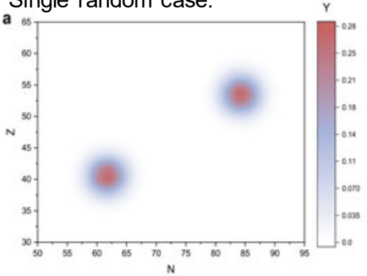


Initial configuration variation+ dynamical transition+ fission channel extraction

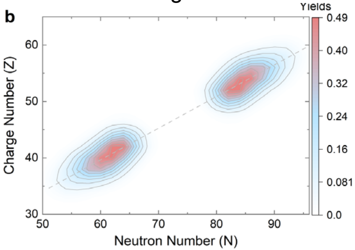
Yu Qiang, Junchen Pei, Kyle Godbey, Phys. Lett.B 2025,861(139248)

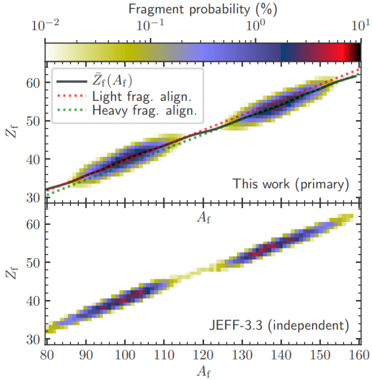
N-Z distribution

Single random case:



all cases merged:





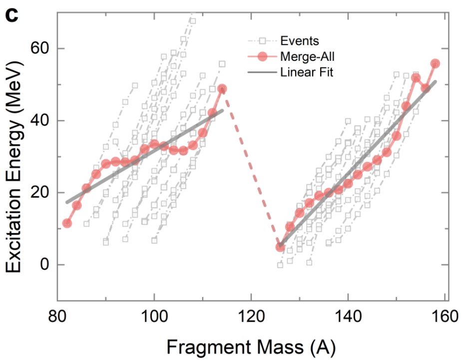
M. Verriere, N. Schunck, D. Regnier Phys. Rev. C 103, 054602 (2021)

Adding fluctuation :
more elongated and flattened distribution

Fission fragment excitation energy

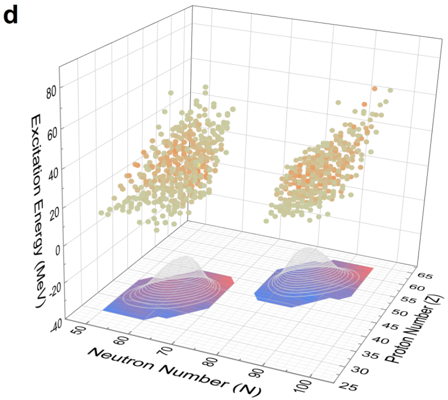
Projected energy calculation:

weighted average



Dynamical fluctuation:
Reduce the slope of sawtooth structure

Event by event distribution:



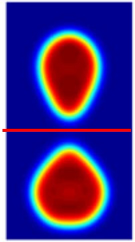
Fission simulator

The other observables

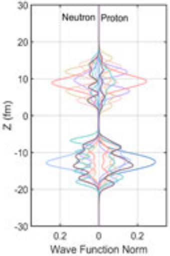
Fragment entanglement

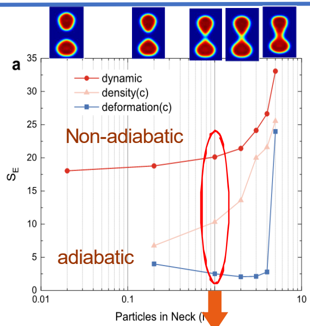
Entanglement entropy: S_E

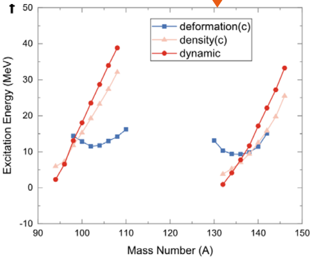
Final density dist.



Final s.p. states





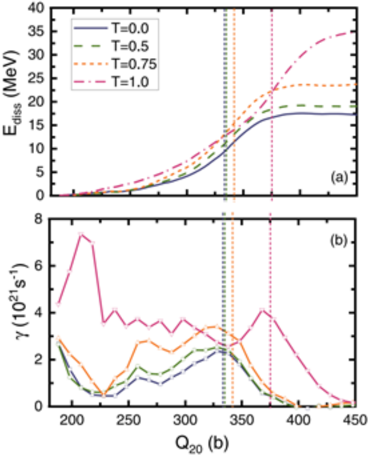


Yu Qiang, Junchen Pei, Kyle Godbey, Phys. Lett.B 2025,861(139248)

Dissipation coefficient & energy

Energy dependence

Deformation dependence



Yu Qiang, J. C. Pei, Phys. Rev. C, 2021, 104(054604).

