

Clusterization in binary fission

H. Paşca, G. Adamian, N. Antonenko

Statistical Scission-point Model

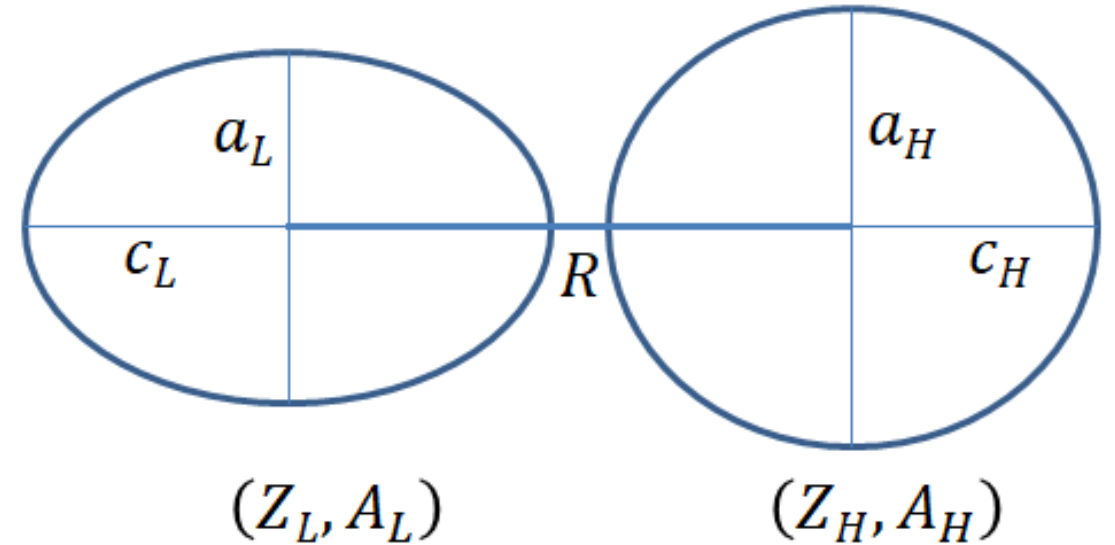
or

Cluster model of fission

- Scission-point model relies on assumption that statistical equilibrium is established at scission where observable characteristics of fission are formed.

Model

Coordinates Z_i , A_i , $\beta_i = c_i/a_i$ ($i = L, H$),
 R completely describe the geometry of
system



Total potential energy:

$$U(A_i, Z_i, \beta_i, R) = U_L^{LD}(A_L, Z_L, \beta_L) + \delta U_L^{shell}(A_L, Z_L, \beta_L, E_L^*) + U_H^{LD}(A_H, Z_H, \beta_H) \\ + \delta U_H^{shell}(A_H, Z_H, \beta_H, E_H^*) + V^C(A_i, Z_i, \beta_i, R) + V^N(A_i, Z_i, \beta_i, R)$$

$$V = V^C + V^N - \text{interaction potential}$$

Model

$$\omega(A_i, Z_i, \beta_i, E^*) = N_0 \exp \left[-\frac{U(A_i, Z_i, \beta_i, R_b)}{T} \right]$$

The different yields can be calculated by integrating over the deformations:

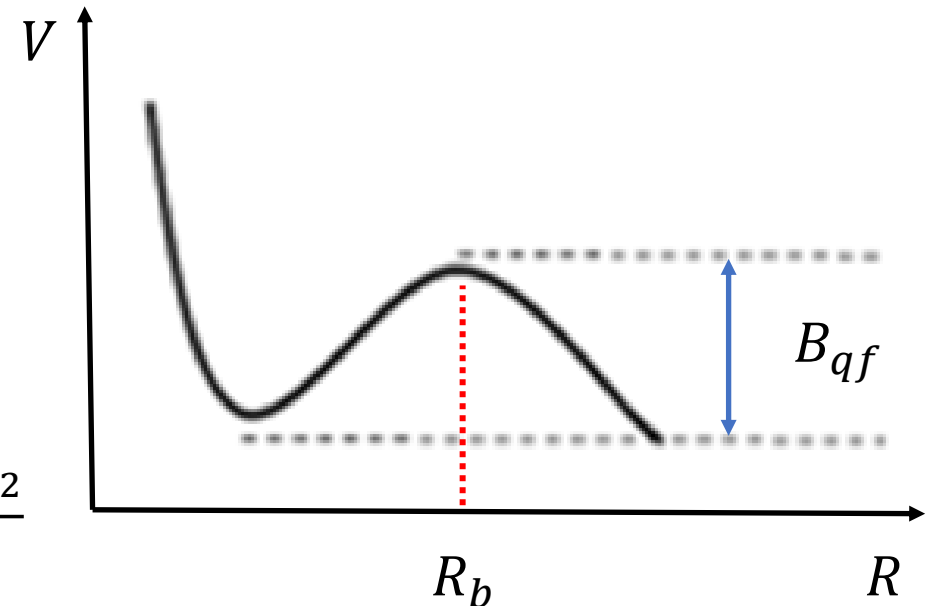
$$Y(A_i, Z_i) = \int \int \omega(A_i, Z_i, \beta_i, E^*) d\beta_1 d\beta_2$$

$$Y(A_i) = \sum_{Z_i} \int \int \omega(A_i, Z_i, \beta_i, E^*) d\beta_1 d\beta_2$$

$$Y(Z_i) = \sum_{A_i} \int \int \omega(A_i, Z_i, \beta_i, E^*) d\beta_1 d\beta_2$$

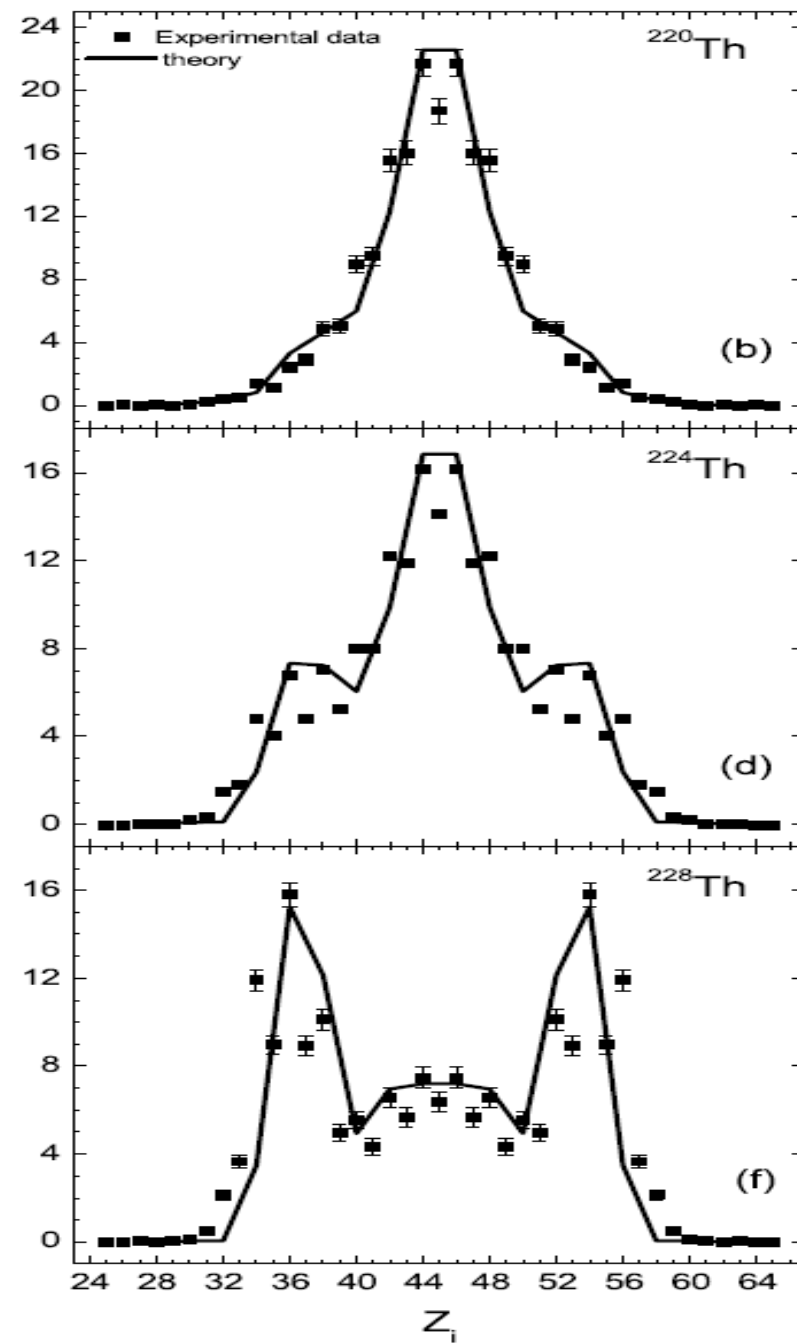
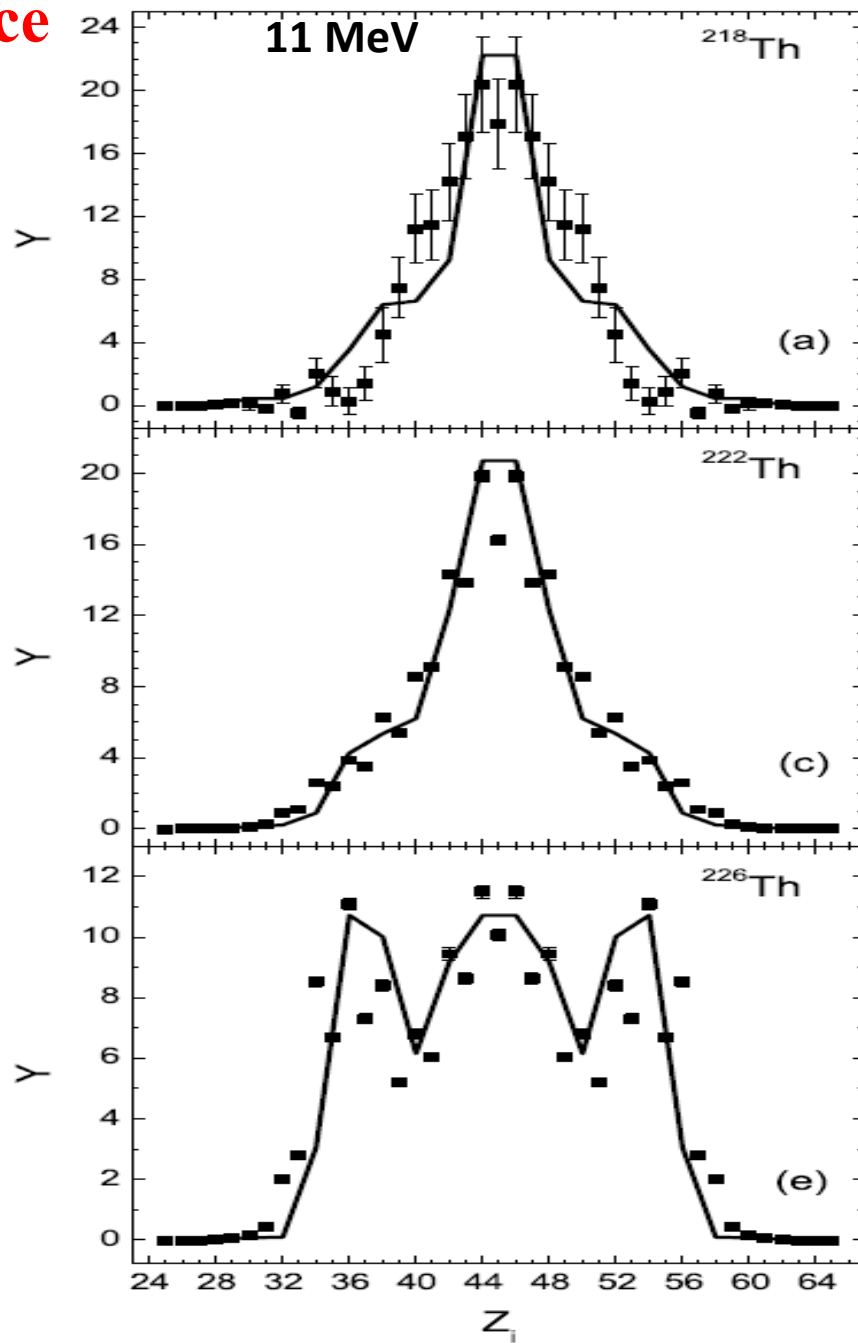
$$TKE(A_i, Z_i, \beta_i, R_b) = V_c(A_i, Z_i, \beta_i, R_b) + V_n(A_i, Z_i, \beta_i, R_b)$$

$$\langle TKE \rangle (A_i) = \frac{\sum_{Z_i} \int TKE(A_i, Z_i, \beta_i, R_b) \omega(A_i, Z_i, \beta_i, E^*) d\beta_1 d\beta_2}{\sum_{Z_i} \int \omega(A_i, Z_i, \beta_i, E^*) d\beta_1 d\beta_2}$$

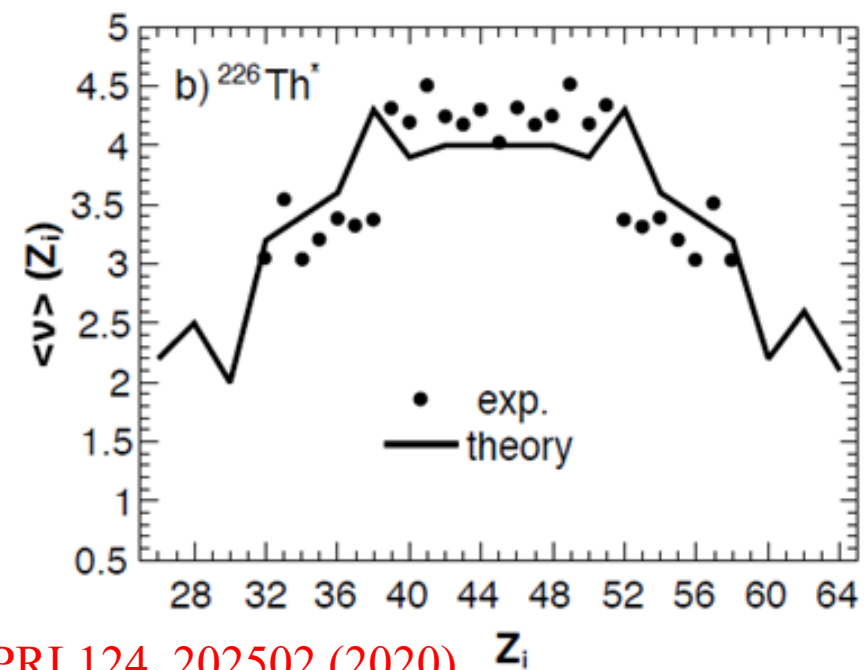
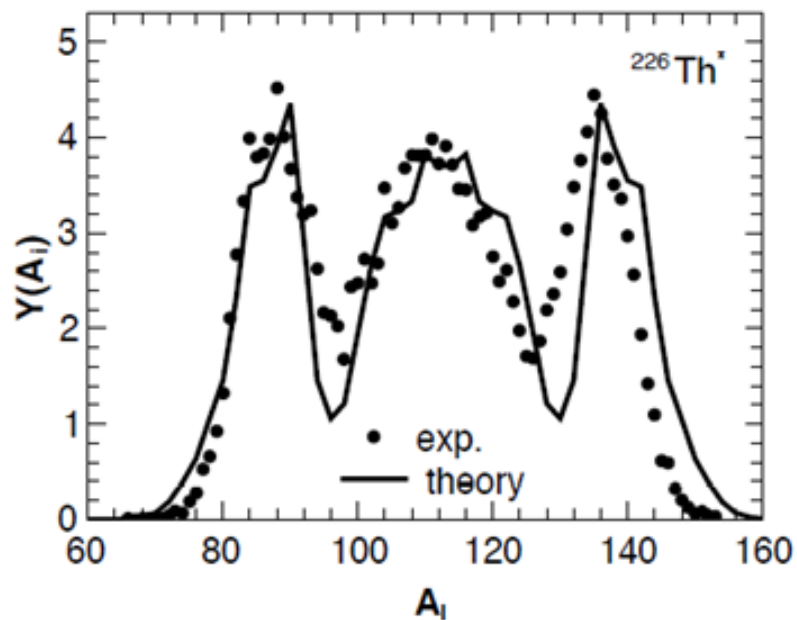
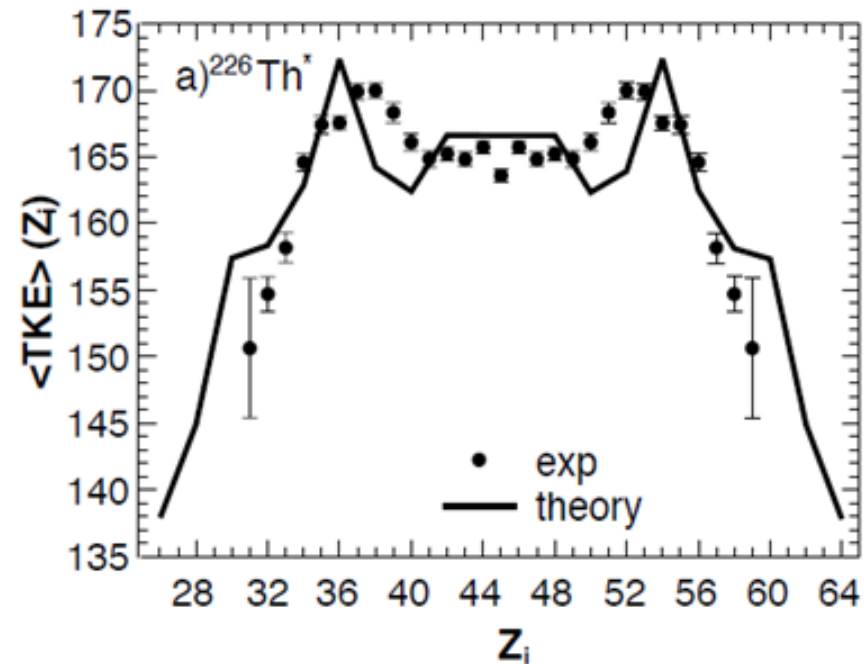
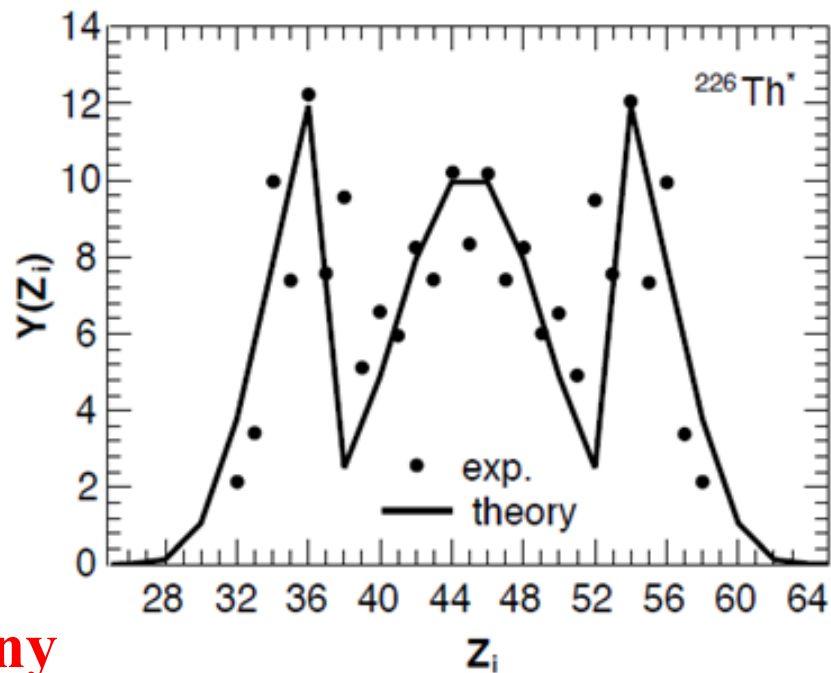


What advantages does the scission model have?

