Nuclear Fission Dynamics within the Langevin Framework

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Workshop on Fission Dynamics 2026
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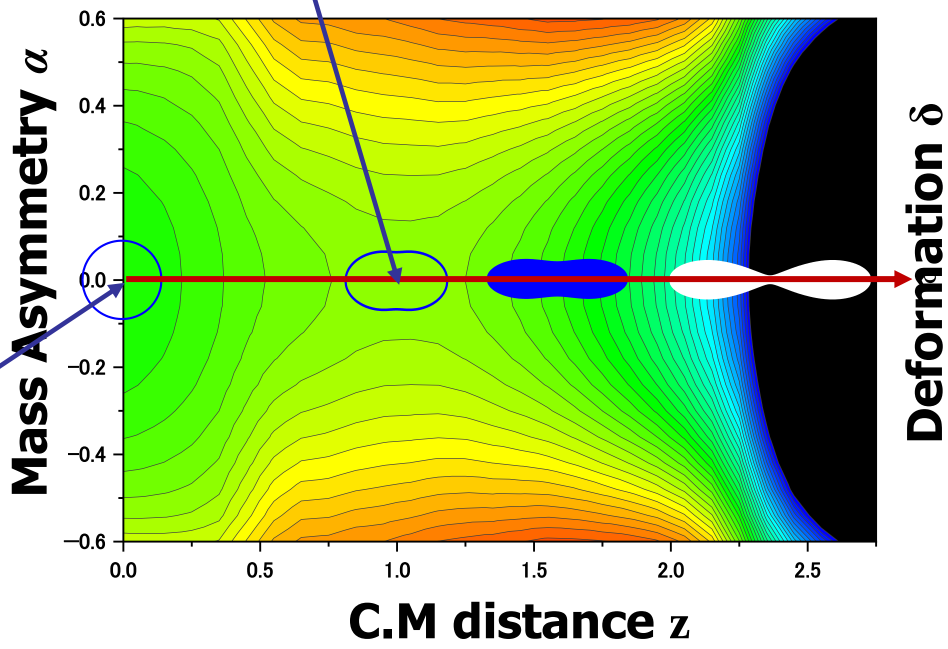
Chongqing Chaotianmen Voco Hotel
Chongqing, China

We would like to start this talk with a question

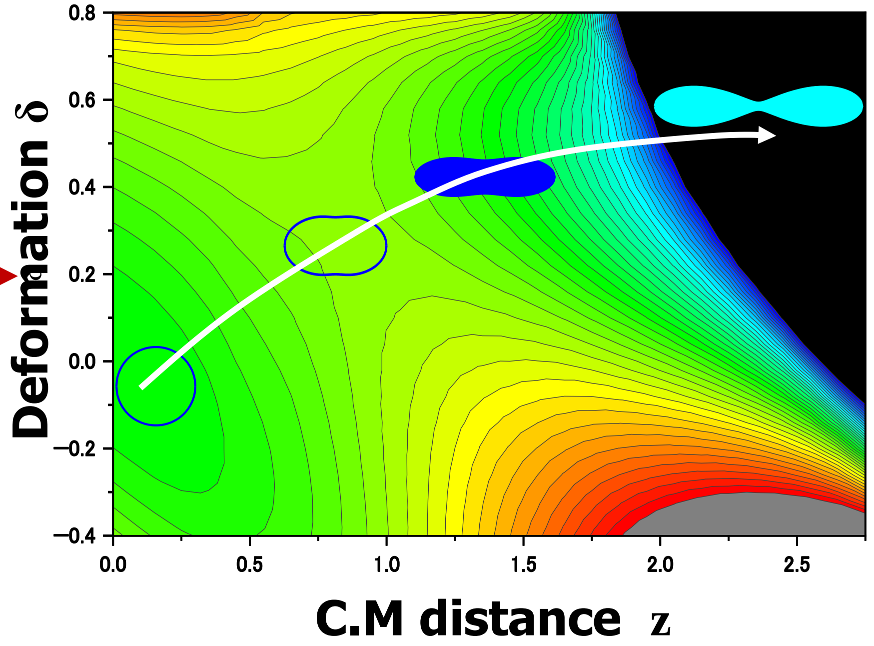
Fission Path on LDM potential energy ^{236}U

Are we sure this is correct?

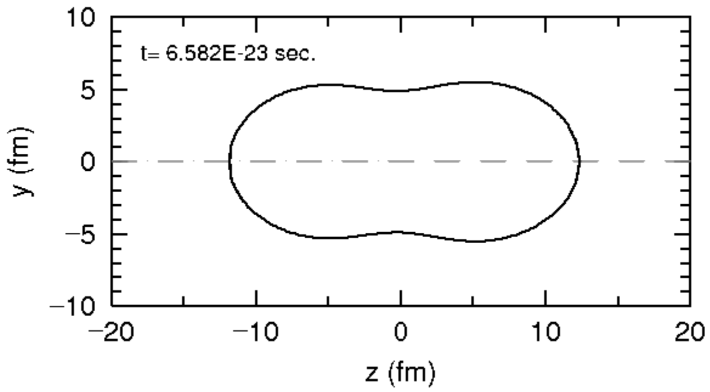
Saddle point



Ground state



Draw fission path….
In general,



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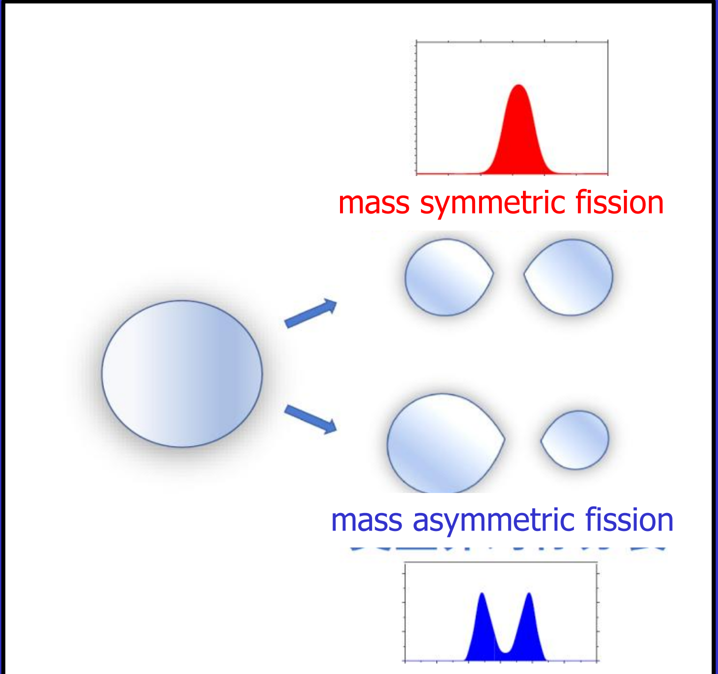
    Microscopic and Macroscopic properties

2. Model
    Dynamical model
        Two-center shell model + Langevin equation
        Analysis by trajectory calculations

3. Fission processes
    Role of transport coefficients

4. Summary



Mass distribution of fission fragments

Nuclear Physics　Theoretical research interests

Microscopic feature

Progress
Develop

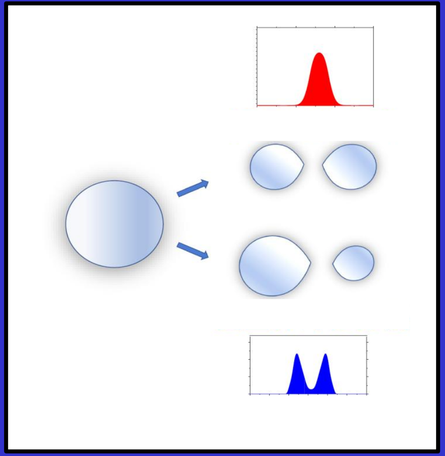
As discussed at this workshop until yesterday

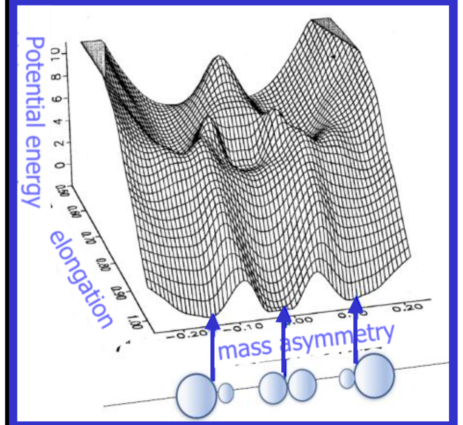
Mass-asymmetric fission

Shell structure

One of the features that appear in finite quantum many-body systems

Quantum Mechanics





Macroscopic feature

Nuclear Physics　Theoretical research interests

Microscopic feature

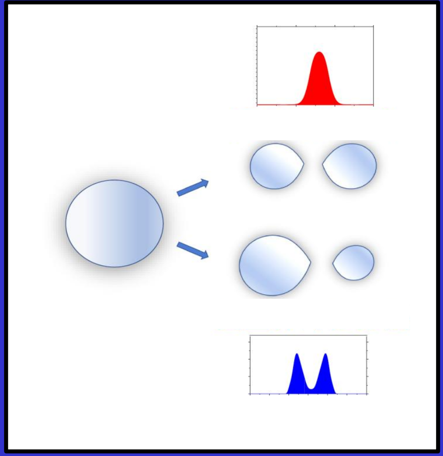
As discussed at this workshop until yesterday

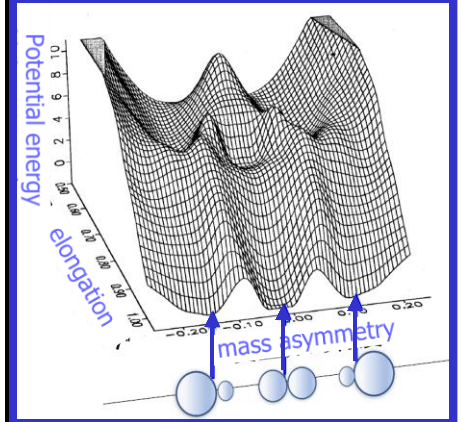
Mass-asymmetric fission

Shell structure

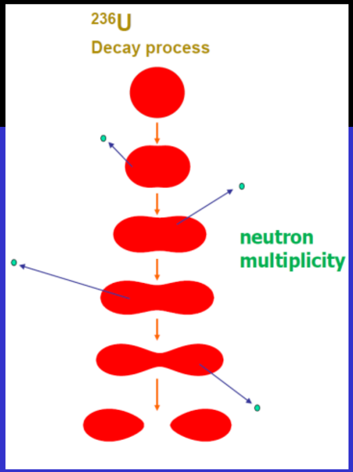
One of the features that appear in finite quantum many-body systems