Fission of rapidly rotating compound nuclei

Rotating procedure

The Skyrme-type single particle Hamiltonian:

$$\hat{h}_q = U_q(\vec{r}) - \nabla\cdot[B_q(\vec{r})\nabla] + i\vec{W}_q\cdot(\vec{\sigma}\times\nabla) + \vec{S}_q\times\vec{\sigma} - \frac{i}{2}[(\nabla\times\vec{A}_q) + 2\vec{A}_q\cdot\nabla]$$

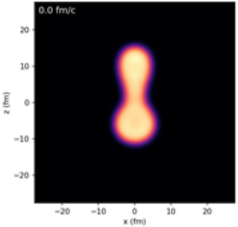
$[U_q, B_q, \vec{W}_q, \vec{S}_q, \vec{A}_q]$ Mean-field potential, effective mass…

Rotating each term

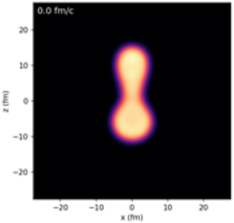
$$\vec{r}(t_i) = (x_i, y_i, z_i) \rightarrow (x_i + z_i cos(\omega\Delta t), y_i, z_i - x_i sin(\omega\Delta t))$$

Initial rotation: 30fm/c, different rotation speed ω

L_{CN}=45

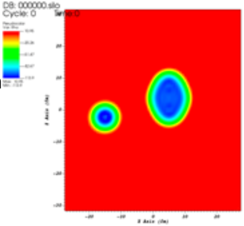


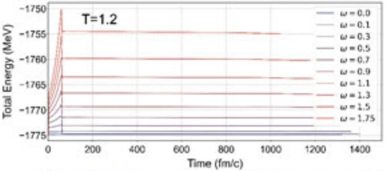
L_{CN}=95



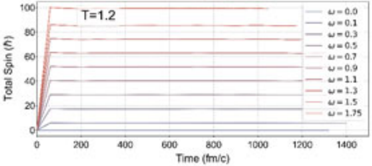
$U_q(\vec{r})$

in Collision reaction

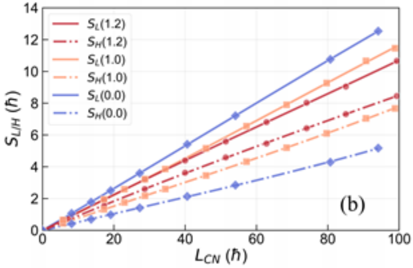




E_{tot} and L_{CN} conserved, after initial rotation



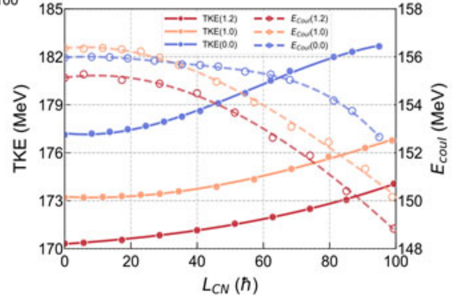
Fission fragment spin & TKE



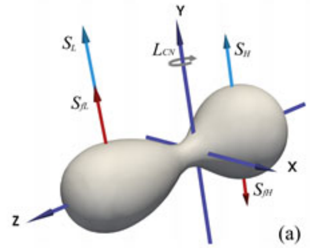
$$S_{H/L} = \langle\Psi|\hat{J}_y^{H/L}|\Psi\rangle \quad L_{CN} = \langle\Psi|\hat{J}_y^{tot}|\Psi\rangle$$

Compound nuclei rotating faster:
$S_L > S_H$
$S_{L/H}$ increase linearly
The ratio S_L/S_H constant with L_{CN}
;Decrease as temperature increases.

