Workshop on Fission Dynamics
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Microscopic Theory of Angular Momentum Distributions Across the Full Range of Fission Fragments

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Outline

1. Introduction
2. Theoretical Framework
3. n-Induced Fission of ^{235}U and ^{239}Pu
4. Conclusion

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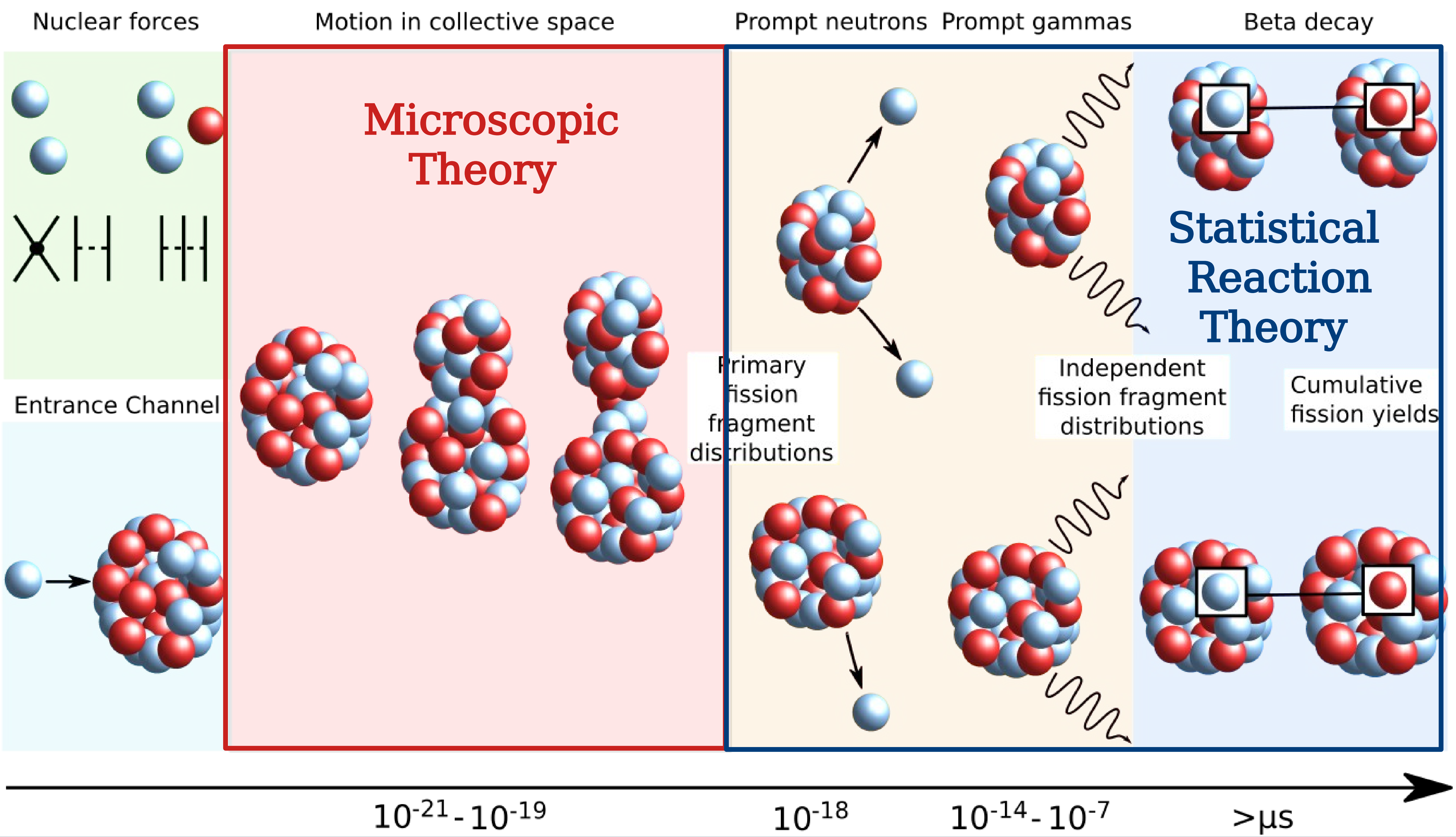
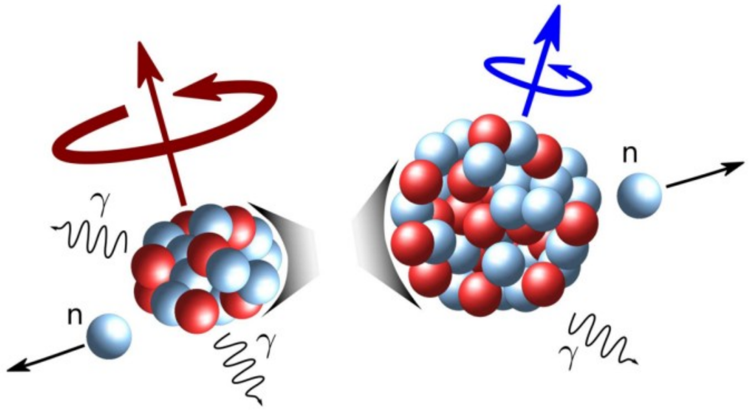
Introduction
Microscopic Approach to Nuclear Fission

- Microscopic theory = using our best knowledge of internucleon forces and many-body techniques (DFT)



Introduction
Angular Momentum of Fission Fragments (FFs)

- FFs at scission are hot, deformed, and rotating
  - High excitation energy
  - Large deformation, typically different than the g.s.
  - Distribution of angular momentum (AM)



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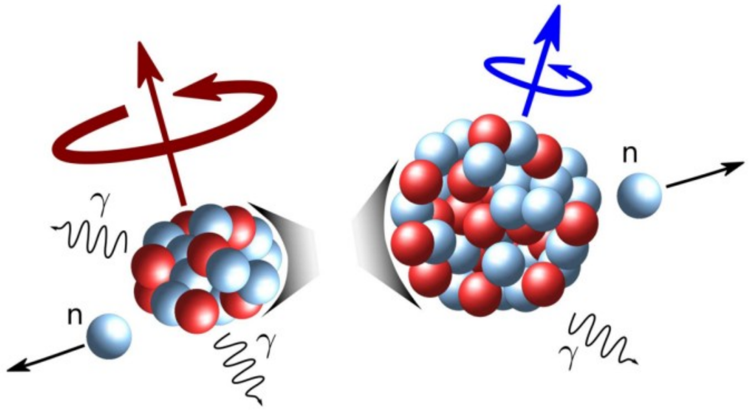
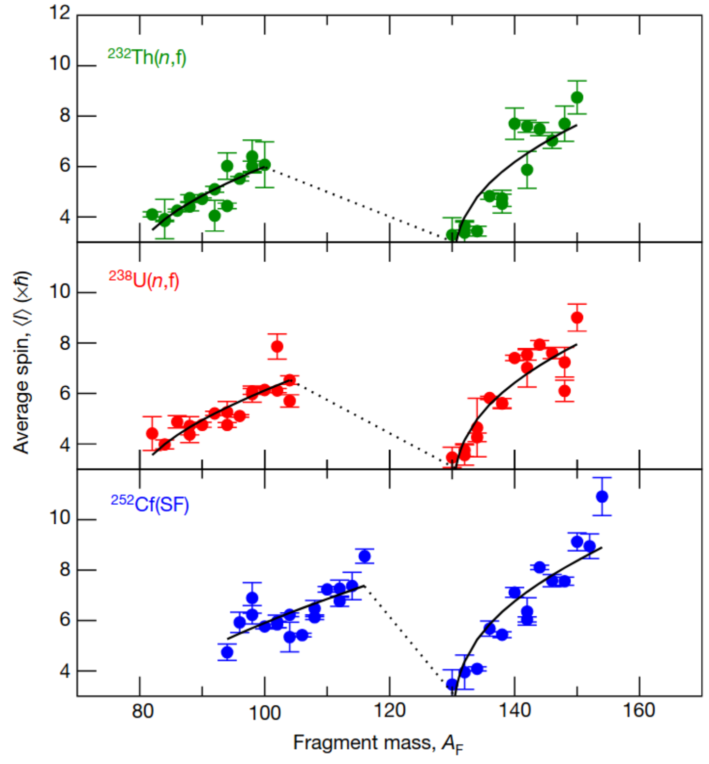


- Recently, the study of AM of FFs experienced a *renaissance*

Wilson *et al.*, Nature **590**, 566 (2021).



- A. Bulgac *et al.*, PRL **126**, 142502 (2021).
- P. Marević *et al.*, PRC **104**, L021601 (2021).
- J. Randrup and R. Vogt, PRL **127**, 062502 (2021).
- I. Stetcu *et al.*, PRL **127**, 222502 (2021).
- A. Bulgac *et al.*, PRL **128**, 022501 (2022).
- J. Randrup, PRC **106**, L051601 (2022).
- G. Scamps, PRC **106**, 054614 (2022).
- G. Scamps *et al.*, PRC 108, L061602 (2023).
- T. Dossing *et al.*, PRC **109**, 034615 (2024).
- G. Scamps *et al.*, arXiv:2512.02207v1 (2025).
- ... and others

Most studies still focus on the **most probable fragmentation**.

Introduction
Statistical Decay of Fission Fragments

- FF initial conditions are essential inputs to decay models

Inputs from Data/Models

- (N_F, Z_F), Binding Energies
- Primary Yields $Y(N_F, Z_F)$
- Excitation E^* of FFs
- $|c(J^\pi)|^2$ Distributions
- Level Densities $\rho(E^*, J^\pi)$
- Transmission Coeff. $T_a(\varepsilon)$
- γ Strength Function
- β^- decay half-lives

→

Statistical Modeling

CGMF
FREYA
FIFRELIN
YAHFC
GEF
...

→

Correlated Observables

- n and γ Multiplicities
- n and γ Spectra
- n and γ Angular Corr.
- Independent Fission Yields
- Cumulative Product Fission Yields
- ...