Quantum Mechanics





Macroscopic feature



Fission process

time scale

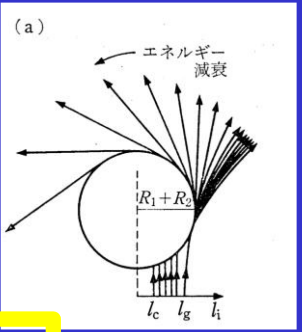
viscosity → friction coefficient

Easiness of deformation→ inertia mass

Fusion process

Scattering angle of grazing collision

Dissipation of kinetic energy



Liquid Drop Model

Classical Mechanics

Phenomenological model

Advantage of using Macroscopic model

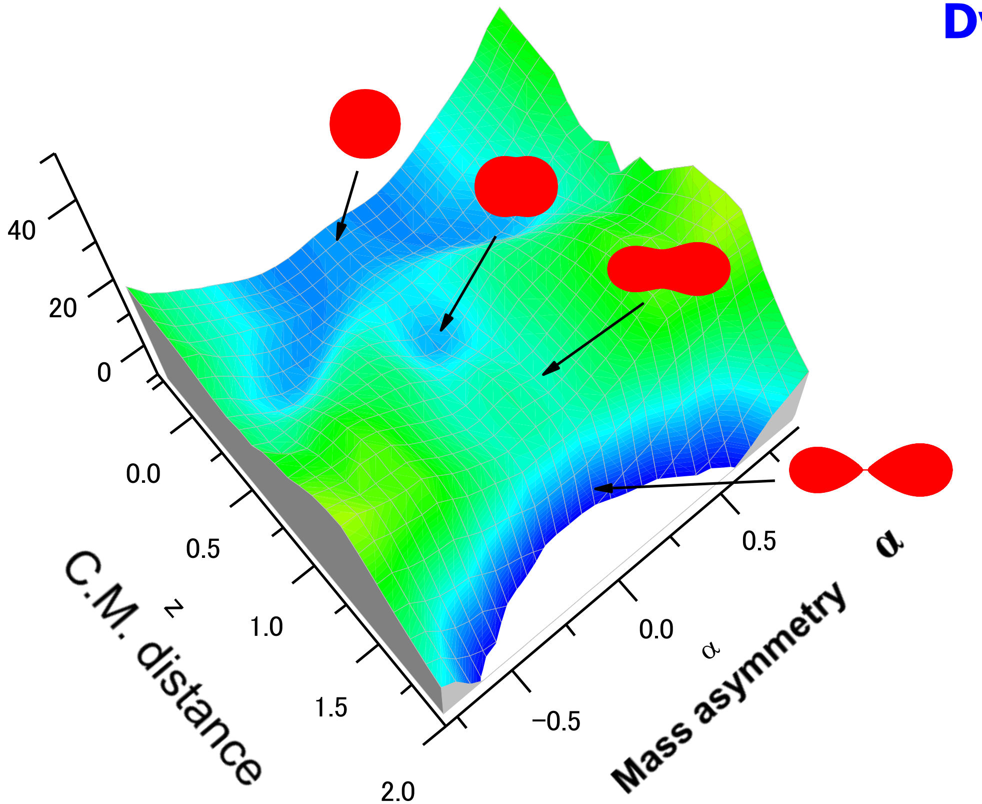
Model

Dynamical Model

Two-center shell model

Langevin equation

Fission process ^{236}U E* < 20 MeV



Dynamical calculation

Time-evolution of nuclear shape in fission process

Two Items

1. Potential energy surface

2. Trajectory ← described by Equation of Motion

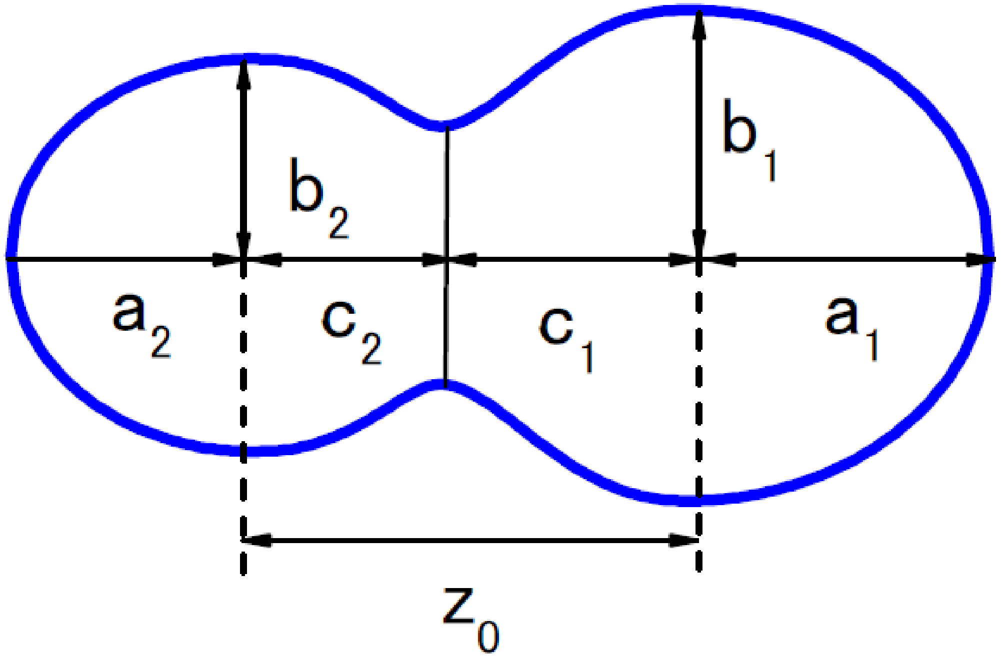
Trajectory on potential energy surface

Nuclear Shape

two-center parametrization (z,δ,α)

(Maruhn and Greiner,
Z. Phys. 251(1972) 431)

$$q(z,\delta,\alpha)$$



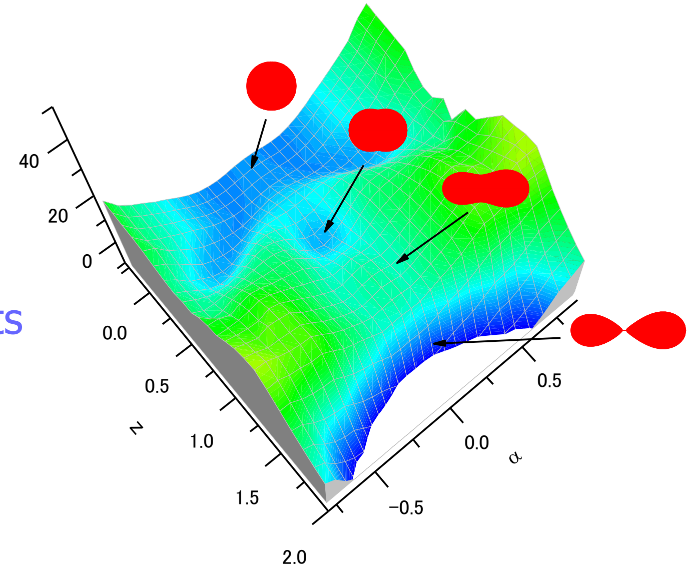
$z = \frac{z_0}{BR}$ Distance between center of mass

$B = \frac{3+\delta}{3-2\delta}$

R: Radius of the spherical compound nucleus

$\delta = \frac{3(a-b)}{2a+b}$ (δ1=δ2) Deformation of fragments

$\alpha = \frac{A_1 - A_2}{A_{CN}}$ Mass asymmetry

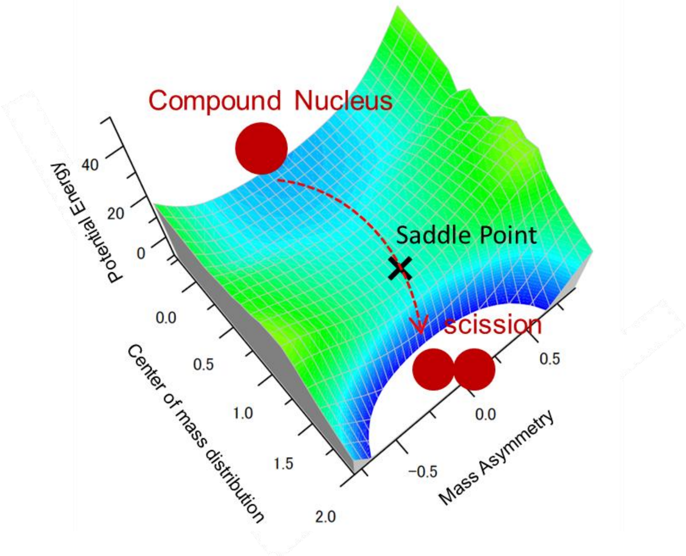


Potential energy

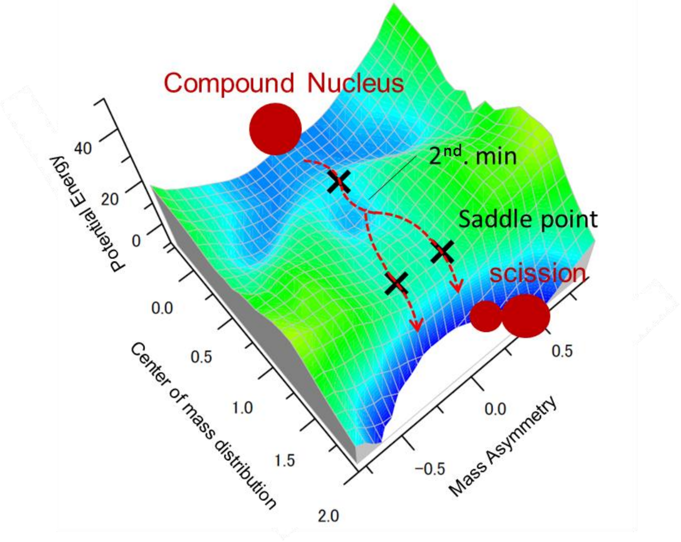
$$V(q,T) = V_{LD}(q) + V_{SH}(q,T)$$

Microscopic model part

Liquid Drop Model



Liquid Drop Model
+ Shell correction energy



Multi-dimensional Langevin Equation

$$\frac{dq_i}{dt} = \left(m^{-1}\right)_{ij} p_j$$

$$\frac{dp_i}{dt} = -\frac{\partial V}{\partial q_i} - \frac{1}{2}\frac{\partial}{\partial q_i}\left(m^{-1}\right)_{jk} p_j p_k - \gamma_{ij}\left(m^{-1}\right)_{jk} p_k + g_{ij} R_j(t)$$

Friction dissipation — Random force fluctuation

Newton equation

ordinary differential equation

$$\langle R_i(t)\rangle = 0,\ \langle R_i(t_1) R_j(t_2)\rangle = 2\delta_{ij}\delta(t_1 - t_2) : \text{white noise (Markovian process)}$$

$$\sum_k g_{ik} g_{jk} = T\gamma_{ij}$$

Einstein relation — Fluctuation-dissipation theorem

q_i : deformation coordinate (z, δ, α) (nuclear shape)

two-center parametrization (Maruhn and Greiner, Z. Phys. 251(1972) 431)

p_i : momentum

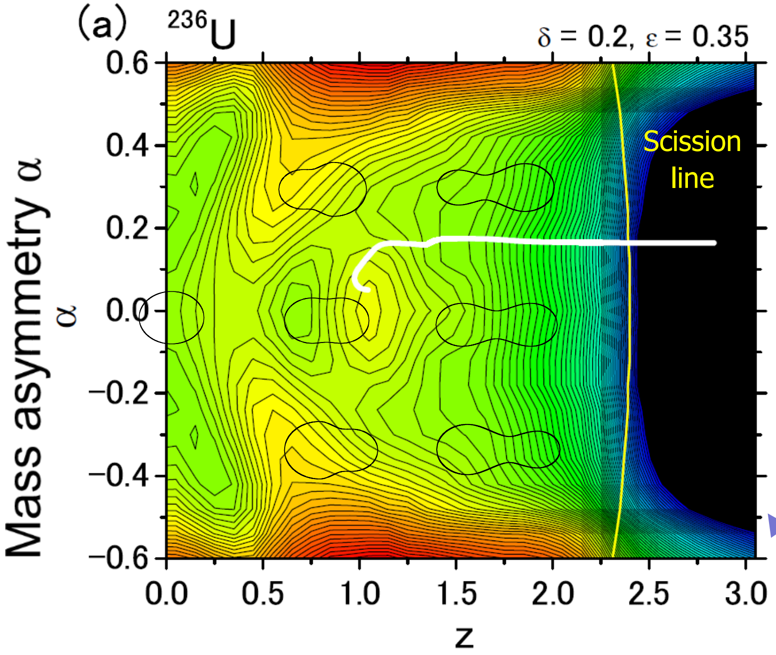
m_{ij} : Hydrodynamical mass (inertia mass)

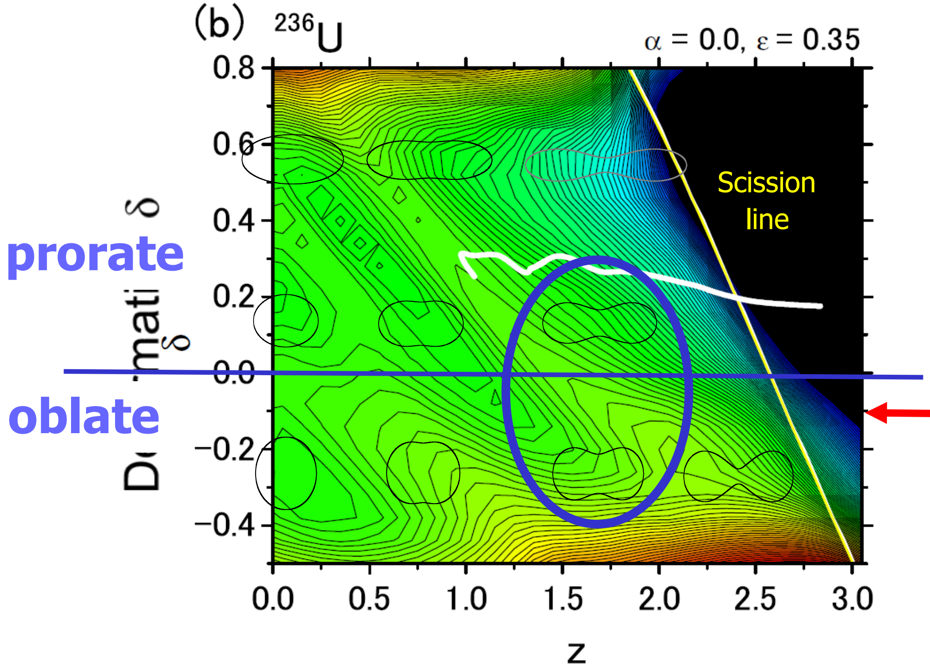
γ_{ij}: Wall and Window (one-body) dissipation (friction)