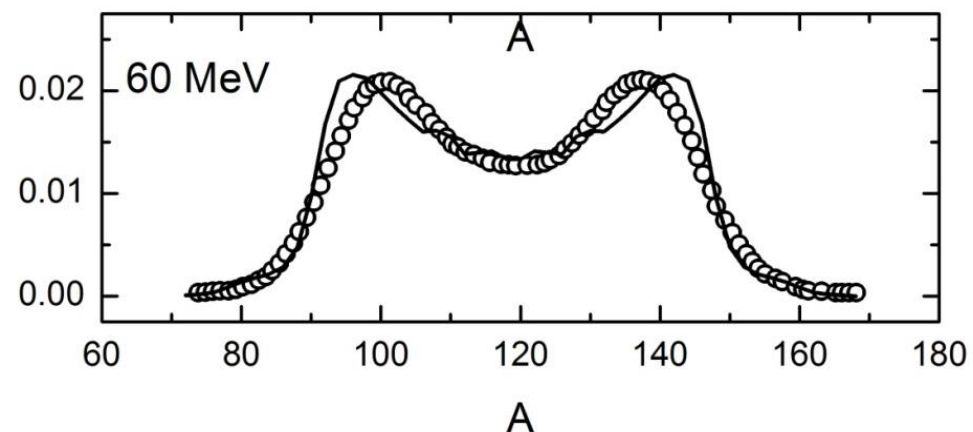
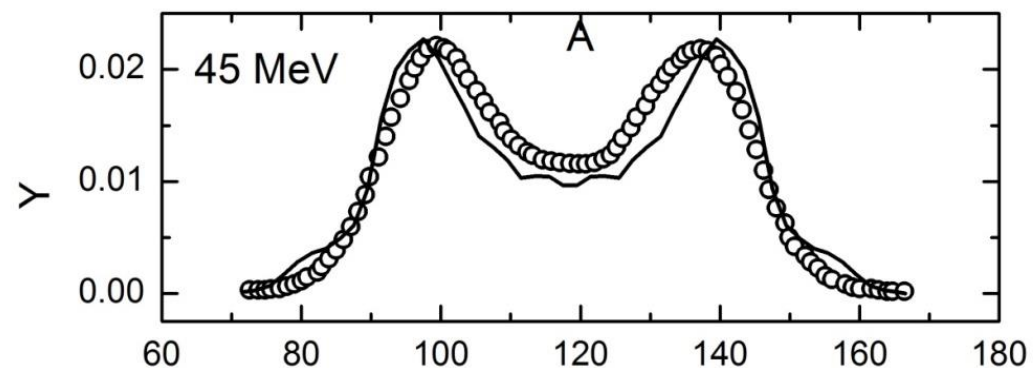
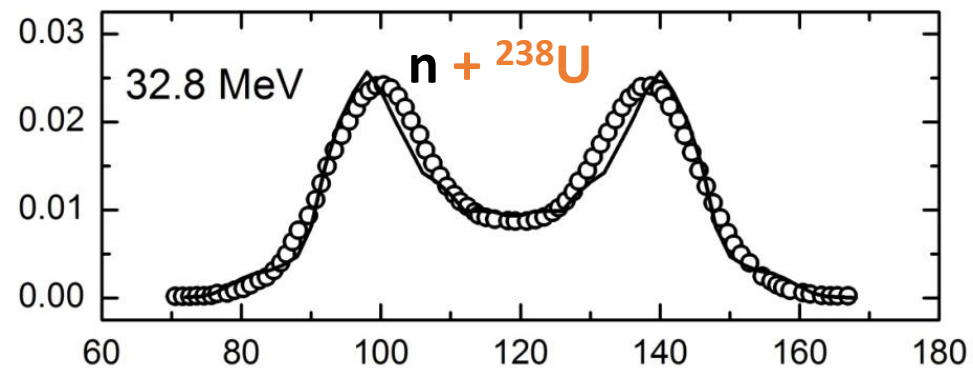
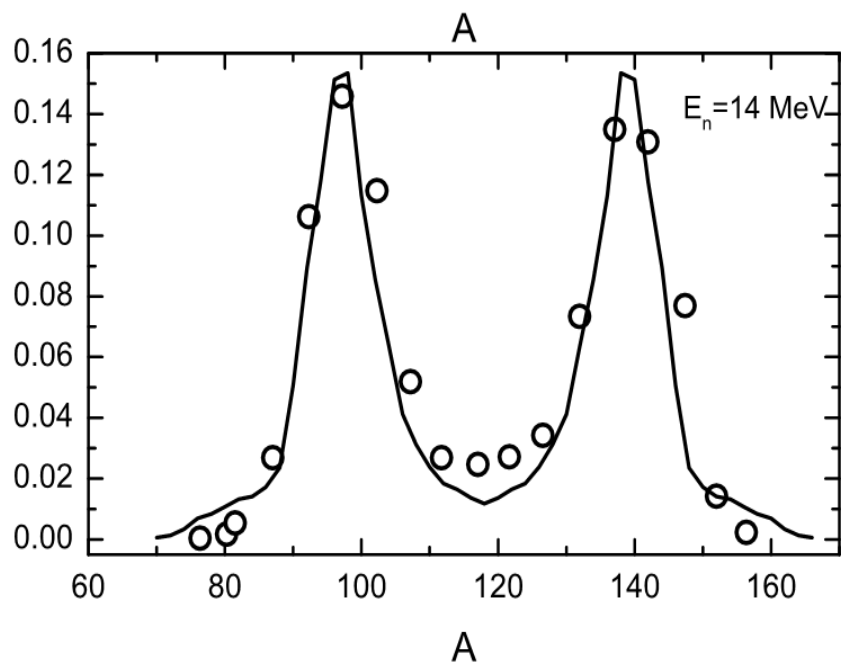
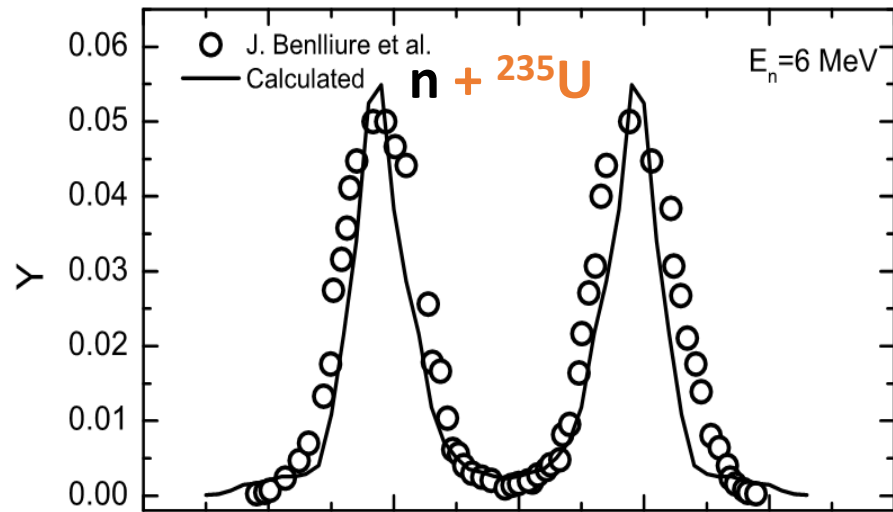
The dependence on isospin



The simultaneous description of many observables

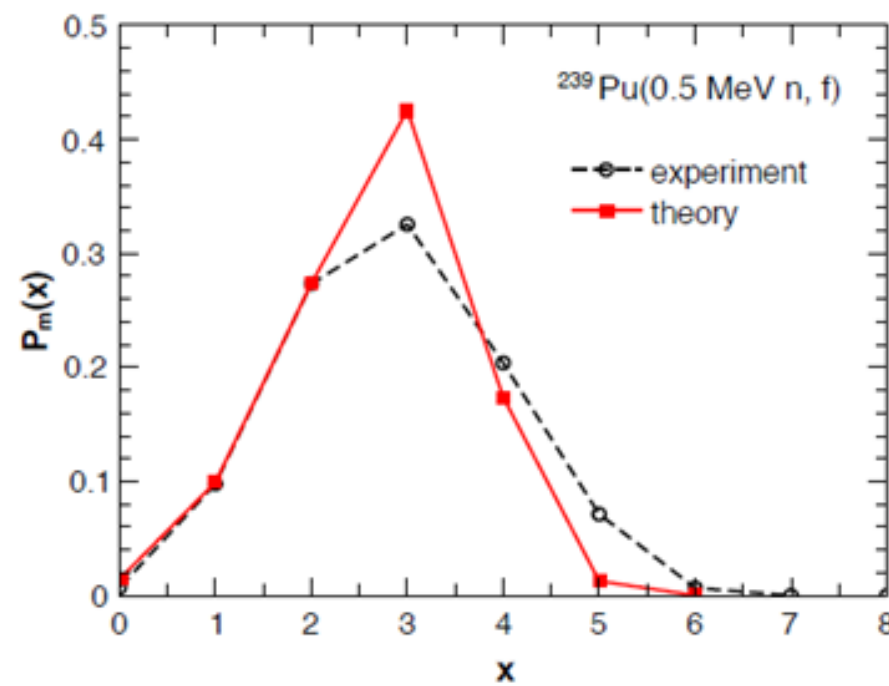
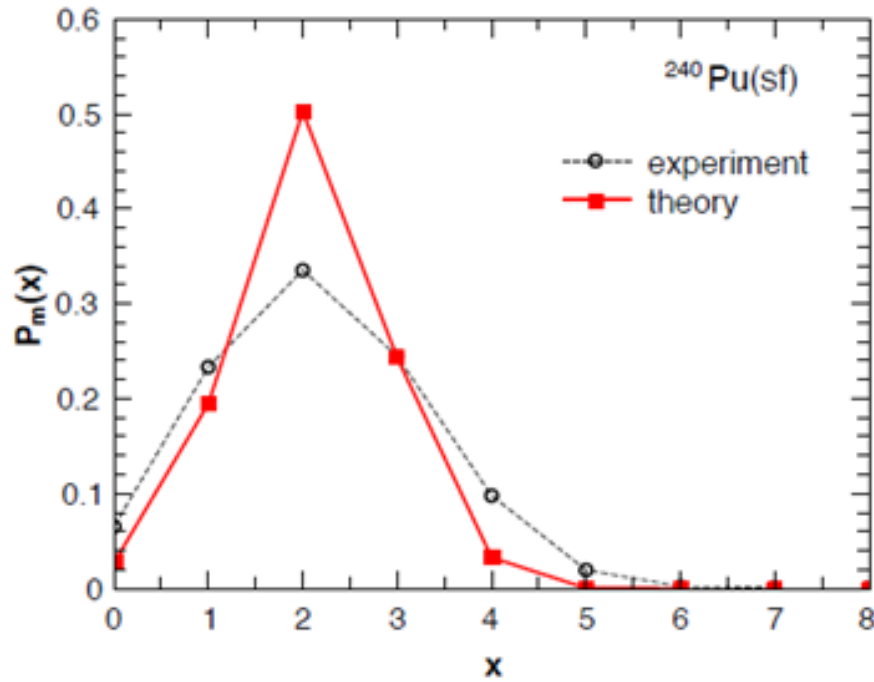


Exp.: NPA665, 221 (2000); 693, 169 (2001); PRL124, 202502 (2020)



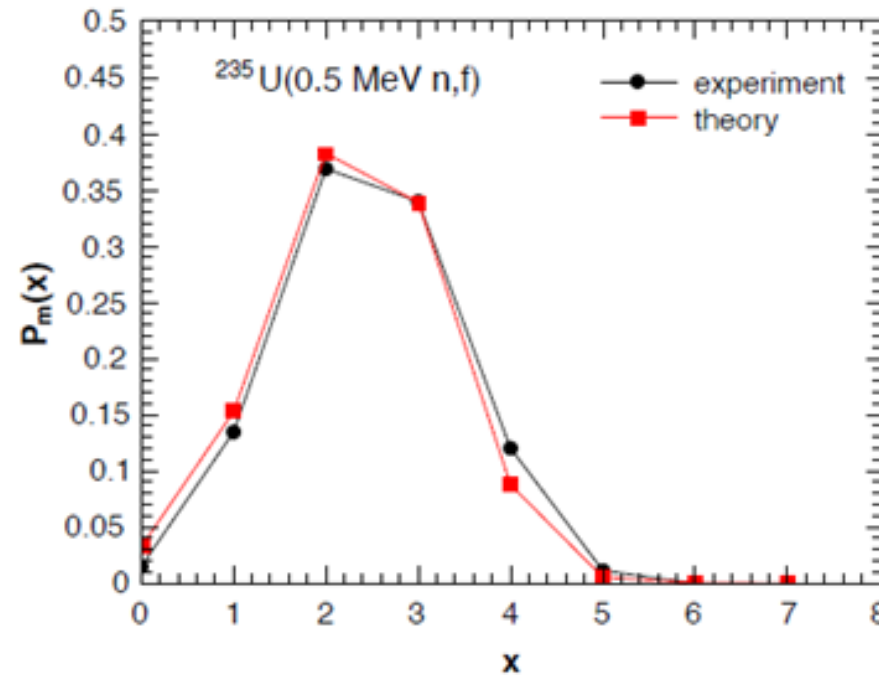
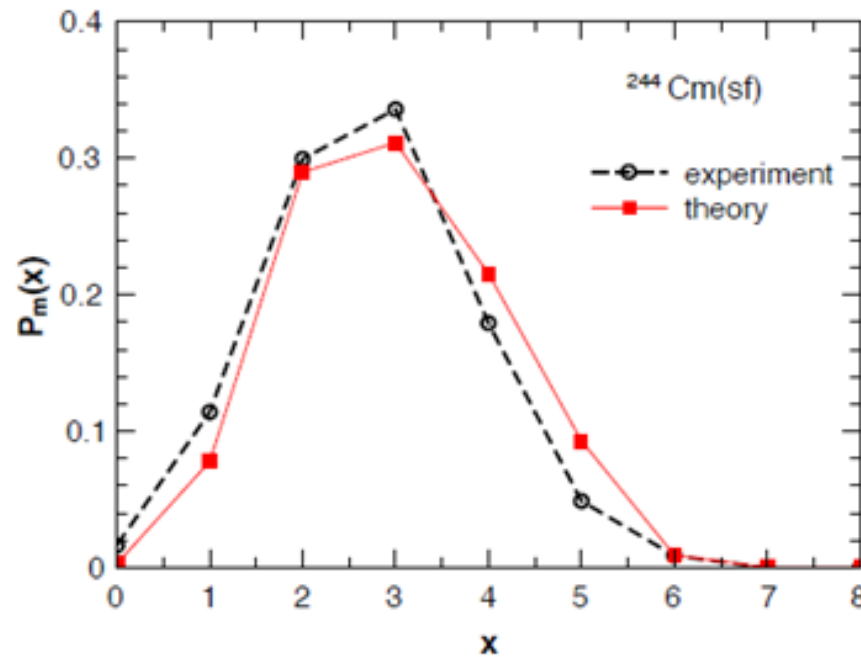
The dependence on excitation energy

Neutron Multiplicity



B.C. Diven et al.,
Phys. Rev. 101, 1012 (1956)

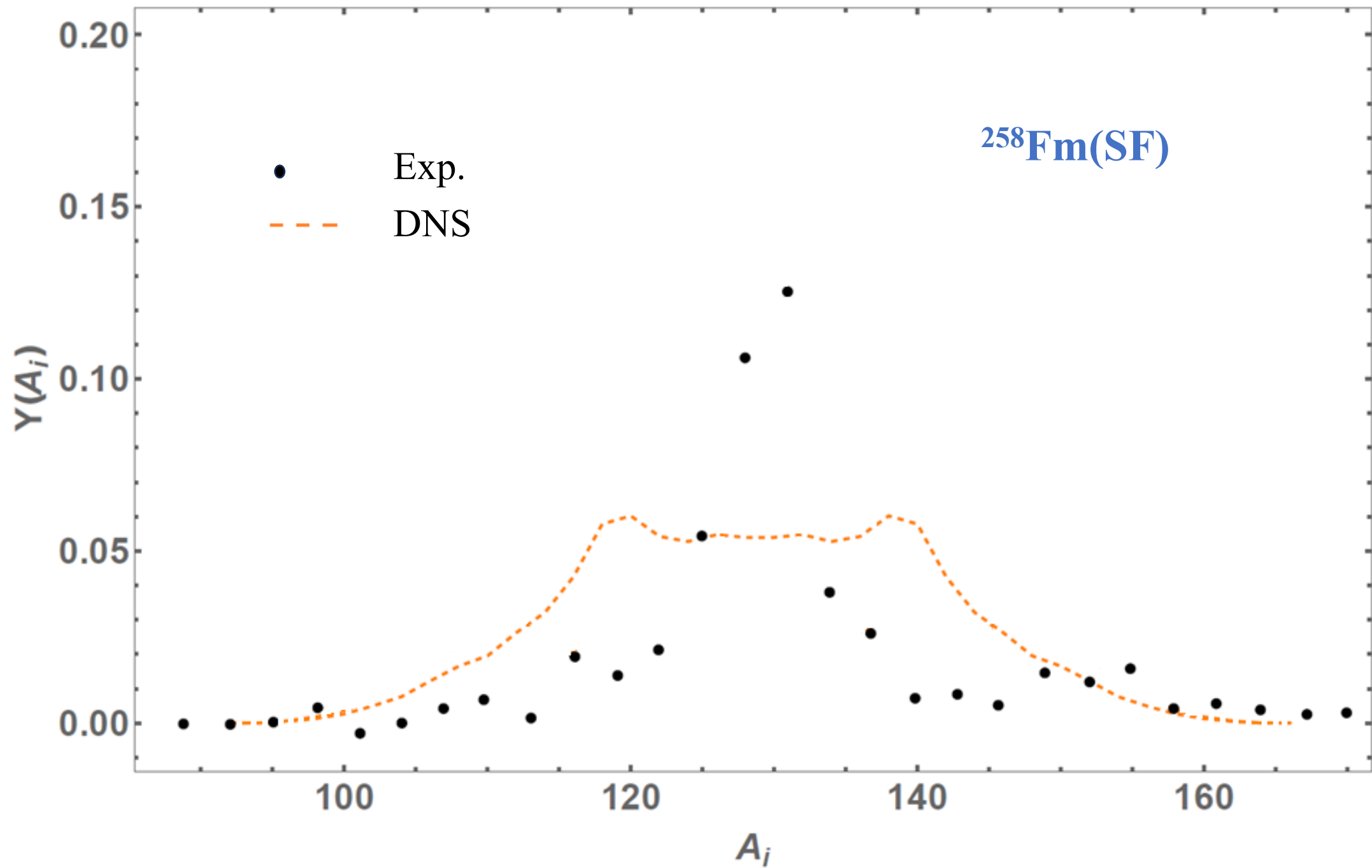
N.E. Holden et al.,
BNL-NCS-35513 (1985)



H.-Q. Zhang et al.,
Nucl. Sci. Eng. 86, 315
(1984)

Not all aspects of the binary SF can be simply understood on the basis of pure binary fragmentation (DNS) of the fissioning nucleus