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Correlated Observables
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- Traditionally, inputs are taken from experiments and empirical models
- We've entered an era where **microscopic theory is becoming useful**
  - Binding energy tables (~2000)
  - Level Densities (~2001) and γ strength function (~2002)
  - Primary FF yields for even (~2005) and odd (~2022) actinides
  - Excitation energies (~2016, but for most likely FFs)
  - **Angular momentum distributions (~2021)**

Introduction
What Are We Trying to Achieve?

Objectives:

1) Establish a microscopic framework for predicting AM distributions in the **full range** of FF masses and charges.

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4) Use the results to **inform phenomenological models** (deformation effects, isobaric dependence, ...).

5) Assess how microscopic AM distributions **impact the FF decay process** (neutron and photon spectra).

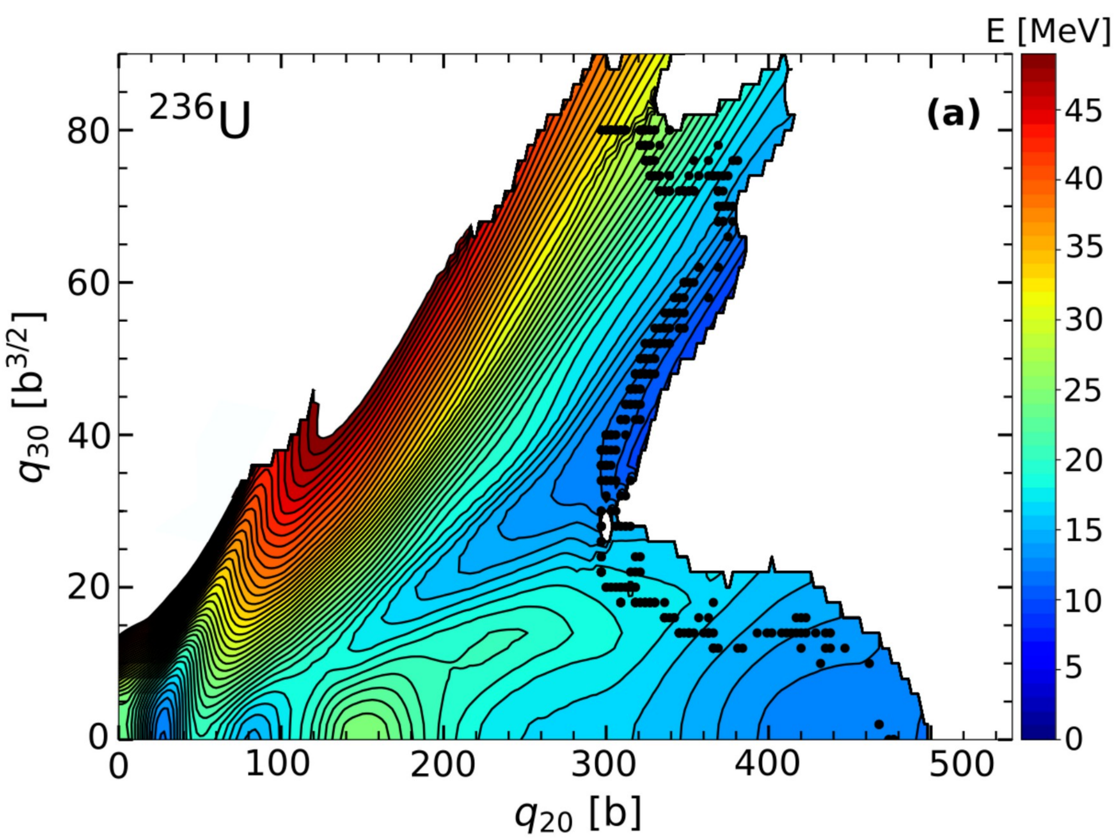
Outline

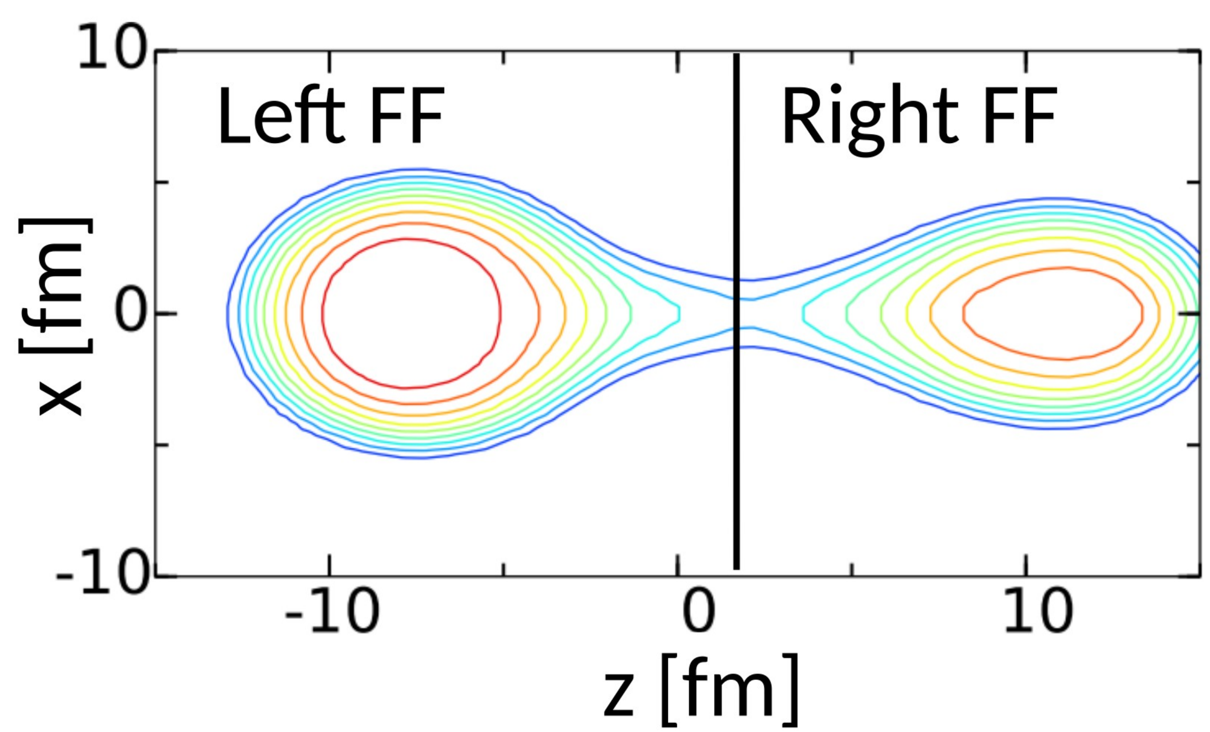
Theoretical Framework

Potential Energy Surface and Scission Configurations

- **Neutron-induced fission**: even-Z and odd-N nucleus absorbs a neutron
- HFB model determines the **potential energy surface** of a fissioning system that contains **scission configurations**
  - Skyrme EDF and zero-range pairing
  - Wfs expanded in the HO basis
  - Axial and time-reversal symmetry assumed
  - Constraints imposed on q_{20}, q_{30}, and q_N values
  - Neck position is the isoscalar density minimum

HFBTHO program
P. Marević *et al.*, CPC **276** (2022) 108367





Theoretical Framework
Quantum Number Distributions in Scission Configurations

- Quantum number distributions for both FFs in each scission configuration are extracted through **projection techniques**

$$\mathbb{P}_F(J_F, N_F, Z_F, N, Z|q) = \langle \Phi_q^S | \hat{P}^{J_F} \hat{P}^{N,Z}_{N_F,Z_F} | \Phi_q^S \rangle$$

  - Good particle number in the total system (N, Z)
  - Good particle number in FFs (N_F, Z_F)
  - Good angular momentum in FFs (J_F)

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- In practice, we use 1D-AMP and Fomenko expansion for PNP

$$\mathbb{P}_F(J_F, N_F, Z_F, N, Z|q) = \frac{2J_F + 1}{2} \frac{1}{N_\varphi^4} \int_0^\pi d\beta \sin\beta d_{00}^{J_F}(\beta) \times \sum_{l_N, l_Z = 1}^{N_\varphi} e^{-iN_F \varphi_{l_N}} e^{-iZ_F \varphi_{l_Z}} \sum_{k_N, k_Z = 1}^{N_\varphi} e^{-iN\tilde{\varphi}_{k_N}} e^{-iZ\tilde{\varphi}_{k_Z}} \times \mathcal{N}_q^F(\beta, \varphi_{l_N}, \varphi_{l_Z}, \tilde{\varphi}_{k_N}, \tilde{\varphi}_{k_Z})$$

Theoretical Framework

Quantum Number Distributions in Scission Configurations

- Norm kernel = overlap between the initial and rotated state

$$\mathcal{N}_q^F(\beta,\vec{\varphi}) = \langle \Phi_q^S | \hat{R}_y^F(\beta) \hat{R}_{N_F,Z_F}^{N,Z}(\vec{\varphi}) | \Phi_q^S \rangle$$

- Overlaps are separable in isospin

$$\mathcal{N}_q^F(\beta,\vec{\varphi}) = \mathcal{N}_q^{F(\tau=n)}(\beta,\varphi_{l_N},\tilde{\varphi}_{k_N}) \times \mathcal{N}_q^{F(\tau=p)}(\beta,\varphi_{l_Z},\tilde{\varphi}_{k_Z})$$

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- They are calculated with Onishi formula (taking care of non-unitarity)

$$\mathcal{N}_q^{F(\tau)}(\beta,\varphi,\tilde{\varphi}) = \sqrt{\det[A_q^{F(\tau)}(\beta,\varphi,\tilde{\varphi})]\det[\mathcal{R}^F(\beta,\varphi,\tilde{\varphi})]}$$

$$A_q^{F(\tau)}(\beta,\varphi,\tilde{\varphi}) = U_q^{(\tau)T}\left[\left(\mathcal{R}^F(\beta,\varphi,\tilde{\varphi})\right)^T\right]^{-1} U_q^{(\tau)*} + V_q^{(\tau)T}\mathcal{R}^F(\beta,\varphi,\tilde{\varphi})V_q^{(\tau)*}$$

