Workshop on Fission Dynamics

11-15 May 2026 Chongqing - China

Angular Momentum Dynamics in Fission

Effect of nucleon exchange on fission fragment angular momenta

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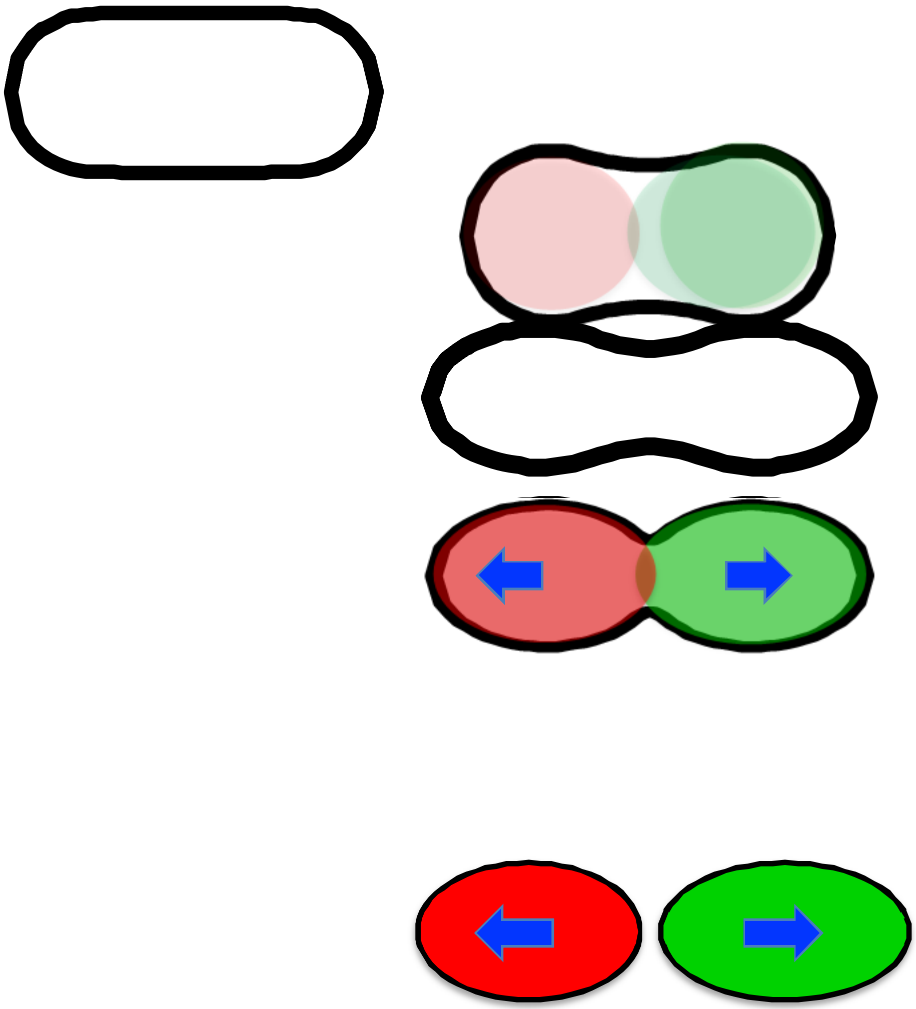
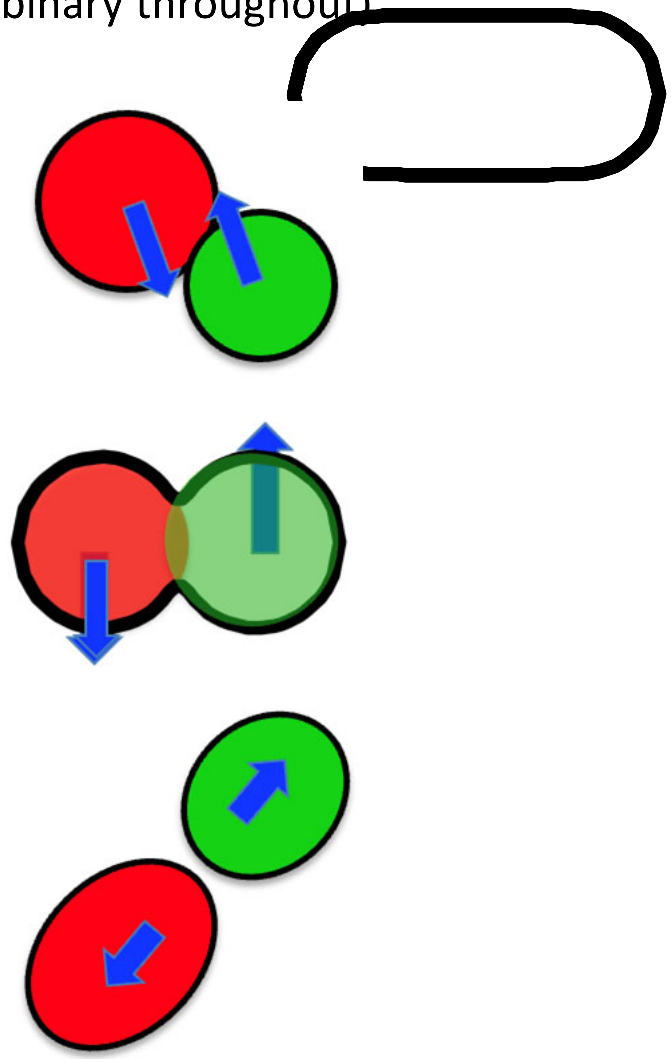
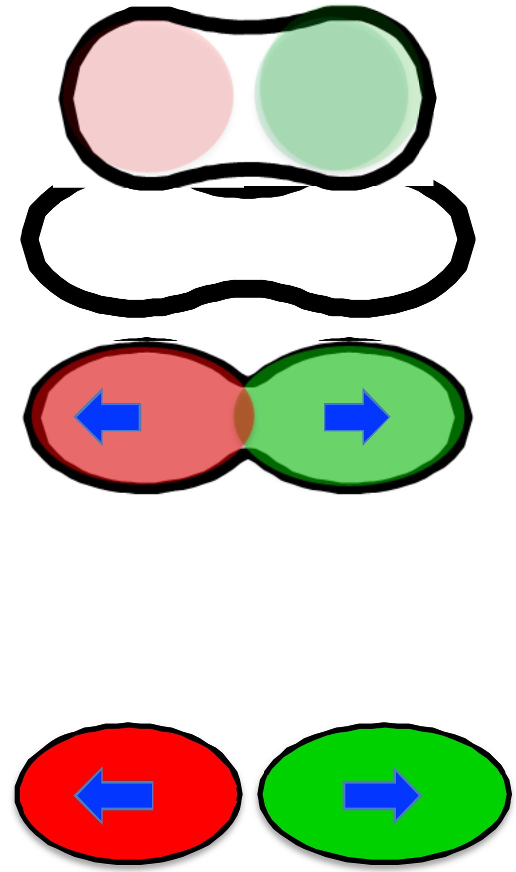
Saddle to scission



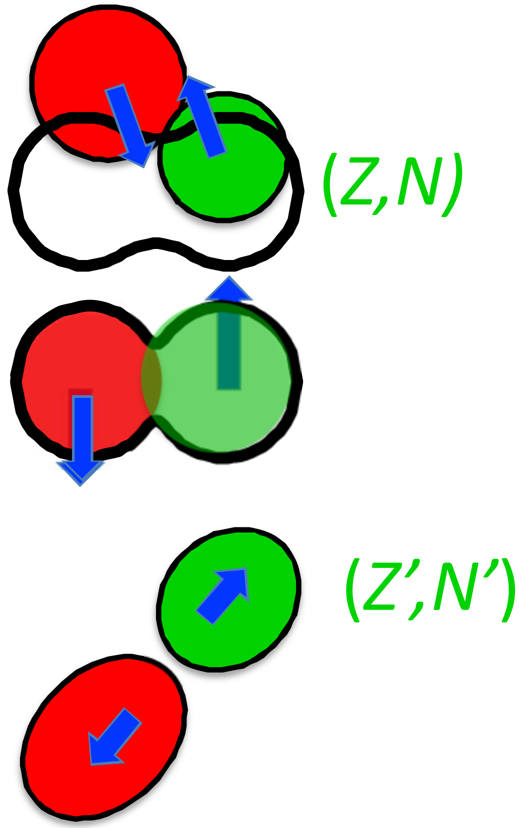
Damped reaction
(binary throughout)



Saddle to scission
(becomes binary)