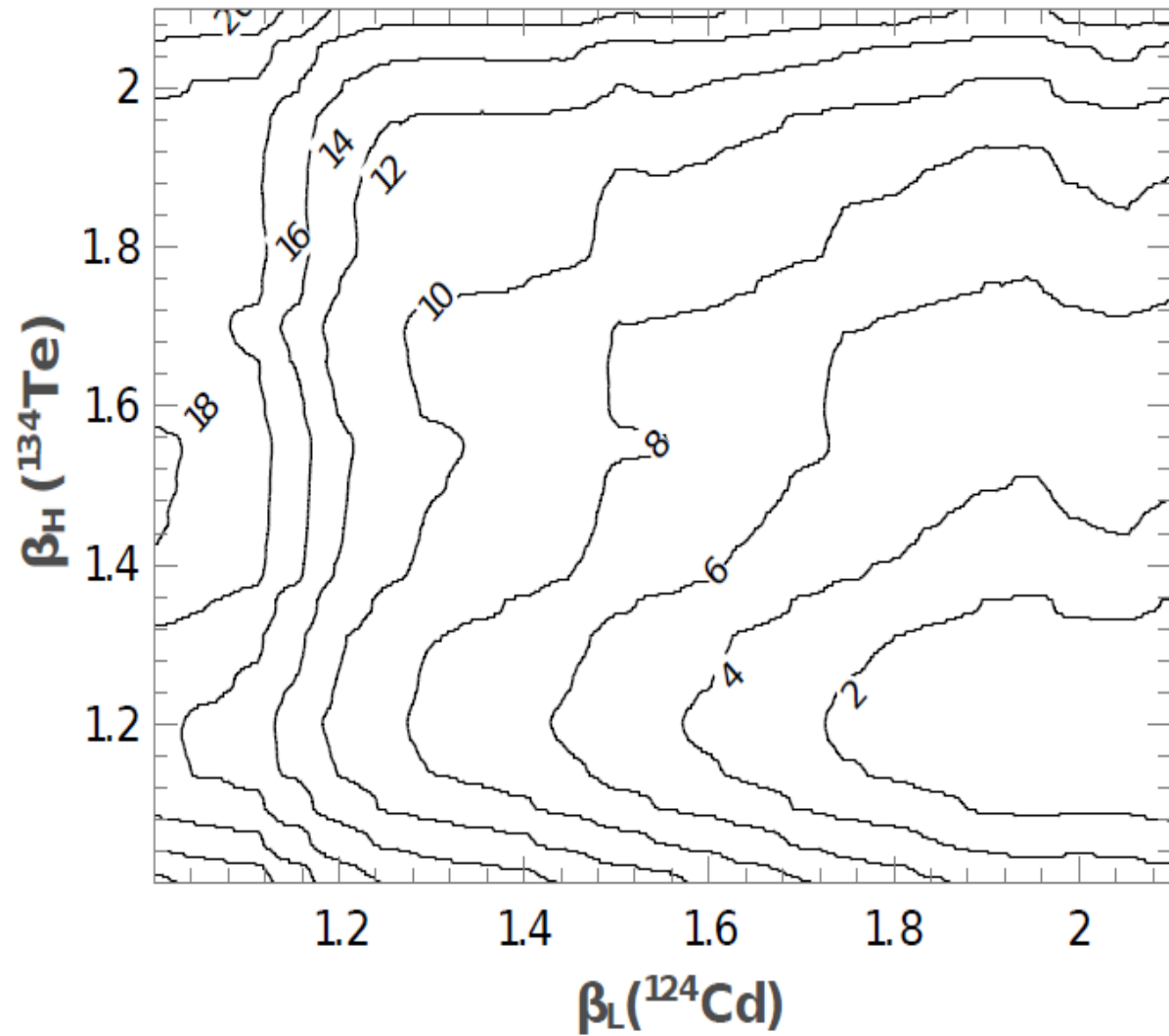
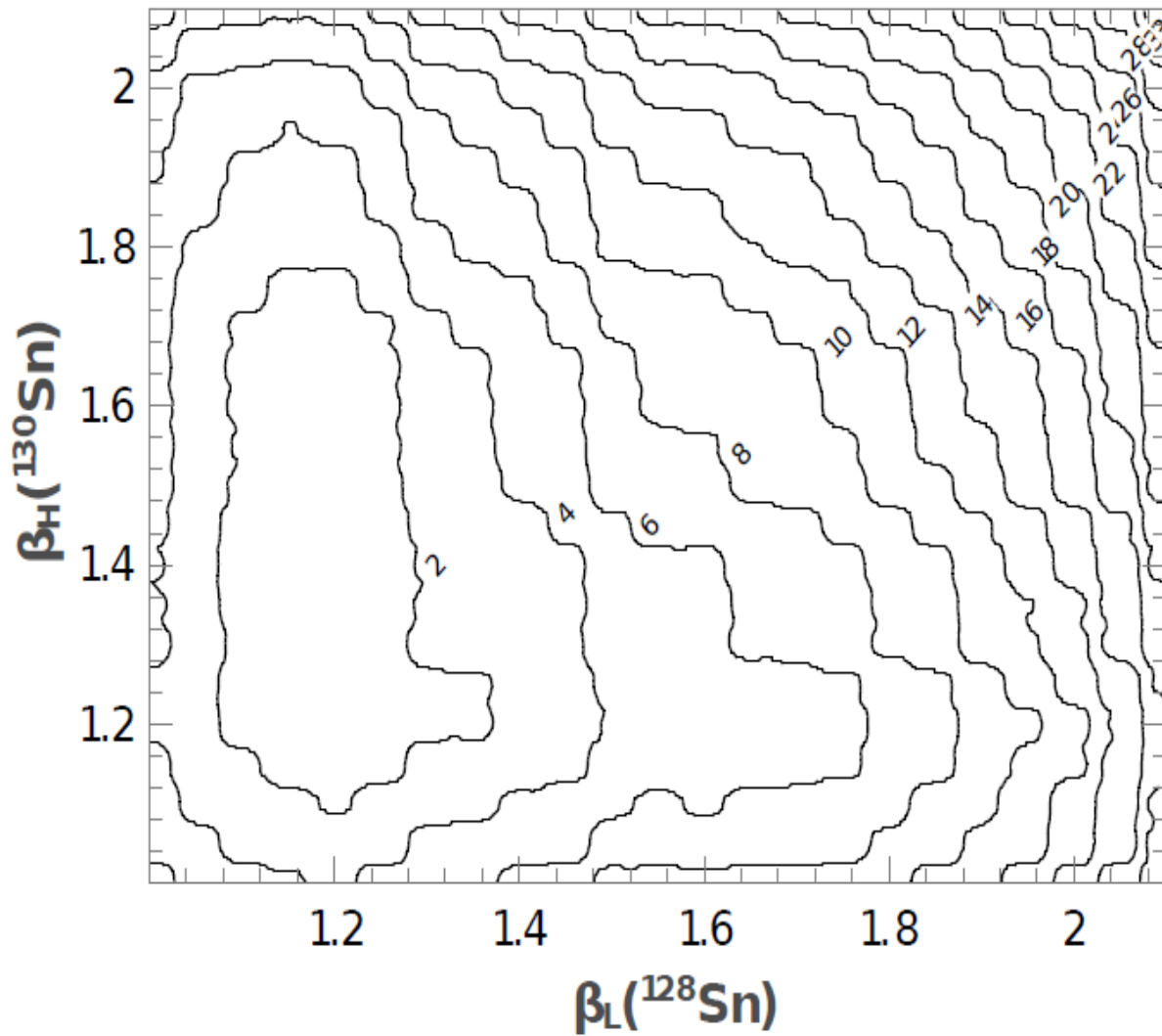
Focus: $^{258}\text{Fm}(\text{SF})$

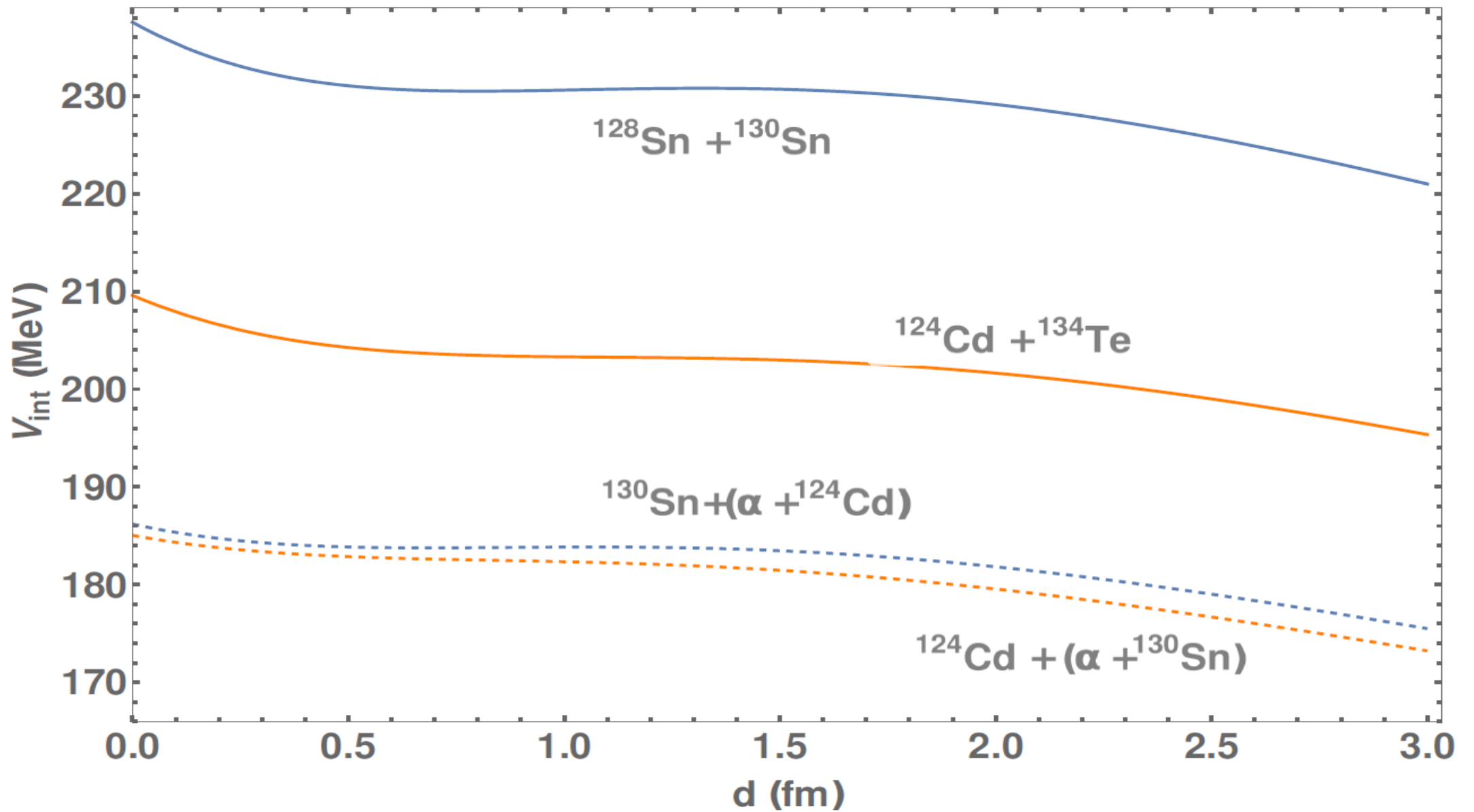


The most probable fragmentations

$^{128}\text{Sn} + ^{130}\text{Sn}$

$^{124}\text{Cd} + ^{134}\text{Te}$





One can explain the 2 peaks in the **TKE**, but the **mass yields** can not be described

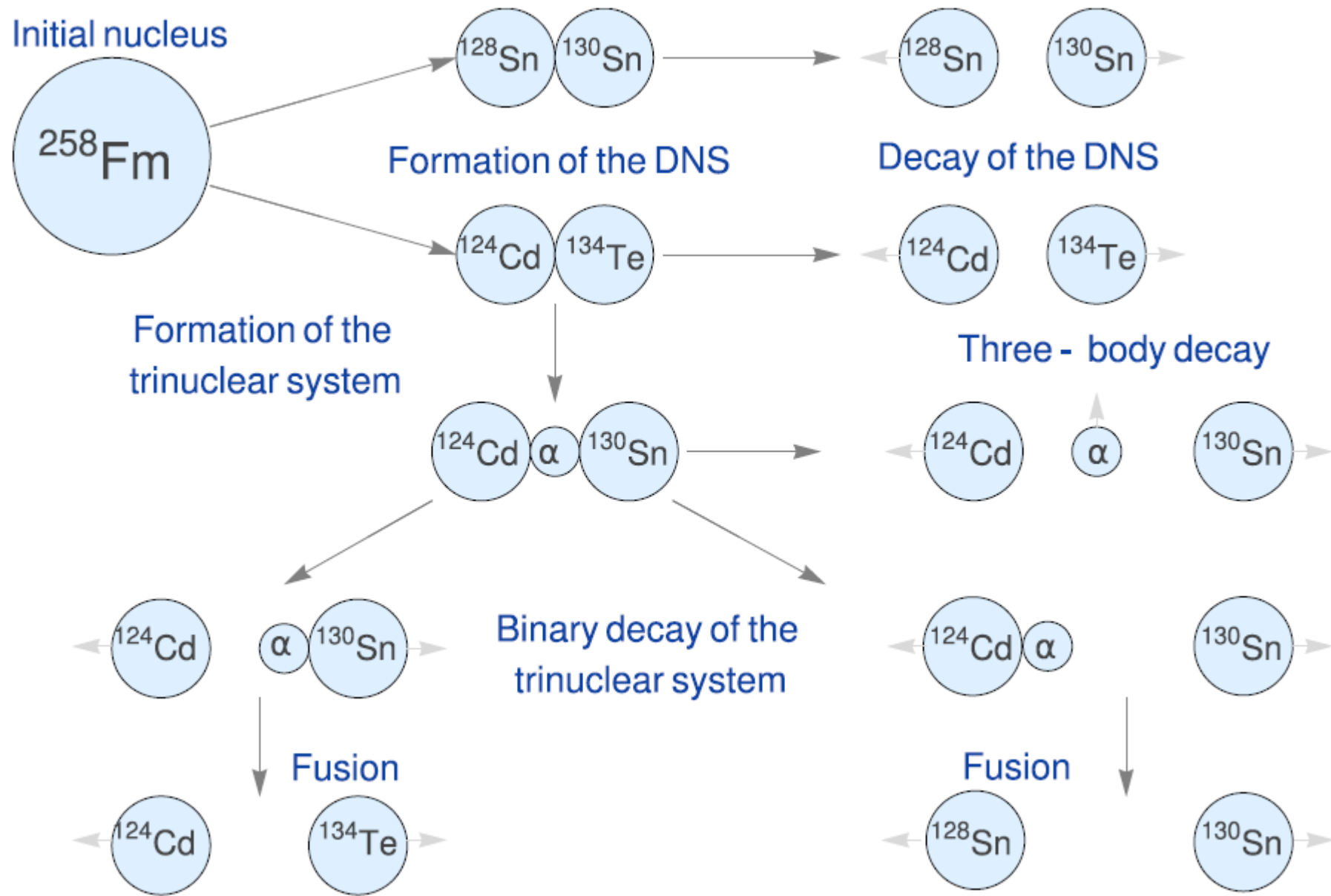
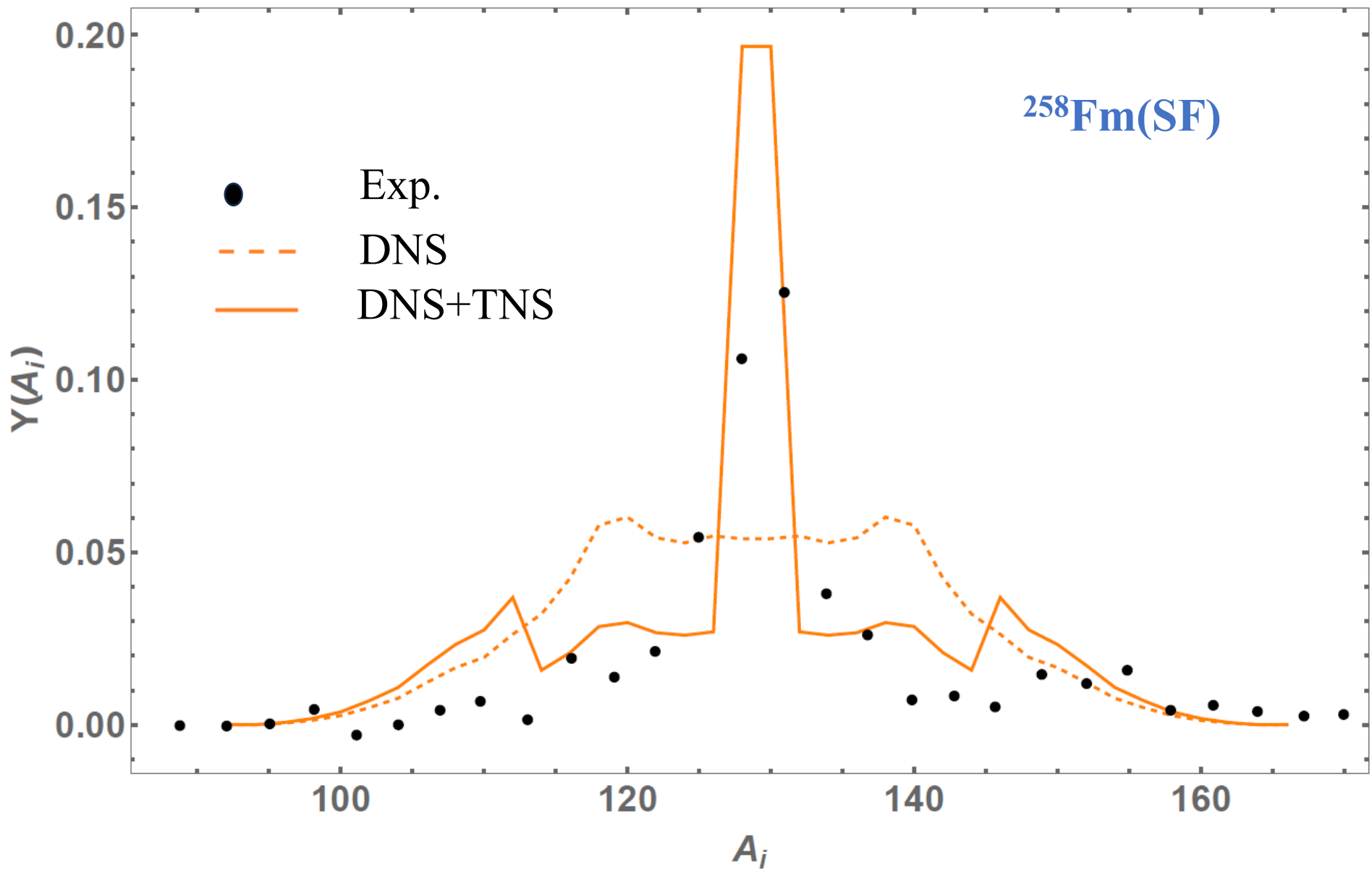
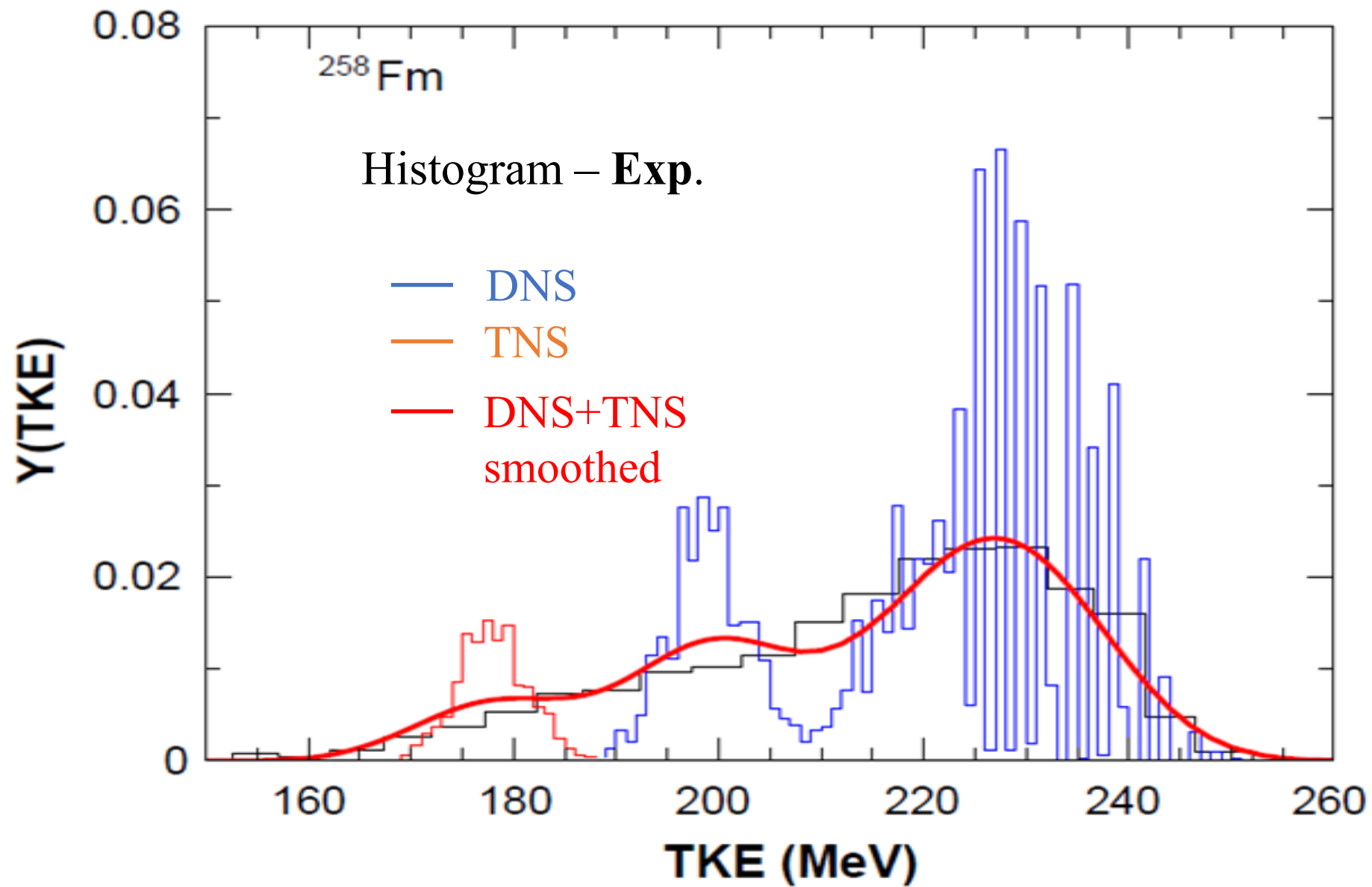