Theoretical Framework

Quantum Number Distributions in Scission Configurations

- Finally we extract the component with good total number of particles

$$\mathbb{P}_F(J_F, N_F, Z_F | N_0, Z_0, q) = \langle \Phi_q^S | \hat{P}^{J_F} \hat{P}^{N_0, Z_0}_{N_F, Z_F} | \Phi_q^S \rangle / \langle \Phi_q^S | \hat{P}^{N_0} \hat{P}^{Z_0} | \Phi_q^S \rangle$$

- The procedure is repeated for all configurations (hundreds of them)

Theoretical Framework

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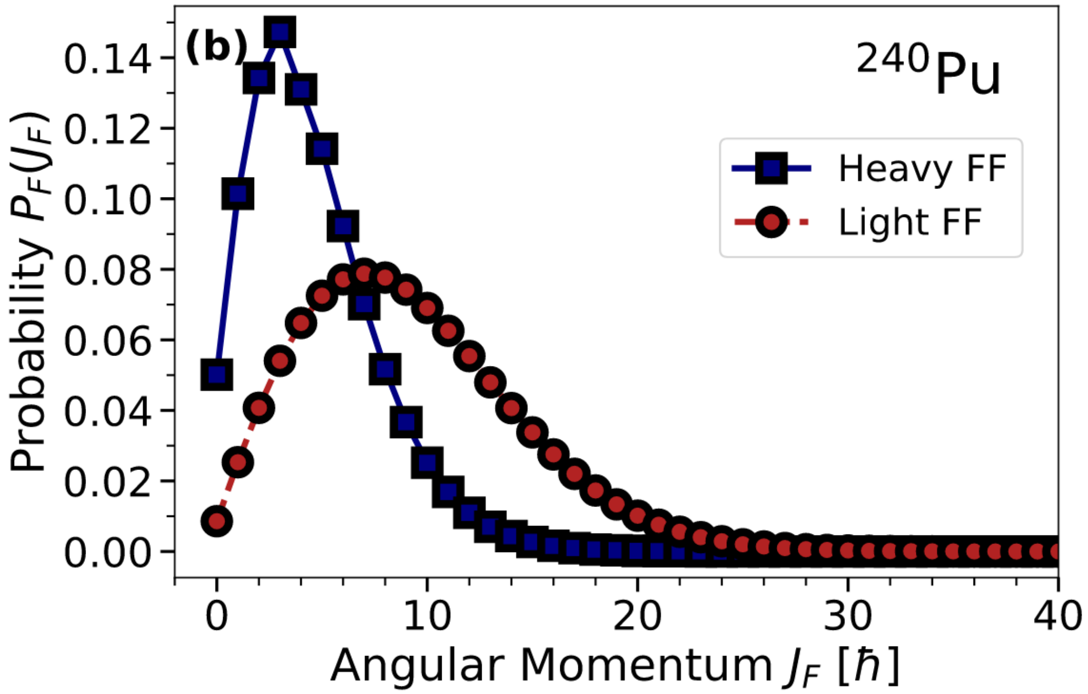
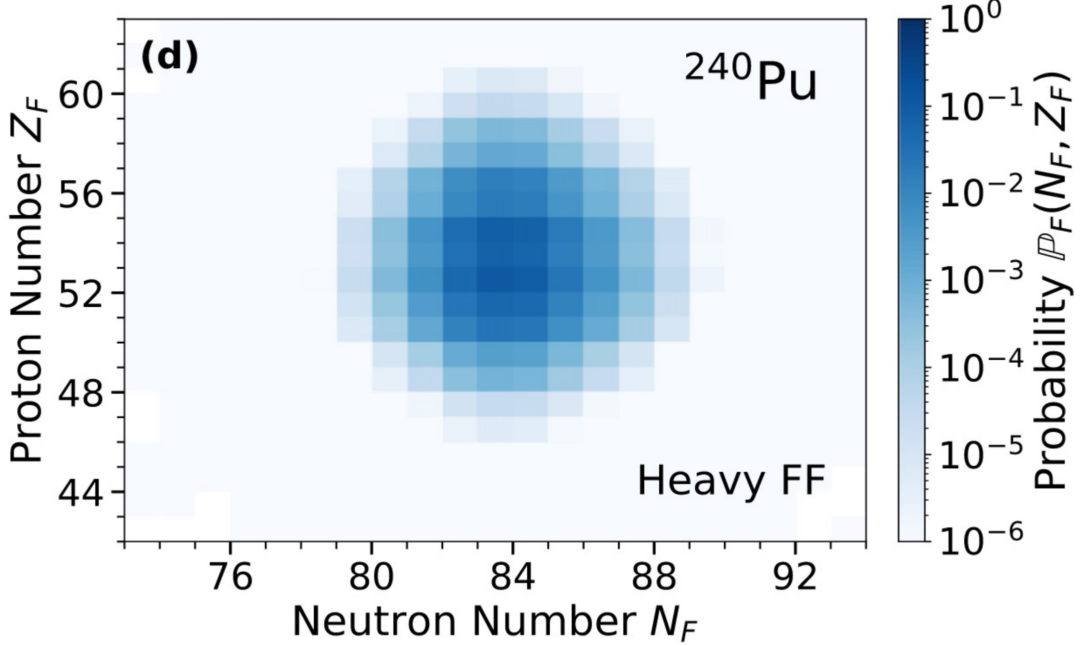
- The procedure is repeated for all configurations (hundreds of them)
- Each scission configuration is characterized by a **full distribution**

Distributions near the most likely fragmentation in ^{240}Pu*

$$\mathbb{P}_F(J_F | q) = \sum_{N_F, Z_F} \mathbb{P}_F(J_F, N_F, Z_F | q) \qquad \mathbb{P}_F(N_F, Z_F | q) = \sum_{J_F} \mathbb{P}_F(J_F, N_F, Z_F | q)$$





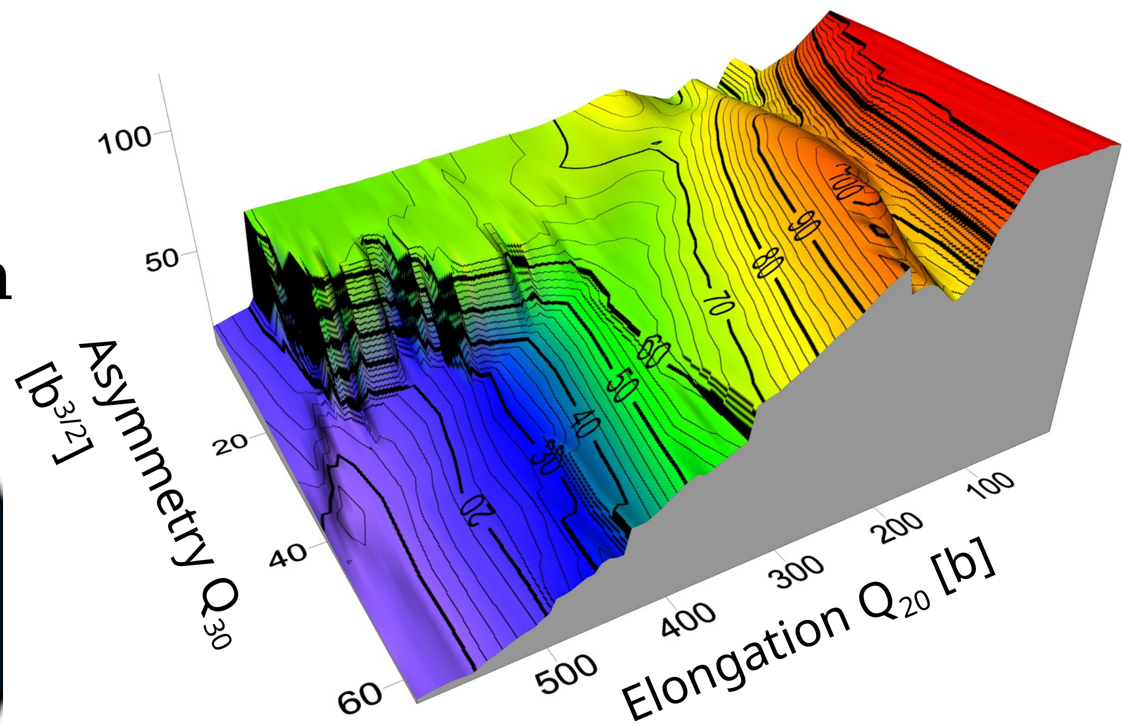
Theoretical Framework
Probability of Populating Scission Configurations

- **Population probability** for each conf. is estimated with **TDGCM+GOA**
- Time-dependent wave function is a superposition of HFB states

$$|\Psi(t)\rangle = \int dq f_q(t)|\Phi_q\rangle$$

- Variational principle and GOA lead to a Schrödinger-like equation

$$i\hbar\frac{\partial}{\partial t}g_q(t) = \left[\mathcal{H}_q^{coll} + i\mathcal{A}_q^{coll}\right]g_q(t)$$



- Probability that the wave packet exits through any particular point is proportional to the time-integrated flux density

$$F(q) = \lim_{t\to\infty}\int_{\tau=0}^{\tau=t} d\tau\phi(q,\tau)$$

FELIX Finite Element Solver
D. Regnier *et al.*, CPC **225** (2018) 180–191

Theoretical Framework

Final Distribution

- Final distribution is obtained by folding the two results

$$\mathbb{P}_F(J_F, N_F, Z_F | N_0, Z_0) = \int dq F(q) \mathbb{P}_F(J_F, N_F, Z_F | N_0, Z_0, q)$$

$$\mathbb{P}(J_F, N_F, Z_F | N_0, Z_0) = \mathbb{P}_l(J_F, N_F, Z_F | N_0, Z_0) + \mathbb{P}_r(J_F, N_F, Z_F | N_0, Z_0)$$