TKE increase
E_{coul} decrease
Collective rotation energy transfer
Elongate with Temperature



Fission mode analysis



$E^*(T)$	L_{CN}	S_{fH}	S_{fL}	P_{fH}	P_{fL}
1.8(0.0)	27.6	-1.04	1.47	70.1%	41.8%
10.9(0.0)	68.6	-2.67	3.42	78.3%	40.0%
21.0(0.0)	95.2	-3.41	4.64	70.3%	40.2%
20.9(1.0)	28.8	-0.46	1.03	20.6%	34.4%
30.7(1.0)	68.7	-1.14	2.73	22.0%	37.0%
41.1(1.0)	99.3	-1.44	4.07	19.2%	38.2%
34.4(1.2)	28.8	-0.03	0.91	1.1%	32.3%
40.6(1.2)	63.0	-0.51	1.83	9.4%	30.7%
52.6(1.2)	99.3	-0.80	3.24	9.6%	33.5%

$$\frac{L_{CN}}{\mathcal{I}_{CN}} = \frac{S_{rH}}{\mathcal{I}_H} = \frac{S_{rL}}{\mathcal{I}_L}$$

$$S_{f(H/L)} = S_{H/L} - S_{r(H/L)}$$

Two components:

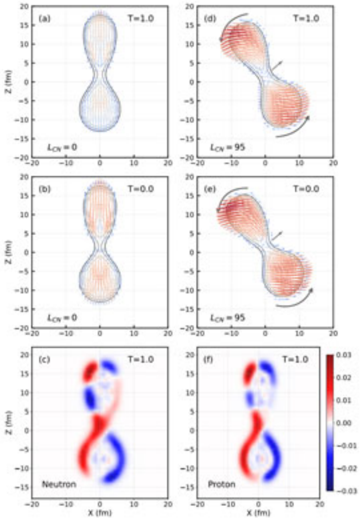
- inherited part $S_{r(H/L)}$
- fluctuation part $S_{f(H/L)}$

$S_{f(H/L)}$ Without AMP

reflects the bias in the occurrence probabilities of different fission modes

Bending mode is more probable
Bias decrease with increasing temperature

Scission configuration



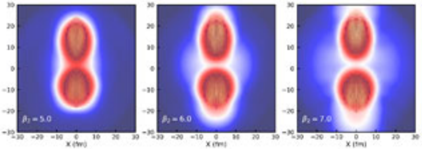
$\Delta\rho$

β_2=4.0

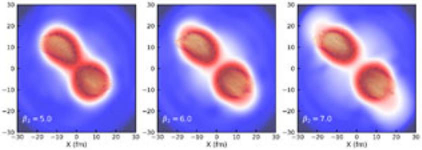
Under Rotation:

- neck become thicker
- non-axially symmetric deformation
- perpendicular scission neutron emission
- Induce large variation than temperature

L_{CN}=0



L_{CN}=95



Logarithmic Density distribution

Summary & Outlooks

Summary:

1. extended TD-DFT framework (TD-BCS+RT plus PNP)

- Fragment mass distribution comparable with experiments
- Fragment excitation energy sawtooth structure
- The dynamical fluctuation can increase the fission observables distribution width and reduce the slope of the sawtooth structure.

2. initial rotation on the s.p. Hamiltonian (rapidly rotating fission)

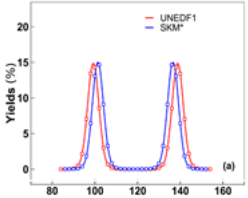
- The spin ratio of light to heavy fragments remains constant with LCN but decreases with temperature.
- The scission neck becomes thicker as L_{CN} increase.
- Although E_{coul} decrease, more collective rotation energy of CN transfer to the fragments TKE.

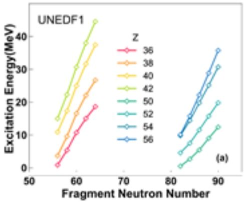
Outlooks:

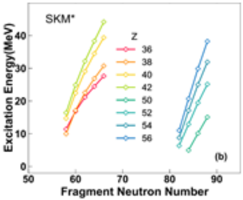
- developing finite temperature PNP and AMP.
- developing the simultaneously PNP and AMP on both light and heavy fragment.
- developing more self-consistent form of random transition.

Interaction dependence

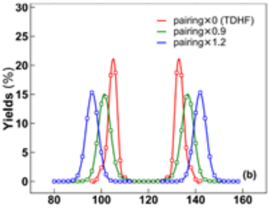
Different nuclear interaction: similar results

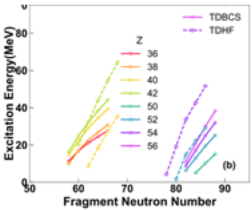


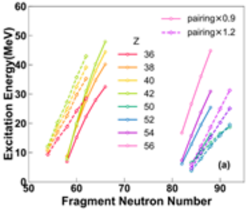




Different pairing strength: Pairing can alleviate the associate slope







Hao-Yu Shang, Yu Qiang, Junchen Pei, Nucl. Sci. Tech. (2025) 36:211