Figure schematically depicts the possible SF pathways

The formation of TNS from **Cd+Te** and its binary decay enhance the yields of **Sn** and decrease the yields of **Cd** and **Te**

Spectroscopic factor for ***α***-cluster is equal to **0.1**



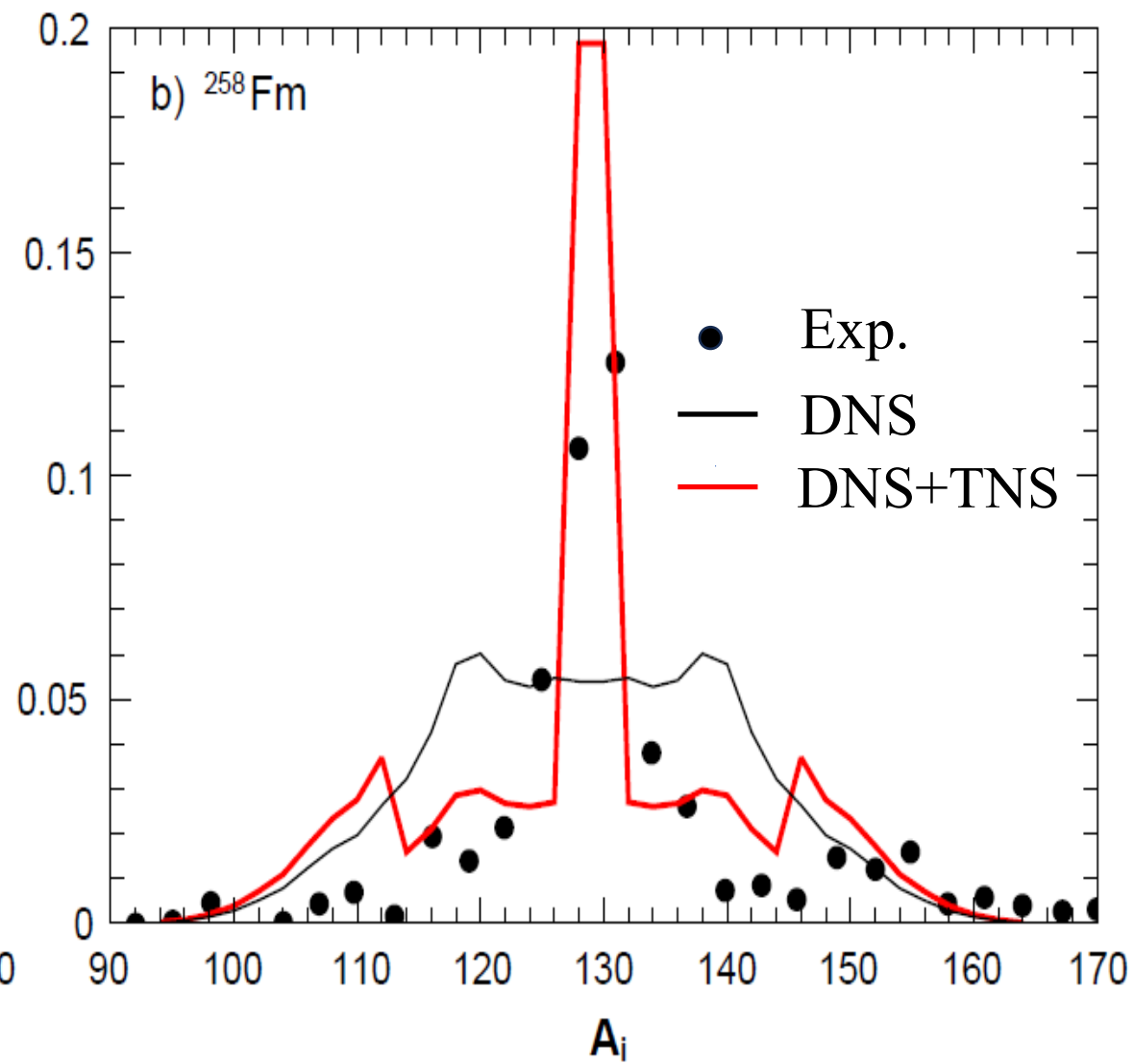
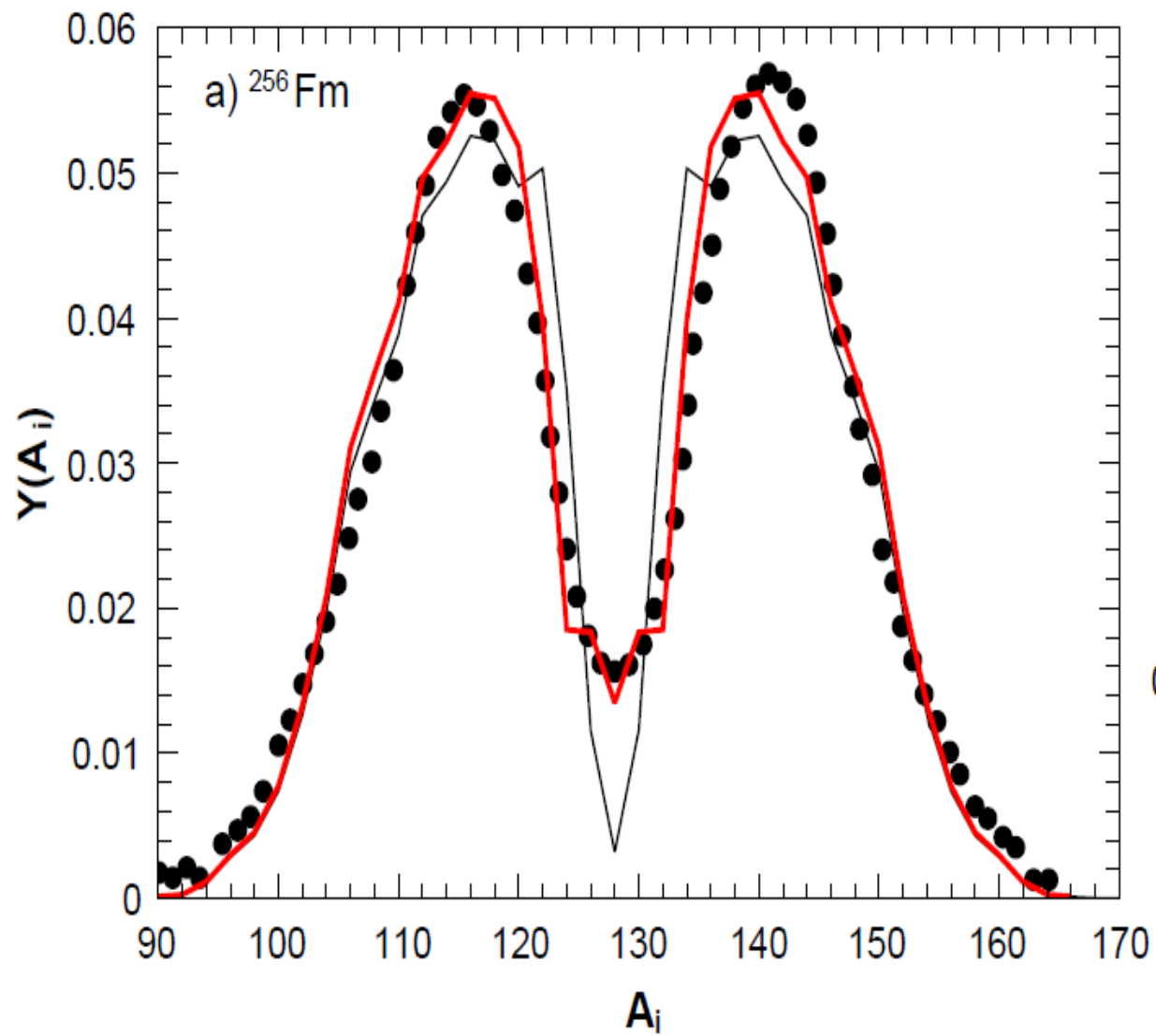


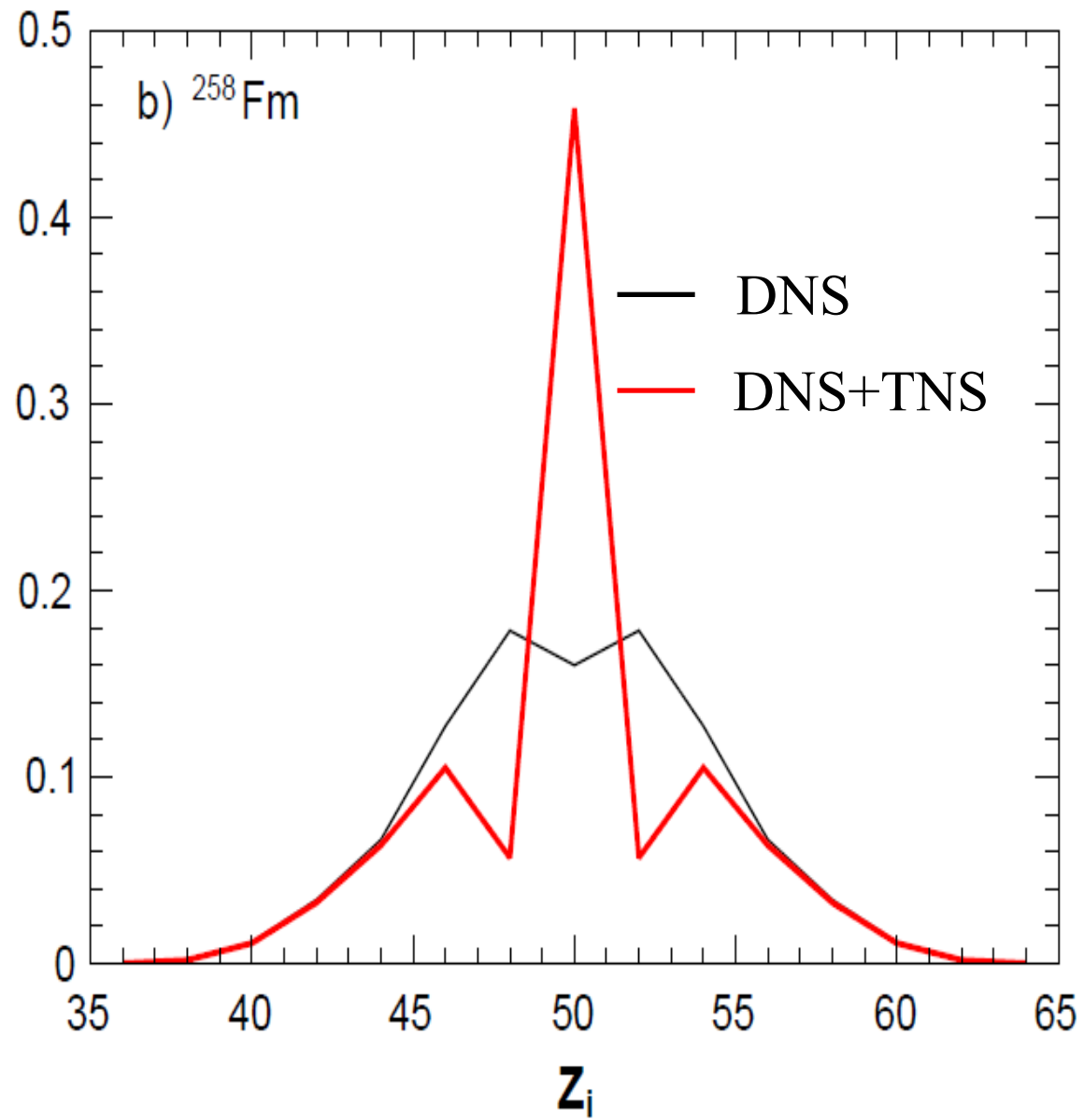
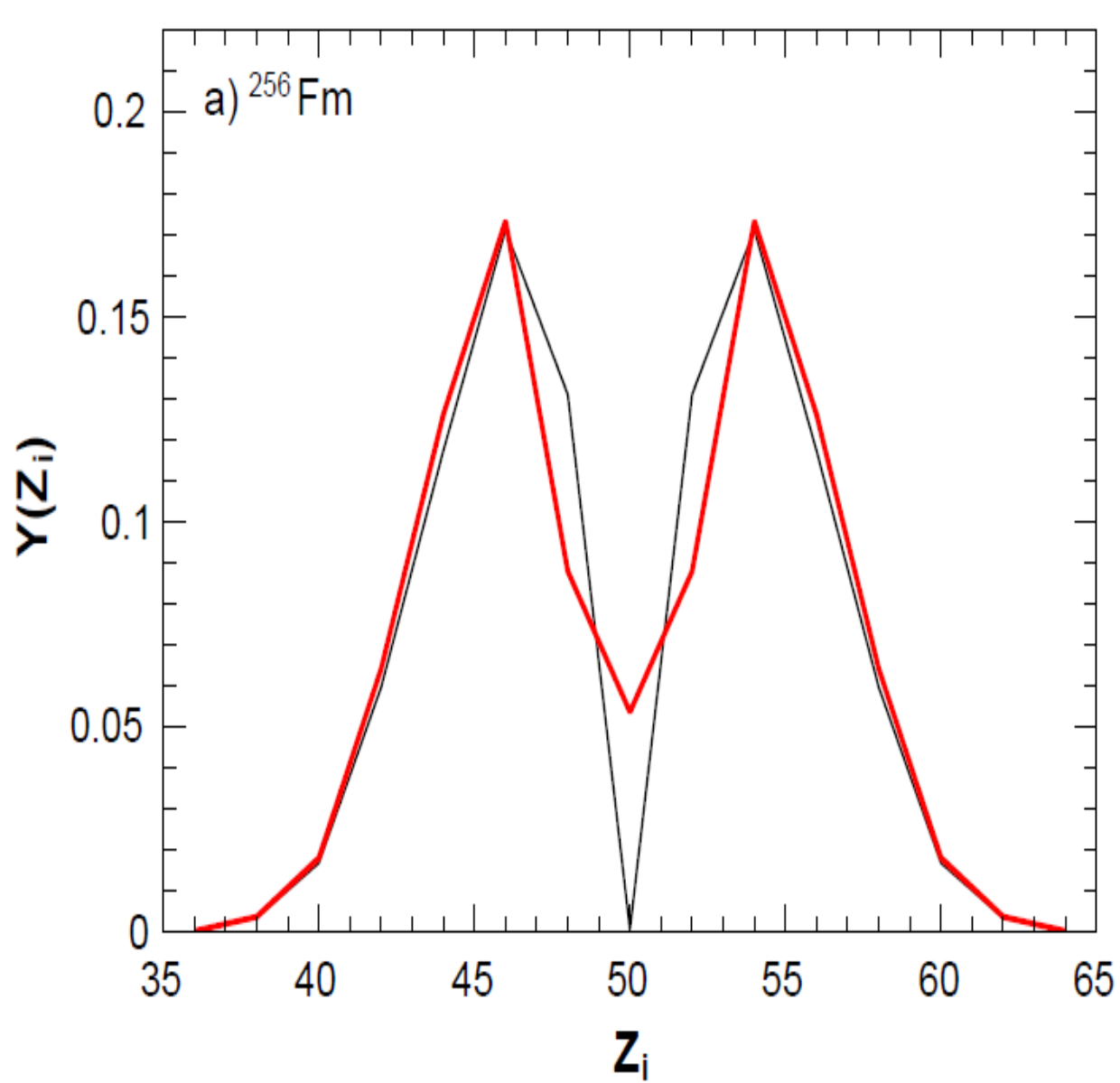
Trimodal structure of TKE distribution instead of bimodal one!

Is the α -clustering mechanism generally applicable in the SF of other actinides and super-heavies?