  - Distribution covers the full range of N_F and Z_F
  - Fixing (N_F, Z_F) and renormalizing gives the J_F distribution in particular FF
  - Marginalization over J_F gives pre-neutron mass and charge yields
  - Initial conditions partially simulate different incident neutron energies

$$\mathbb{P}(J_F, N_F, Z_F | N_0, Z_0) \rightarrow \mathbb{P}(J_F, N_F, Z_F | N_0, Z_0, E_n)$$

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$$\mathbb{P}(J_F, N_F, Z_F|N_0, Z_0) \rightarrow \mathbb{P}(J_F, N_F, Z_F|N_0, Z_0, E_n)$$

- Some limitations of the current model:
  - Only K = 0 components in AMP
  - No intrinsic excitations, only deformation energy
  - Influence of relative motion disregarded
  - No joint projection in the two FFs

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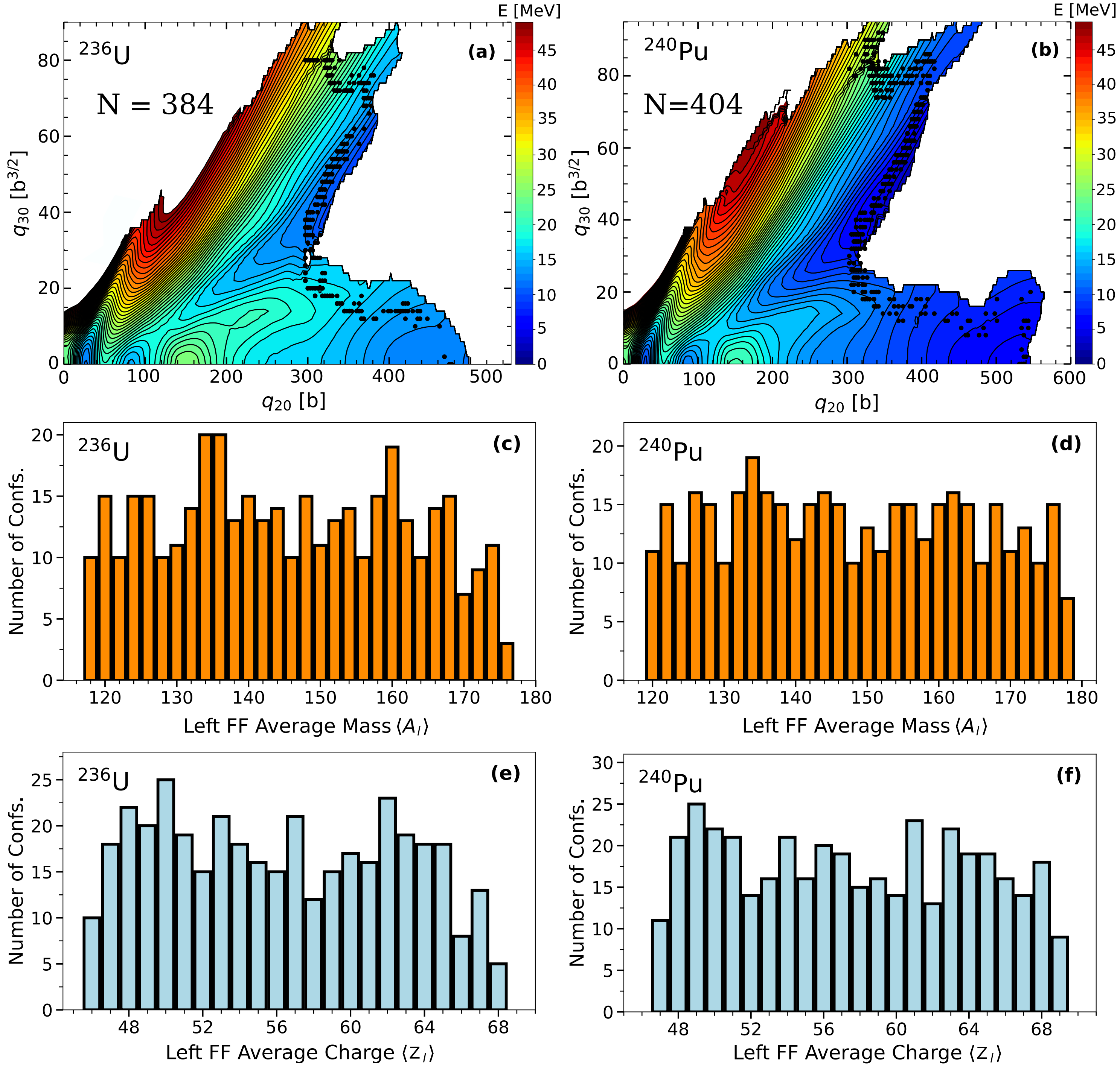
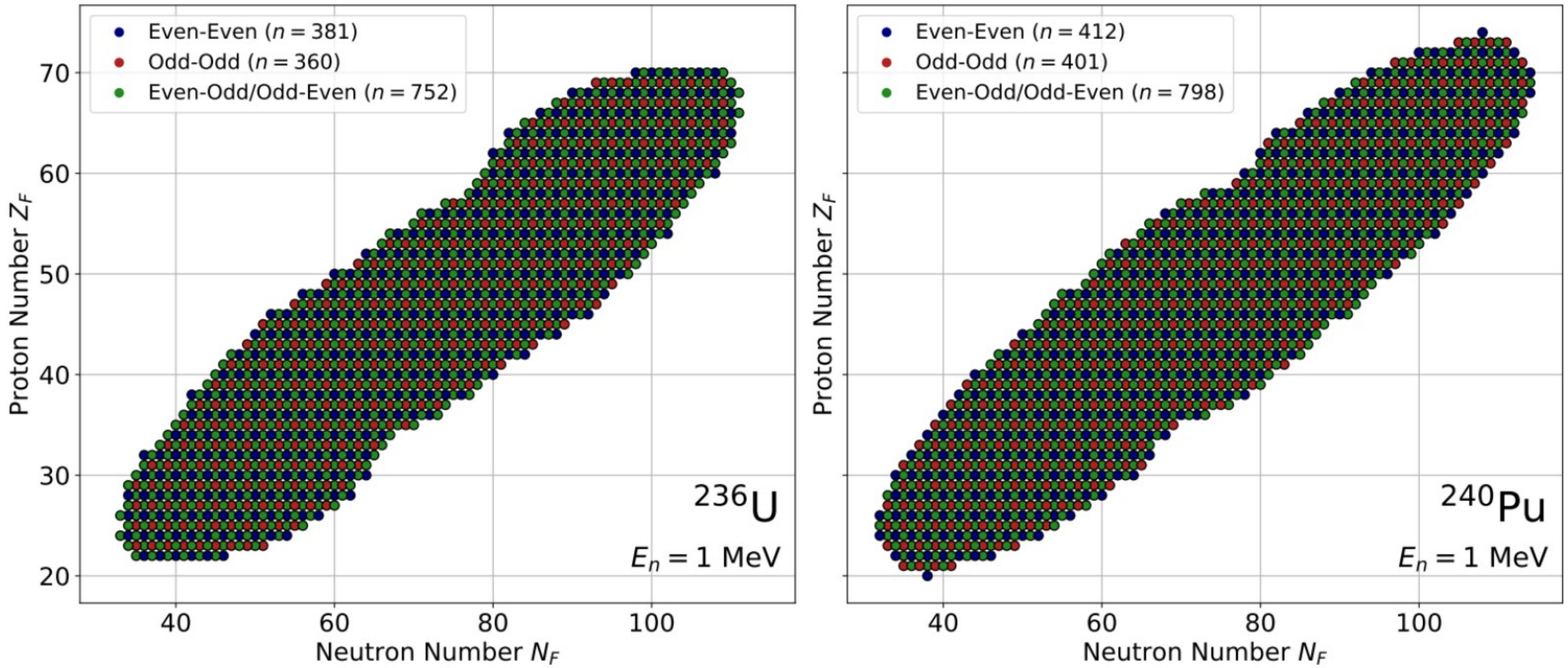
n-induced Fission of ^{235}U and ^{239}Pu
Properties of Scission Configurations

- We consider a very wide range of scission configurations



n-induced Fission of ^{235}U and ^{239}Pu
Range of Distributions

- First microscopic prediction of J_F distributions for the **full range of FFs**



PHYSICAL REVIEW C **113**, 014612 (2026)

Editors' Suggestion

Microscopic theory of angular momentum distributions across the full range of fission fragments

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Marc Verriere [‡]
Nuclear data and Theory Group, Nuclear and Chemical Science Division, Lawrence Livermore National Laboratory, Livermore, California 94550, USA and CEA, DAM, DIF, 91297 Arpajon, France

Microscopic Angular Momentum Distributions in Fragments for Neutron-Induced Fission of U-235 and Pu-239

https://zenodo.org/records/17303186

This dataset provides microscopic angular momentum distributions in even-even fission fragments for neutron-induced fission of ^{235}U and ^{239}Pu.

It was produced within the "*Angular momentum predictions for low-energy induced fission*" (AMPLIFI) project, funded by the European Union's Horizon Europe research and innovation programme under the Marie Skłodowska-Curie Actions Grant Agreement No. 101149053.