$$E_{\text{int}} = E^* - \frac{1}{2}\left(m^{-1}\right)_{ij} p_i p_j - V(q)$$

E_{int} : intrinsic energy, E^* : excitation energy

Without fluctuation





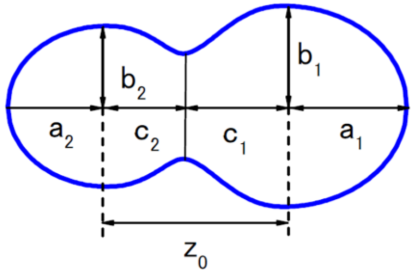
$$z = \frac{z_0}{BR}$$

$$B = \frac{3+\delta}{3-2\delta}$$

R: Radius of the spherical compound nucleus

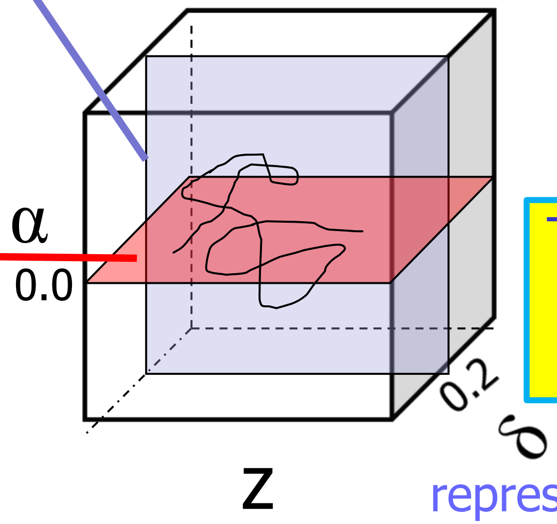
$$\delta = \frac{3(a-b)}{2a+b}$$

$$\alpha = \frac{A_1 - A_2}{A_{CN}}$$



Projection on two-dim. plane

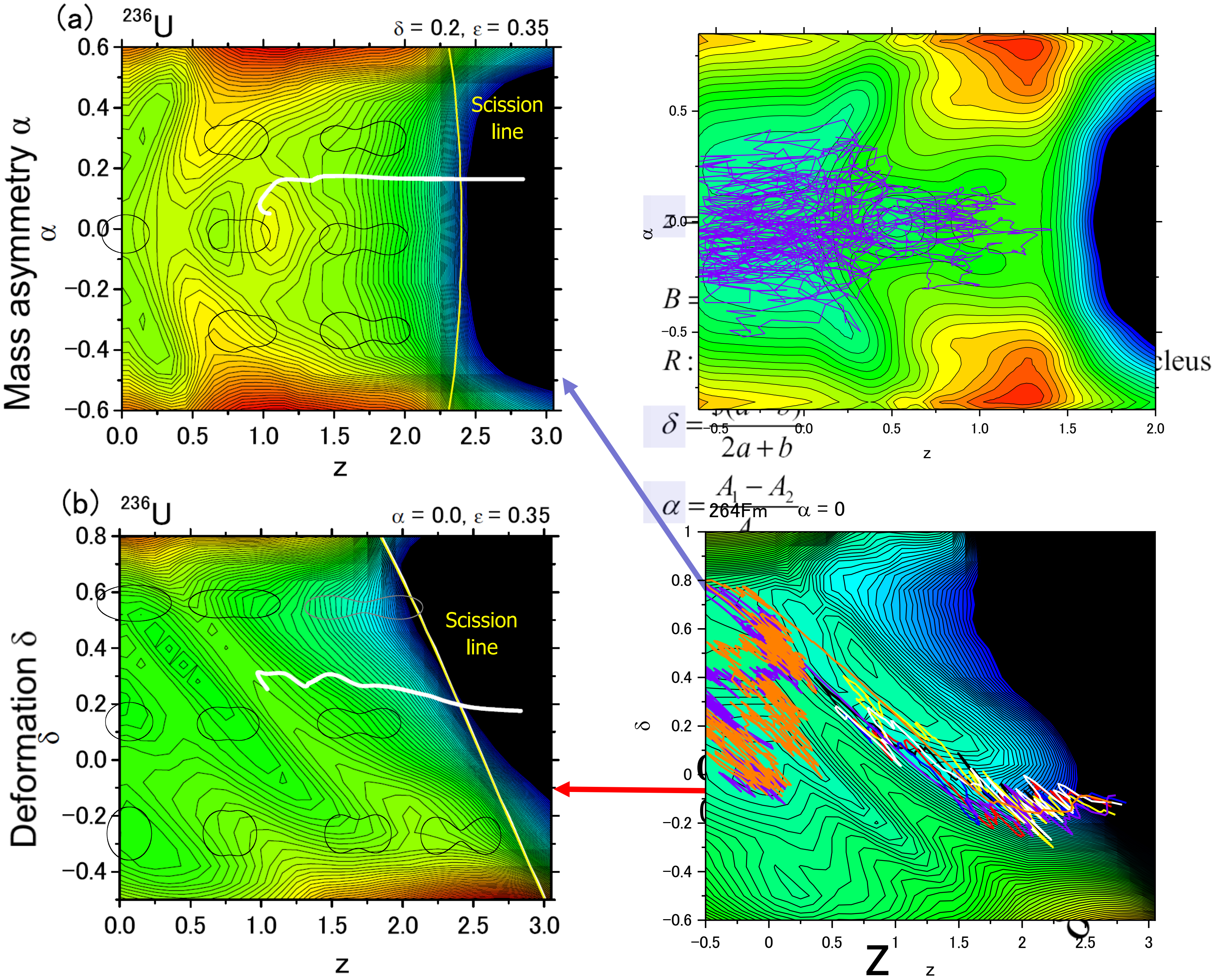
3D coordinate space



To perform trajectory analysis

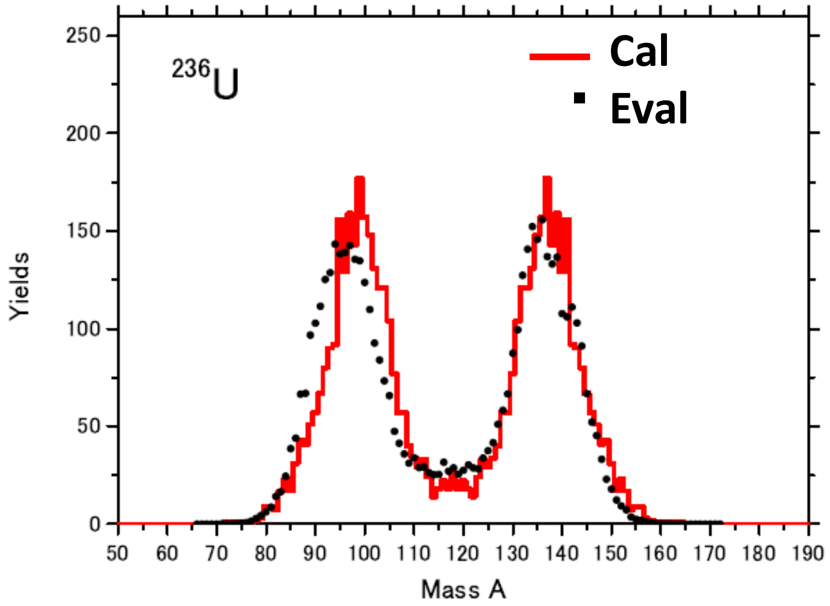
representative point

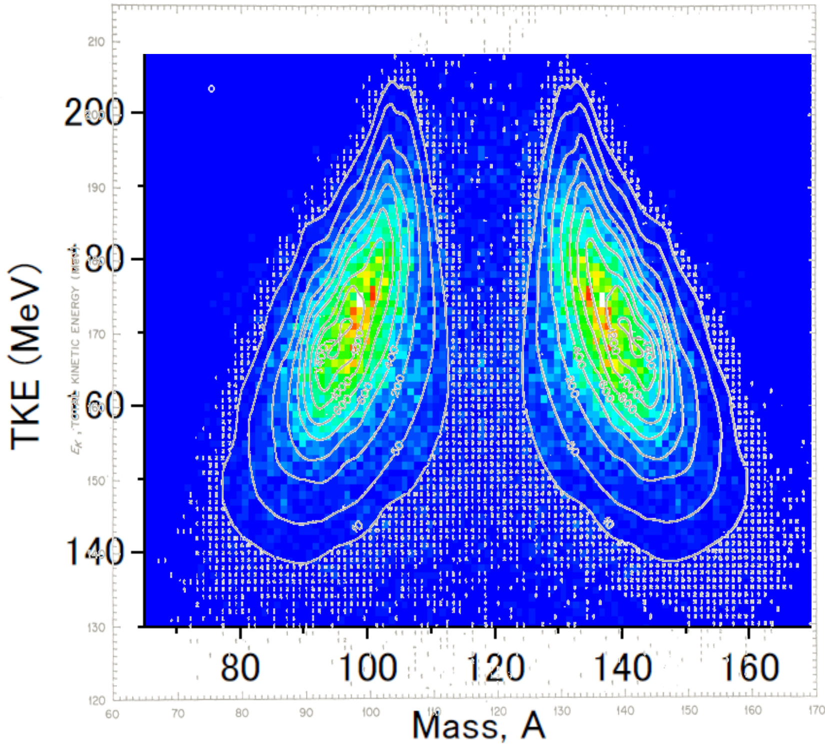
Without fluctuation



Projection on two-dim. plane

$^{236}U^*$ (E^* = 20MeV)





evaluation value	J.Katakura, JENDL FP Decay Data File 2011 and Fission Yields
calculation	Y. Aritomo and S. Chiba, PRC 88, 044614(2013)

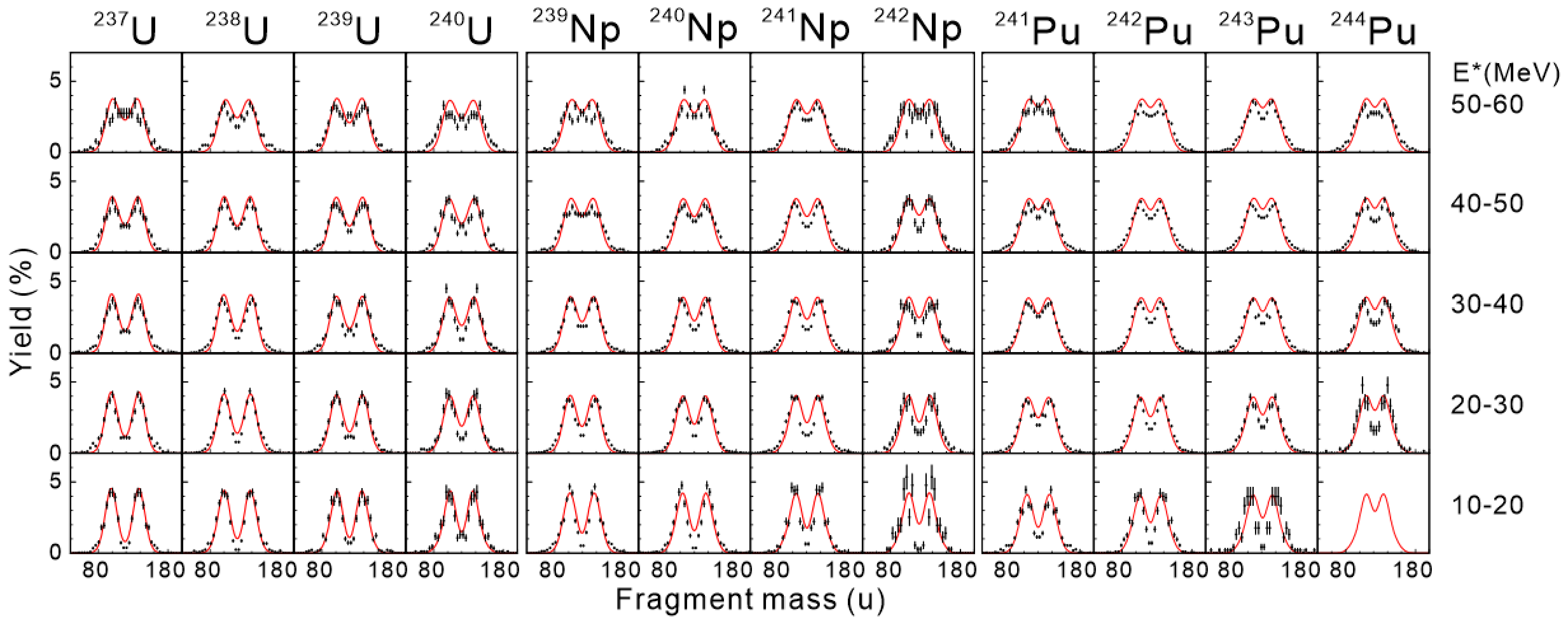
Exp. Phys.Rev. **141**(1966)1146

$\langle TKE\rangle_{cal}$=171.8 MeV

$\langle TKE\rangle_{exp}$=168.2~171.3 MeV (E*=21 MeV)

As Systematic calculation,

Langevin Calculation (Multi-chance Fission)



With
Multi-chance Fission

Calculated by S. Tanaka (Riken)

K. Hirose, K. Nishio, S. Tanaka, et al Phys. Rev. Lett. 119, 222501 (2017)

S. Tanaka, et al.
Phys. Rev. C 100, 064605 (2019)

Research Trigger

Good agreement with
the experimental data

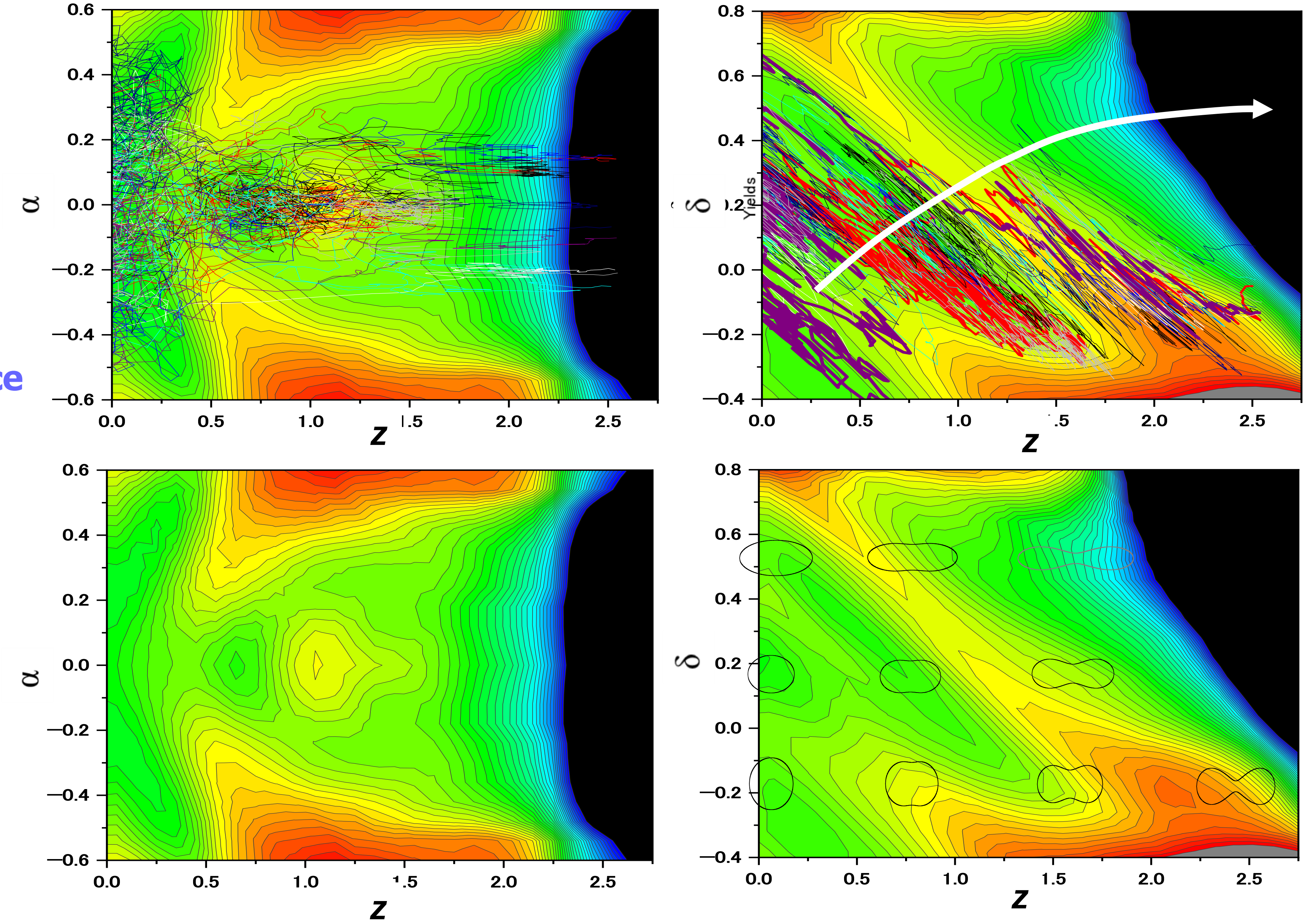
We obtain calculation results,
Based on the trajectory behavior