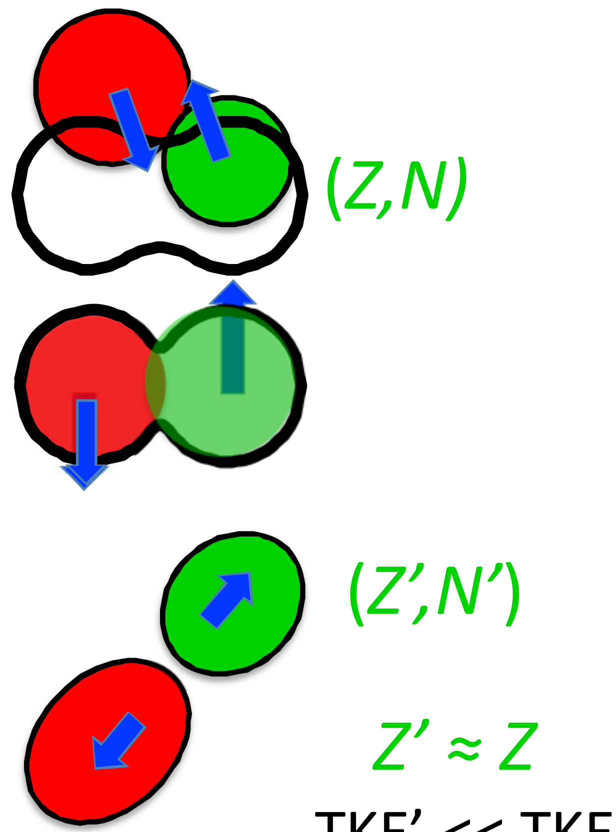


Damped Nuclear Reactions

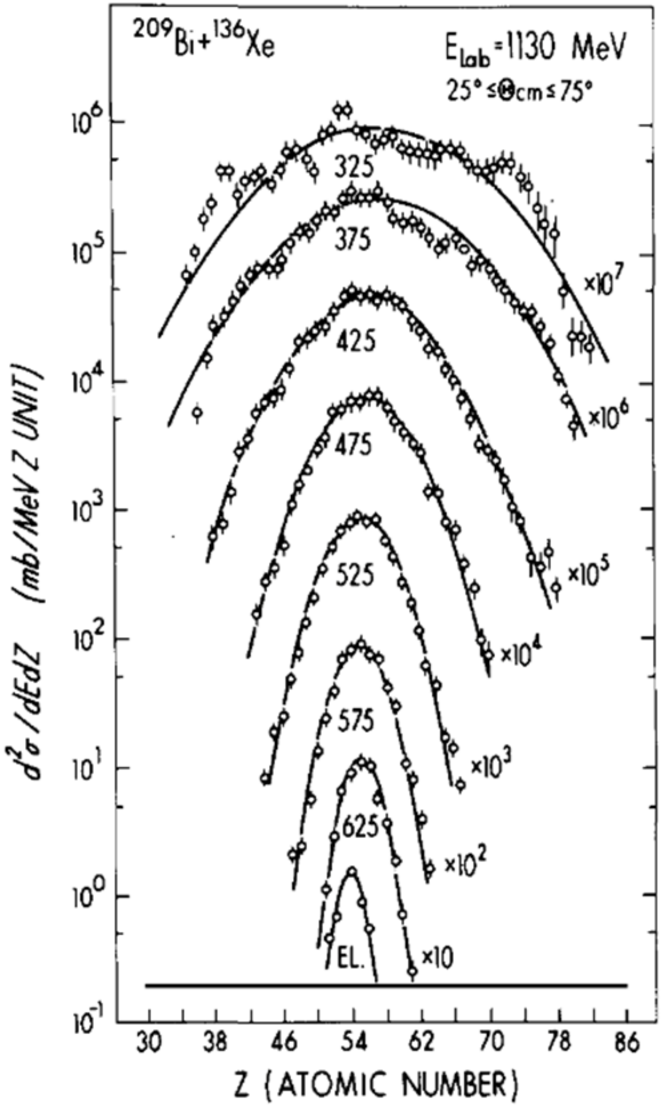


Strongly Damped Nuclear Reactions

Large loss of kinetic energy



Damped Nuclear Reactions



Nucleon exchange

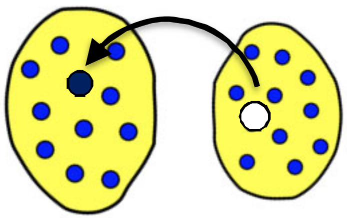
was found to be the

dominant dissipation mechanism

W.U. Schröder and J.R. Huizenga,
Damped Nuclear Reactions,
Treatise on Heavy-Ion Science II, p. 113,
(1984) Plenum Press, Ed: D.A. Bromley

W.U. Schröder, J.R. Birkelund, J.R. Huizenga, K.L. Wolf, J.P. Unik, V.E. Viola, Phys. Rev. Lett. **36**, 514 (1975)

Damped Nuclear Reactions



The participating nucleons
are *near the Fermi surface*

=> The dissipation is *strong!*

W.U. Schröder, J.R. Birkelund, J.R. Huizenga,
W.W. Wilcke, J. Randrup PRL **44**, 308 (1980)

Multiple nucleon transfers
produce a dissipative force
affecting the *linear* and *angular*
momenta of the binary partners:

window formula

$Z_H, N_H, \boldsymbol{P}_H, \boldsymbol{S}_H, E_H^*, Z_L, N_L, \boldsymbol{P}_L, \boldsymbol{S}_L, E_L^*$

Nucleon Exchange Transport Theory

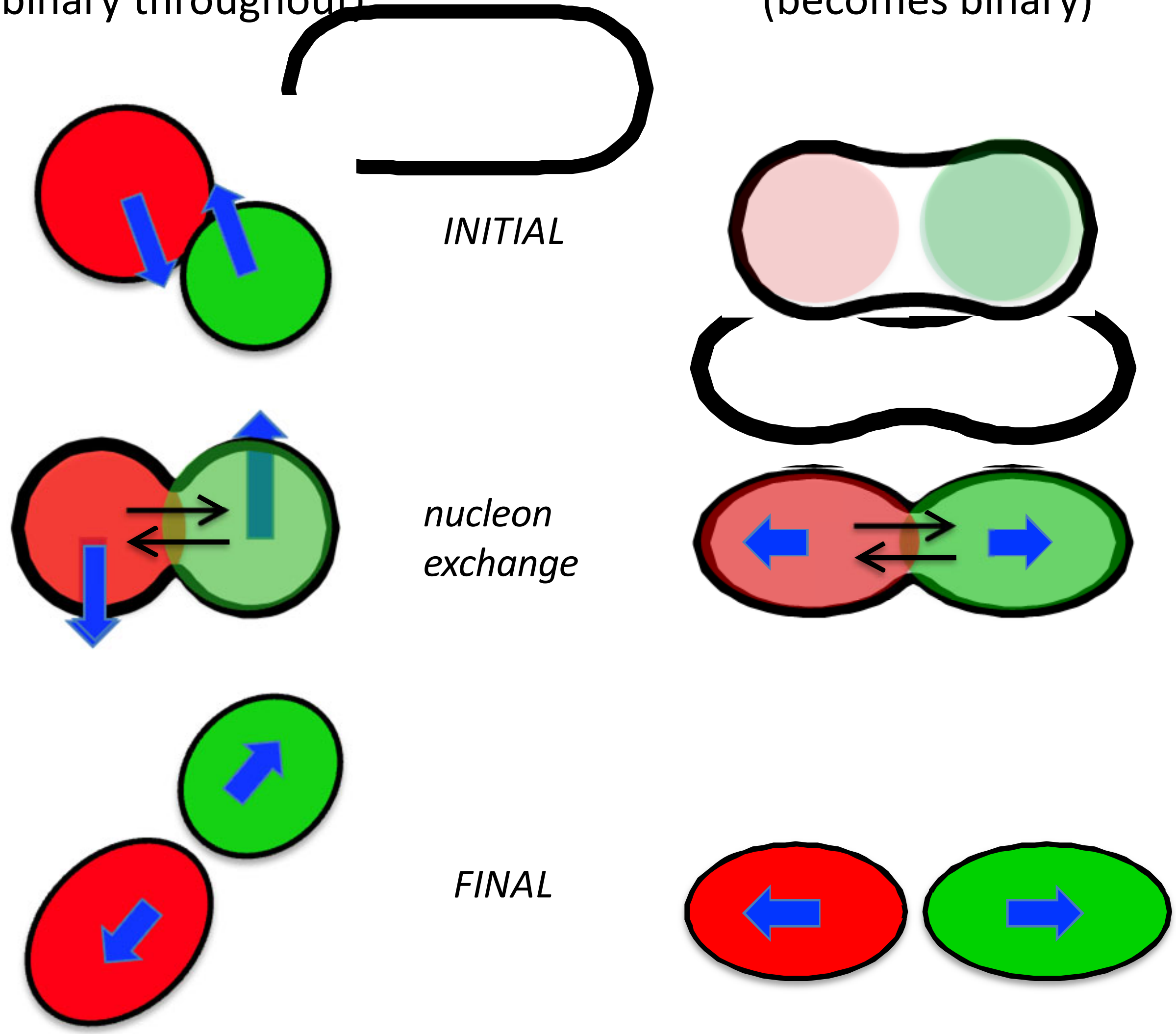
J. Randrup, Nucl. Phys. A **327**, 490 (1979)
J. Randrup, Nucl. Phys. A **383**, 468 (1983)
T. Døssing & J. Randrup, NPA **433**, 215 (1985)

Damped reaction

(binary throughout)

Saddle to scission

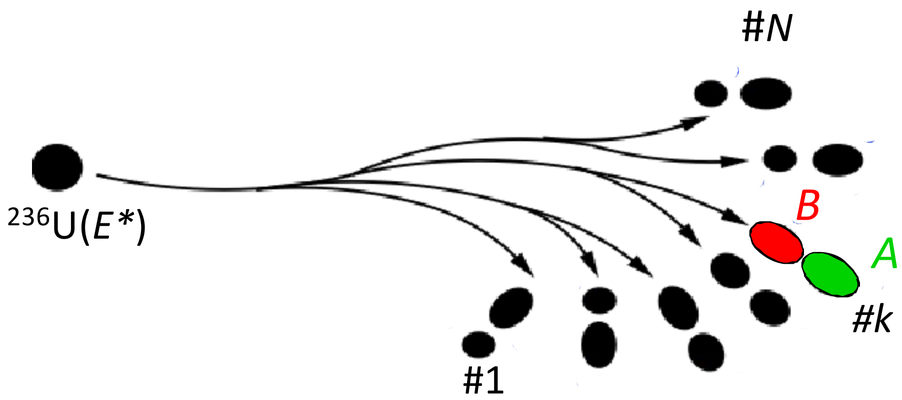
(becomes binary)