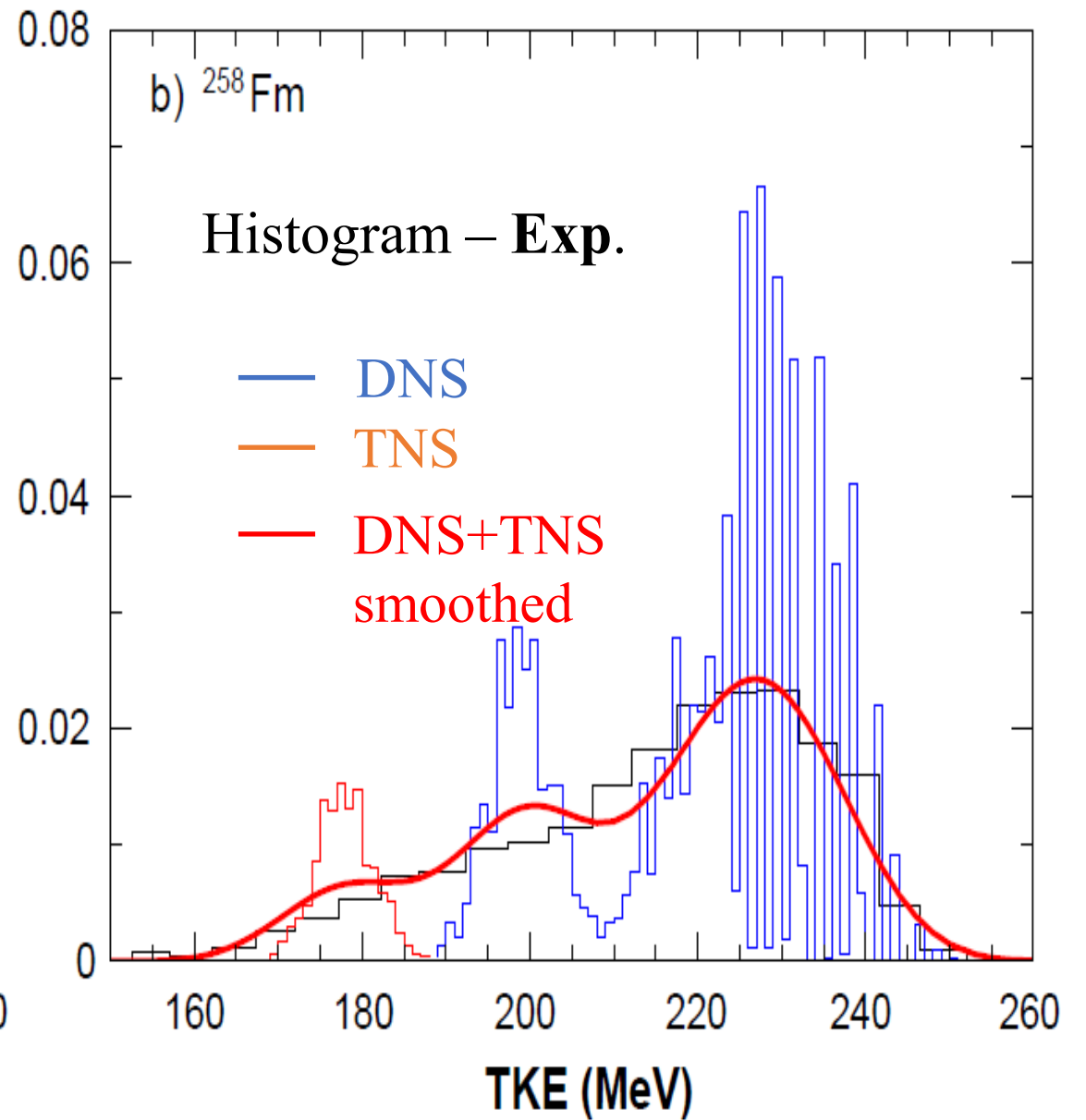
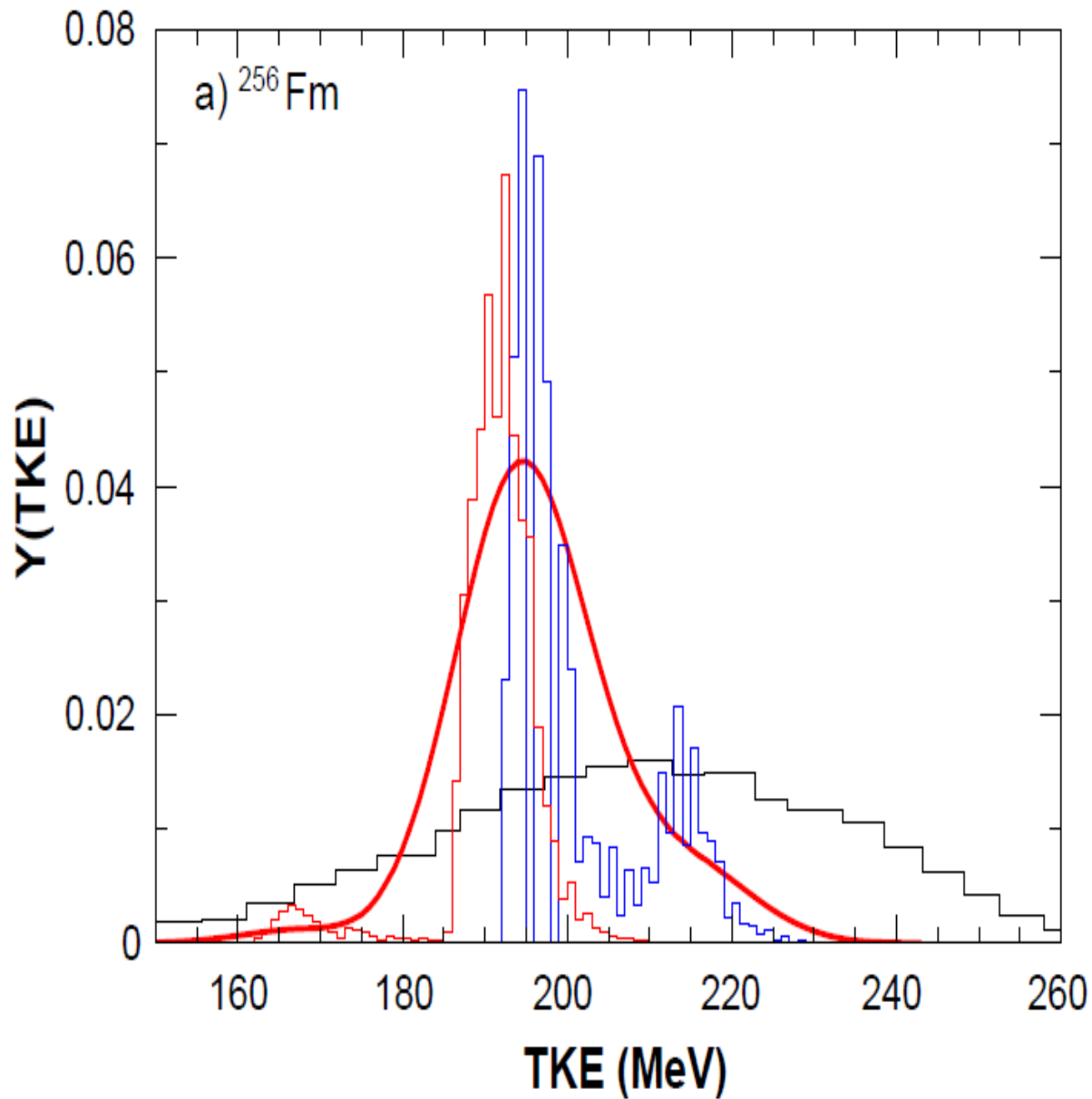
Focus: $^{256}\text{Fm}(\text{SF})$

The characteristics of SF of ^{256}Fm and ^{258}Fm are completely different!

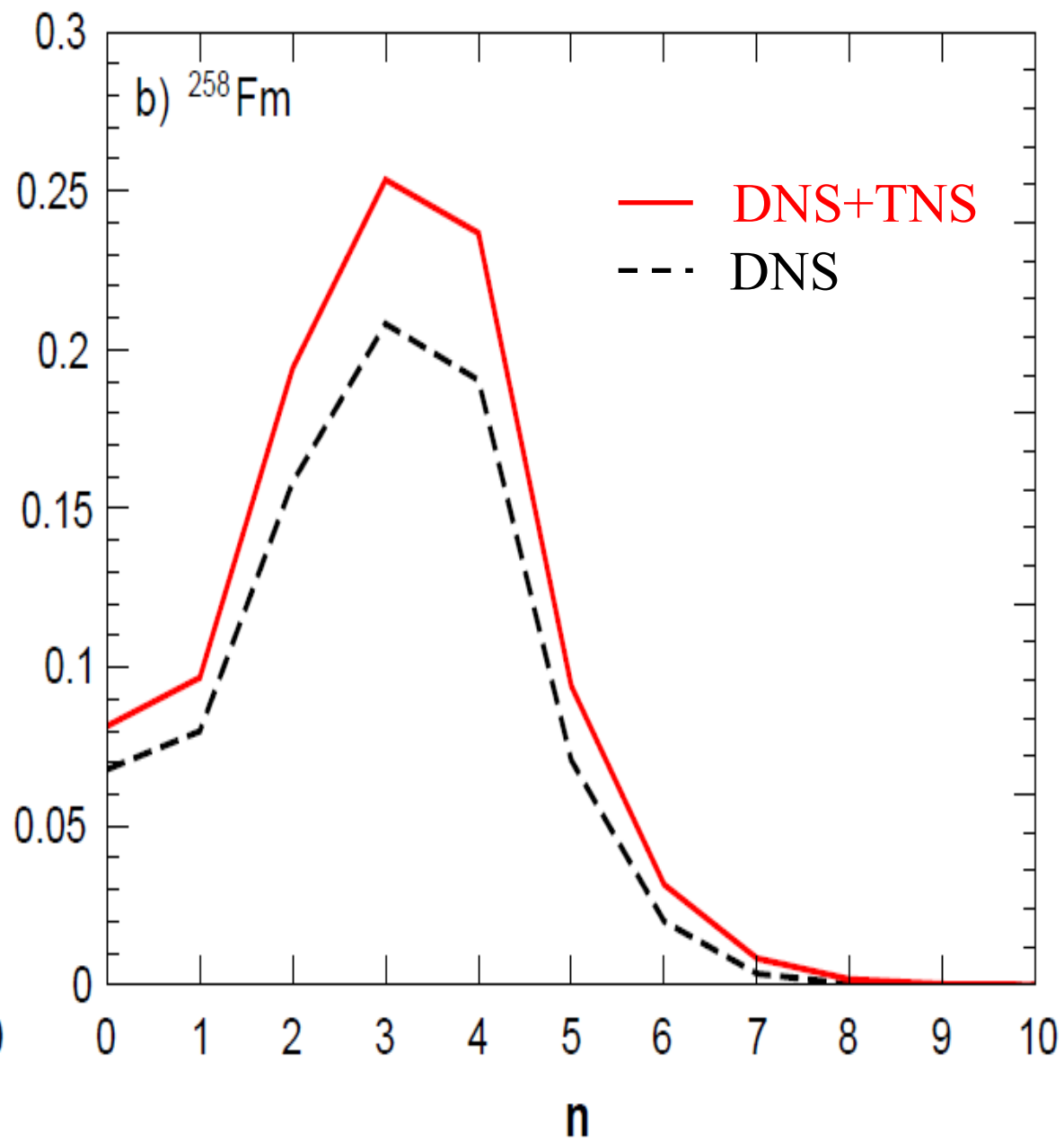
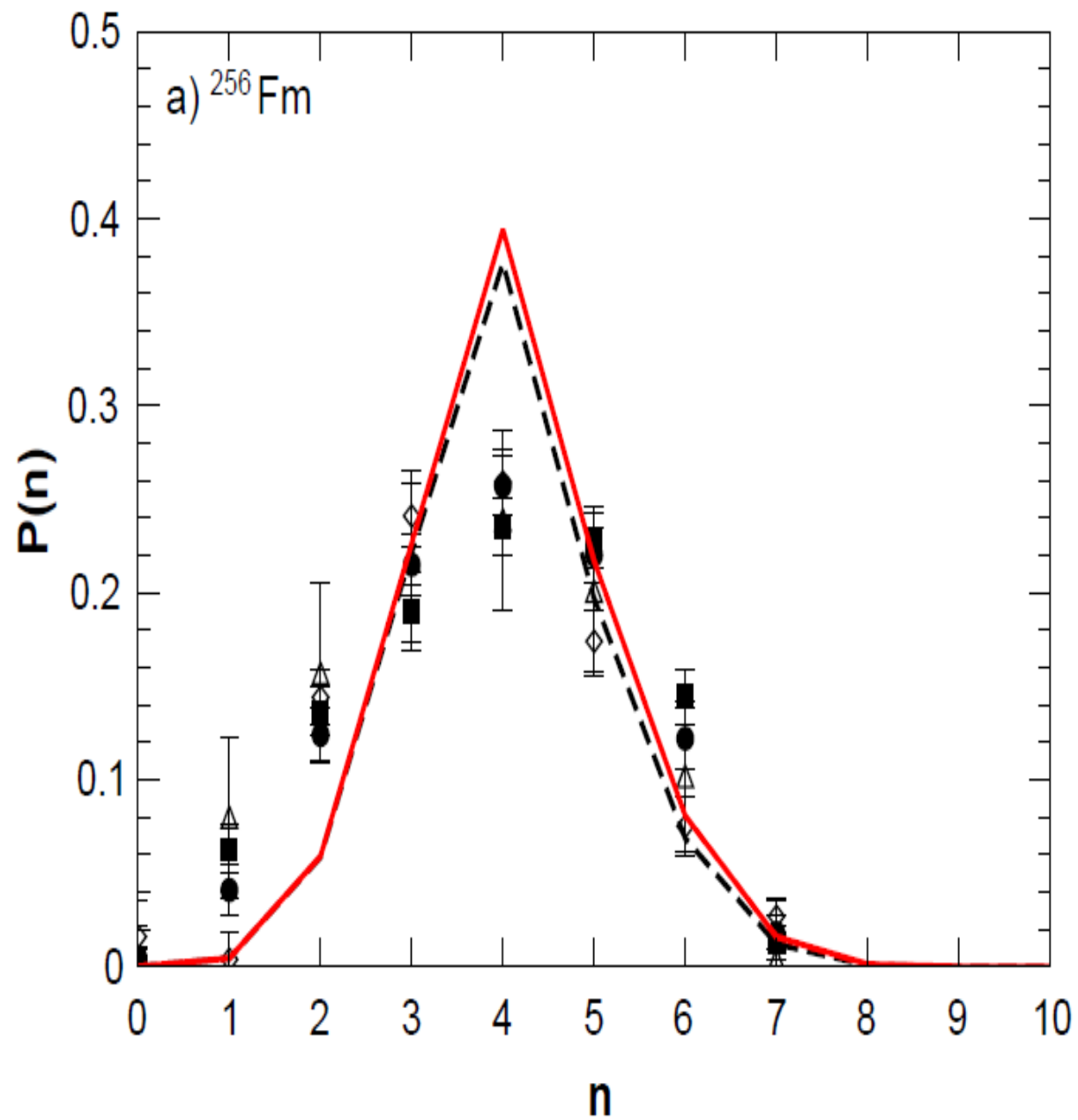




Predictions of charge yields



The large variation of **TKE** distribution is explained by the existence of binary fission pathways through TNS



The question arises whether heavier cluster than α -particle may play an important role in binary fission

Focus: $^{252}\text{Cf}(\text{SF})$

Focus: $^{252}\text{Cf}(\text{SF})$

Two fission modes?

Ba-Mo: 1st mode: $\langle TKE \rangle = 189 \pm 1 \text{ MeV}$

$\langle \nu \rangle = 3 - 4$

2nd mode: $\langle TKE \rangle = 153 \pm 3 \text{ MeV}$

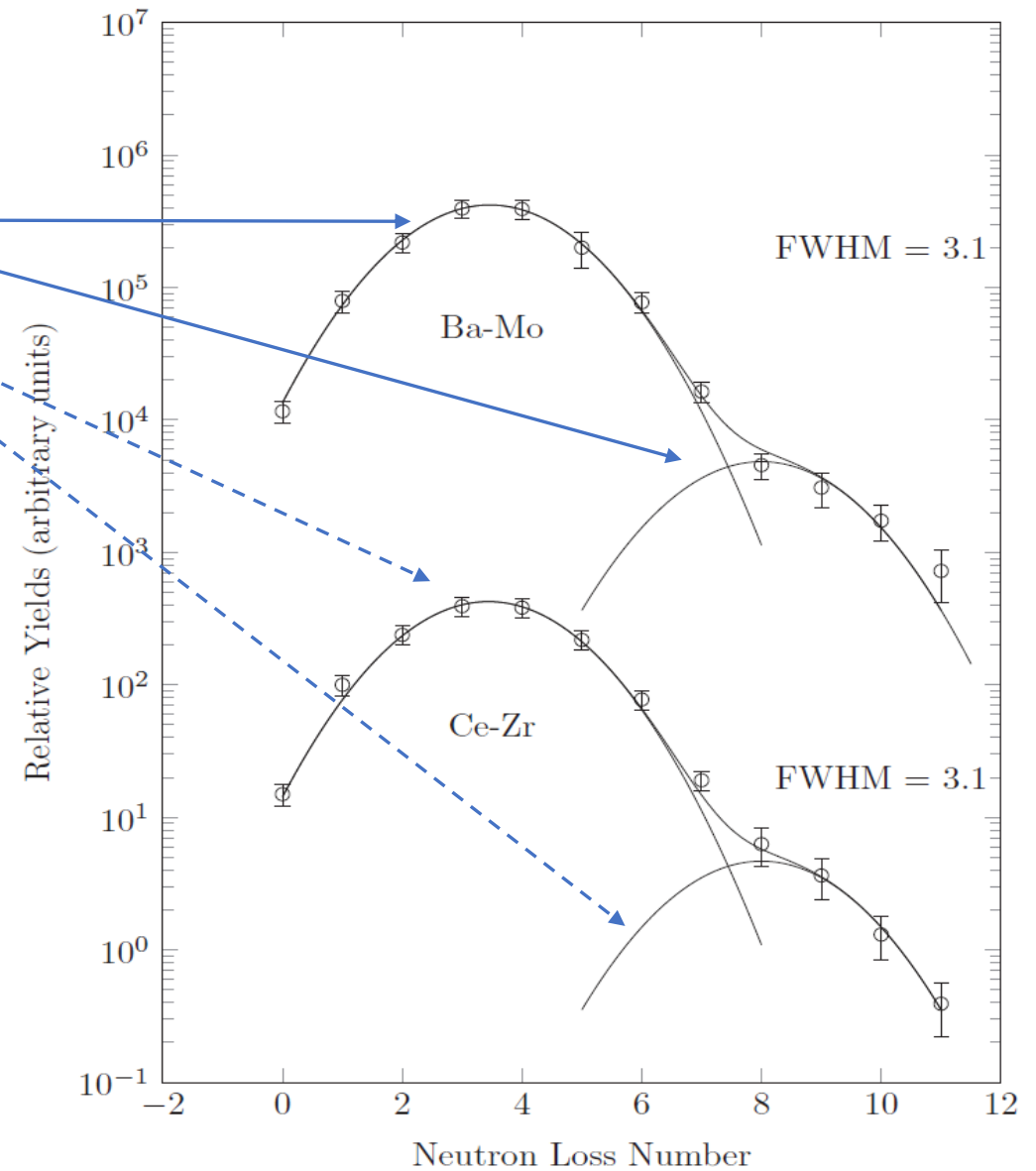
$\langle \nu \rangle \approx 8$

G.M. Ter-Akopian et al., *PRC55*, 1146 (1997): “(. . .) data clearly indicate that most of the neutrons in the high neutron emission events come out of the **Ba** fragments-not the **Mo** fragments.”

Theory: for **Ba-Mo** TKE ($\beta_L = \beta_H = 2$) = 173.4 MeV

and for **Ce-Zr** TKE ($\beta_L = \beta_H = 2$) = 171.0 MeV

To obtain $TKE \sim 153 \text{ MeV}$ one needs $\beta_{\text{Ba,Mo}} > 3$
(*PRC55*, 1146 (1997)) – unrealistic!