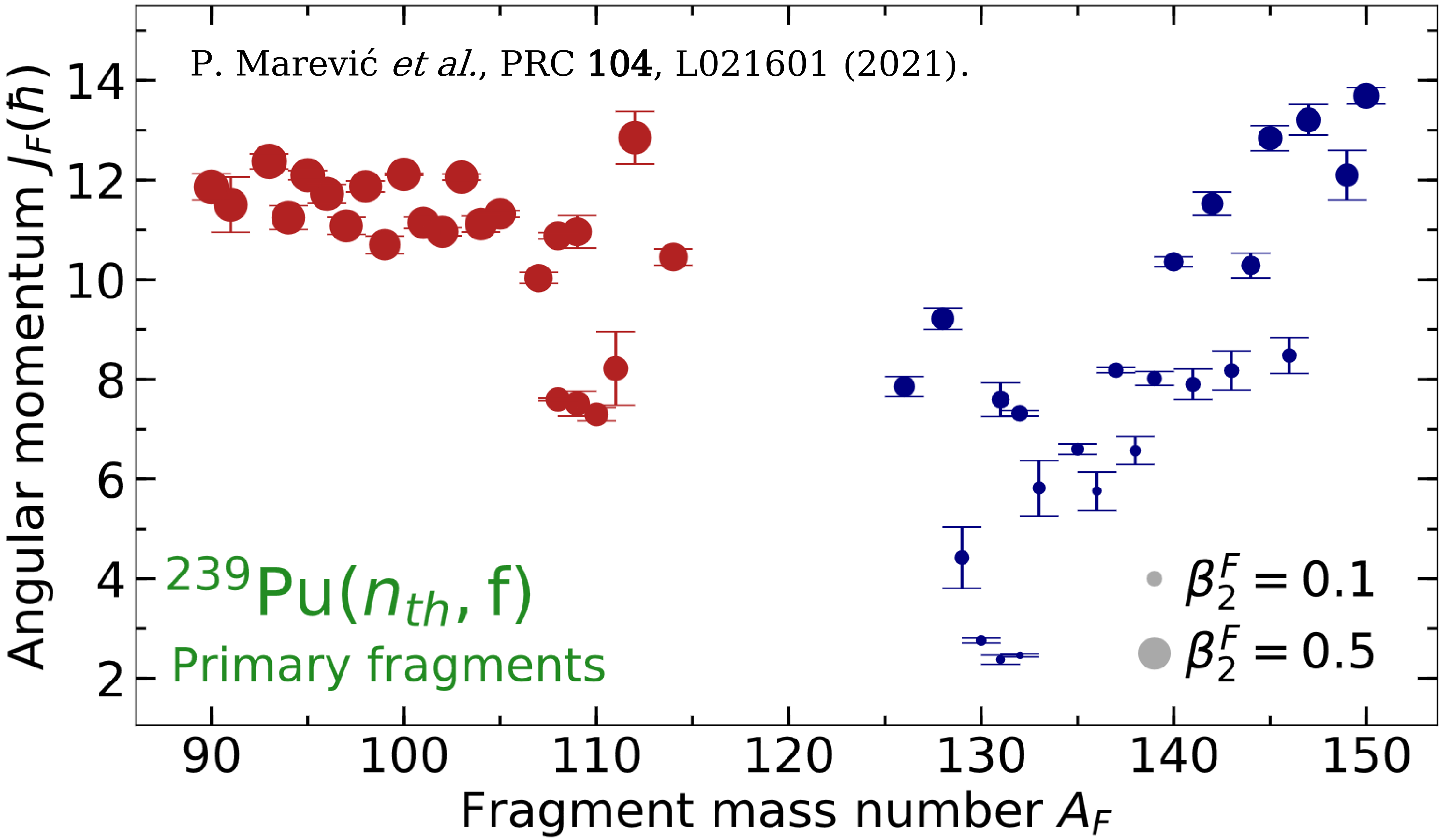
Reference: P. Marević, N. Schunck, M. Verriere, "*Microscopic theory of angular momentum distributions across the full range of fission fragments*", Phys. Rev. C **113**, 014612 (2026).

For a full description of the framework, see the publication or the arXiv preprint.

n-induced Fission of ^{235}U and ^{239}Pu

The Sawtooth Pattern

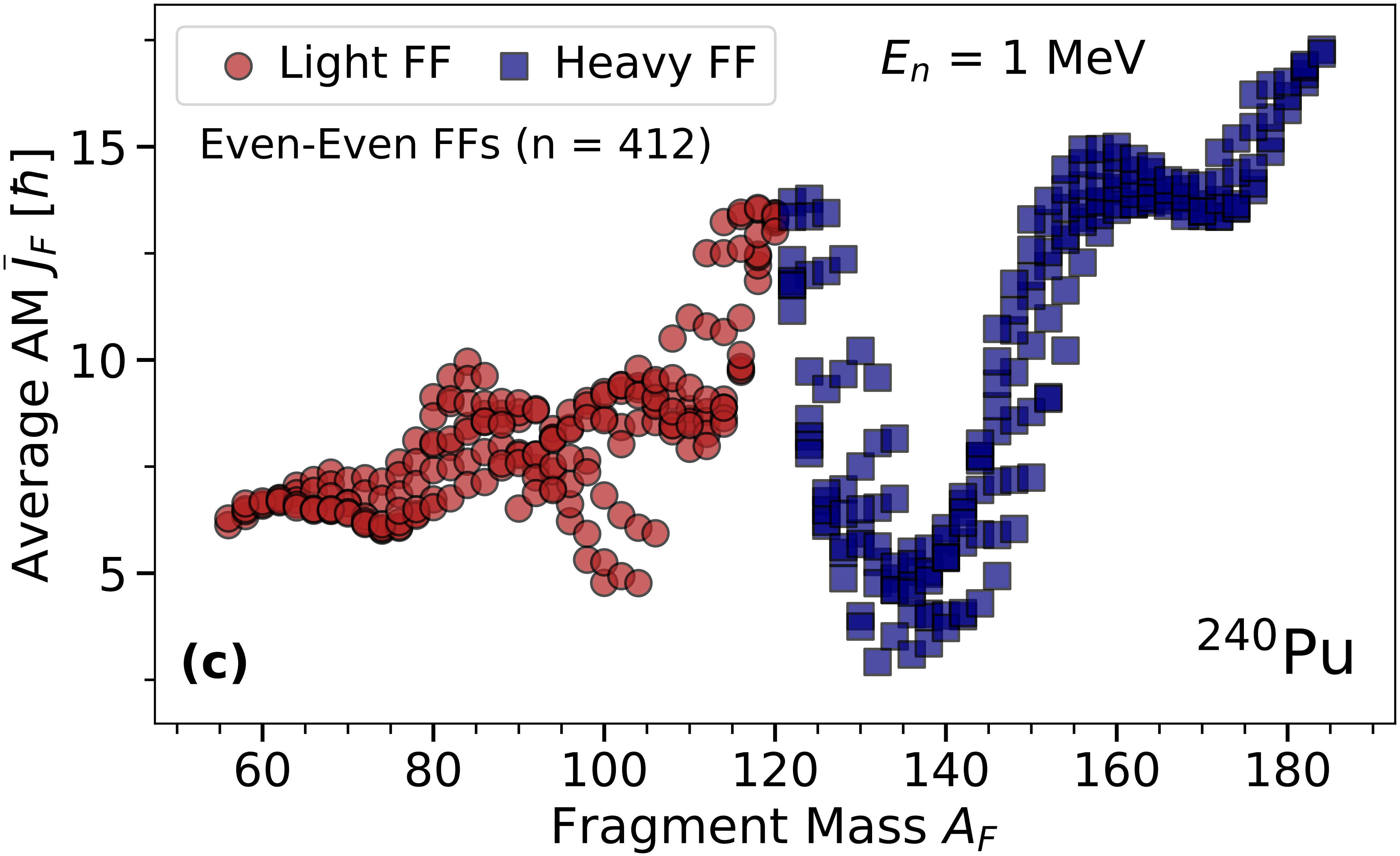
- First analysis (2021) was consistent with a sawtooth pattern