Go back to the fission path

^{236}U

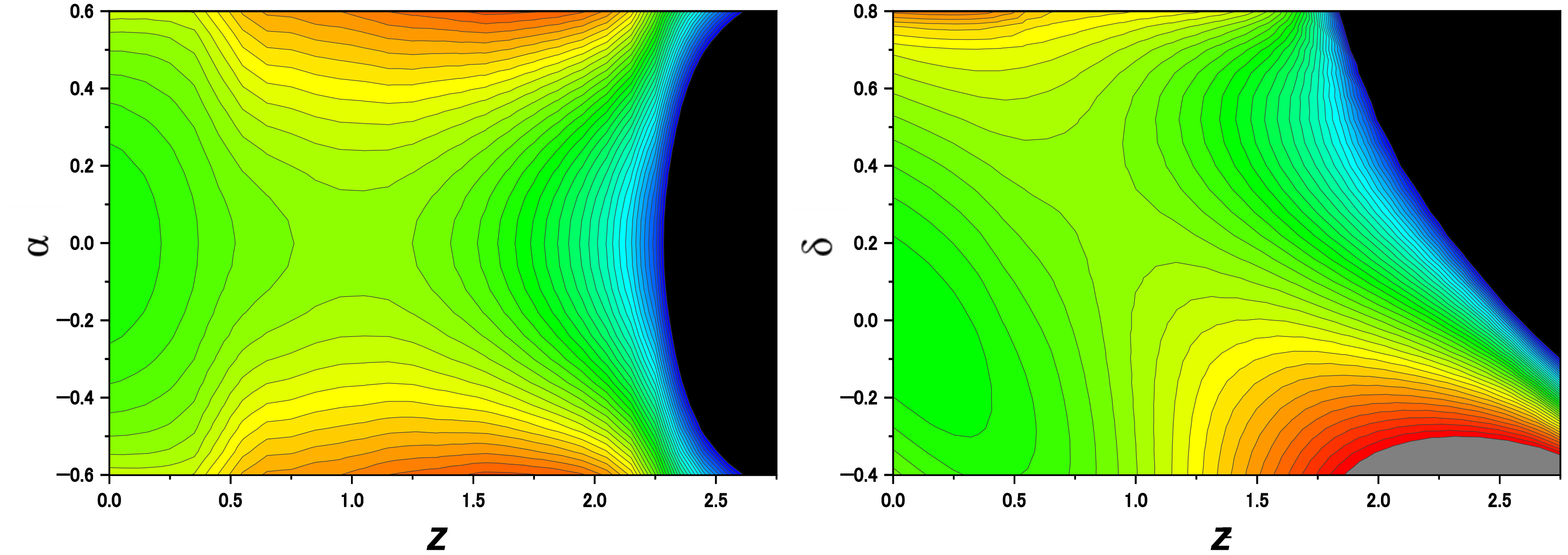
LDM+shell

Potential energy surface

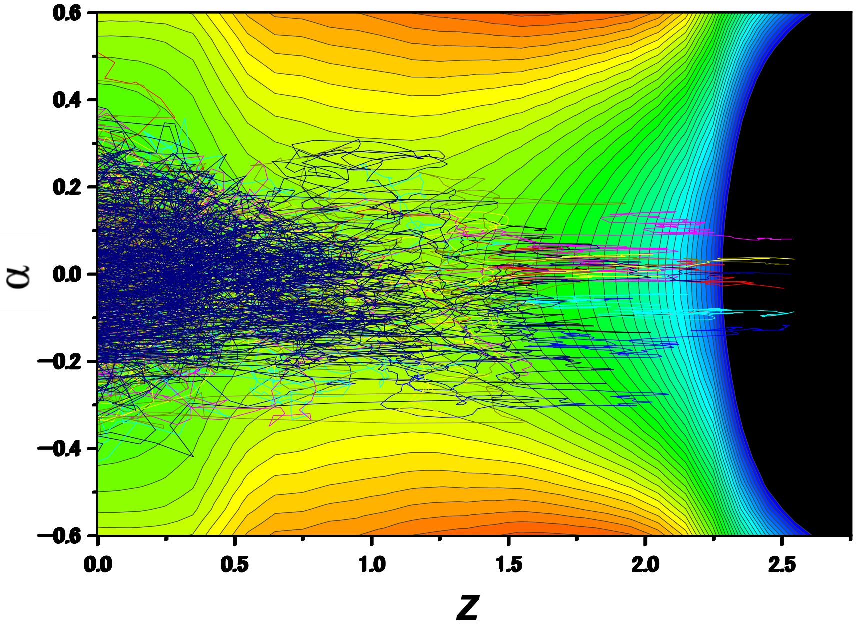


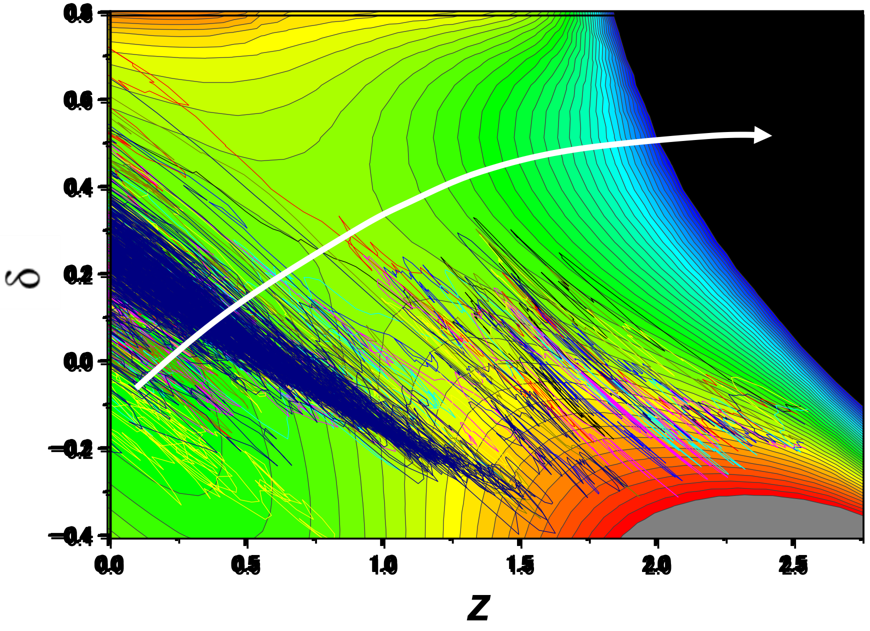
Such trajectory behavior is not observed

What happens with the LDM potential? ^{236}U



What happens with the LDM potential? ^{236}U





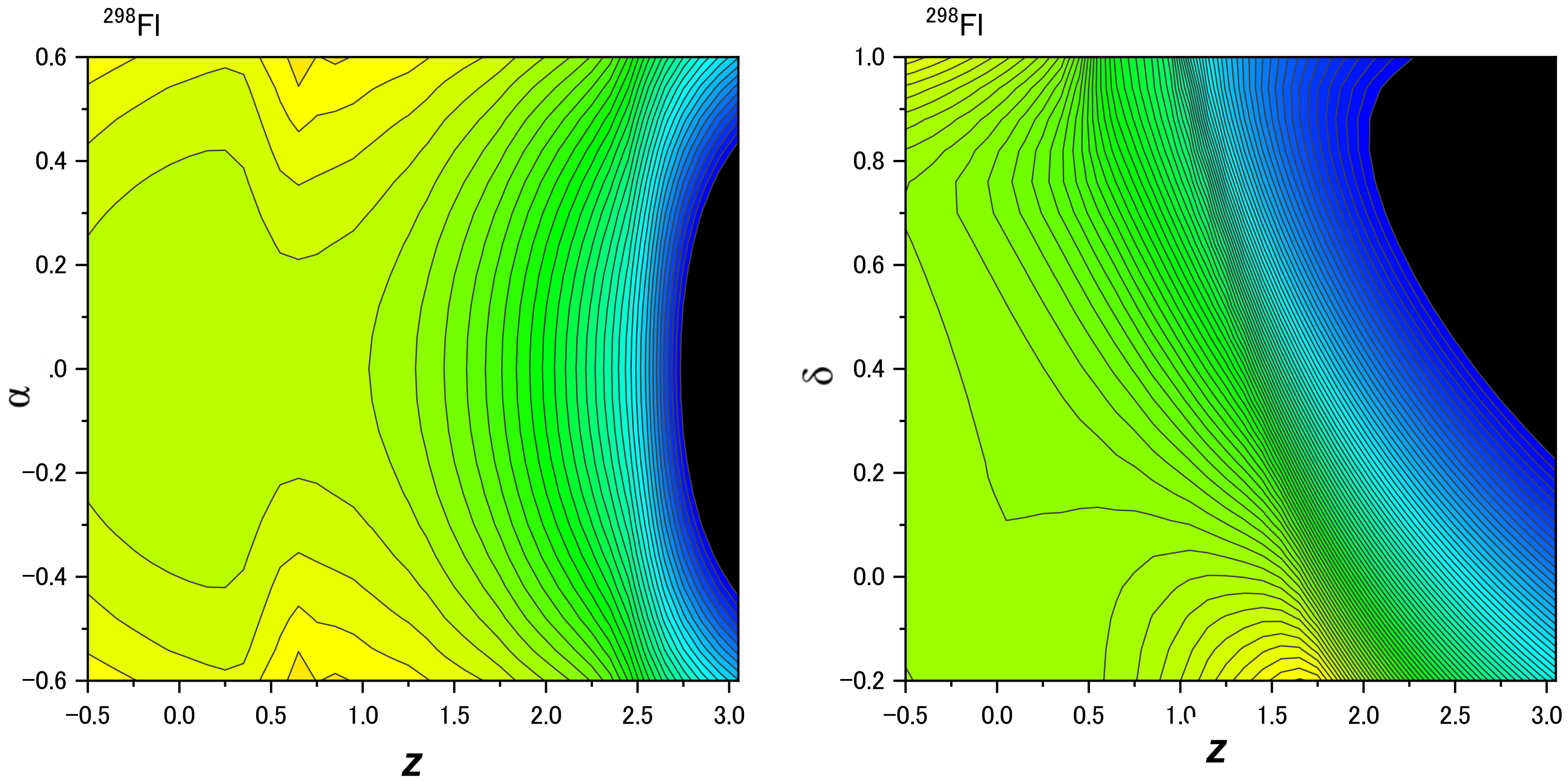
verification

Does the trajectory fluctuate in a particular direction?

Test on a flat surface
← unaffected by potential

^{298}Fl

V_{DM} : Liquid Drop Model



Transport coefficient

Important physical quantities in trajectory analysis

Conditions

① Equation

$$\frac{dq_i}{dt} = \left(m^{-1}\right)_{ij} p_j$$

$$\frac{dp_i}{dt} = -\frac{\partial V}{\partial q_i} - \frac{1}{2}\frac{\partial}{\partial q_i}\left(m^{-1}\right)_{jk} p_j p_k - \gamma_{ij}\left(m^{-1}\right)_{jk} p_k + g_{ij} R_j(t)$$

② Transport coefficients ← Change the appearance of the motion

Includes non-diagonal components (normal calculation case)

$$\begin{bmatrix} \gamma_{zz} & \gamma_{z\delta} & \gamma_{z\alpha} \\ \gamma_{\delta z} & \gamma_{\delta\delta} & \gamma_{\delta\alpha} \\ \gamma_{\alpha z} & \gamma_{\alpha\delta} & \gamma_{\alpha\alpha} \end{bmatrix} \begin{bmatrix} m_{zz} & m_{z\delta} & m_{z\alpha} \\ m_{\delta z} & m_{\delta\delta} & m_{\delta\alpha} \\ m_{\alpha z} & m_{\alpha\delta} & m_{\alpha\alpha} \end{bmatrix}$$

Without non-diagonal component (test calculation)