B.M. Musangu et al., *PRC 101*, 034610 (2020)

Maximum excitation energy:

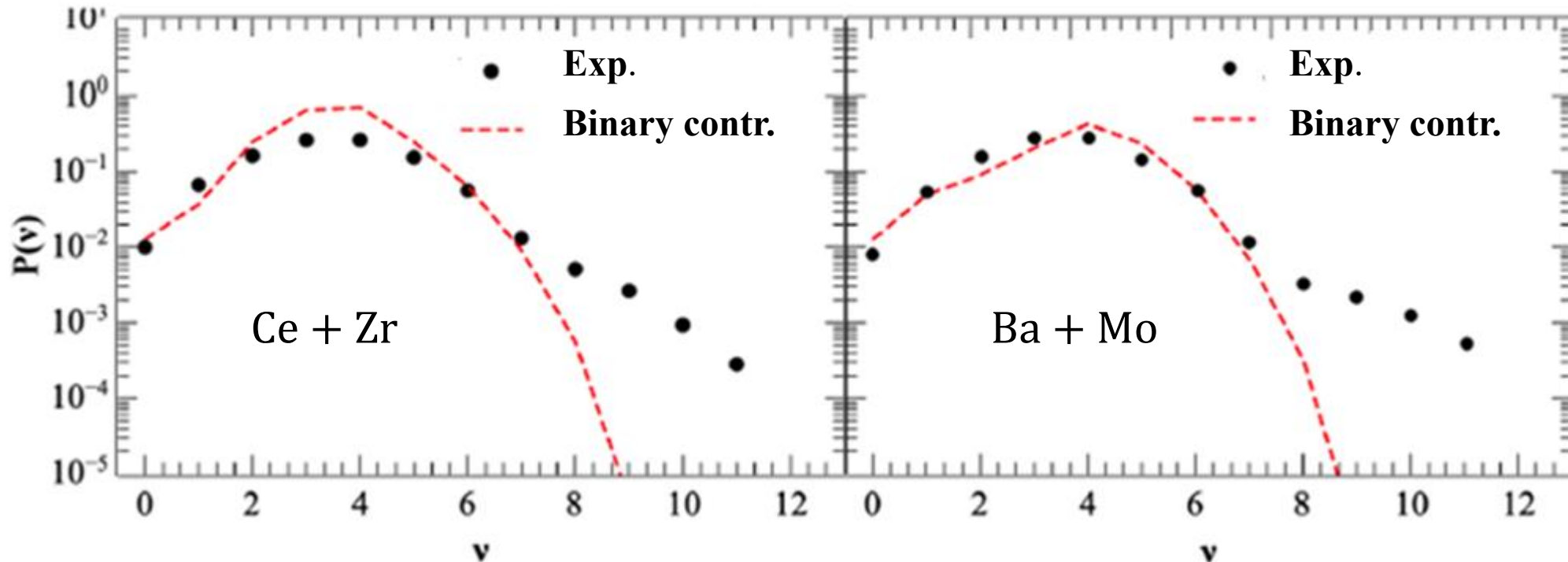
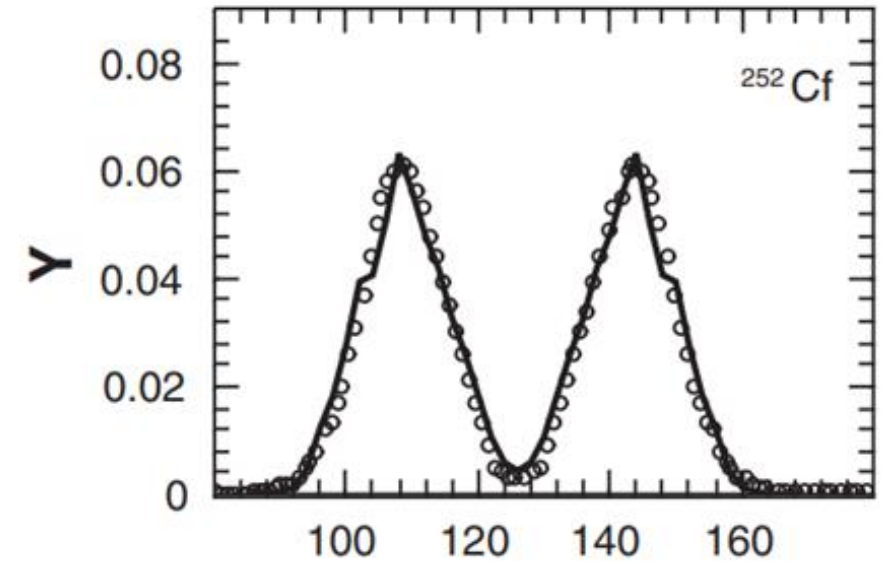
$$\varepsilon_{\text{Ba}}^* = E_{\text{Ba}}^* + U_{\text{Ba}}^{\text{def}} = 28.1 \text{ MeV}$$

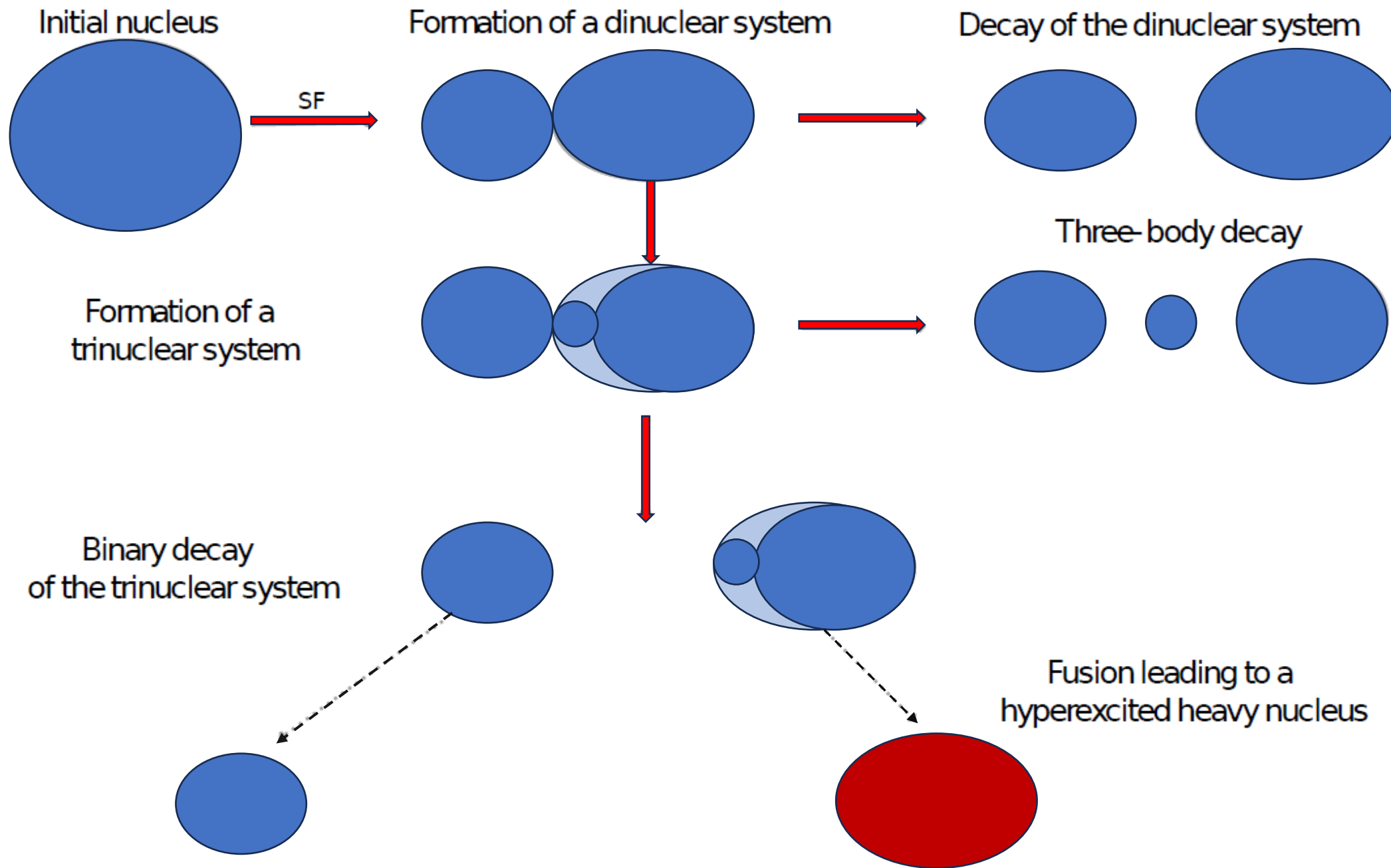
$$\varepsilon_{\text{Ce}}^* = E_{\text{Ce}}^* + U_{\text{Ce}}^{\text{def}} = 24.1 \text{ MeV}$$

Theory: $\langle TKE \rangle = 192.4 \text{ MeV}$ (Ba – Mo)

$\langle TKE \rangle = 189.1 \text{ MeV}$ (Ce – Zr)

$\langle \nu \rangle \simeq 3.5$





Two important aspects are:

1. The reduction of the interaction potential V after the formation of a TNS. The decrease in the V is restored as the internal excitation energy of system.
2. Complete fusion Q -value gives contribution to the internal excitation energy.

What kind of clusters can be formed?

Competing mechanism with **true ternary fission**.

Idea supported by: I. Tsekhanovich et al. PRC**67**, 034610 (2003)

Absolute yields of third clusters in $^{249}\text{Cf}(n_{th}, f)$:

Cluster	Yield
^{10}Be	$(3.8 \pm 0.7) \times 10^{-5}$
^{14}C	$(1.3 \pm 0.2) \times 10^{-5}$
^{20}O	$(2.5 \pm 0.7) \times 10^{-6}$
^{24}Ne	$(2.4 \pm 0.6) \times 10^{-7}$

Probability for formation of ternary system (TNS)

The probability of emitting exactly ν neutrons contains DNS and TNS contributions:

$$P(\nu) = \lambda_d P_\nu(A_i, Z_i, \beta_i, \varepsilon^*) + \lambda_t P_\nu(A_i, Z_i, \beta_i, \varepsilon^{**})$$

formation probability of DNS formation probability of TNS

$$P_\nu(A_i, Z_i, \beta_i, \varepsilon) = \sum_{\nu_L=0}^{\nu} \int_0^{\varepsilon - U_H^{def} - U_L^{def}} dE P_c(E) P_{\nu_L}(U_L^{def} + E) P_{\nu - \nu_L}(\varepsilon - U_L^{def} - E)$$

The probability of emitting the third cluster is the product of the probabilities for formation of ternary system λ_t and three body decay.

λ_t = the experimental probability of emitting the cluster / the probability of 3-body decay.

Focus: ^{252}Cf

Final excitation energy of the heavy fragment

is: $\varepsilon_H^{**} = \varepsilon_H^* + \Delta V - Q$

where $\varepsilon_H^* = E_H^* + U_H^{def}$

ΔV – the reduction of the interaction potential due to third cluster

Q is the fusion Q –value

Large variations in ΔV , especially for heavy third cluster.

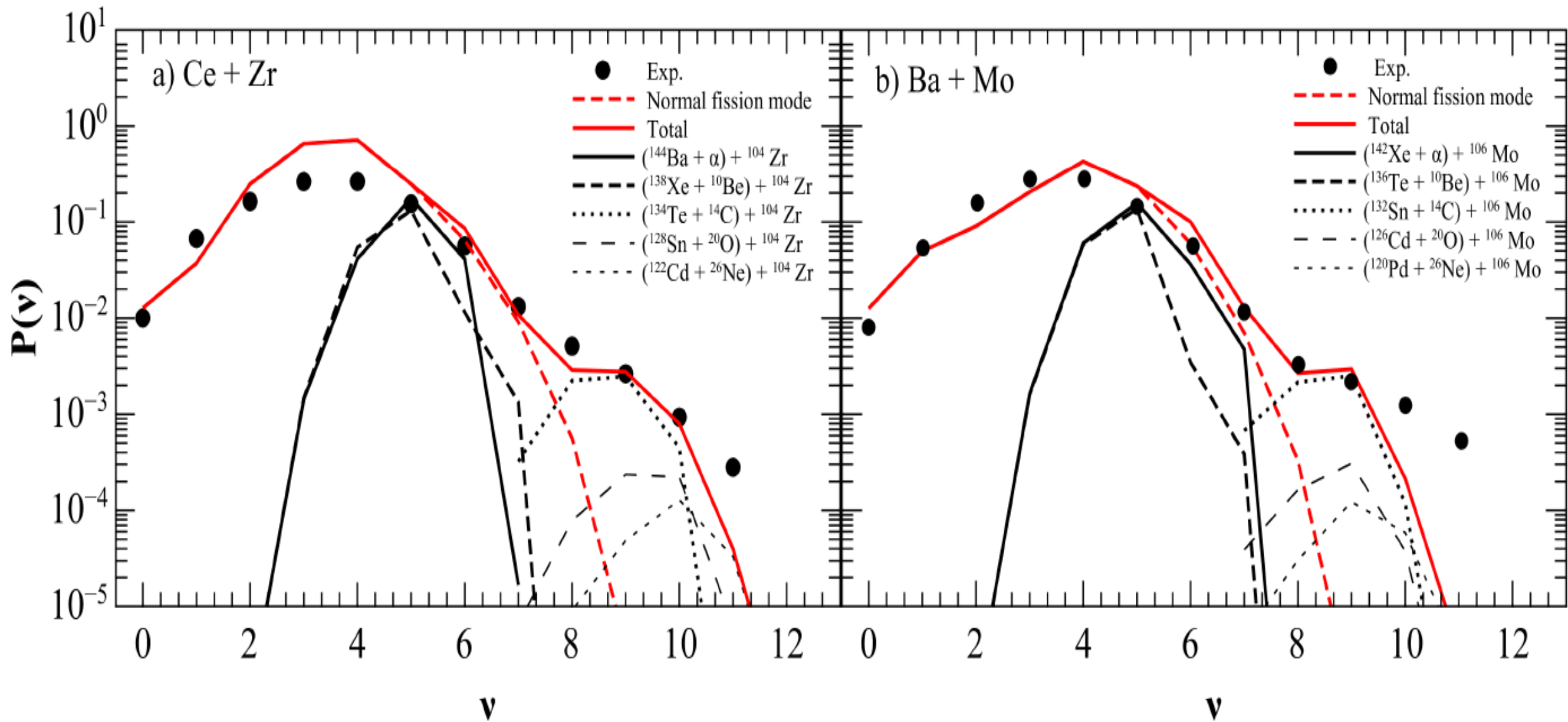
Large negative Q – values enhance ε_H^{**} , especially for heavy third cluster

Extremely large excitation energy for the heavy fragment after cluster fusion

$$^{148}\text{Ce} + ^{104}\text{Zr} \quad \varepsilon_H^* = 24.1 \text{ MeV}$$

$$^{146}\text{Ba} + ^{106}\text{Mo} \quad \varepsilon_H^* = 28.1 \text{ MeV}$$

$^{148}\text{Ce} + ^{104}\text{Zr}$	ΔV	Q	ε_H^{**}	TKE
$(^{144}\text{Ba} + \alpha) + ^{104}\text{Zr}$	8.3	1.1	31.3	162.0
$(^{138}\text{Xe} + ^{10}\text{Be}) + ^{104}\text{Zr}$	13.9	2.9	35.1	140.5
$(^{134}\text{Te} + ^{14}\text{C}) + ^{104}\text{Zr}$	30.8	-9.2	64.0	150.3
$(^{128}\text{Sn} + ^{20}\text{O}) + ^{104}\text{Zr}$	39.6	-9.2	72.7	153.6
$(^{122}\text{Cd} + ^{26}\text{Ne}) + ^{104}\text{Zr}$	47.4	-9.9	81.4	144.5
$(^{124}\text{Cd} + ^{24}\text{Ne}) + ^{104}\text{Zr}$	45.8	-11.0	80.9	144.2
$^{146}\text{Ba} + ^{106}\text{Mo}$	ΔV	Q	ε_H^{**}	TKE
$(^{142}\text{Xe} + \alpha) + ^{106}\text{Mo}$	12.7	2.0	38.8	166.6
$(^{136}\text{Te} + ^{10}\text{Be}) + ^{106}\text{Mo}$	13.7	3.2	38.6	144.4
$(^{132}\text{Sn} + ^{14}\text{C}) + ^{106}\text{Mo}$	30.5	-8.5	67.1	155.6
$(^{126}\text{Cd} + ^{20}\text{O}) + ^{106}\text{Mo}$	39.0	-3.5	70.6	160.6
$(^{120}\text{Pd} + ^{26}\text{Ne}) + ^{106}\text{Mo}$	45.0	-4.7	77.8	157.1
$(^{122}\text{Pd} + ^{24}\text{Ne}) + ^{106}\text{Mo}$	44.8	-5.6	78.5	158.0



Ba+Mo:

Total kinetic energy:

1st mode: $\text{TKE}_{\text{exp}} = 189 \pm 1 \text{ MeV}, \text{TKE}_{\text{th}} = 192 \text{ MeV},$

2nd mode: $\text{TKE}_{\text{exp}} = 153 \pm 3 \text{ MeV}, \text{TKE}_{\text{th}} = 156 \text{ MeV},$

Neutron multiplicity:

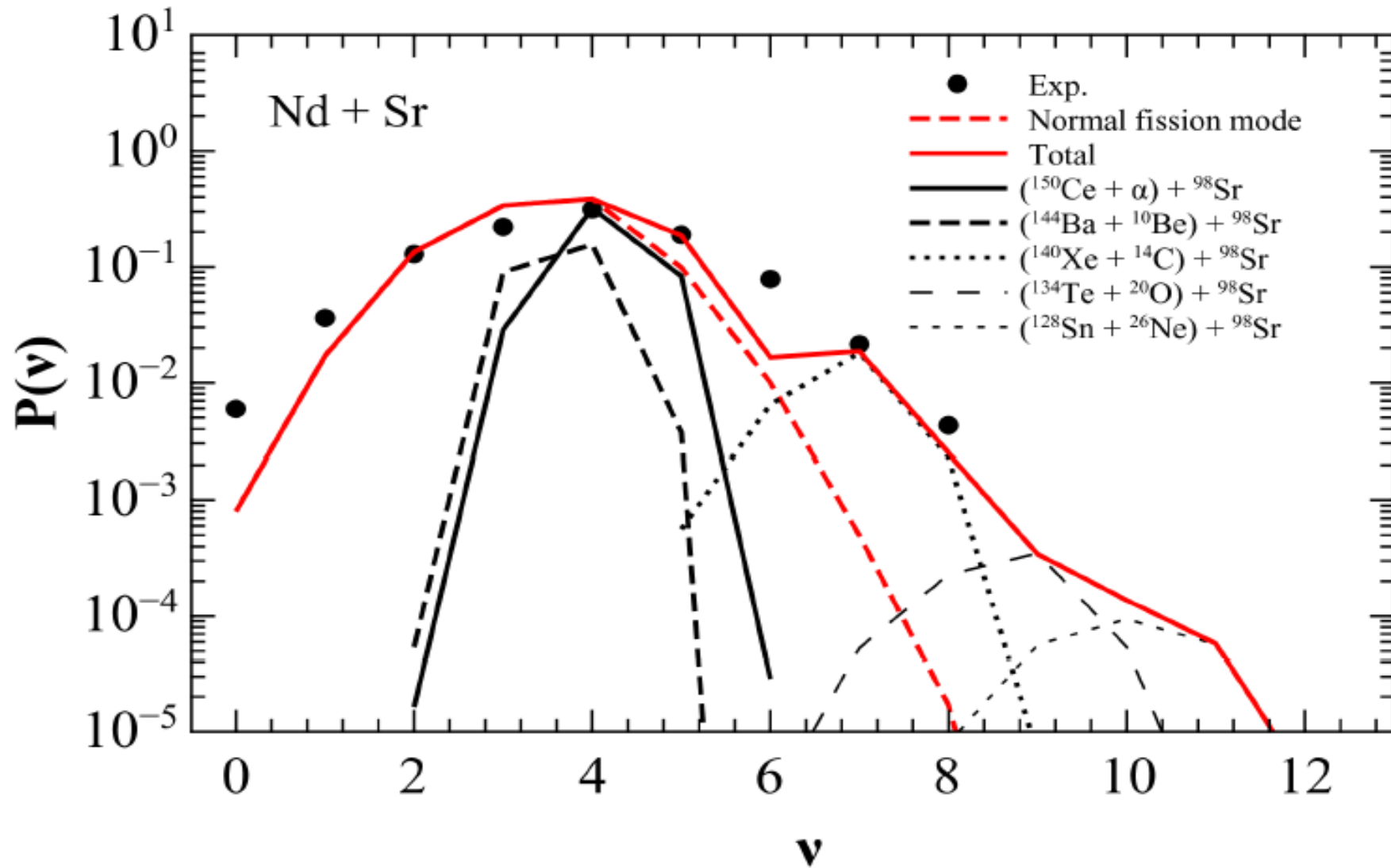
$$\langle \nu \rangle_{\text{exp}} = 3 - 4, \langle \nu \rangle_{\text{th}} = 3.5$$

$$\langle \nu \rangle_{\text{exp}} = 8, \langle \nu \rangle_{\text{th}} = 9$$

Excitation energy:

$$\left(\frac{E_{\text{Ba},\text{mode}2}^*}{E_{\text{Ba},\text{mode}1}^*} \right)_{\text{exp}} = 2.6$$