Calculational strategy

We consider a variety of cases, such as ^{236}U(E^*=46 MeV)

For each case, $N = 10^4$ different shape evolutions, $k = 1,\ldots,N$, are generated by *Langevin* simulation



For each trajectory, k, the evolution of the correlated fragment spin distribution, $P^{(k)}(\boldsymbol{S}_A, \boldsymbol{S}_B; t)$, is calculated with the *Nucleon Exchange transport theory*

Generate fission events by Langevin simulation

G.D. Adeev, A.V. Karpov, P.N. Nadtochy, and D.V. Vanin,
Physics of Particles and Nuclei **36,** 378 (2005)

8 cases:
$N = 10^4$

- ^{236}U at $E^* = 46$ & 70 MeV with $k_s = 1$ & 0.25
- ^{202}Po at $E^* = 46$ MeV with $k_s = 1$ & 0.25
- ^{182}Hg at $E^* = 46$ MeV with $k_s = 1$ & 0.25

One-body dissipation: $$\dot{Q}_{\rm wall} = k_{\rm s} \times \underbrace{m\rho\bar{v} \oint \dot{n}^2 d^2\sigma}_{\text{Standard } wall\ formula}$$

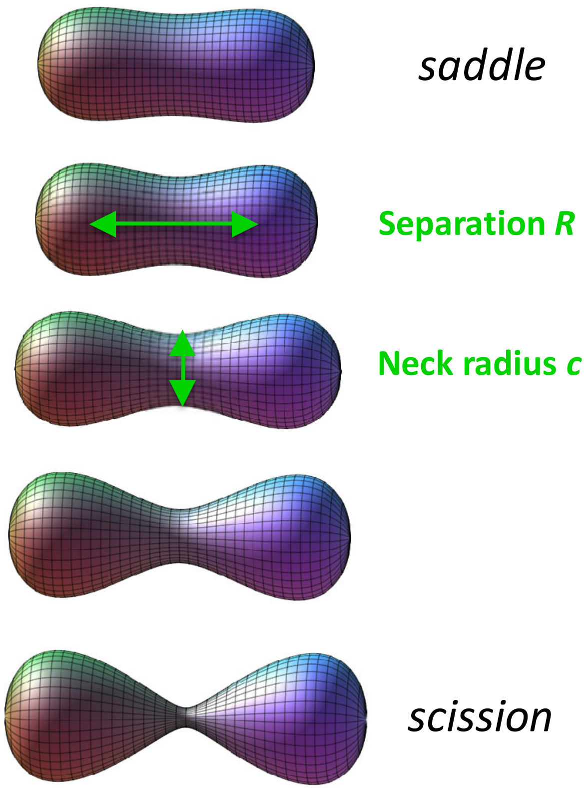
Standard *wall formula*

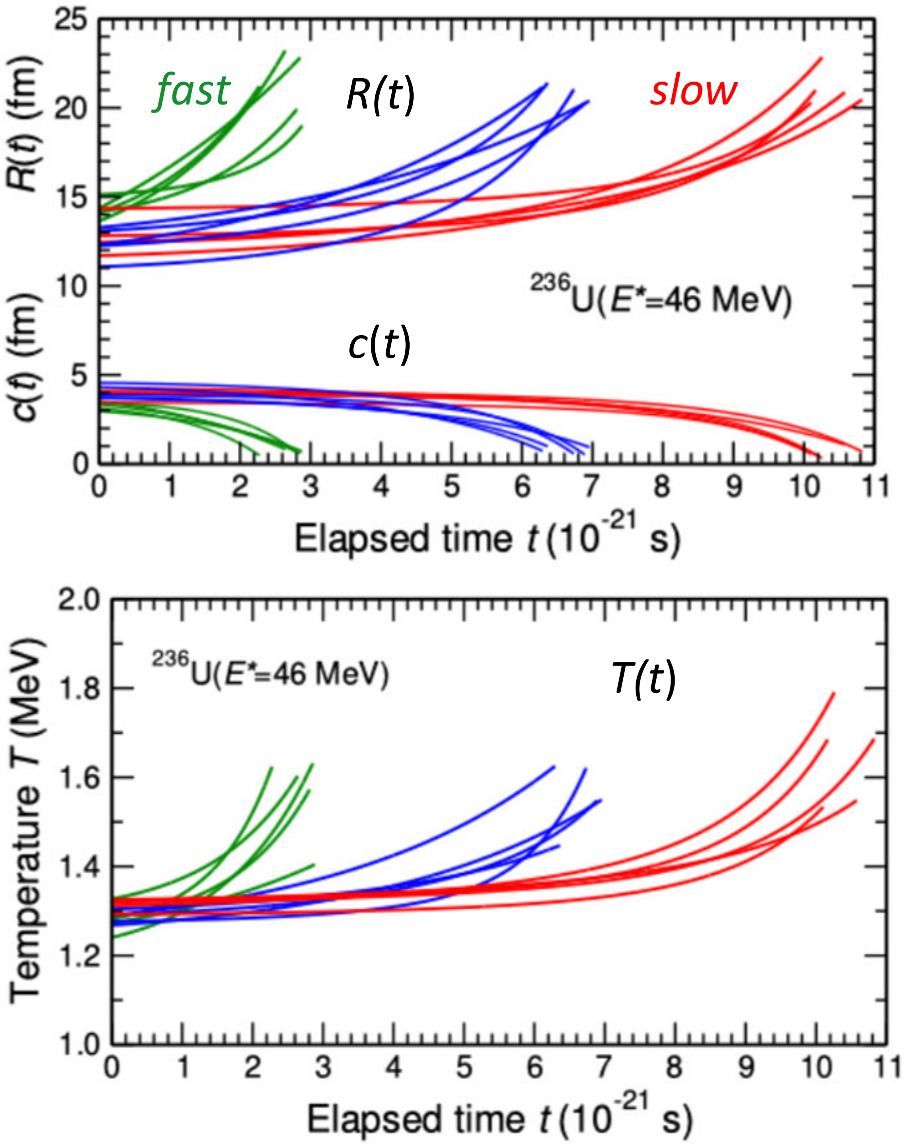
$k_s = 1$: *Standard value*

$k_s = 0.25$: *Fitted value*

J. Blocki, Y. Boneh, J.R. Nix. J. Randrup, M. Robel,
A.J. Sierk, W.J. Swiatecki, Ann. Phys. **113**, 330 (1978)

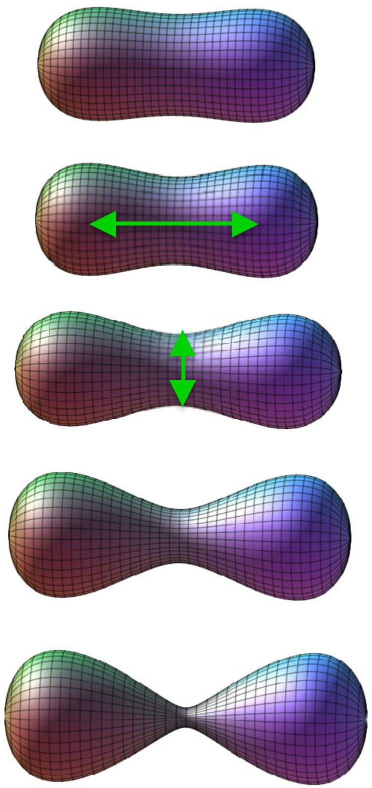
Dynamical evolution: $R(t)$ & $c(t)$ & $\alpha(t)$ - as well as $T(t)$