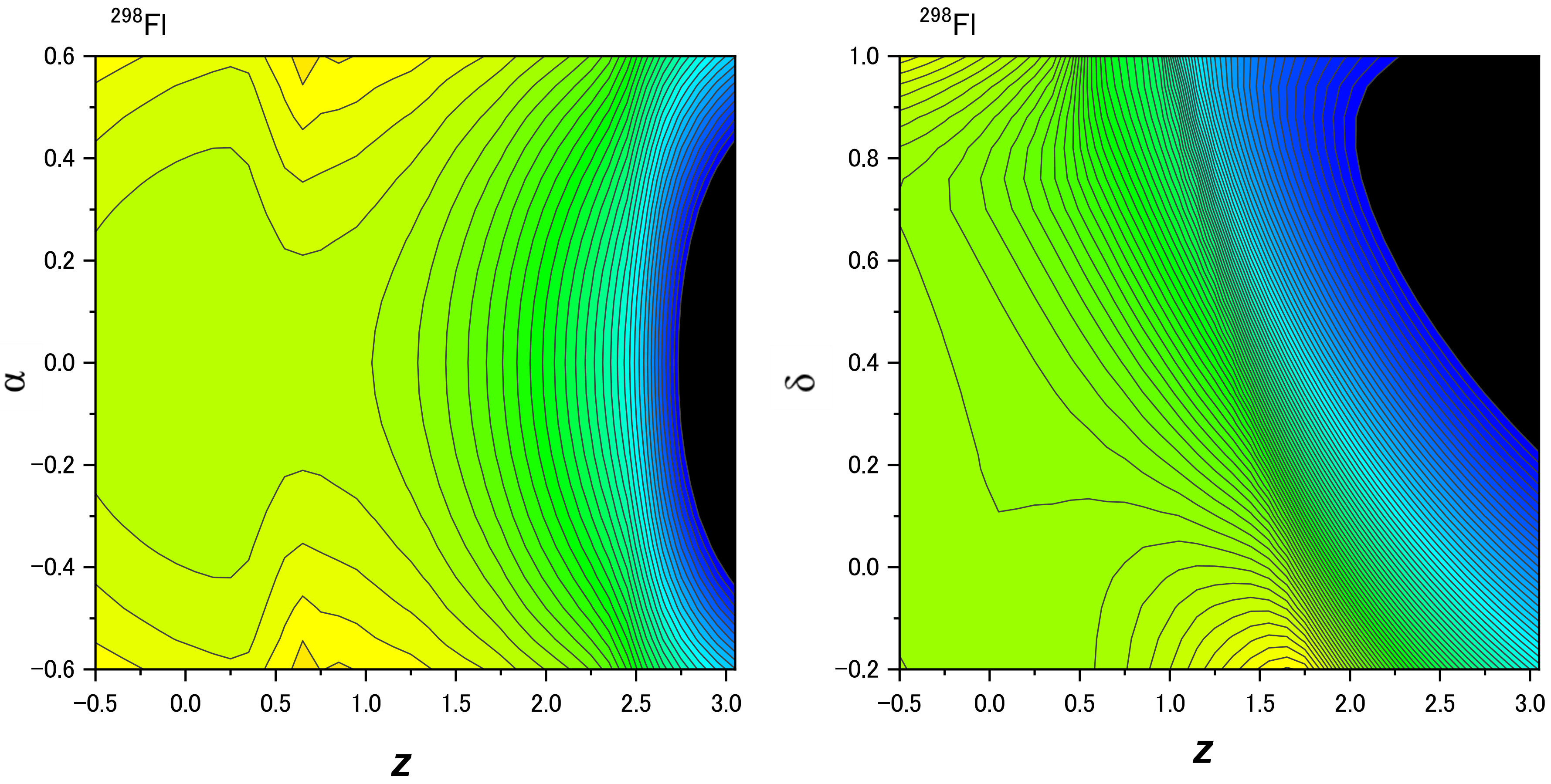
$$\begin{bmatrix} \gamma_{zz} & 0 & 0 \\ 0 & \gamma_{\delta\delta} & 0 \\ 0 & 0 & \gamma_{\alpha\alpha} \end{bmatrix} \begin{bmatrix} m_{zz} & 0 & 0 \\ 0 & m_{\delta\delta} & 0 \\ 0 & 0 & m_{\alpha\alpha} \end{bmatrix}$$