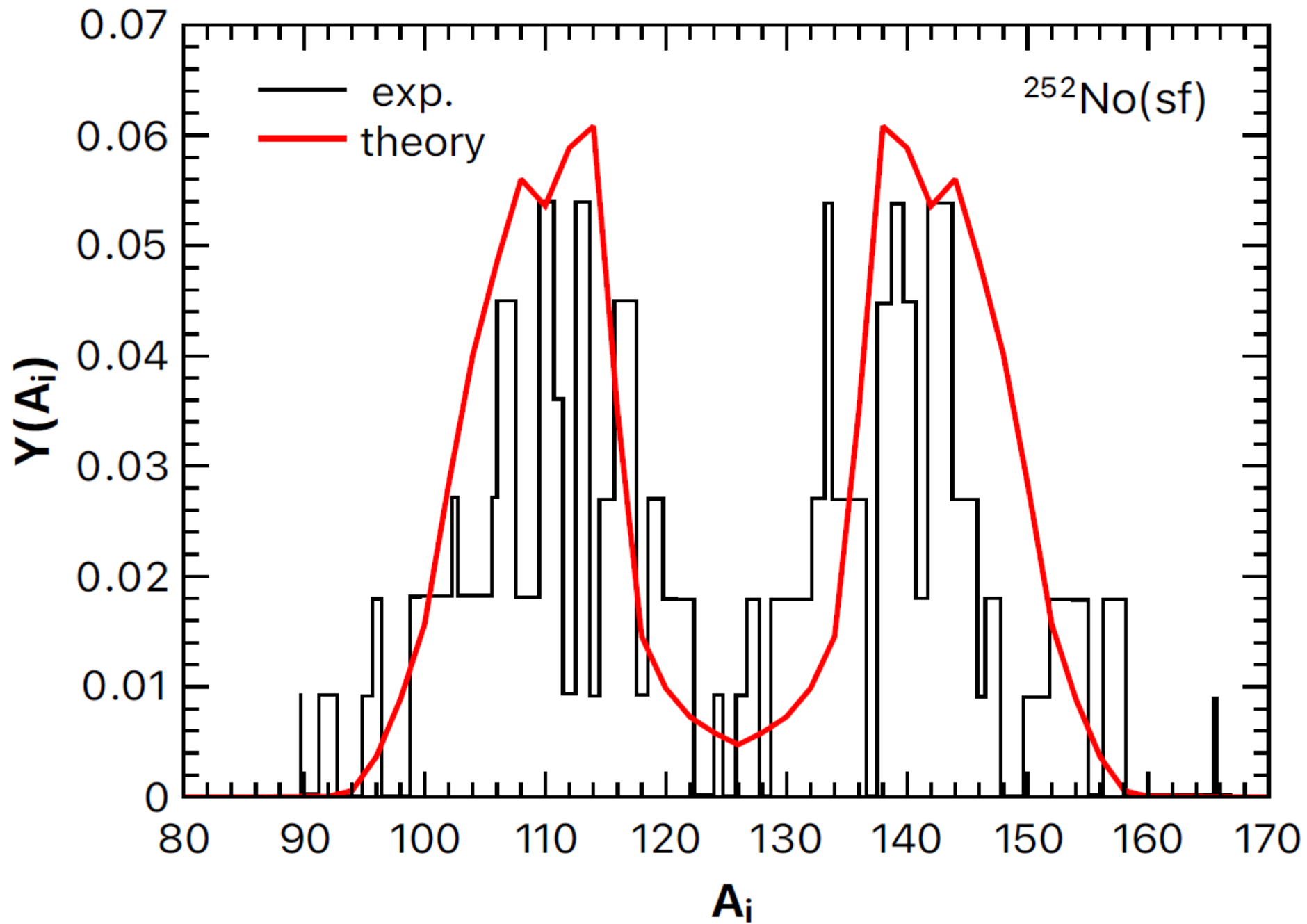
$$\left(\frac{E_{\text{Ba},\text{mode}2}^*}{E_{\text{Ba},\text{mode}1}^*} \right)_{\text{th}} = 2.4$$



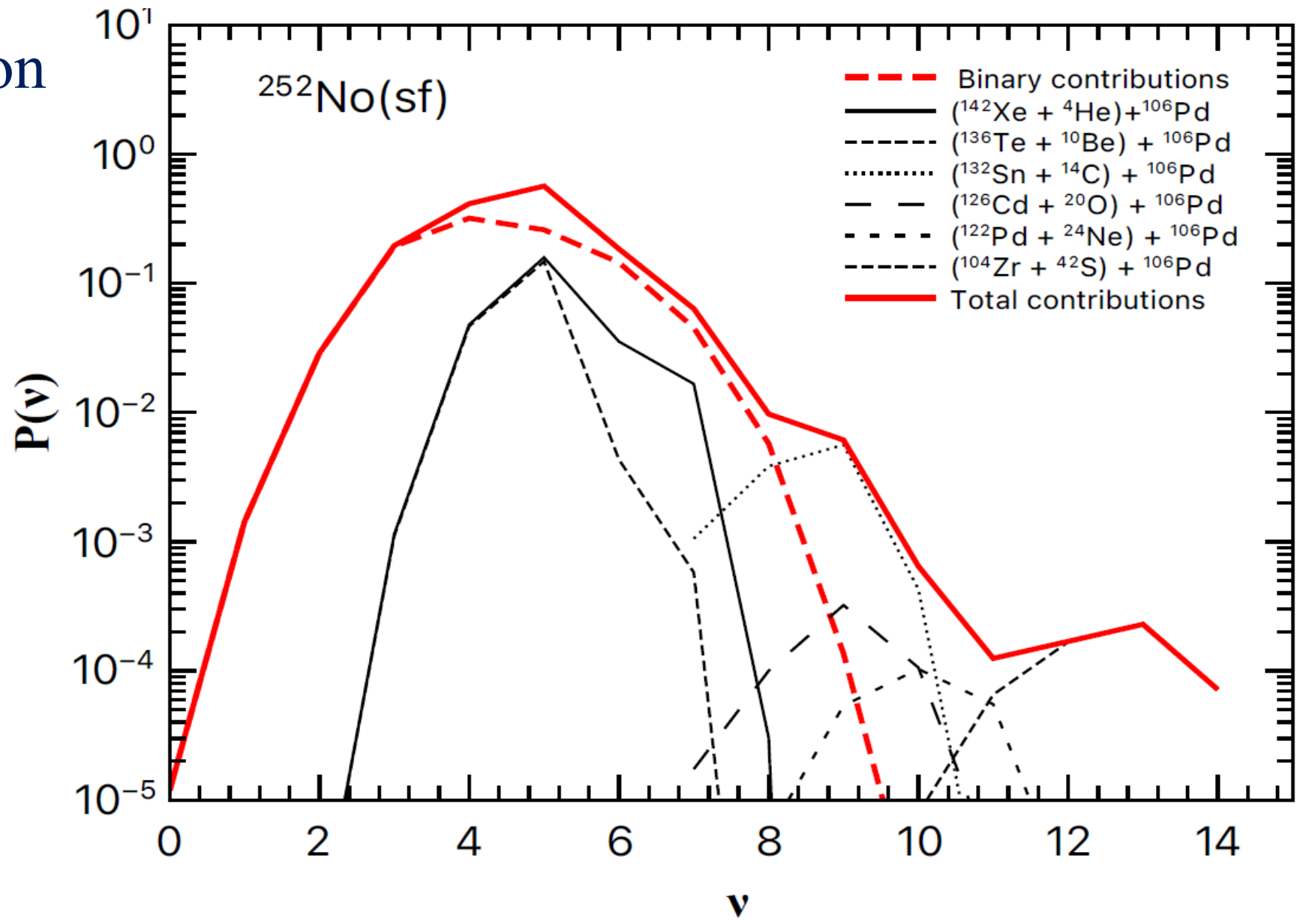
Because $\varepsilon_{\text{Nd}}^{**}(\text{C}) = 50.9 \text{ MeV}$ is smaller than $\varepsilon_{\text{Ba}}^{**}(\text{C}) = 67.1 \text{ MeV}$ and $\varepsilon_{\text{Ce}}^{**}(\text{C}) = 64 \text{ MeV}$, the contributions from normal and abnormal SF modes overlap.

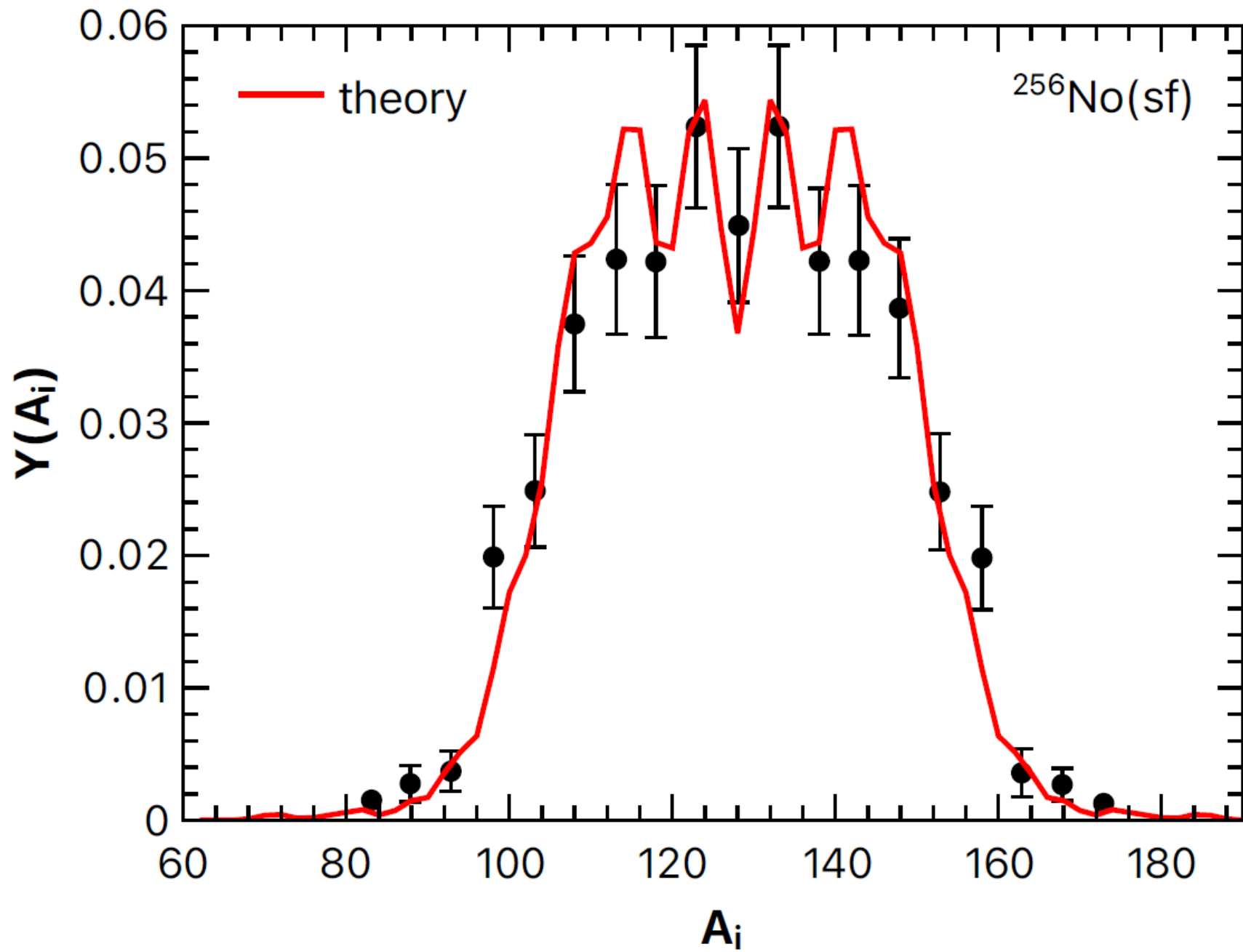
Can an abnormal mode exist in other nuclei?