Dynamical evolution: $R(t)$ & $c(t)$ & $\alpha(t)$ - as well as $T(t)$

One shape evolution



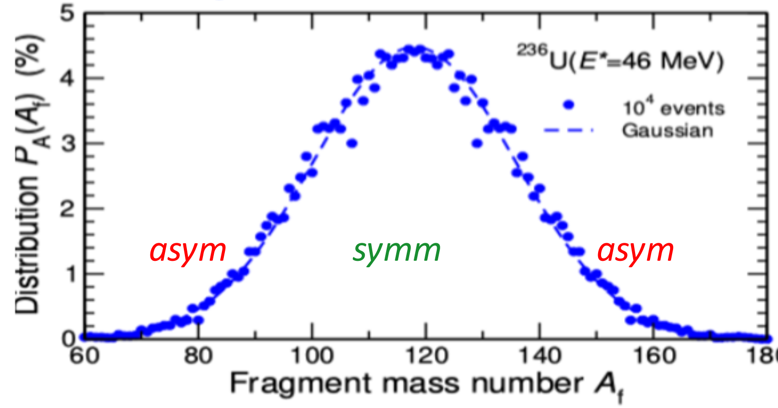
saddle

Separation *R*

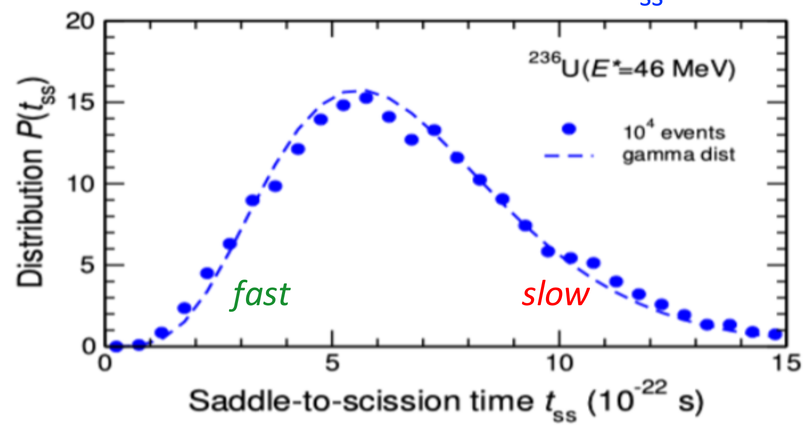
Neck radius *c*

scission

Fragment mass distribution

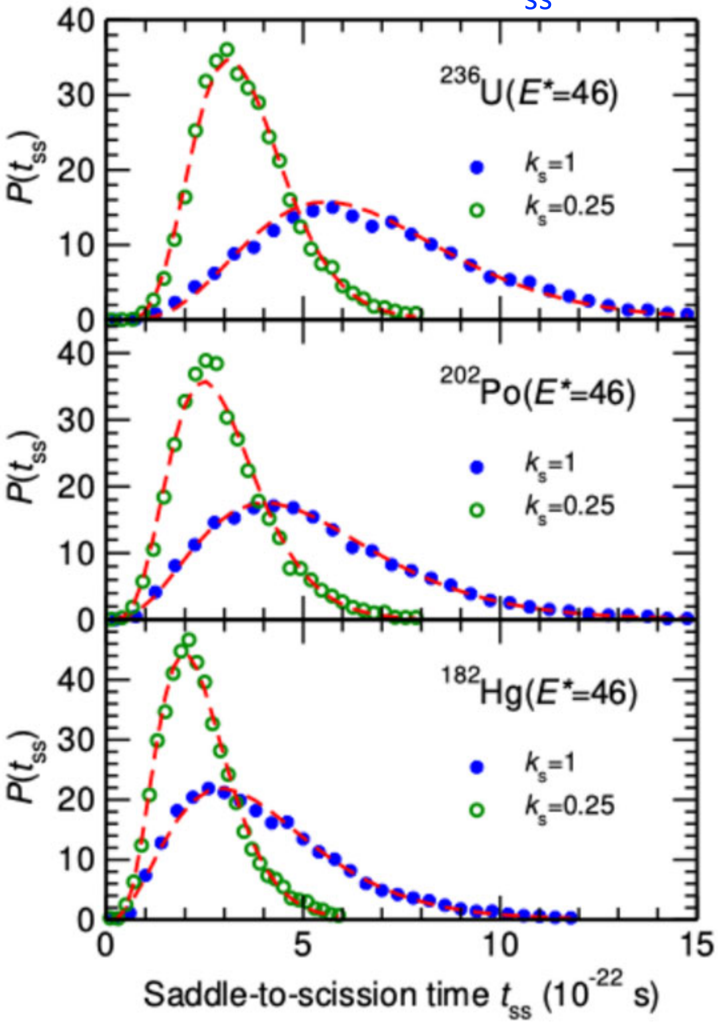


Saddle-to-scission time t_{ss}



Saddle-to-scission time t_{ss}

Distribution of t_{ss}



Average: $\bar{t}_{ss}$

Dispersion: σ_{ss}

P_{ss} (t_{ss}) is a *gamma* distribution:

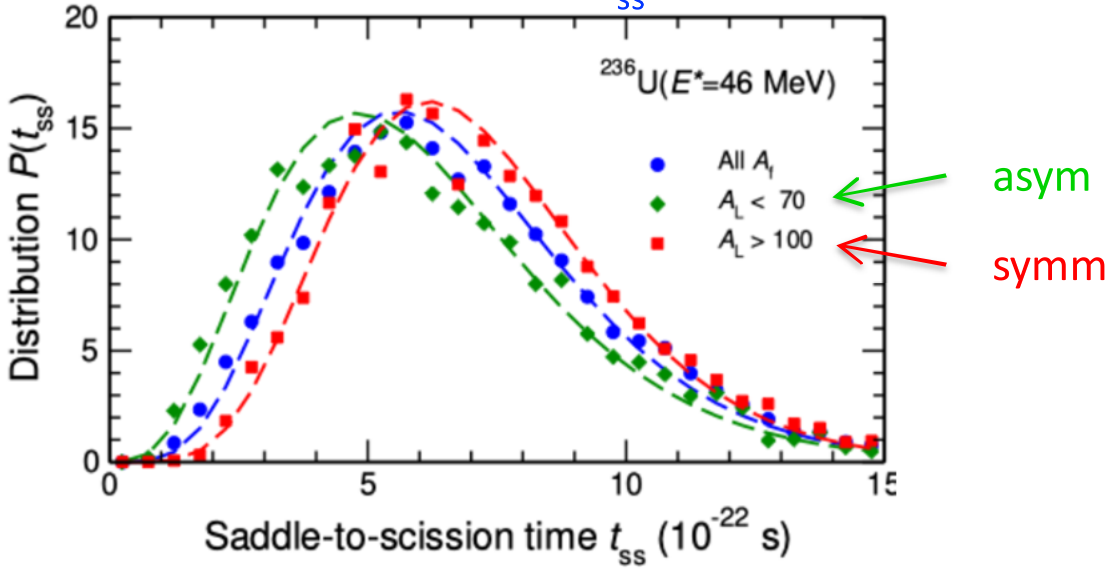
$$P_{ss}(t_{ss}) \approx \frac{1}{\theta_{ss}} \frac{1}{\Gamma(\alpha_{ss})} \left(\frac{t_{ss}}{\theta_{ss}}\right)^{\alpha_{ss}-1} e^{-t_{ss}/\theta_{ss}}$$

Scale parameter: $\theta_{ss} = \sigma_{ss}^2 / \bar{t}_{ss}$

Shape parameter: $\alpha_{ss} = \bar{t}_{ss}^2 / \sigma_{ss}^2 - 1$

Dependence of t_{ss} on mass asymmetry

Distribution of t_{ss}



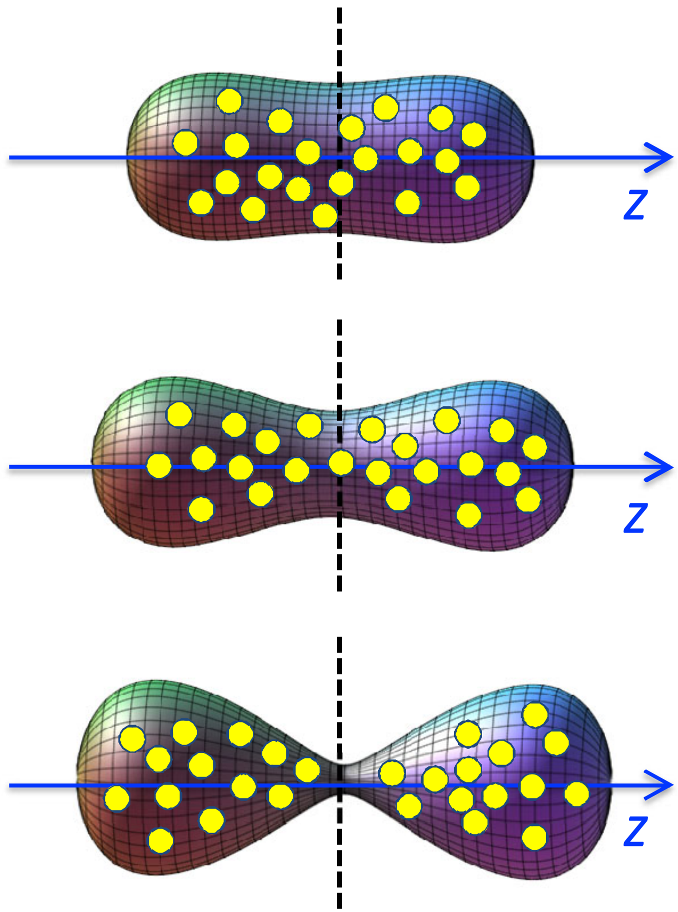
The nuclear system is made of nucleons

Linear momentum
of the left part:

$\boldsymbol{P}_B = \Sigma\, \boldsymbol{p}_n \quad z_n < 0$

Angular momentum
of the left part:

$\boldsymbol{S}_B = \Sigma\, \boldsymbol{j}_n \quad z_n < 0$



Linear momentum
in the right part:

$\boldsymbol{P}_\mathrm{A} = \Sigma\, \boldsymbol{p}_n \quad z_n > 0$

Angular momentum
in the right part:

$\boldsymbol{S}_\mathrm{A} = \Sigma\, \boldsymbol{j}_n \quad z_n > 0$

The relative angular momentum is $\boldsymbol{L} = \boldsymbol{R}_{AB} \times \boldsymbol{P}_\mathrm{AB}$