n-induced Fission of ^{235}U and ^{239}Pu

The Sawtooth Pattern

- Unequivocal microscopic evidence of a universal sawtooth pattern



n-induced Fission of ^{235}U and ^{239}Pu
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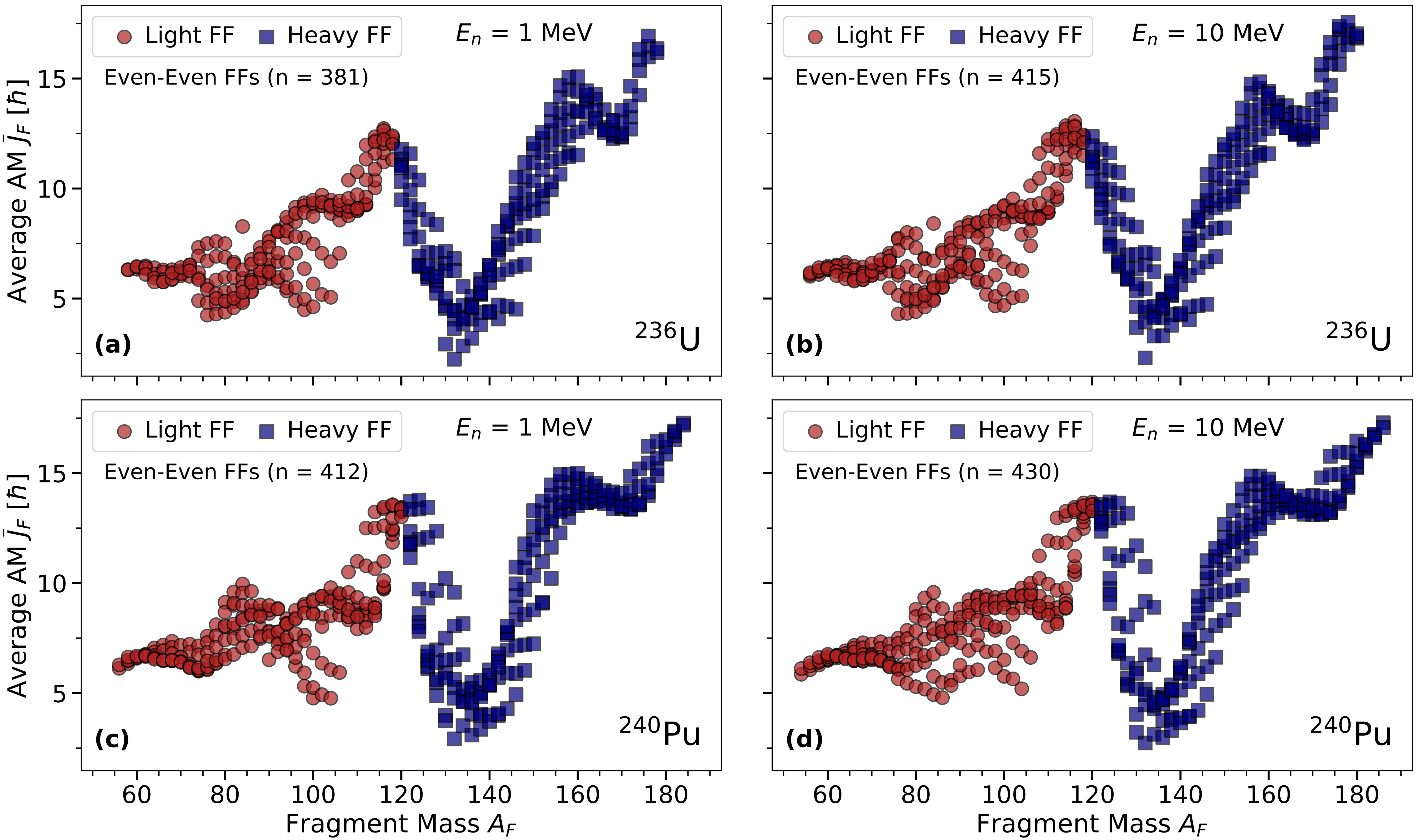
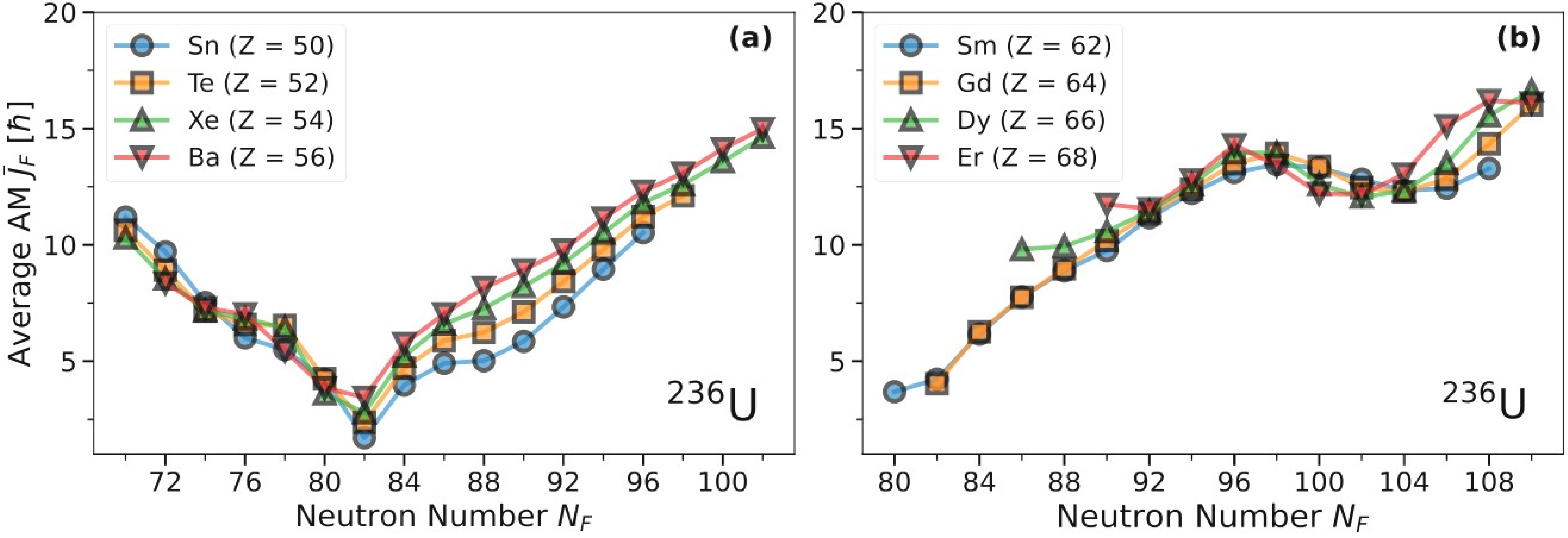


n-induced Fission of ^{235}U and ^{239}Pu
Shell Effects

- Shell effects hinder the ability of FFs to carry angular momentum
  - N=82 shell closure has a decisive impact
  - Local minimum in the N = 100-104 region → deformed shell closure?



n-induced Fission of ^{235}U and ^{239}Pu

Isobaric Dependence

Phenomenological formula: $$p(J_F) \propto (2J_F + 1) \exp\Big(- \frac{1}{2} \frac{J_F(J_F + 1)}{B^2(Z_F, A_F, T_F)} \Big)$$

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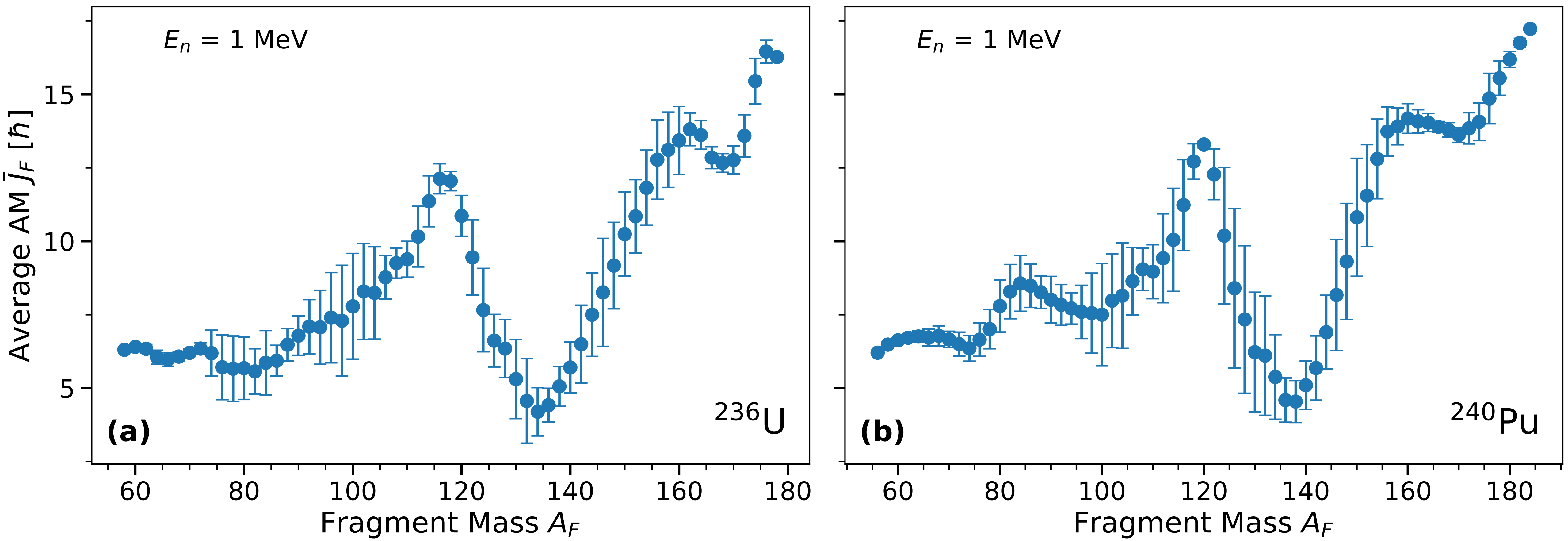
- There is a strong **isobaric dependence** of AM distributions