^{298}Fl

V_{DM} : Liquid Drop Model without a random term

With non-diagonal

----- Without non-diagonal

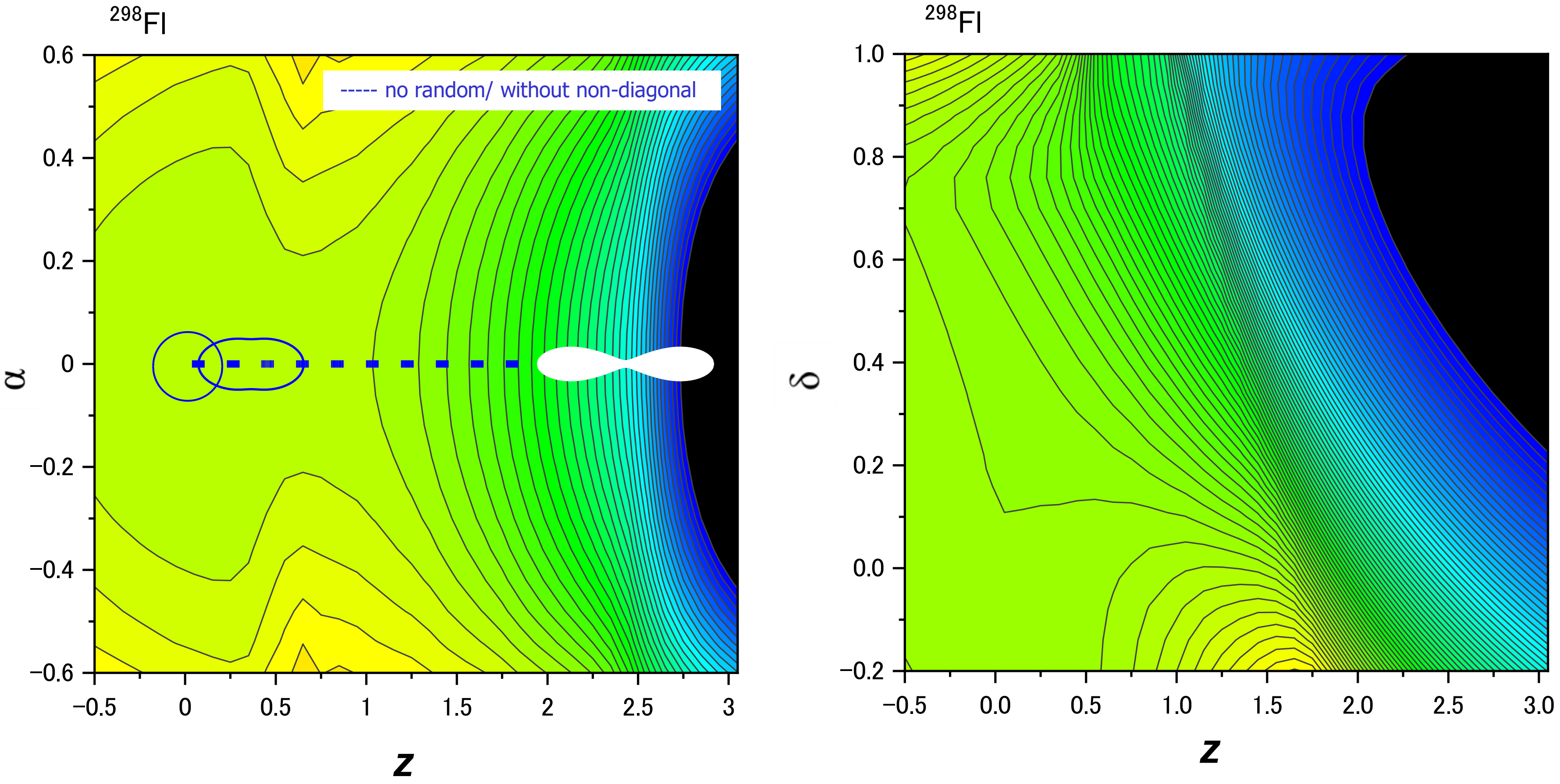


^{298}Fl

V_{DM} : Liquid Drop Model without a random term

With non-diagonal

Without non-diagonal

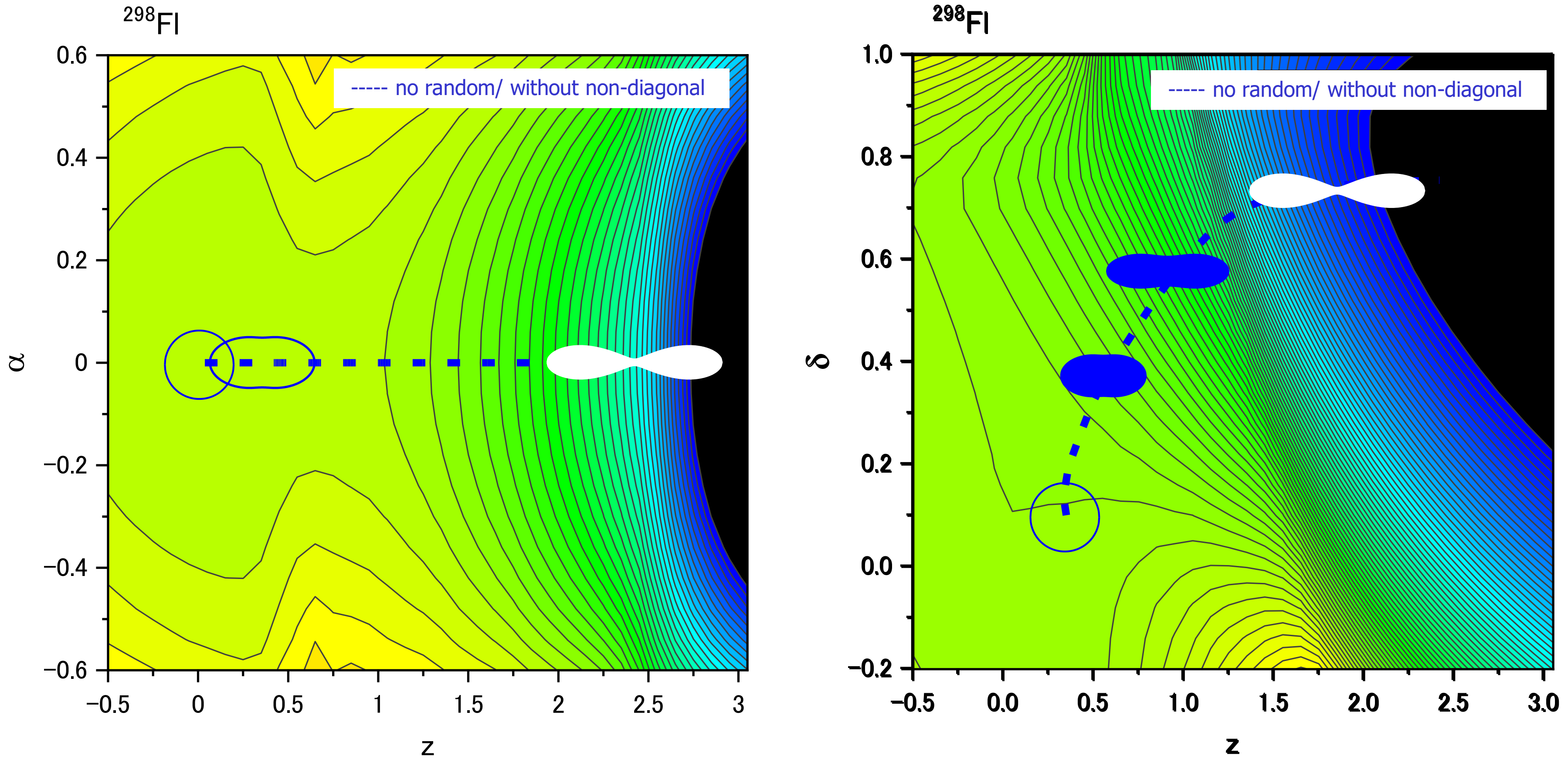


^{298}Fl

V_{DM} : Liquid Drop Model without a random term

With non-diagonal

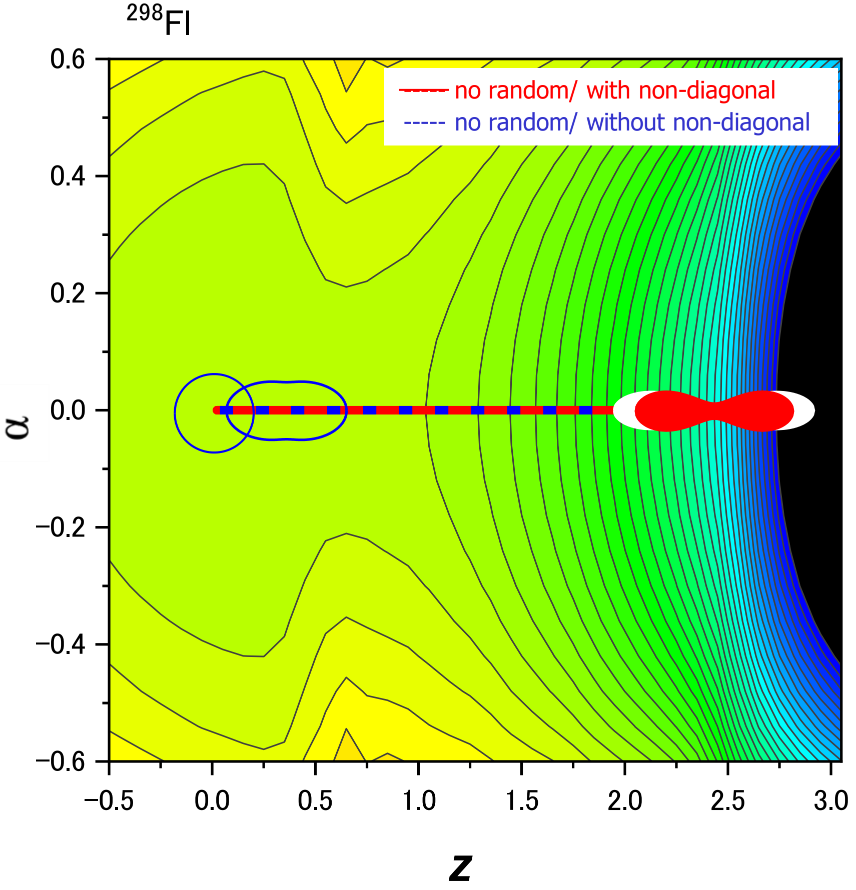
Without non-diagonal

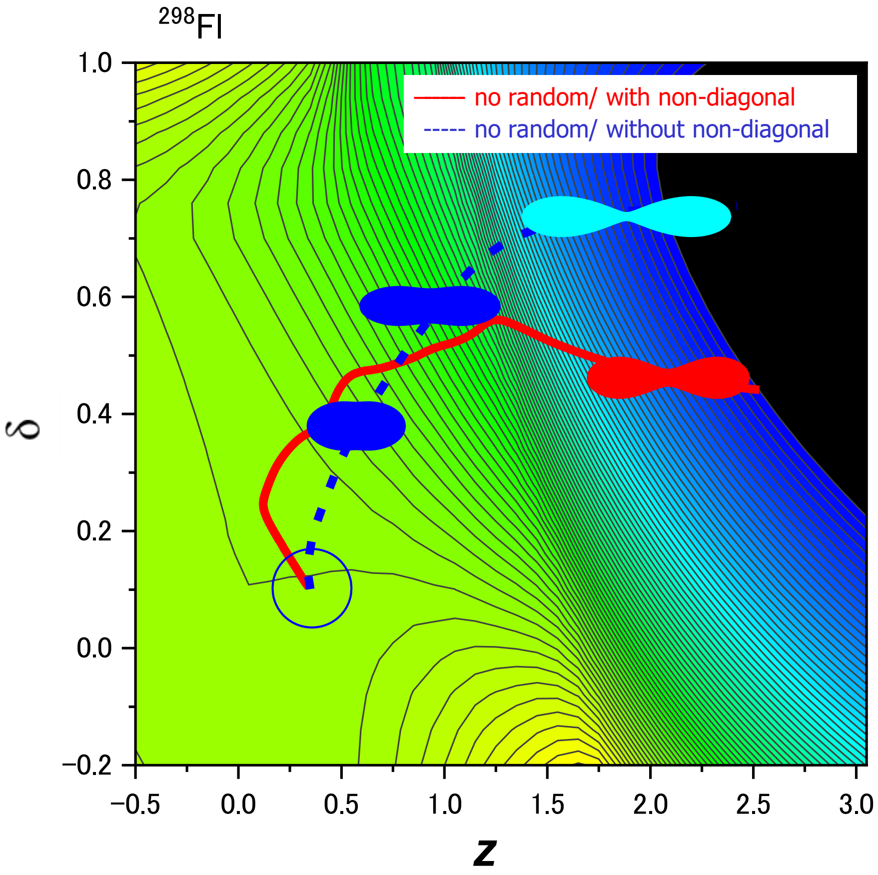


^{298}Fl

V_{DM} : Liquid Drop Model without a random term

With non-diagonal
Without non-diagonal





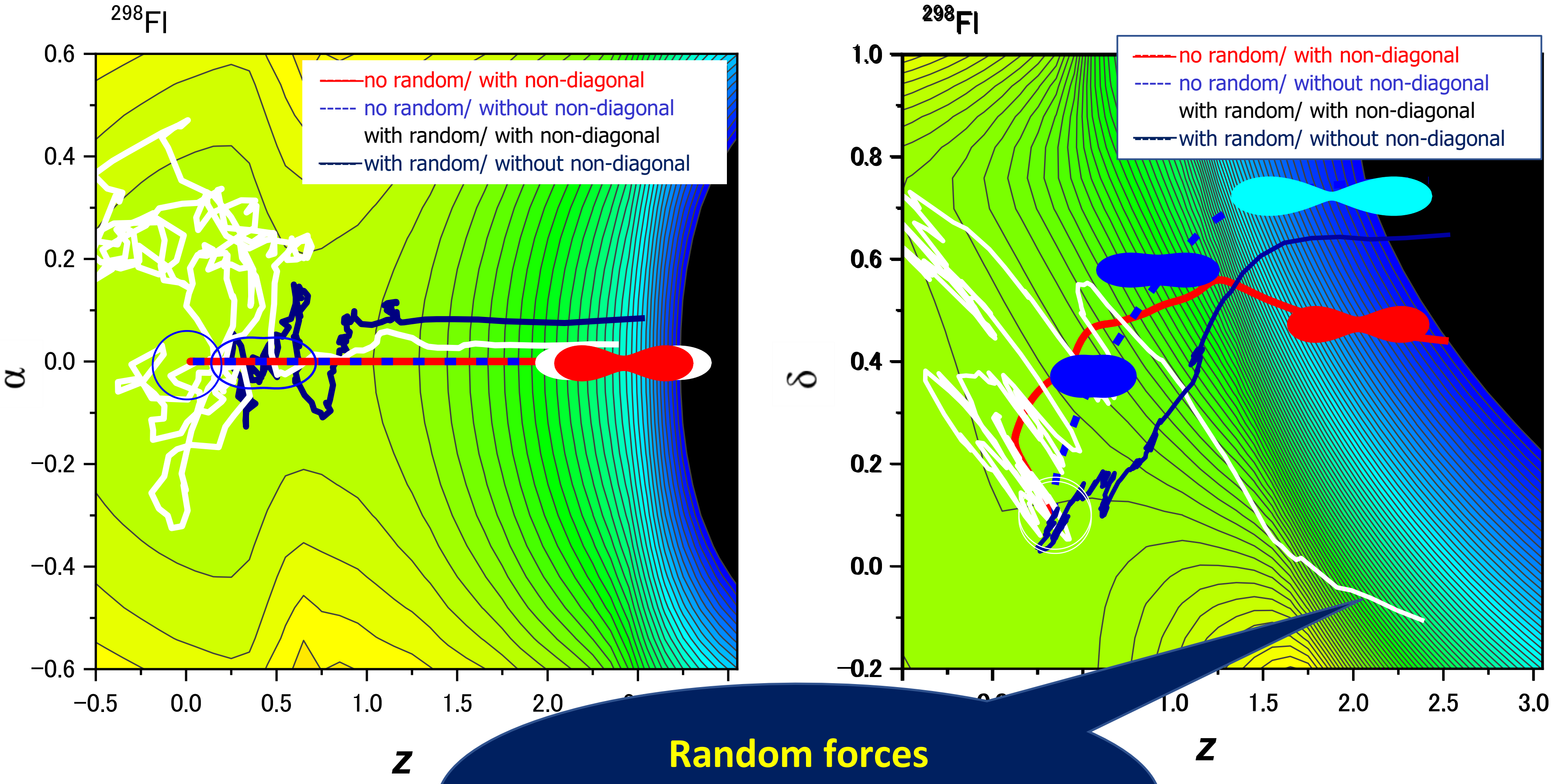
What is the shape?....

^{298}Fl

V_{DM} : Liquid Drop Model with a random term

With non-diagonal

----- Without non-diagonal

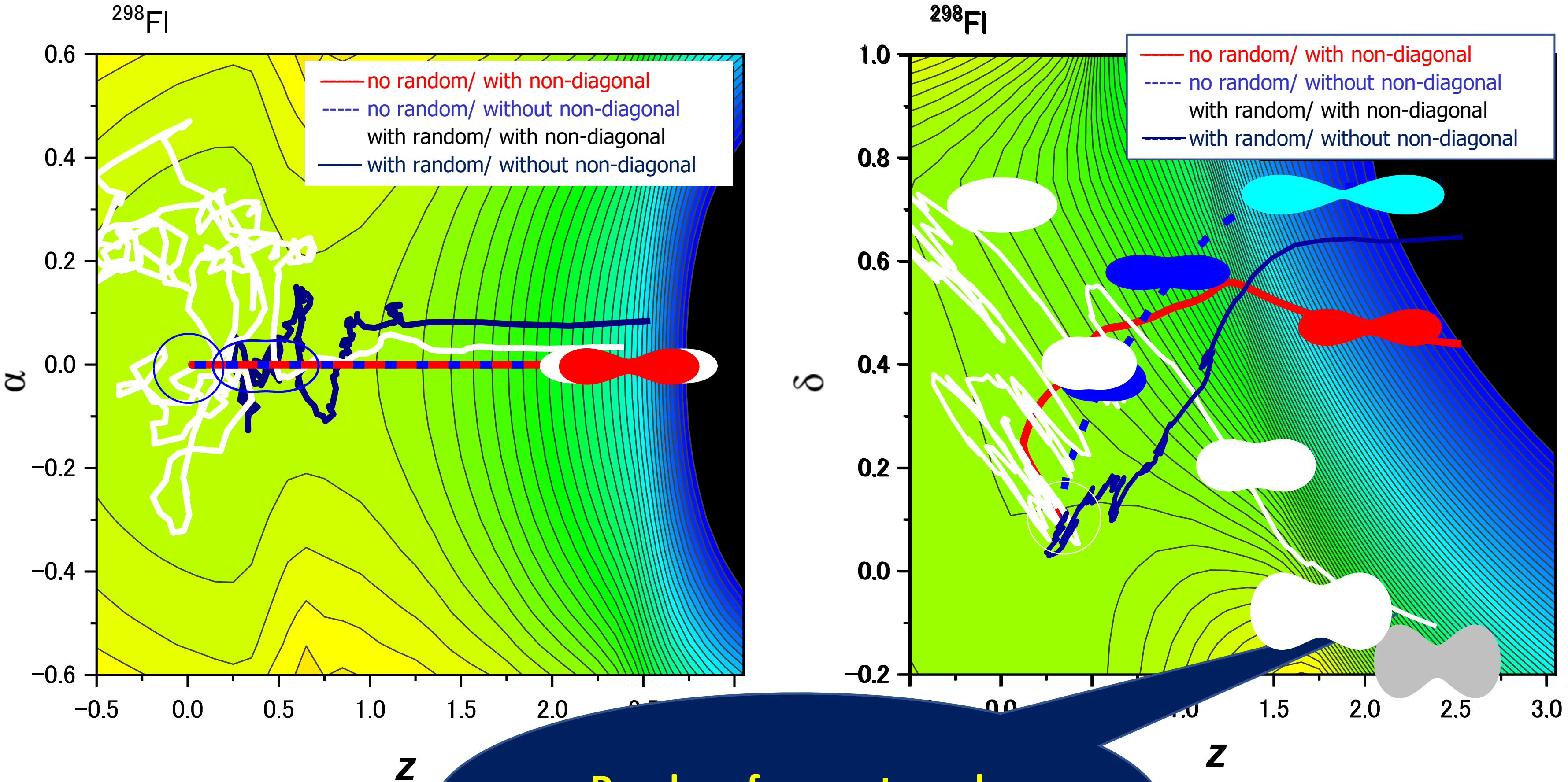


Random forces strongly deflect the trajectory

^{298}Fl

V_{DM} : Liquid Drop Model with a random term

With non-diagonal
Without non-diagonal

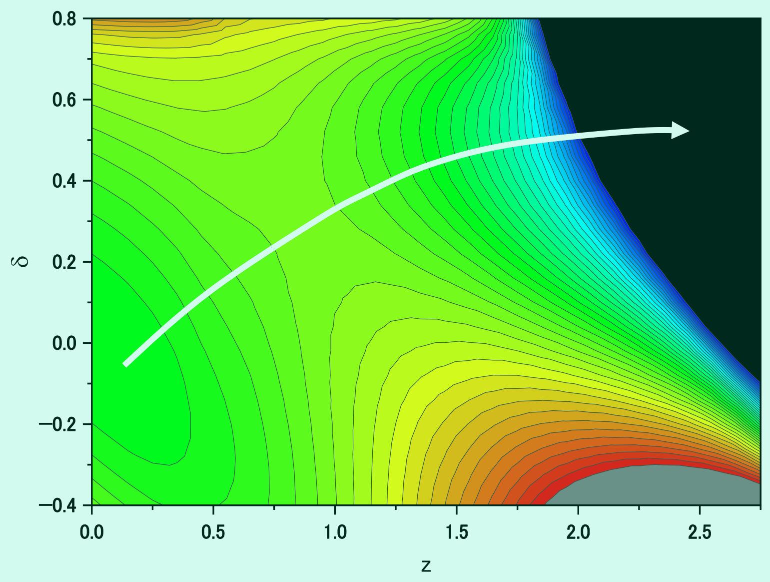


Random forces strongly deflect the trajectory

This is not special case

Transport coefficient

Important physical quantities in trajectory analysis



Influence the motion

② Transport coefficients

Includes non-diagonal components
(normal calculation case)

$$\begin{bmatrix} \gamma_{zz} & \gamma_{z\delta} & \gamma_{z\alpha} \\ \gamma_{\delta z} & \gamma_{\delta\delta} & \gamma_{\delta\alpha} \\ \gamma_{\alpha z} & \gamma_{\alpha\delta} & \gamma_{\alpha\alpha} \end{bmatrix} \begin{bmatrix} m_{zz} & m_{z\delta} & m_{z\alpha} \\ m_{\delta z} & m_{\delta\delta} & m_{\delta\alpha} \\ m_{\alpha z} & m_{\alpha\delta} & m_{\alpha\alpha} \end{bmatrix}$$

Without non-diagonal component
(test calculation)

$$\begin{bmatrix} \gamma_{zz} & 0 & 0 \\ 0 & \gamma_{\delta\delta} & 0 \\ 0 & 0 & \gamma_{\alpha\alpha} \end{bmatrix} \begin{bmatrix} m_{zz} & 0 & 0 \\ 0 & m_{\delta\delta} & 0 \\ 0 & 0 & m_{\alpha\alpha} \end{bmatrix}$$