Angular momentum in a binary system

Total angular momentum $\boldsymbol{J} = \boldsymbol{S}_A + \boldsymbol{S}_B + \boldsymbol{L}$ is conserved (we take ***J*** = **0**)

Excess spin $\boldsymbol{s}_F$: Fragment spin $\boldsymbol{S}_F$ minus what is required for rigid rotation

Angular momentum *parallel* to the symmetry axis $J_{\parallel} = \boldsymbol{J}\cdot\boldsymbol{z}$:

$$E_{\parallel} = \frac{J_{\parallel}^2}{2\mathcal{I}_{\parallel}} + \frac{(s_A^{\parallel})^2}{2\mathcal{I}_A^{\parallel}} + \frac{(s_B^{\parallel})^2}{2\mathcal{I}_B^{\parallel}} = E_{\parallel}^{\text{rigid}} + \frac{s_{\text{twst}}^2}{2\mathcal{I}_{\text{twst}}}$$

$J_{\parallel}$

twisting

Angular momentum *perpendicular* to the symmetry axis $\boldsymbol{J}_{\perp}$:

$J_{\perp}$

$$E_{\perp} = \frac{J_{\perp}^2}{2\mathcal{I}_{\perp}} + \frac{(s_A^{\perp})^2}{2\mathcal{I}_A^{\perp}} + \frac{(s_B^{\perp})^2}{2\mathcal{I}_B^{\perp}} + \frac{(s_A^{\perp} + s_B^{\perp})^2}{2\mathcal{I}_R} = E_{\perp}^{\text{rigid}} + \frac{s_{\text{wrig}}^2}{2\mathcal{I}_{\text{wrig}}} + \frac{s_{\text{bend}}^2}{2\mathcal{I}_{\text{bend}}}.$$

wriggling *bending*

For each shape evolution, we wish to calculate the evolution of the distribution of the correlated excess spins, $P(\boldsymbol{s}_A, \boldsymbol{s}_B)$

Fokker-Planck equation for $P(\boldsymbol{s}_A, \boldsymbol{s}_B; t)$

Nucleon Exchange Transport Theory

J. Randrup, Nucl. Phys. A **327**, 490 (1979)
J. Randrup, Nucl. Phys. A **383**, 468 (1983)
T. Døssing & J. Randrup, NPA **433**, 215 (1985)

$$\frac{\partial}{\partial t}P = -\sum_i \frac{\partial}{\partial s_i} V_i P + \sum_{ij} \frac{\partial^2}{\partial s_i \partial s_j} D_{ij} P$$

Drift coefficient $V_i(\chi) = \Sigma_j M_{ij}(\chi) f_j$ determines the *mean* evolution of $s_i(t)$:

$$\frac{d\bar{s}_i}{dt} = V_i(\bar{\chi})$$

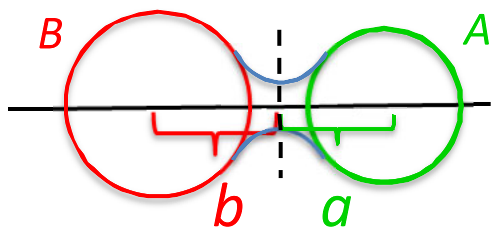
M_{ij}: ***Mobility*** coefficients

Diffusion coefficient $D_{ij}(\chi) = M_{ij}(\chi)\, T$ generates *correlated spin fluctuations* $\sigma_{ij}(t)$:

$$\sigma_{ij} \equiv \langle (s_i - \bar{s}_i)(s_j - \bar{s}_j) \rangle :$$

$$\frac{d}{dt}\sigma_{ij} = 2D_{ij}(\bar{\chi}) - \sum_k [\nu_{ik}\sigma_{kj} + \nu_{jk}\sigma_{ki}]$$

Mobility coefficients $M_{ij}(\chi)$



$$c_{\rm ave}^2 = \tfrac{1}{2}c^2$$

$$a + b = R$$

Twisting $\quad (s_A^{\parallel}, s_B^{\parallel}) : \quad \boldsymbol{M}_{AB}^{\parallel} = \tfrac{1}{4}m\rho\bar{v}\,\pi c^2 \begin{pmatrix} c_{\rm ave}^2 & -c_{\rm ave}^2 \\ -c_{\rm ave}^2 & c_{\rm ave}^2 \end{pmatrix}$

Wriggling
Bending $\quad (s_{\rm wrig}, s_{\rm bend}) : \quad \boldsymbol{M}_{\pm} = \tfrac{1}{4}m\rho\bar{v}\,\pi c^2 \begin{pmatrix} R^2 & R\delta \\ R\delta & \delta^2 + c_{\rm ave}^2 \end{pmatrix}$

$$\delta = \frac{a\mathcal{I}_B - b\mathcal{I}_A}{\mathcal{I}_A + \mathcal{I}_B}$$

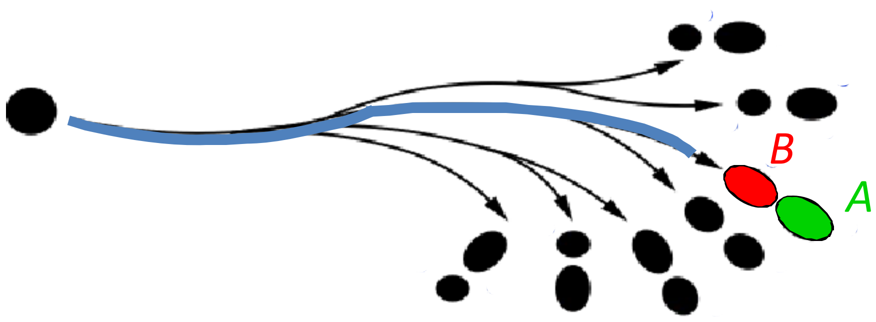
Individual spins $\quad (s_A^{\perp}, s_B^{\perp}) : \quad \boldsymbol{M}_{AB}^{\perp} = \tfrac{1}{4}m\rho\bar{v}\,\pi c^2 \begin{pmatrix} a^2 + c_{\rm ave}^2 & ab - c_{\rm ave}^2 \\ ab - c_{\rm ave}^2 & b^2 + c_{\rm ave}^2 \end{pmatrix}$

$$\boldsymbol{s}_A = \frac{\mathcal{I}_A^{\perp}}{\mathcal{I}_A^{\perp} + \mathcal{I}_B^{\perp}}\boldsymbol{s}_{\rm wrig} + \boldsymbol{s}_{\rm bend}$$

$$\boldsymbol{s}_B = \frac{\mathcal{I}_B^{\perp}}{\mathcal{I}_A^{\perp} + \mathcal{I}_B^{\perp}}\boldsymbol{s}_{\rm wrig} - \boldsymbol{s}_{\rm bend}$$

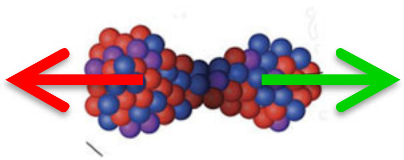
J. Randrup, Nucl. Phys. A **327** (1979) 490; **383** (1983) 468
T. Døssing and J. Randrup, Nucl. Phys. A **433** (1985) 215

Results for a single shape evolution



Results for a single shape evolution:

Spin components *along* the fission axis



Twisting

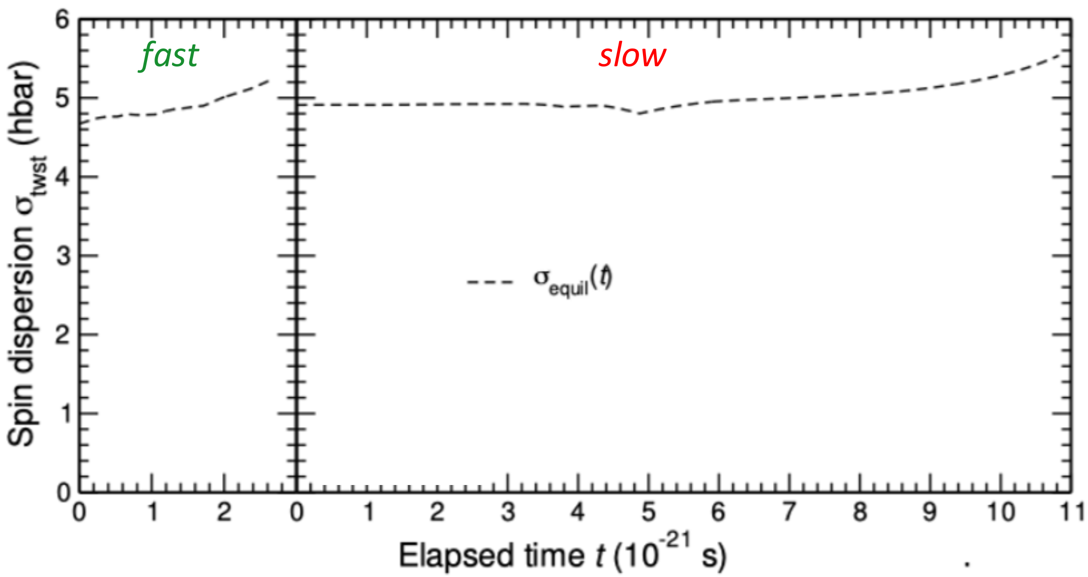
Moment of inertia: $\mathcal{I}_{\rm twst} = \frac{\mathcal{I}_A^{\|}\mathcal{I}_B^{\|}}{\mathcal{I}_A^{\|}+\mathcal{I}_B^{\|}}$