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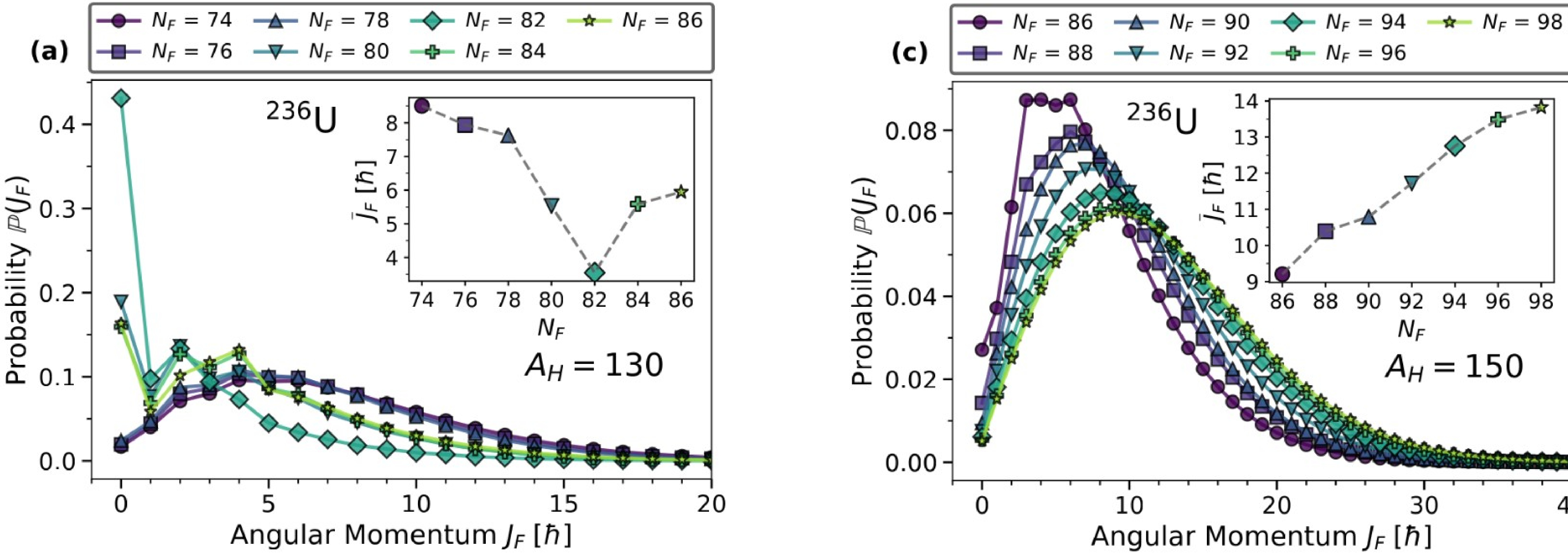
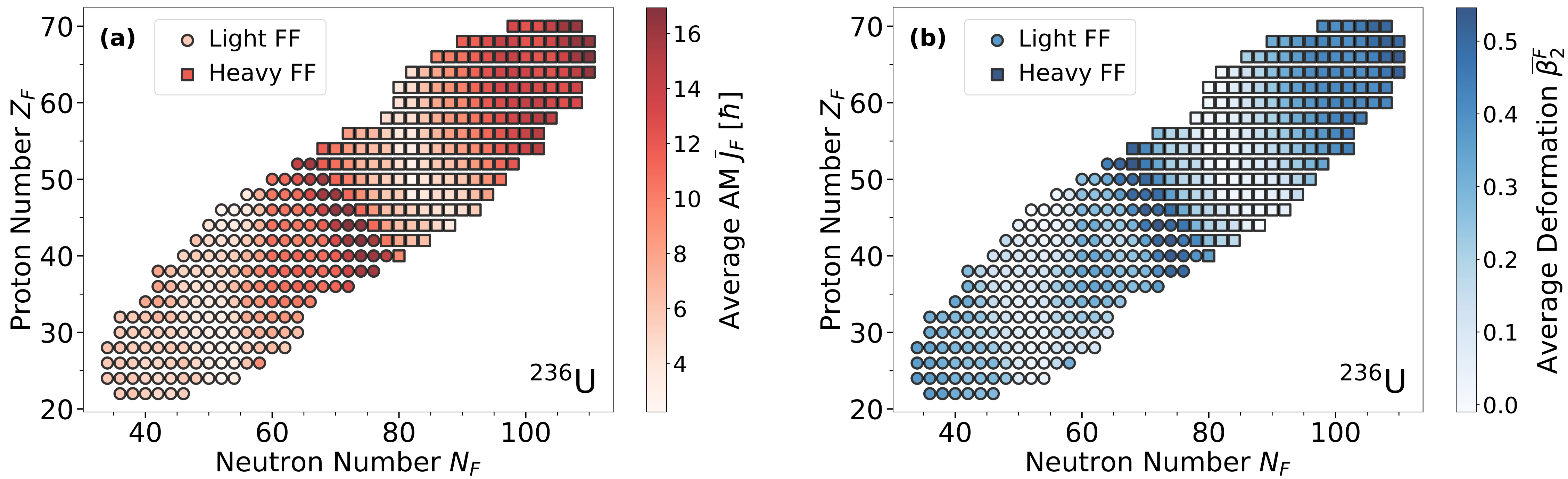


n-induced Fission of ^{235}U and ^{239}Pu
Fragment Deformations

- FF deformations and AM are strongly correlated ($\rho_{\text{Pear}} = 0.67$ for ^{236}U)



$$\bar{\beta}_2^F(N_F, Z_F) = \frac{\int dq F(q) \beta_2(N_F, Z_F, q)}{\int dq F(q) s(N_F, Z_F, q)}$$

$$\beta_2(N_F, Z_F, q) = \sum_{f=l,r} \mathbb{P}_f(N_F, Z_F | N_0, Z_0, q) \beta_2^f(q)$$

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n-induced Fission of ^{235}U and ^{239}Pu

Fragment Deformations

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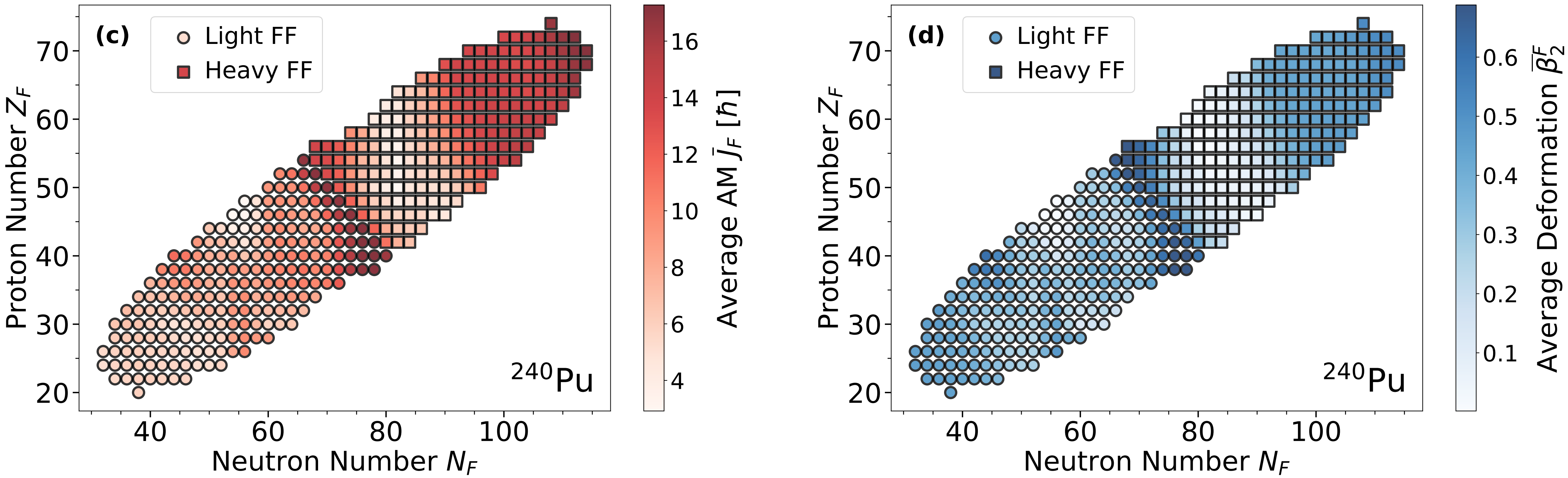

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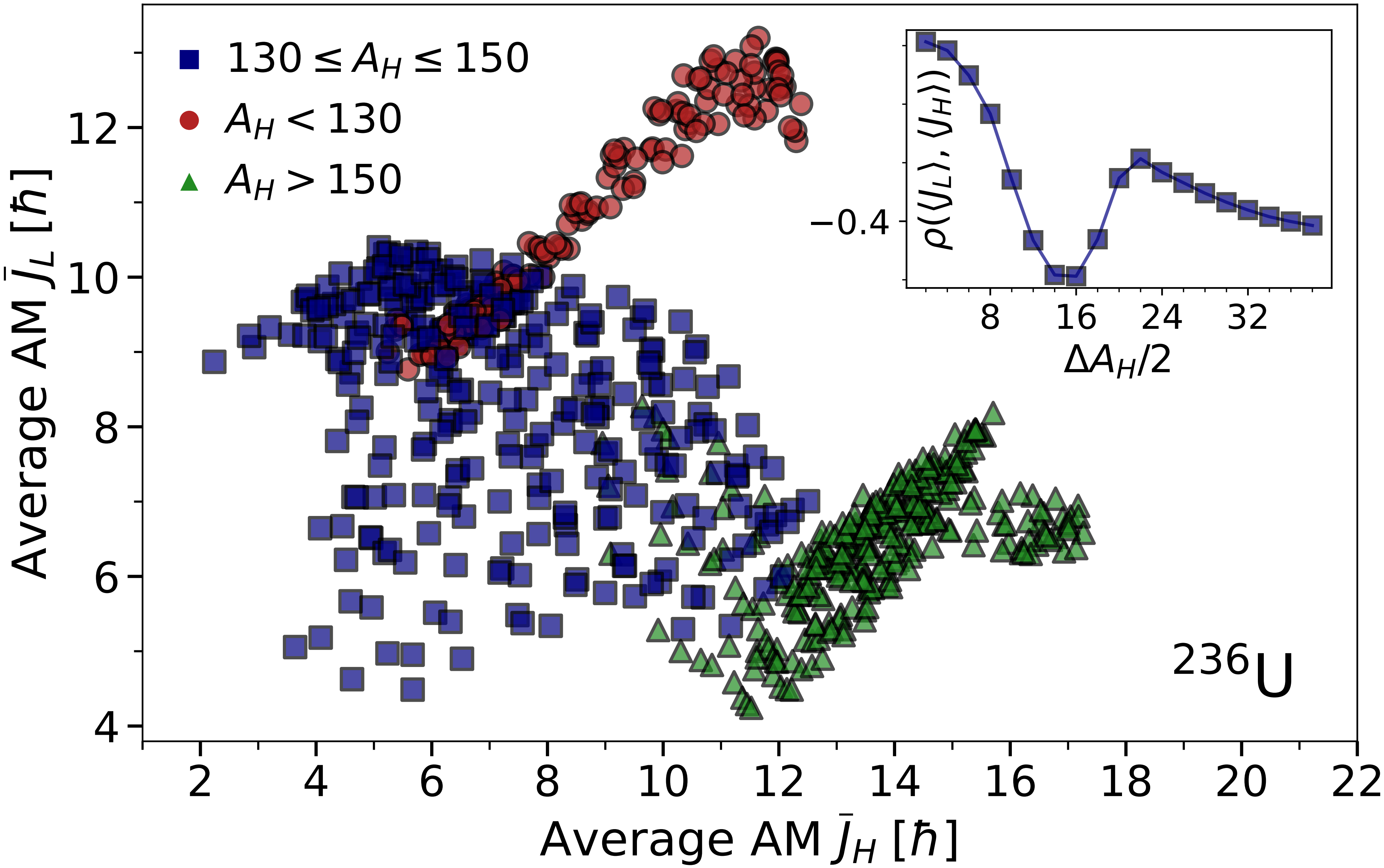
n-induced Fission of ^{235}U and ^{239}Pu
Correlation in Magnitude of FF Angular Momenta