Fission process

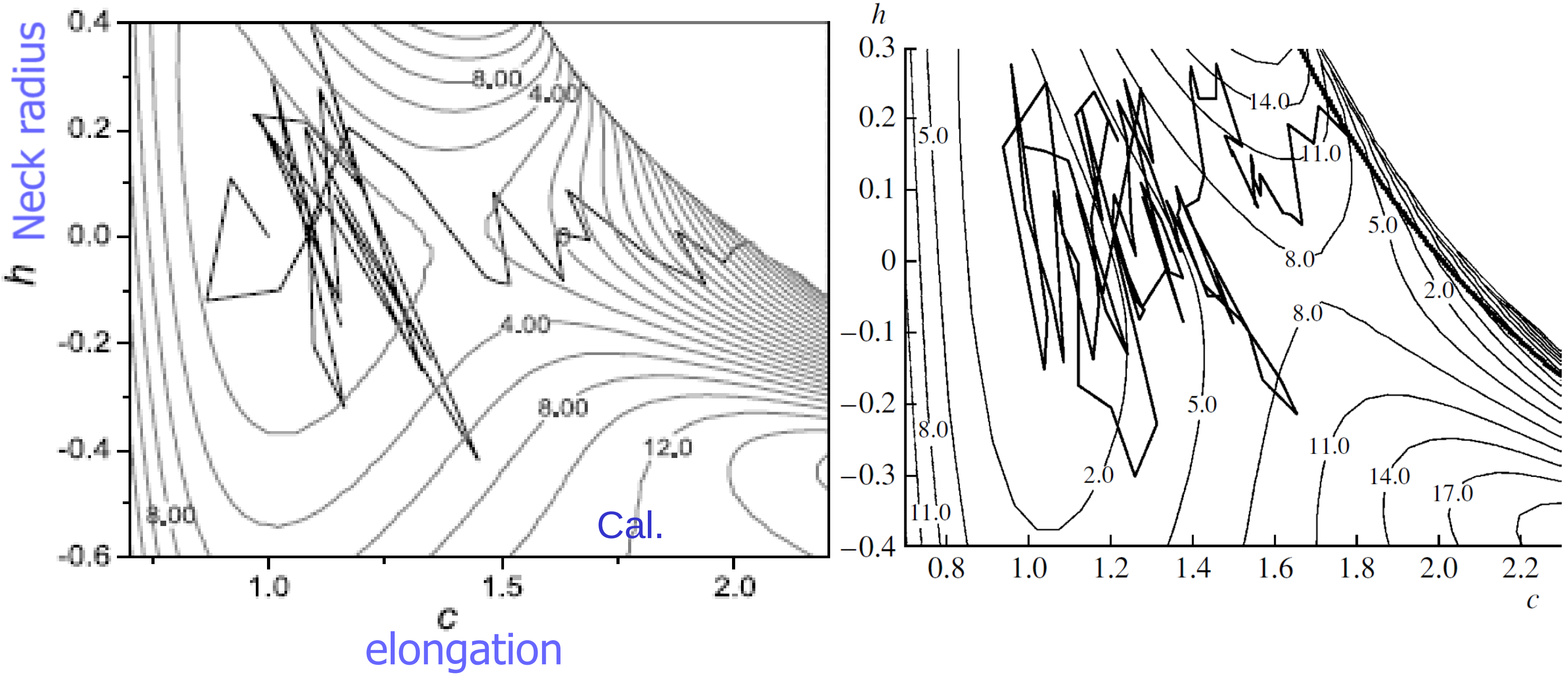
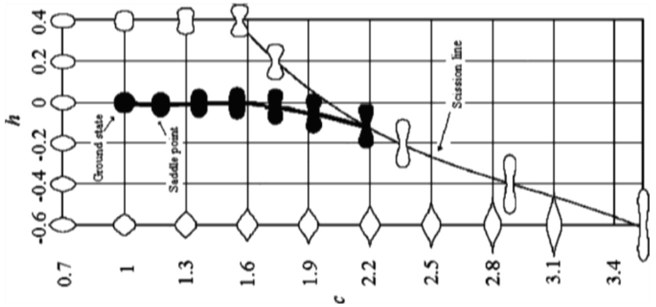
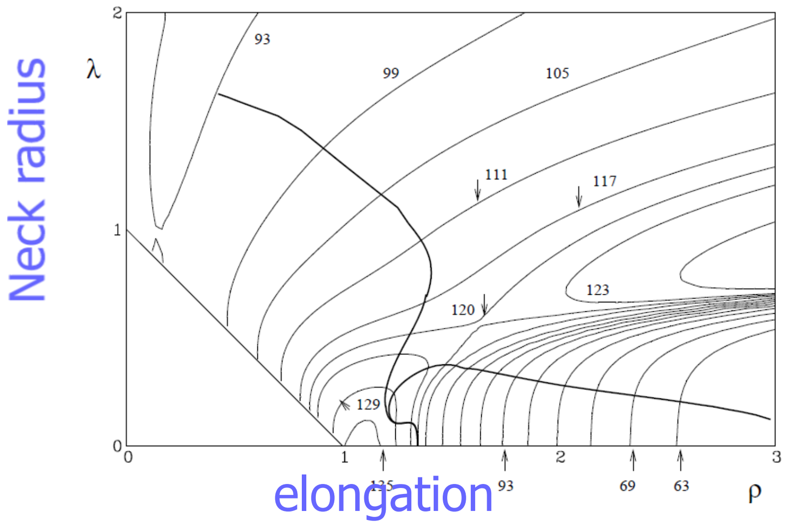


Figure 1. Stochastic Langevin trajectory in the space of the collective coordinates $(c, h, \alpha' = 0)$ is shown against the potential-energy background. The numbers at the isolines specify the values of the potential energy in MeV. The solid line in the right upper corner of the figure is the scission line. The trajectory given in this figure represents a fission event.



A.V. Karpov,
P.N. Nadtochy,
D.V. Vanin, and G.D. Adeev,
PRC 63 (2001) 054610

Fission process



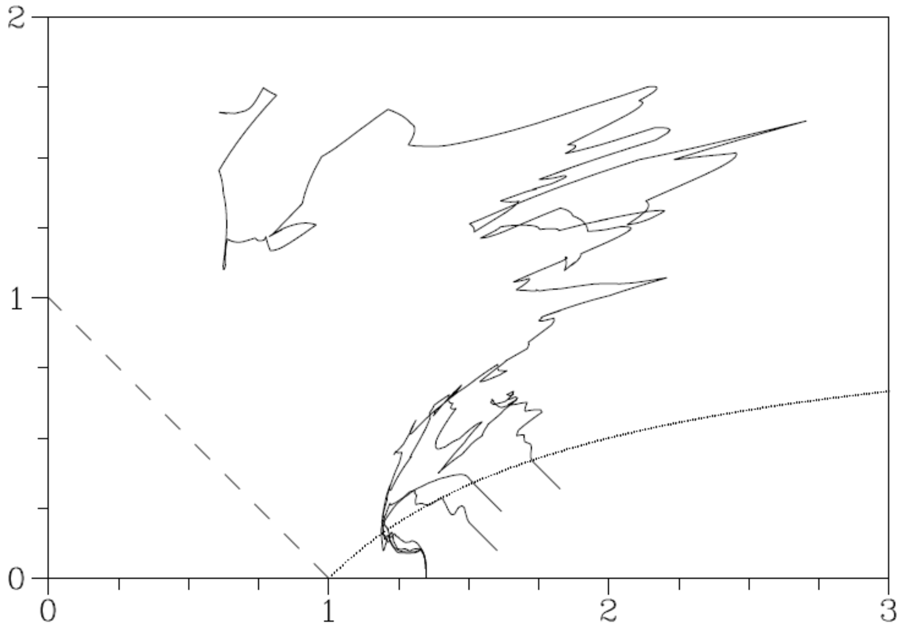
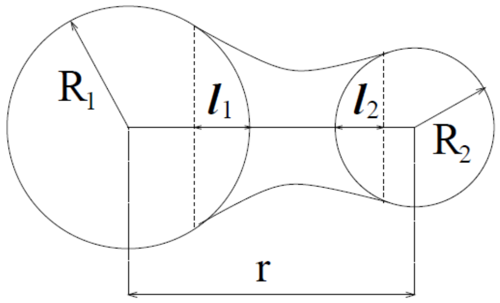


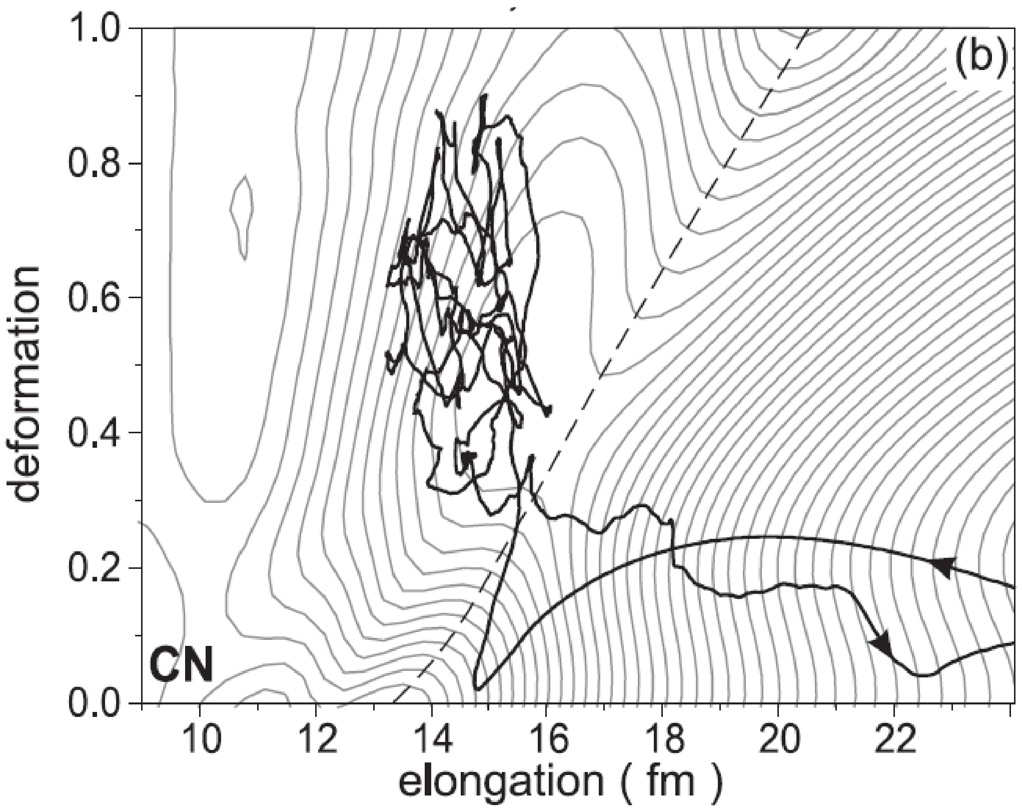
Fig. 6. Two deterministic trajectories for the ^{86}Kr+^{70}Ge colliding system, starting with initial kinetic energy 132 and 134 MeV and plotted on the potential energy map in (ρ, λ)-space. The energy contours are in MeV.

1) the distance variable, $\rho = r/(R_1 + R_2)$,
2) the neck variable, $\lambda = (l_1 + l_2)/(R_1 + R_2)$,
3) the asymmetry variable, $\Delta = (R_1 - R_2)/(R_1 + R_2)$



Langevin Dynamics in 4-dimensional Model of Nucleus-Nucleus Collisions
J. Blocki, O. Mazonka, J. Wilczynski, Z. Sosin, and A. Wieloch
Acta Physica Polonica B 31 1513 (2000)

Fusion-Fission process

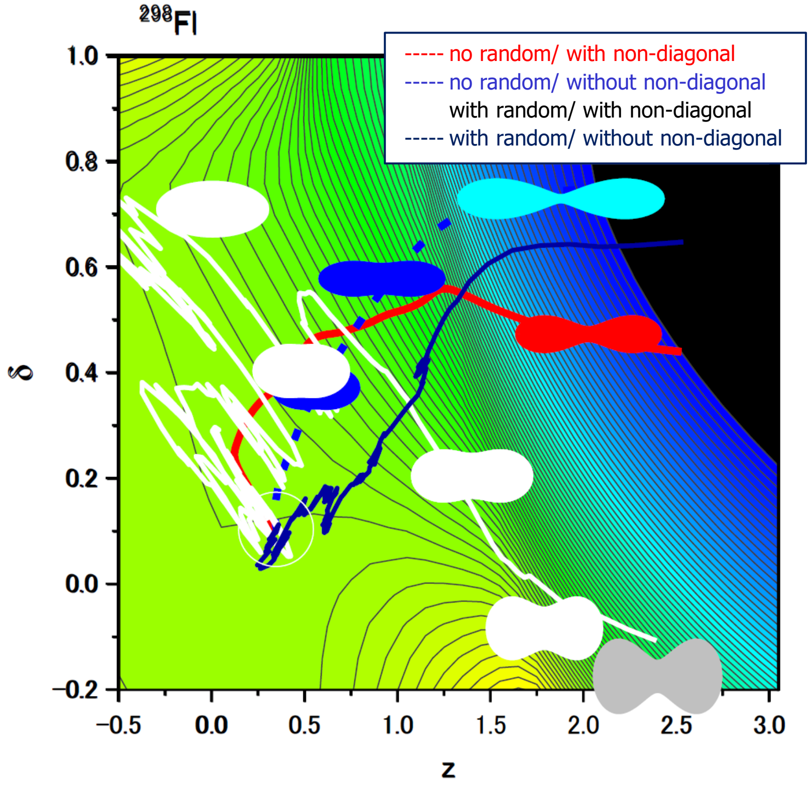


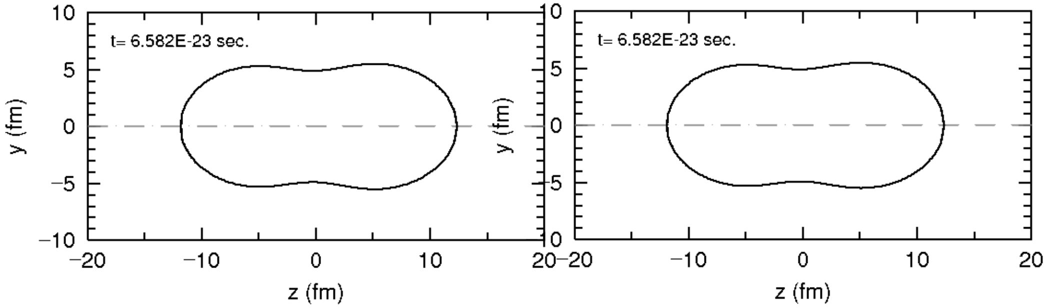
V. Zagrebaev and W. Greiner
J. Phys. G. 31, 825-844 (2005)