Equilibrium variance: $\tilde{\sigma}^2_{\rm twst} = \mathcal{I}_{\rm twst}T$

Individual fragments:

$s_A = s_{\rm twst}$

$s_B = -s_{\rm twst}$



$\sigma_A = \sigma_B = \sigma_{\rm twst}$

$\sigma_{AB} = -\sigma_{\rm twst}^2$

$$M_{\rm twst} = \tfrac{1}{4}m\rho\bar{v}\,\pi c^2 c^2_{\rm ave} \qquad \boldsymbol{M}^{\|}_{AB} = \tfrac{1}{4}m\rho\bar{v}\,\pi c^2 \begin{pmatrix} c^2_{\rm ave} & -c^2_{\rm ave} \\ -c^2_{\rm ave} & c^2_{\rm ave} \end{pmatrix}$$

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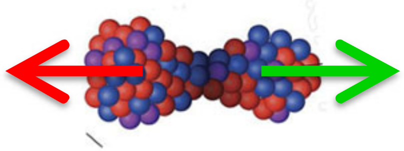

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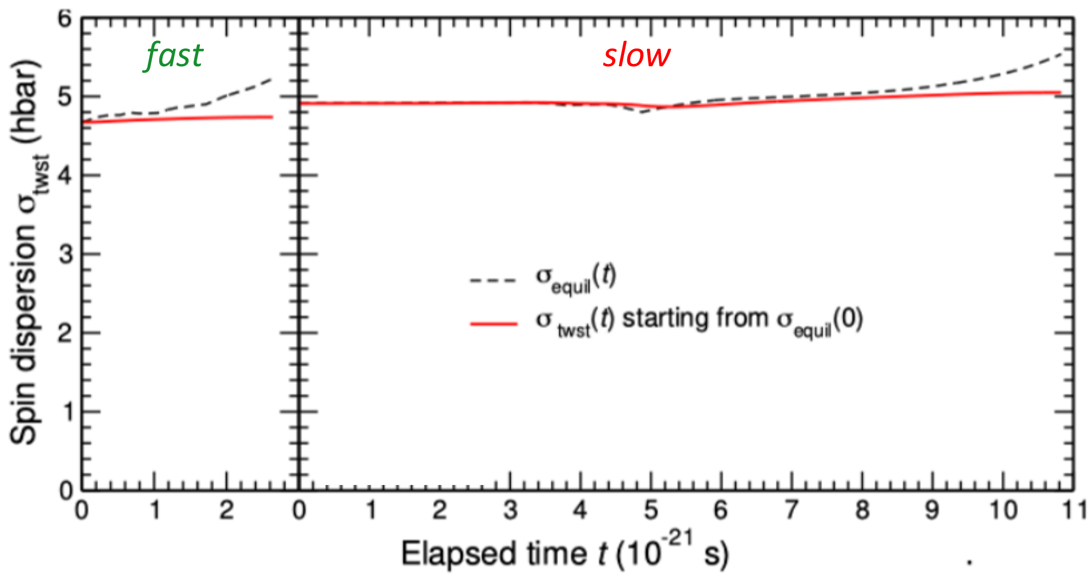

$\sigma_A = \sigma_B = \sigma_{\rm twst}$

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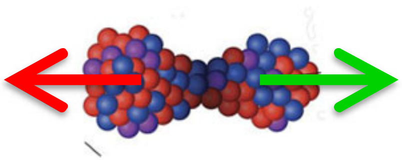
Spin components *along* the fission axis



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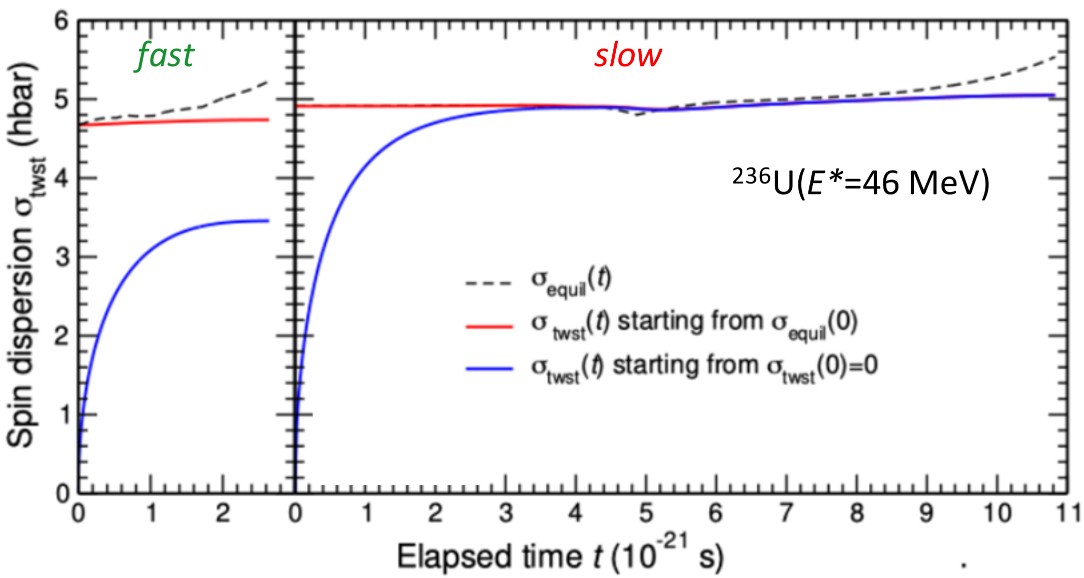

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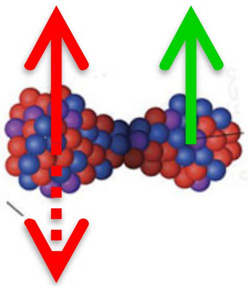


Results for a single shape evolution:

Spin components *perpendicular* to the fission axis

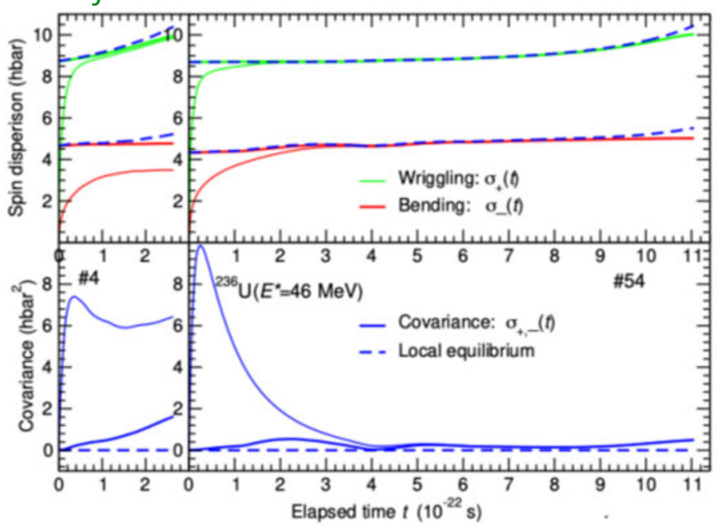
Wriggling & bending

$$\mathcal{I}_{\rm wrig} = \frac{(\mathcal{I}_A^\perp + \mathcal{I}_B^\perp)\mathcal{I}_R}{\mathcal{I}_A^\perp + \mathcal{I}_B^\perp + \mathcal{I}_R} \qquad \tilde{\sigma}^2_{\rm wrig} = \mathcal{I}_{\rm wrig} T$$

$$\tilde{\sigma}_{\rm wrig,bend} = 0$$

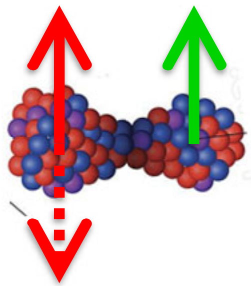
$$\mathcal{I}_{\rm bend} = \frac{\mathcal{I}_A^\perp \mathcal{I}_B^\perp}{\mathcal{I}_A^\perp + \mathcal{I}_B^\perp} \qquad \tilde{\sigma}^2_{\rm bend} = \mathcal{I}_{\rm bend} T$$

fast *slow*



$$\boldsymbol{M}_\pm = \tfrac{1}{4} m\rho\bar{v}\,\pi c^2 \begin{pmatrix} R^2 & R\delta \\ R\delta & \delta^2 + c^2_{\rm ave} \end{pmatrix}$$

$$\delta = (a\mathcal{I}_B - b\mathcal{I}_A)/(\mathcal{I}_A + \mathcal{I}_B)$$ small



Results for a single shape evolution:

Spin components *perpendicular* to the fission axis

Wriggling & bending

$$\mathcal{I}_{\rm wrig} = \frac{(\mathcal{I}_A^\perp + \mathcal{I}_B^\perp)\mathcal{I}_R}{\mathcal{I}_A^\perp + \mathcal{I}_B^\perp + \mathcal{I}_R}$$

$$\mathcal{I}_{\rm bend} = \frac{\mathcal{I}_A^\perp \mathcal{I}_B^\perp}{\mathcal{I}_A^\perp + \mathcal{I}_B^\perp}$$

$$\tilde{\sigma}^2_{\rm wrig} = \mathcal{I}_{\rm wrig} T$$

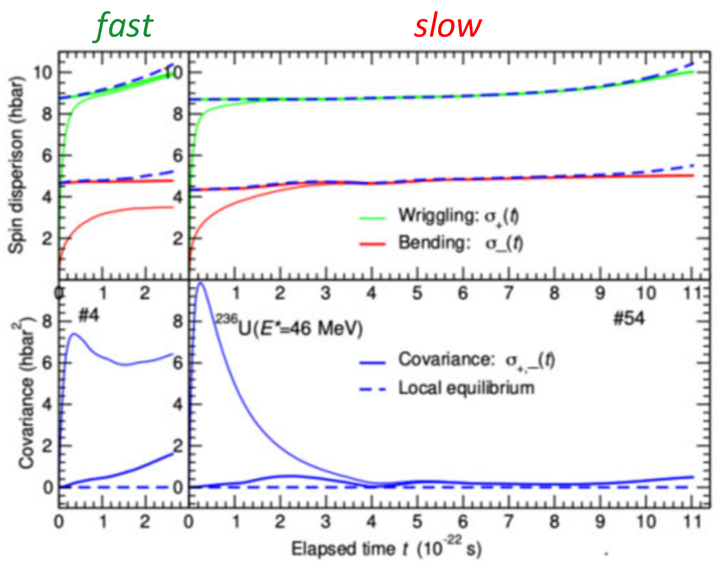
$$\tilde{\sigma}_{\rm wrig,bend} = 0$$

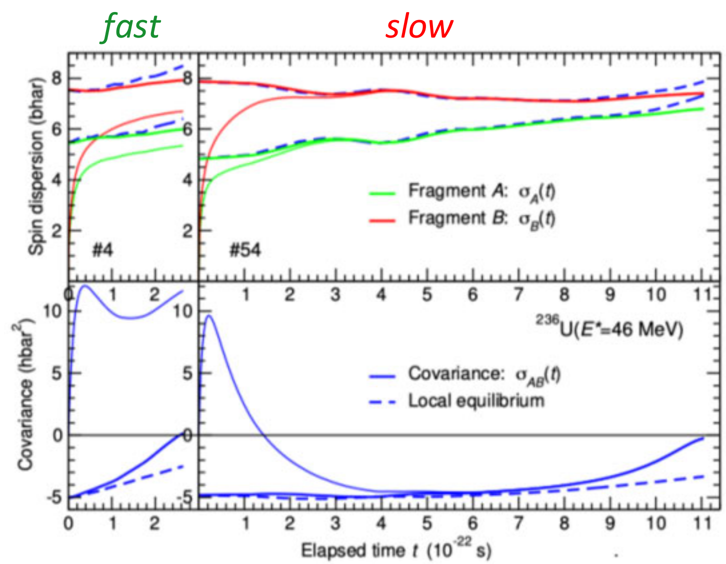
$$\tilde{\sigma}^2_{\rm bend} = \mathcal{I}_{\rm bend} T$$

Individual fragments:

$$\boldsymbol{s}_A = \frac{\mathcal{I}_A^\perp}{\mathcal{I}_A^\perp + \mathcal{I}_B^\perp}\boldsymbol{s}_{\rm wrig} + \boldsymbol{s}_{\rm bend}$$

$$\boldsymbol{s}_B = \frac{\mathcal{I}_B^\perp}{\mathcal{I}_A^\perp + \mathcal{I}_B^\perp}\boldsymbol{s}_{\rm wrig} - \boldsymbol{s}_{\rm bend}$$



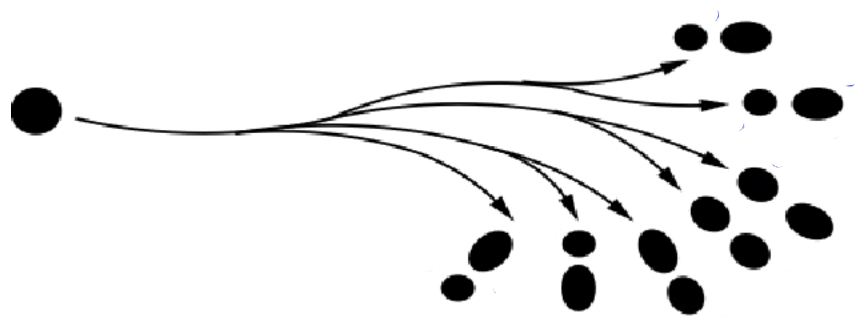


$$\boldsymbol{M}_{\pm} = \tfrac{1}{4} m\rho\bar{v}\,\pi c^2 \begin{pmatrix} R^2 & R\delta \\ R\delta & \delta^2 + c_{\rm ave}^2 \end{pmatrix}$$

$$\boldsymbol{M}_{AB}^{\perp} = \tfrac{1}{4} m\rho\bar{v}\,\pi c^2 \begin{pmatrix} a^2 + c_{\rm ave}^2 & ab - c_{\rm ave}^2 \\ ab - c_{\rm ave}^2 & b^2 + c_{\rm ave}^2 \end{pmatrix}$$

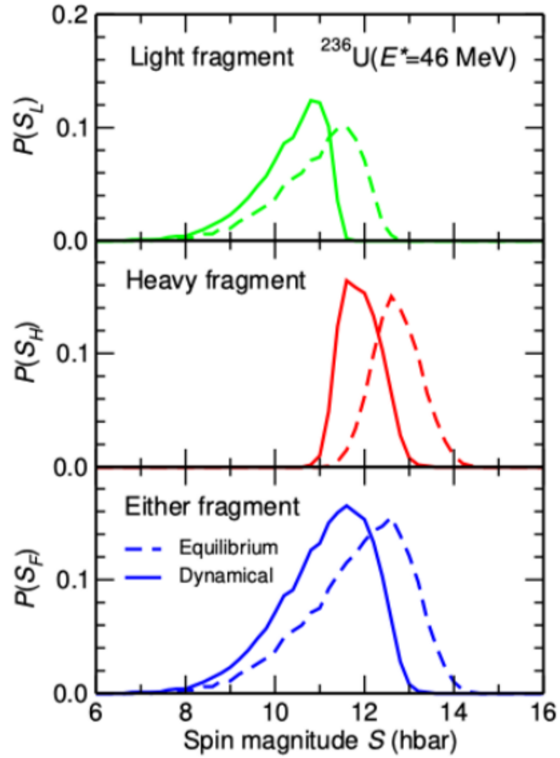
$\delta = (a\mathcal{I}_B - b\mathcal{I}_A)/(\mathcal{I}_A + \mathcal{I}_B)$ small

Results for an ensemble of 10^4 *evolutions*



Results for an ensemble of evolutions:

Fragment spin magnitude

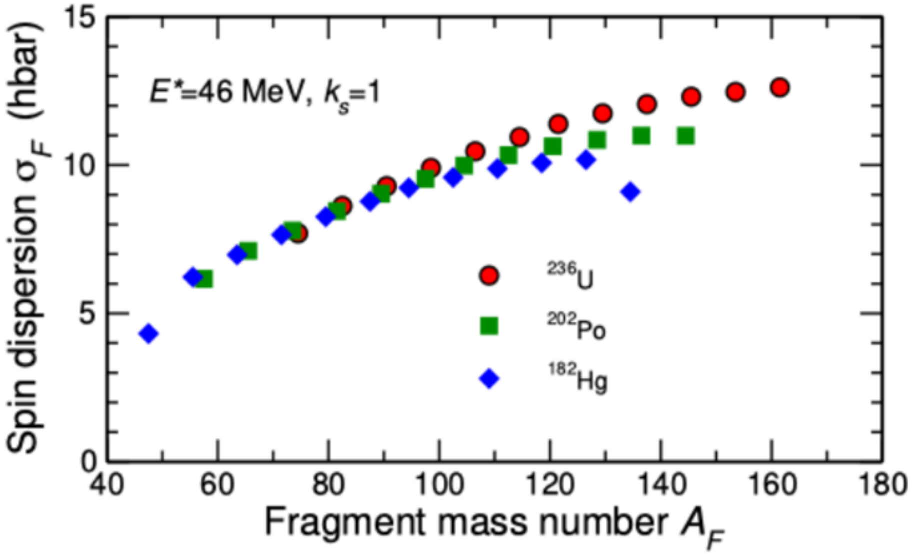


The fragment spins relax quickly to the local equilibrium values throughout most of the evolution from saddle towards scission

But near scission the spin equilibrium magnitudes grow rapidly (due to the increasing temperature), while the mobility coefficients decease (because the neck closes), so the dynamical spin evolution effectively freezes out before scission, leading to smaller magnitudes

Results for an ensemble of evolutions:

Mass dependence of spin magnitude

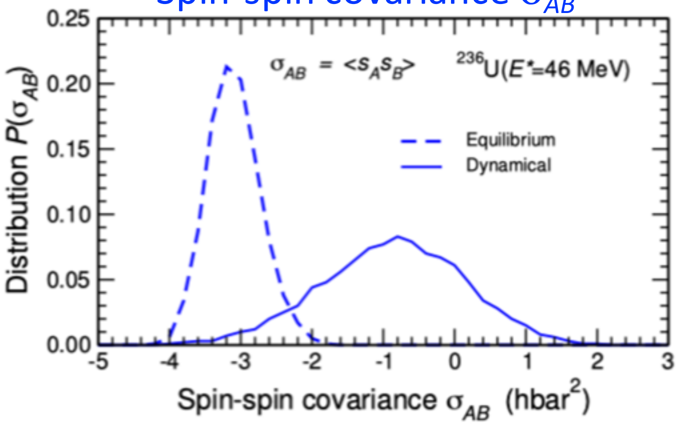


Gating on fragment mass may be informative *Can be measured!*

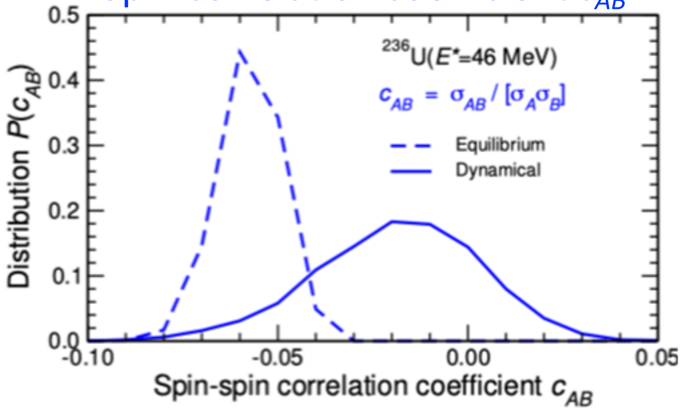
Results for an ensemble of evolutions:

Fragment spin correlation

Spin-spin covariance σ_{AB}



Spin correlation coefficient c_{AB}



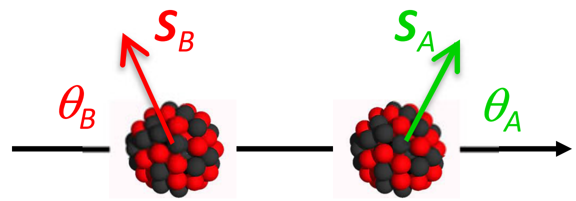
The spin-spin correlation is quite small in equilibrium
and it is even smaller when generated dynamically
=> The fragment spins are practically uncorrelated ←

J. Wilson *et al.*, Nature **590** (2021)

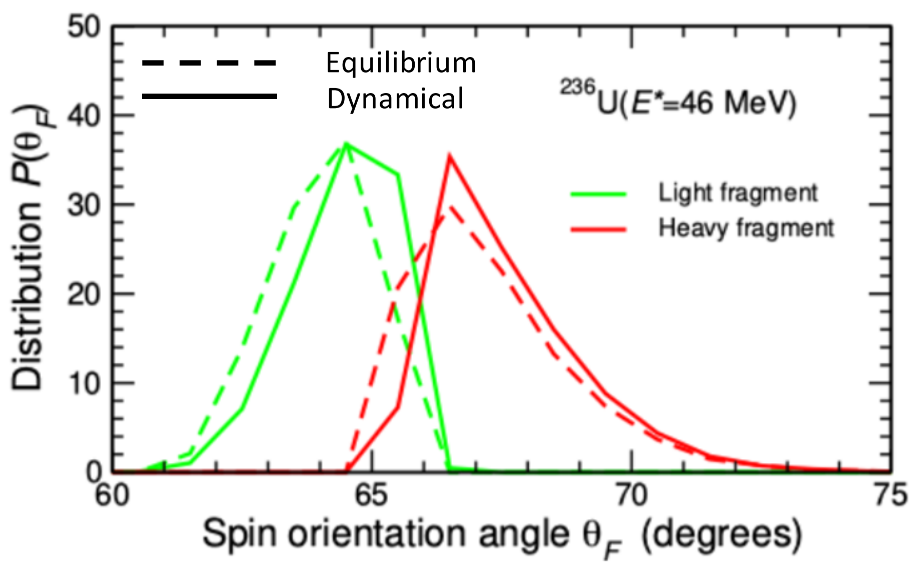
Fragment spin orientation

J.B. Wlhelmy *et al.*,
Phys. Rev. **C 5**, 2041 (1972):

The fragment spins are *approximately perpendicular* to the fission direction



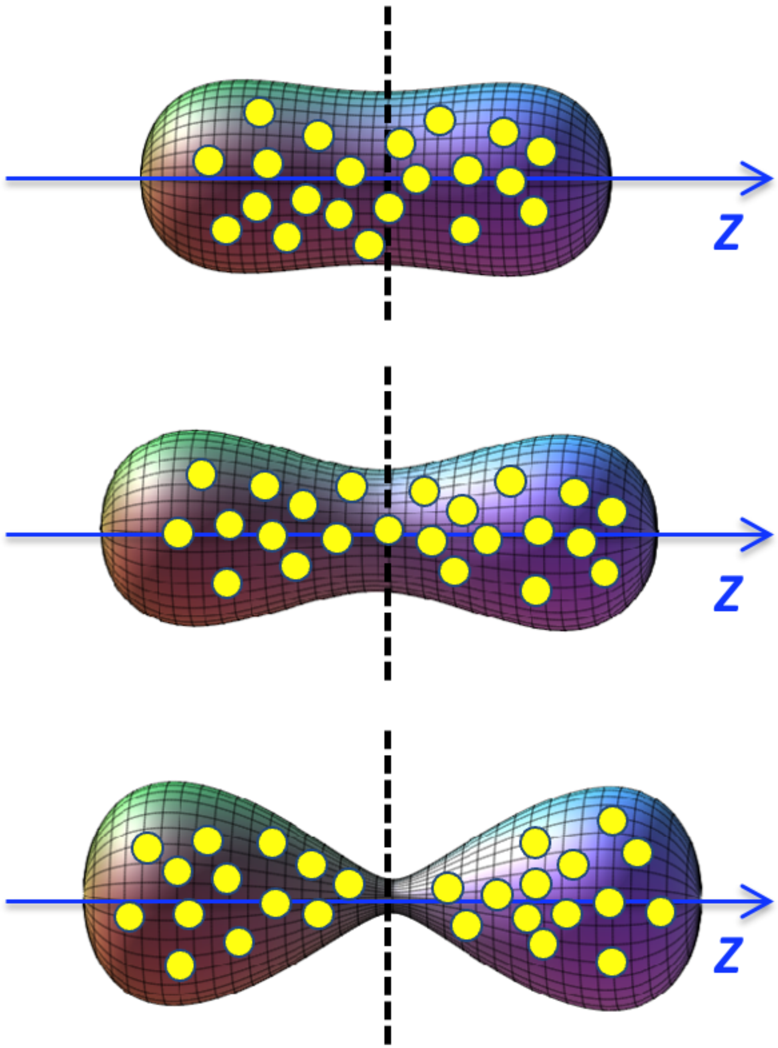
$$\cot^2\theta_F = \frac{(S_F^{\parallel})^2}{(S_F^{\perp})^2} \Rightarrow \cot^2\theta_F^{(k)} \approx \frac{(\sigma_{\parallel}^{(k)})^2}{(\sigma_{\perp}^{(k)})^2}$$



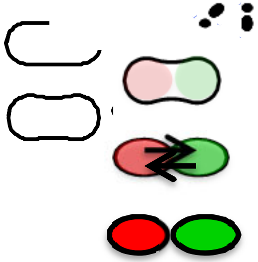
Should be measured ->

While the spin orientation angle θ_F is large, it is significantly smaller than 90°, both in equilibrium and dynamically

Effect of nucleon exchange on fission fragment angular momenta



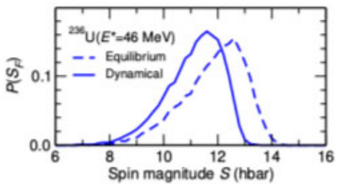
Effect of nucleon exchange on fission fragment angular momenta



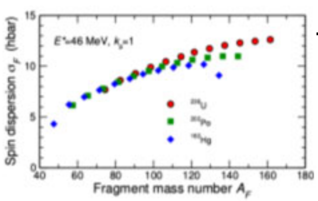
Generate ensembles of shape evolutions by Langevin simulation 10^4

Obtain the correlated fragment spin evolution $P^{(k)}(\boldsymbol{S}_A, \boldsymbol{S}_B;t)$
In each from the *Nucleon Exchange* transport model: $\boldsymbol{M}_{AB}(t)$

Wrigling	fast
Bending	slow
Twisting	slower

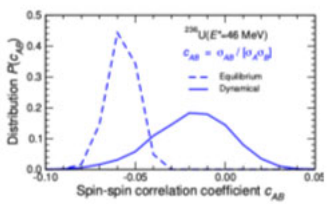


The fragment spins relax to the local equilibrium values throughout most of the evolution from saddle towards scission



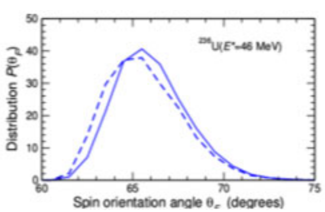
But near scission the spin equilibrium magnitudes grow rapidly (due to the increasing temperature), while the mobility coefficients decrease (because the neck closes), so the dynamical spin evolution effectively freezes out before scission, resulting in smaller magnitudes

Gating on fragment mass may be informative



The spin-spin correlation is quite small in equilibrium and it is even smaller when generated dynamically
=> The fragment spins are practically uncorrelated

J. Wilson *et al.*, Nature **590** (2021)



The spin orientation angle θ_F is about 70°, both in equilibrium and dynamically

Should be measured!

Workshop on Fission Dynamics

11-15 May 2026 Chongqing - China

Angular Momentum Dynamics in Fission

Effect of nucleon exchange on fission fragment angular momenta

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Thank You!