- Weak negative correlation (ρ_{Pear} = -0.33) in AM magnitudes for most strongly populated FFs



n-induced Fission of ^{235}U and ^{239}Pu

Effect on Fission Spectra

$$p(J_F) \propto (2J_F + 1) \exp\Big(- \frac{1}{2} \frac{J_F(J_F + 1)}{B^2(Z_F, A_F, T_F)}\Big)$$

$$B^2(Z_F, A_F, T_F) = \alpha \frac{\mathcal{I}_0(Z_F, A_F)}{\hbar^2} T$$

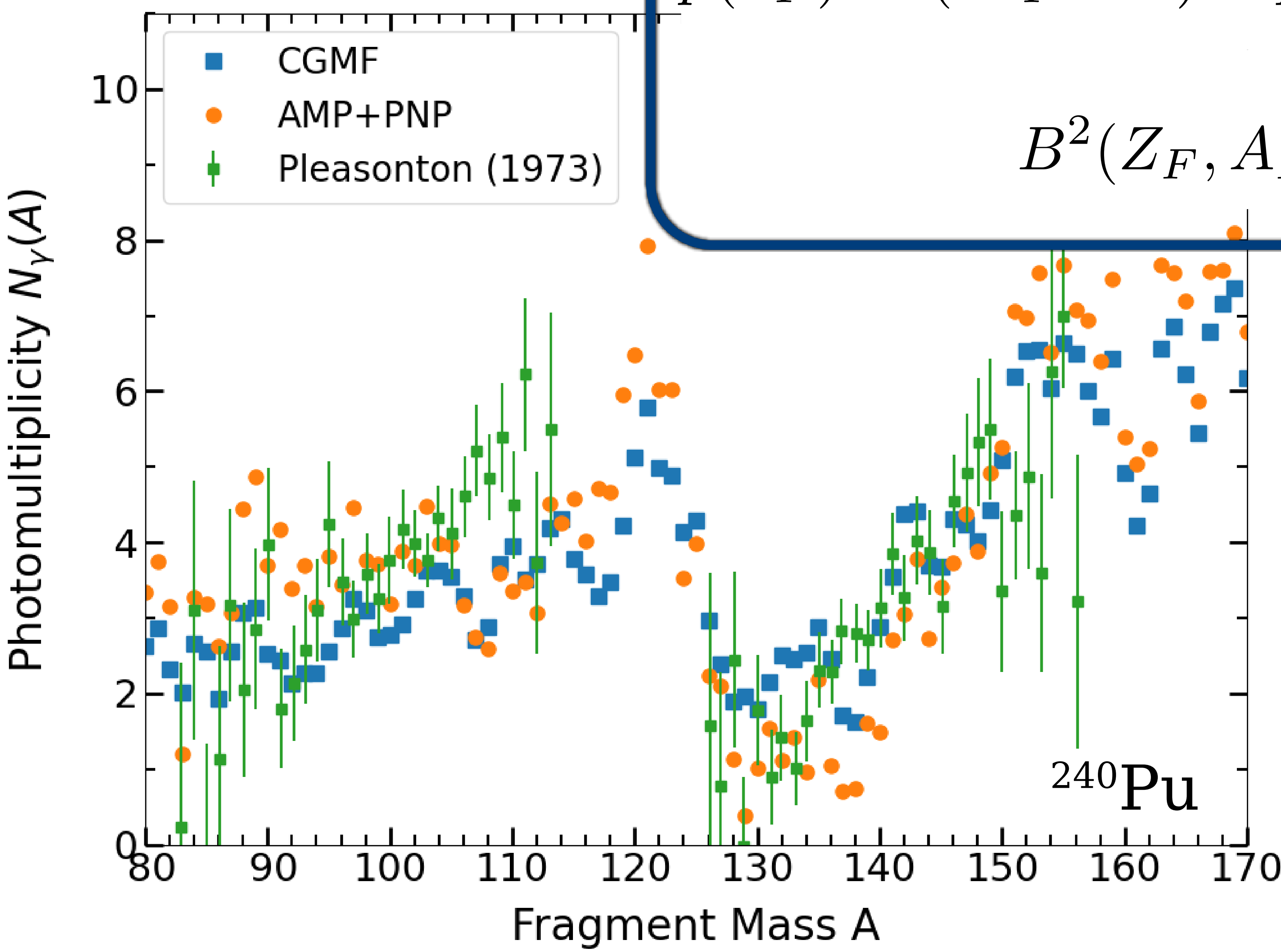
n-induced Fission of ^{235}U and ^{239}Pu

Effect on Fission Spectra

- Photon multiplicities are reproduced **without adjustable parameters** in AM distributions

$$p(J_F) \propto (2J_F+1)\exp\left(-\frac{1}{2}\frac{J_F(J_F+1)}{B^2(Z_F,A_F,T_F)}\right)$$

$$B^2(Z_F,A_F,T_F) = \alpha\frac{\mathcal{I}_0(Z_F,A_F)}{\hbar^2}T$$

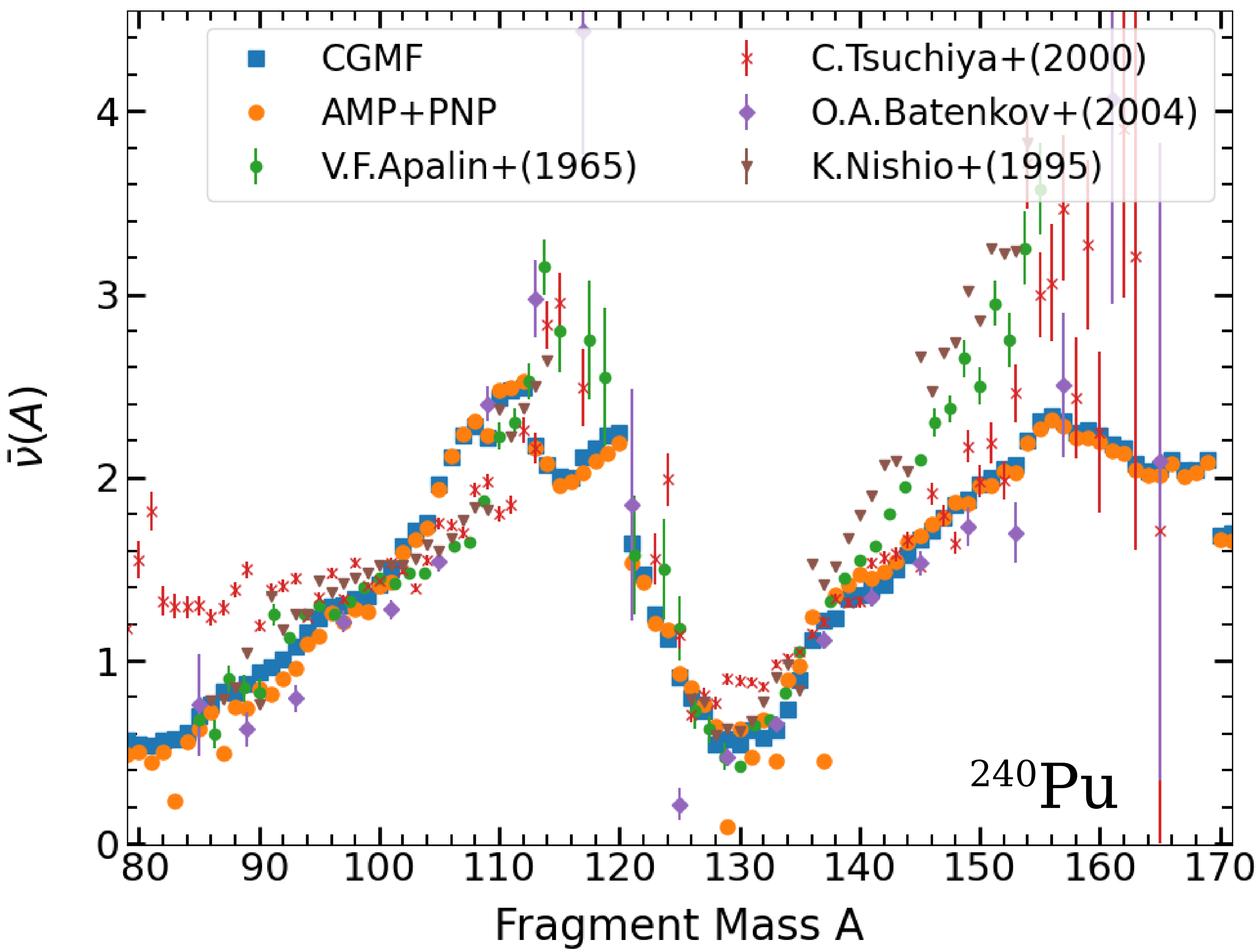


Photon multiplicity

N_γ: 6.24 → 6.08

n-induced Fission of ^{235}U and ^{239}Pu
Effect on Fission Spectra

- Photon multiplicities are reproduced **without adjustable parameters** in AM distributions



Neutron multiplicity

N_ν: 2.86 → 2.80

Outline

1. Introduction
2. Theoretical Framework
3. n-Induced Fission of ^{235}U and ^{239}Pu
4. Conclusion

Conclusion

1. Microscopic fission theory is becoming quantitatively competitive with phenomenological models.

Publication: PM, N. Schunck, M. Verriere, "*Microscopic theory of angular momentum distributions across the full range of fission fragments*", Phys. Rev. C **113**, 014612 (2026).

Database: "*Microscopic Angular Momentum Distributions in Fragments for Neutron-Induced Fission of U-235 and Pu-239*", https://zenodo.org/records/17303186

Funded by the European Union

Conclusion

1. Microscopic fission theory is becoming quantitatively competitive with phenomenological models.

2. We can now predict microscopic AM distributions for all FFs.
   - ✔ Sawtooth pattern
   - ✔ Isobaric dependence
   - ✔ Photon and neutron multiplicities

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3. Exciting developments are underway.
   - Refinement and extension of AM models
   - Predicting other ingredients for FFs decay modeling

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