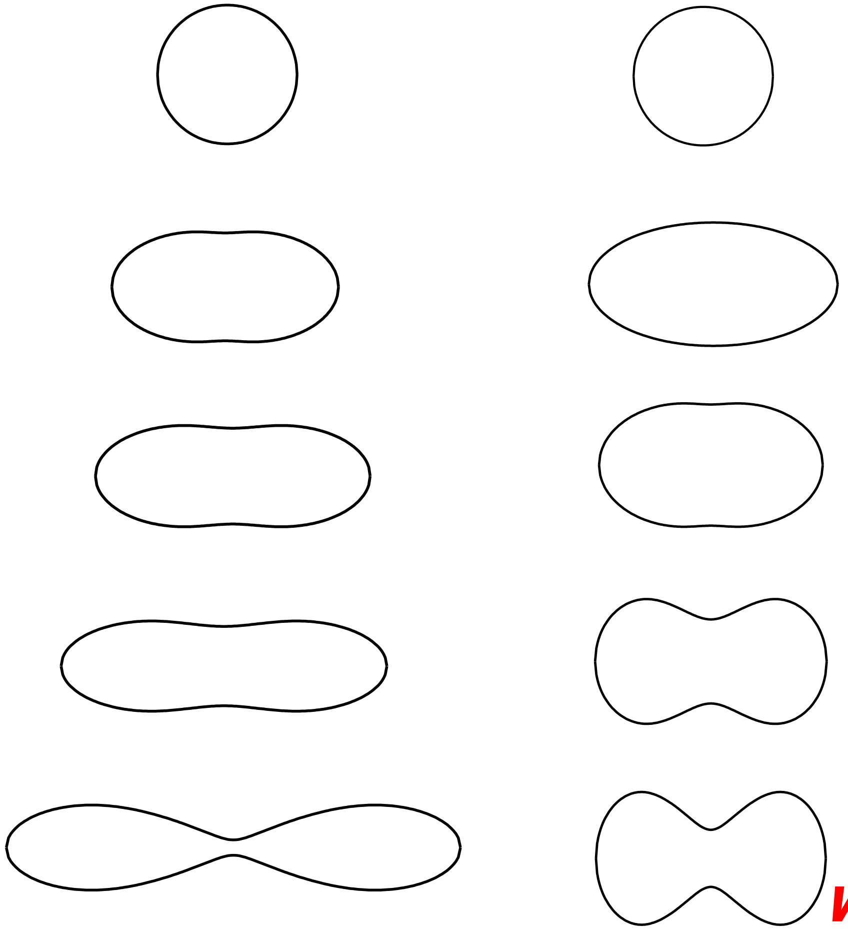
The fluctuation of the trajectory
does not show particular direction

the friction forces which depend generally on the coordinates $\vec{x}$. For the moment, we ignore the non-diagonal terms of the mass and friction parameters. The so-called 'sliding friction' (which

17







Without fluctuation
Without non-diagonal com.

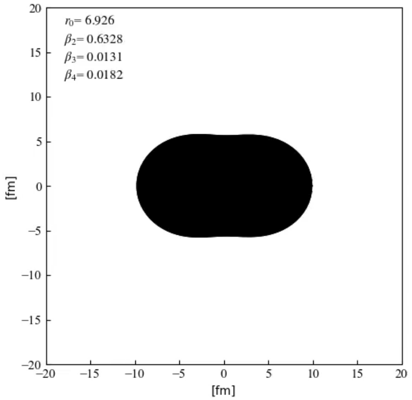
With fluctuation
With non-diagonal com.

What is induced fission?

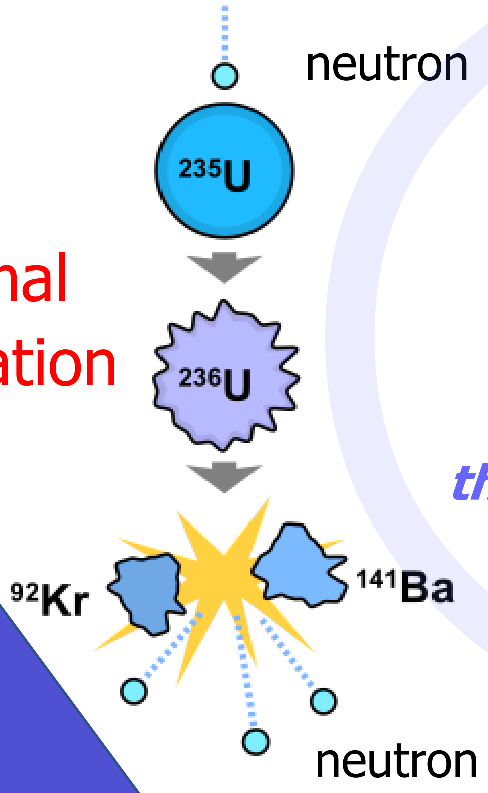
Induced fission occurs by thermal fluctuation of nuclei

To overcome the fission barrier,
Thermal fluctuation of shape is required

Under the strong friction



Thermal Fluctuation

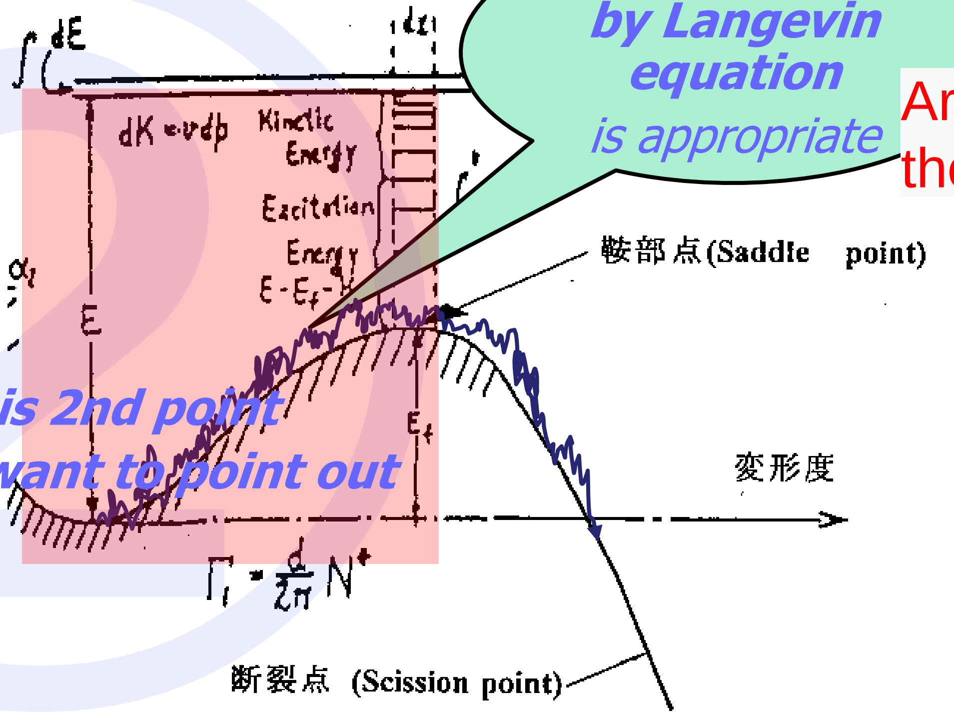


energy exchange
kinetic energy ⟺ intrinsic energy

Description by Langevin equation *is appropriate*

Area inside the fission barrier

This is 2nd point that we want to point out



The Mechanism of Nuclear Fission

Niels Bohr
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(Received June 28, 1939)

Brownian motion

Brownian motion

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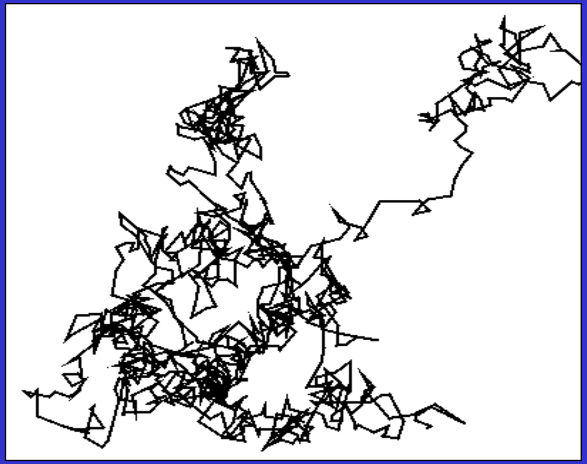
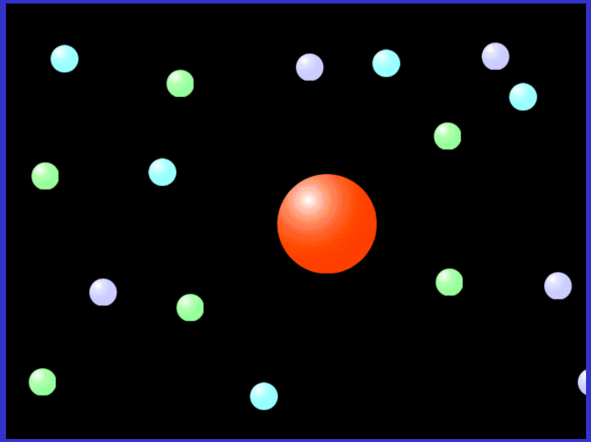


Trajectory of a particle from pollen



…, the general existence of the active molecules in inorganic as well as organic bodies, their apparent indestructibility by heat, …

R. Brown, Philosophical Magazine, vol.4, 1828

Motion of **the tiny particles (0.3μm) from the pollen grains of flowers (30-50μm) on water (0.3nm)**

Microscopic degree of freedom (water molecules)

← Thermal motion

Heat bath

→ Interaction

Macroscopic degree of freedom (particles of pollen)

Brownian motion

$$m\frac{dv}{dt} = -\gamma v + gR(t)$$

$$g = \sqrt{\gamma T}$$

Nuclear Fission　: Employed the concept of Brownian motion

原子核の運動を2つのカテゴリーに分ける

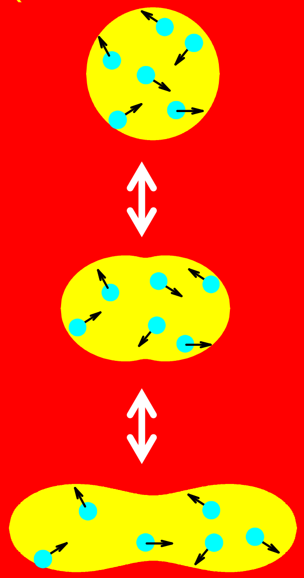
Kramers (1940)

Collective motion {q}
elongation z
mass asymmetry α
deformation of fragments δ

Non-Collective motion
nucleonic degree of freedom

Brownian motion　　coupling　　**Heat bath**

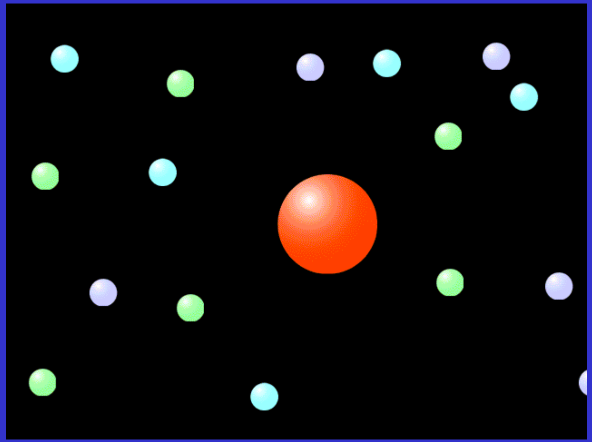
Shape fluctuation
(thermal fluctuation)



{q}

dissipation　*Friction force*

fluctuation　*Random force*



Time scale
nucleonic motion ~10^{-23} sec
collective motion ~10^{-22} sec　{q}

Summary

What governs nuclear fission

Nuclear structure (microscopic properties)

fission valley

Liquid Droplet properties (macroscopic properties)

Characteristics of shape (collective motion) fluctuation

← Requires detailed analysis

inertia mass
friction (viscosity)

tensor ← nuclear density ρ

Affects direction of deformation is important

,which is controlled by strength and direction of random forces

What we want to convey at this workshop

No Induced Fission occurs without fluctuation
(thermal fluctuation)

Fluctuation has two directions, one is easy to deform and the other is hard to deform

It is not a property of potentials
It is the nature of the fluid

It's up to the saddle

Collaborators

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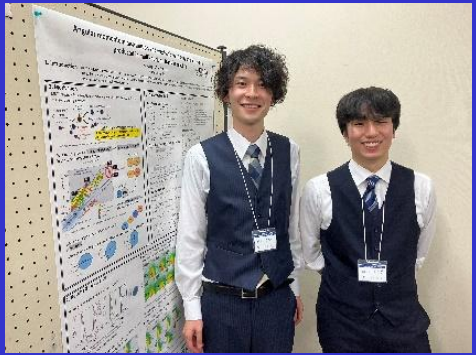

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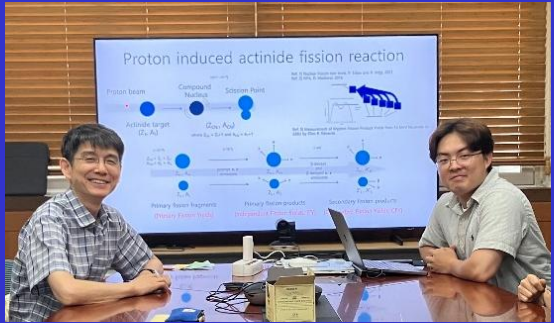

Snapshot of water droplets

No charge