Focus: $^{252,256}\text{No}(\text{SF})$

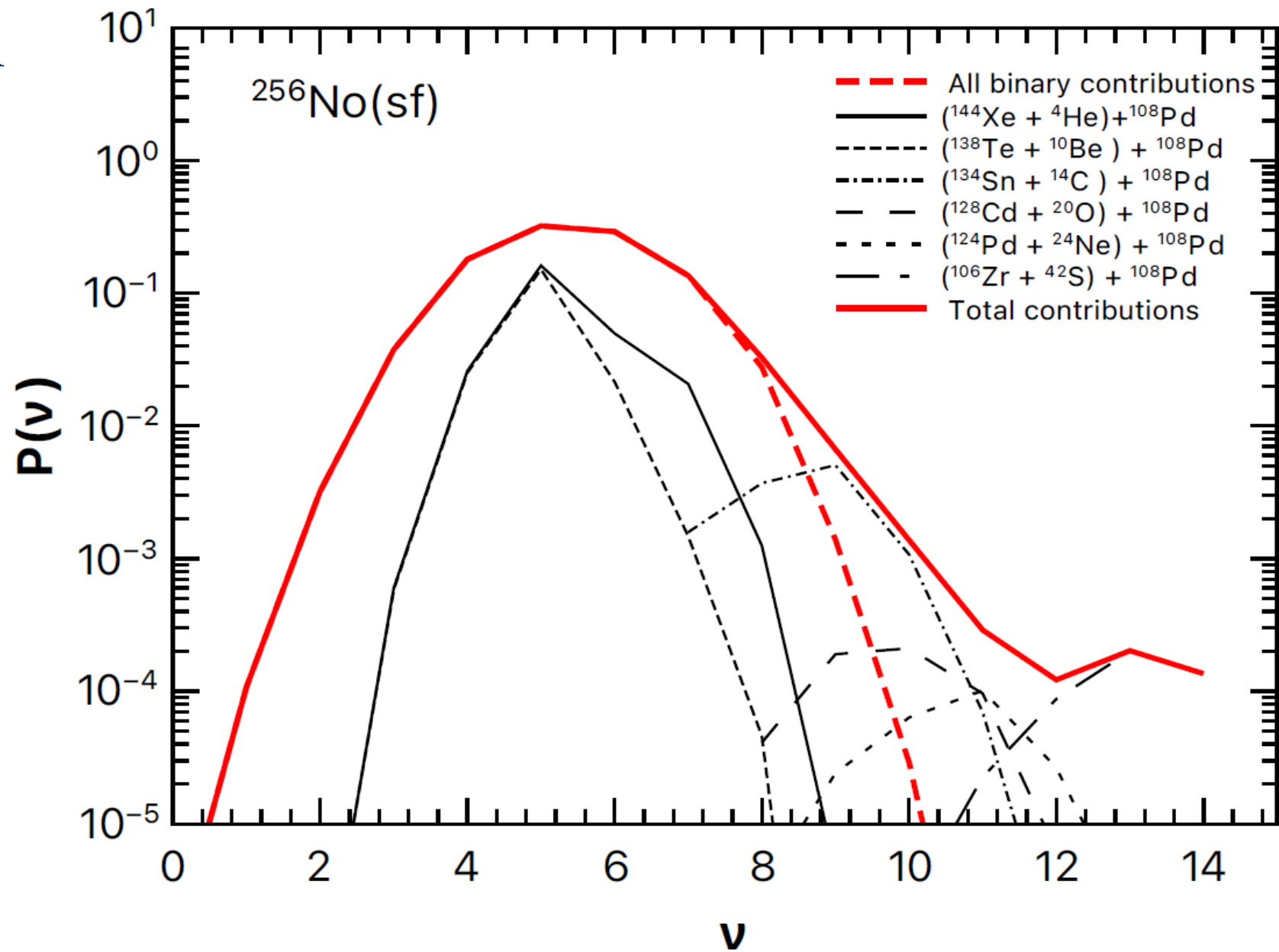


Prediction





Prediction



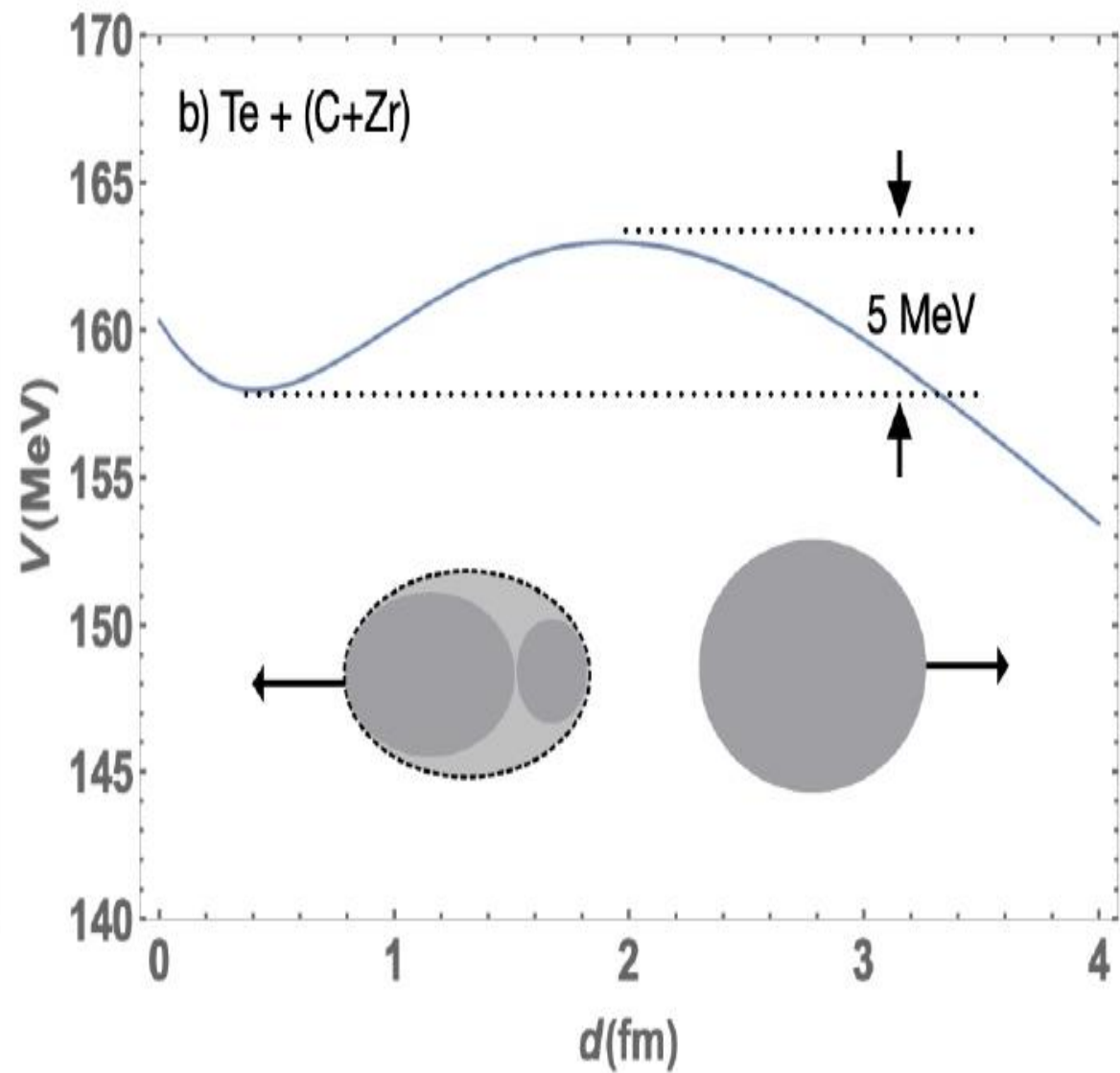
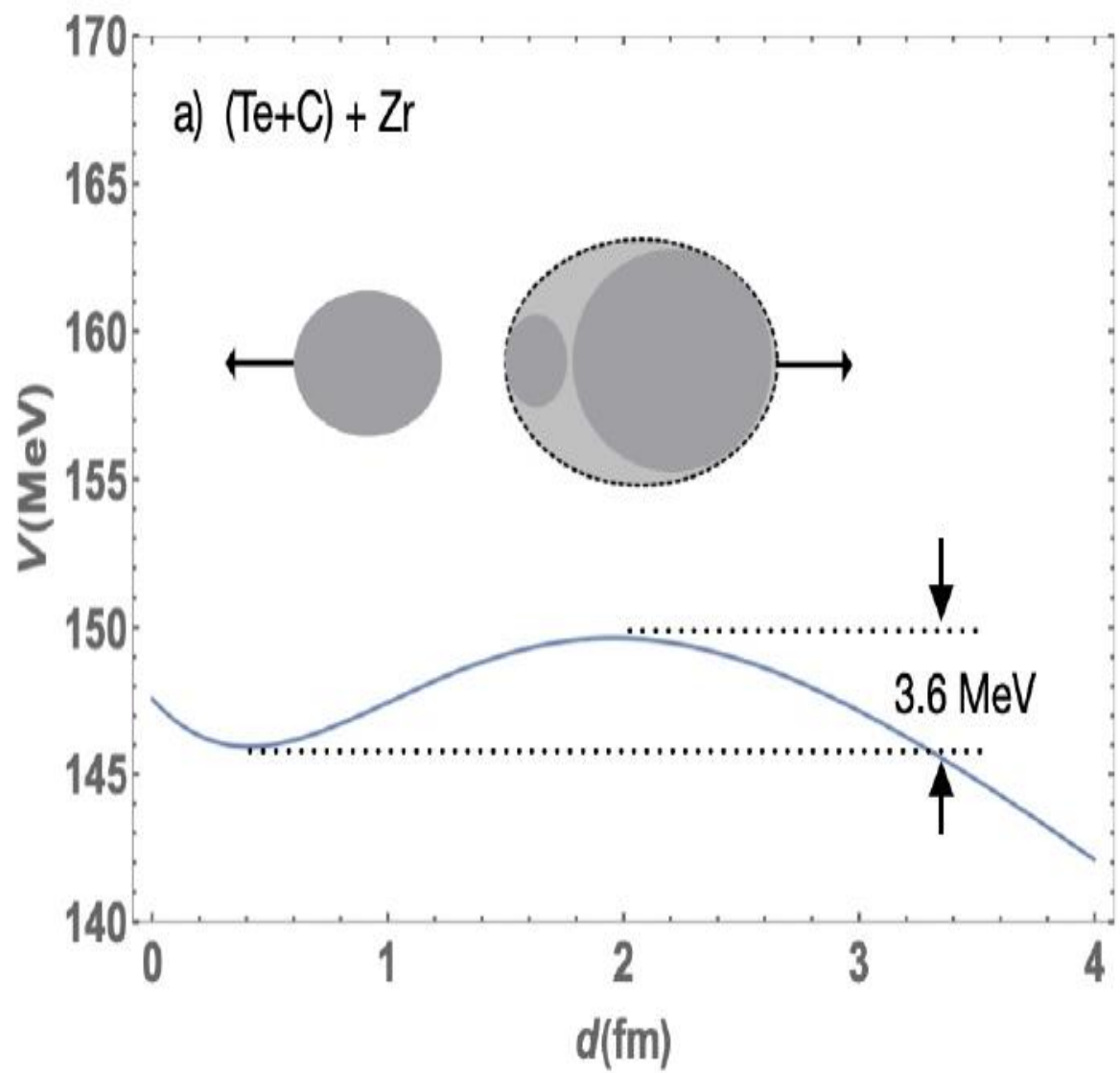
Conclusions

- We demonstrated that SF of ^{252}Cf and $^{256,258}\text{Fm}$ has contributions from fission pathways through the binary decay of trinuclear systems!
- The unusually narrow symmetric distribution of $^{258}\text{Fm}(\text{SF})$ is related to α -clustering of heavy fragment in $\text{Cd}+\text{Te}$ and formation of the trinuclear systems $\text{Cd}+\alpha+\text{Sn}$ and their binary decays leading to the formation of fragments $(\text{Cd}+\alpha)/\text{Sn}\rightarrow\text{Sn}/\text{Sn}$.
- Trimodal structure of the **TKE** distribution.
- Super-long fission mode has α -cluster origin.

Conclusions

- One can think the α -clustering mechanism is generally applicable in the SF of actinides and super-heavies.
- In SF of ^{252}Cf , we explained the abnormal mode with the enormously high neutron multiplicity (8-11 neutrons) by the binary decays of trinuclear systems with C, O, Ne clusters.
- One can conclude the abnormal mode exists for all fissioning nuclei, especially for those nuclei with observed ternary fission.

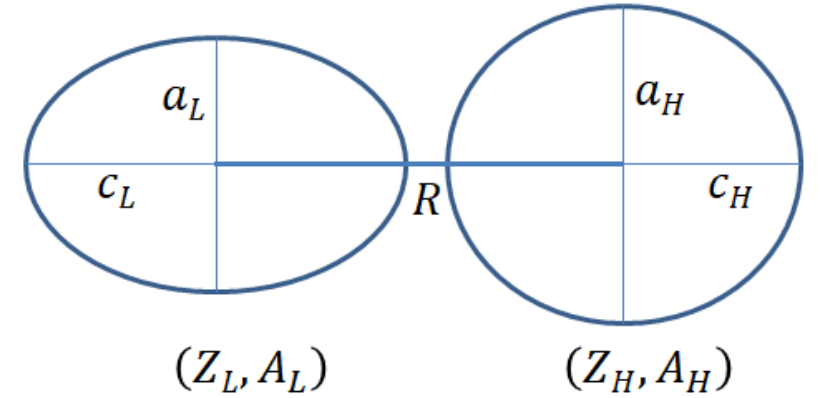
Thank you for your attention !



$^{154}\text{Nd} + ^{98}\text{Sr}$	ΔV	Q	ε_H^{**}	TKE
$(^{150}\text{Ce} + \alpha) + ^{98}\text{Sr}$	9.7	4.1	29.4	172.1
$(^{144}\text{Ba} + ^{10}\text{Be}) + ^{98}\text{Sr}$	14.7	6.5	31.9	139.7
$(^{140}\text{Xe} + ^{14}\text{C}) + ^{98}\text{Sr}$	28.8	-4.3	50.9	137.9
$(^{134}\text{Te} + ^{20}\text{O}) + ^{98}\text{Sr}$	41.8	-13.1	78.6	151.8
$(^{128}\text{Sn} + ^{26}\text{Ne}) + ^{98}\text{Sr}$	56.0	-17.2	97.0	155.6

Model

Total potential energy:



$$U(A_i, Z_i, \beta_i, R) = U_{macro}(A_i, Z_i, \beta_i, R) + \delta U_L^{shell}(A_i, Z_i, \beta_i, R) =$$
$$= \sum_{i=L,H} U_i^{LD}(A_i, Z_i, \beta_i) + \sum_{i=L,H} \delta U_i^{shell}(A_i, Z_i, \beta_i) + V^C(A_i, Z_i, \beta_i, R) + V^N(A_i, Z_i, \beta_i, R)$$

$$U_i^{LD}(A_i, Z_i, \beta_i) = U_i^{surf}(A_i, Z_i, \beta_i) + U_i^C(A_i, Z_i, \beta_i) + U_i^{sym}(A_i, Z_i)$$

Liquid drop terms:

$$U_i^{sym}(A_i, Z_i) = 27.612 \frac{(N_i - Z_i)^2}{A_i}$$

$$U_i^C(A_i, Z_i, \beta_i) = \frac{3}{5} \frac{(Z_i e)^2}{R_{0,i}} \frac{\beta_i^{1/3}}{\sqrt{\beta_i^2 - 1}} \ln \left(\beta_i + \sqrt{\beta_i^2 - 1} \right)$$

Model

Surface energy with variable surface tension:

$$U_i^{surf}(A_i, Z_i, \beta_i) = \sigma_i S_i$$

$$\sigma_i = \sigma_{0,i} \left(1 + k_i (\beta_i - \beta_i^{g.s.})^2 \right)$$

$$\sigma_{0,i} = 0.9517 \left(1 - 1.7826 \left(\frac{(N_i - Z_i)^2}{A_i} \right)^2 \right)$$

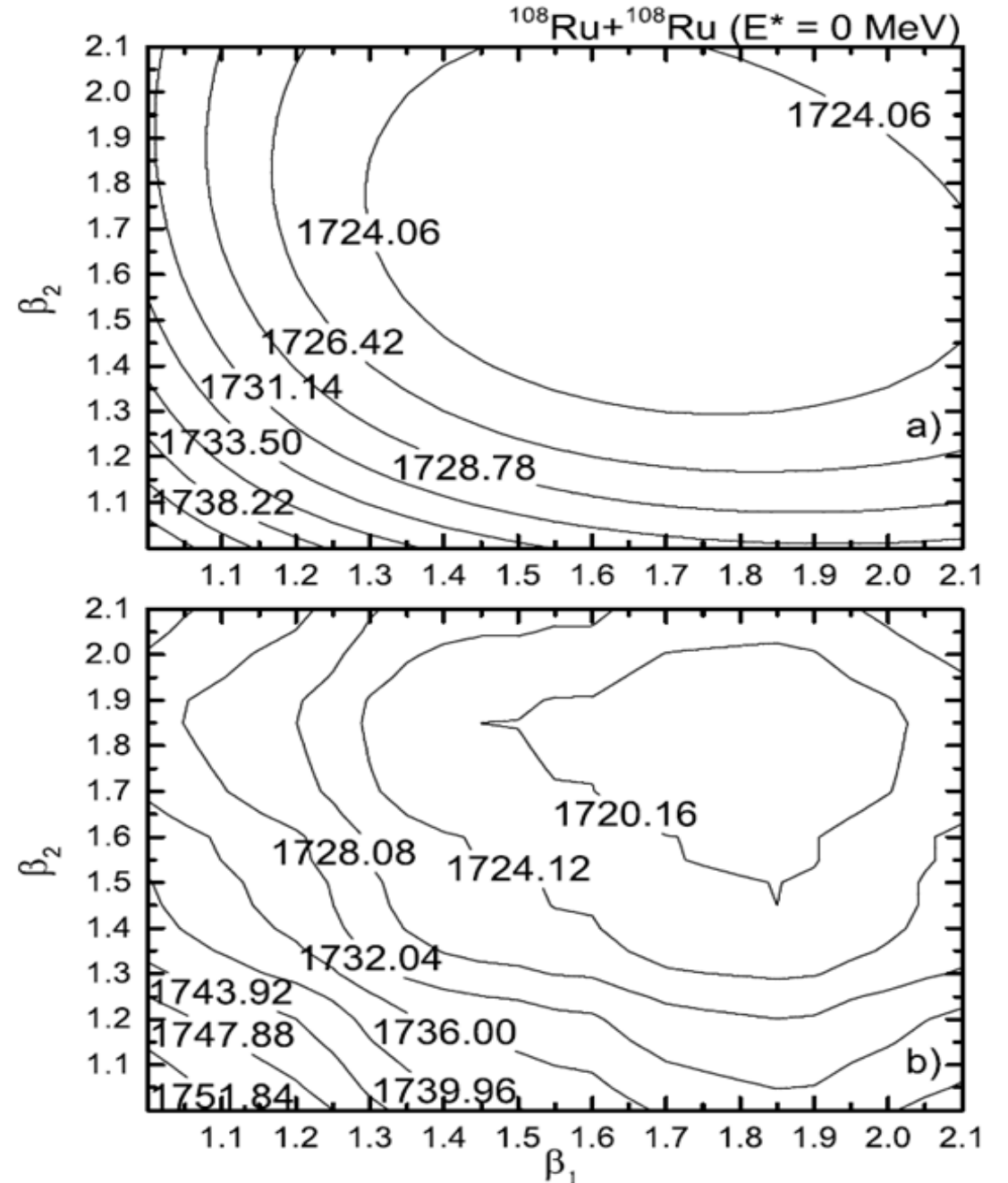
$$k_i = \frac{1}{1 + \exp[-0.063(C_{vib}(A_i, Z_i) - 67)]}$$

$$C_{vib}(A_i, Z_i) = \frac{\hbar\omega_{vib}^i (3Z_i e R_{0,i}^2 / (4\pi))^2}{2B(E2)_{vib}^i}$$

$$B(E2)_{vib}^i = E_{2+}^i + B(E2)_{rot}^i / (\hbar\omega_{vib}^i)$$

Shell corrections are calculated as in:

J. Maruhn and W. Greiner, Z. Phys. **251**, 431 (1972)



Model

Excitation energy of the scission configuration can be calculated as a sum of the initial excitation energy of the fissioning nucleus and the difference of the potential energies of the fissioning nucleus and scission configuration:

$$E^*(A_i, Z_i, \beta_i, R_b) = E_{CN}^* + [U_{CN}(A, Z, \beta) - U(A_i, Z_i, \beta_i, R_b)]$$

$$T_{DNS}(E^*) = \sqrt{E^*/a}, \quad a = A/12 \text{ MeV}^{-1}$$

Temperature dependence of LD terms:

$$U_i^{sym}(A_i, Z_i, T) = U_i^{sym}(A_i, Z_i, T = 0)(1 + 6 \times 10^{-4} E_i^*/A_i)$$

$$U_i^C(A_i, Z_i, \beta_i, T) = U_i^C(A_i, Z_i, \beta_i, T = 0)(1 - 0.12 E_i^*/A_i)$$

$$U_i^{surf}(A_i, Z_i, \beta_i, T) = U_i^{surf}(A_i, Z_i, \beta_i, T = 0)(1 + 0.102 E_i^*/A_i)$$

$$k_i(E_i^*) = k_i \times \exp[-E_i^*/E_k]$$

Shell damping:

$$U_i^{shell}(A_i, Z_i, \beta_i', E_i^*) = \delta U_i^{shell}(A_i, Z_i, \beta_i', E_i^* = 0) \exp[-E_i^*/E_D]$$

The probability of emitting exactly ν neutrons (Jackson model):

$$P_\nu(E_i^*) = P(\nu) - P(\nu + 1)$$

$$P(\nu) = 1 - e^{-\Delta_\nu} \left(1 + \sum_{k=1}^{2\nu-3} \frac{(\Delta_\nu)^k}{k!} \right) \qquad P(\nu + 1) = 1 - e^{-\Delta_{\nu+1}} \left(1 + \sum_{k=1}^{2\nu-1} \frac{(\Delta_{\nu+1})^k}{k!} \right)$$

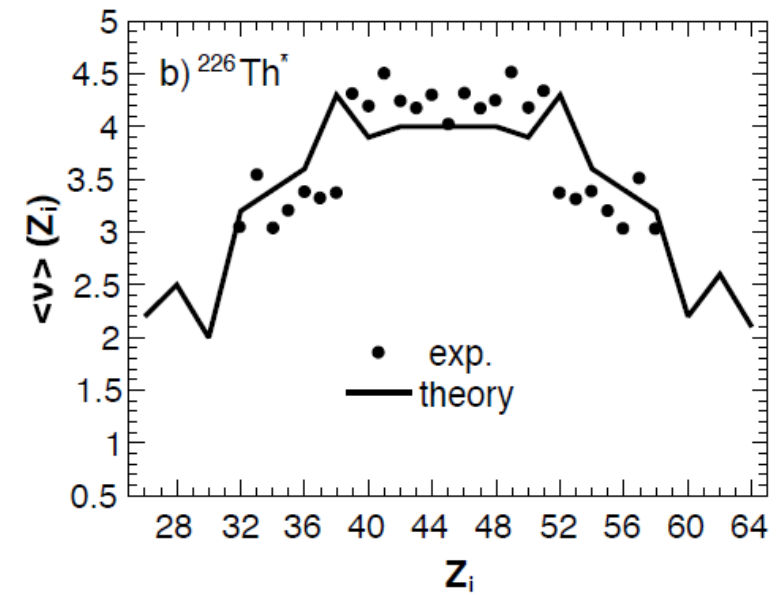
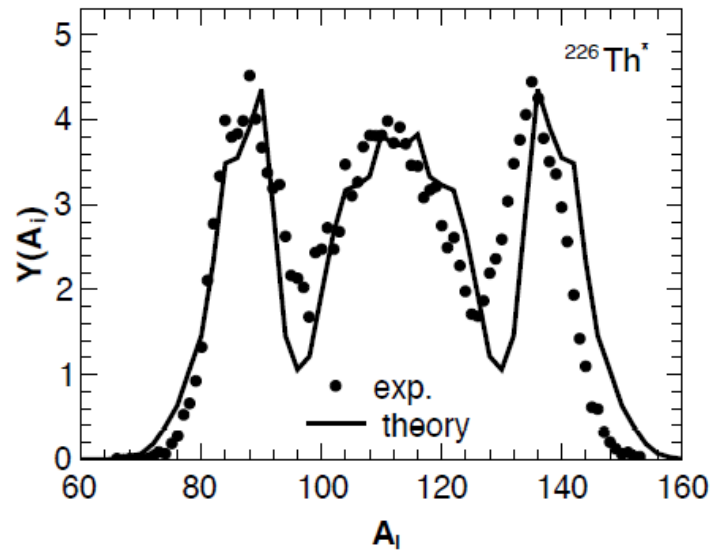
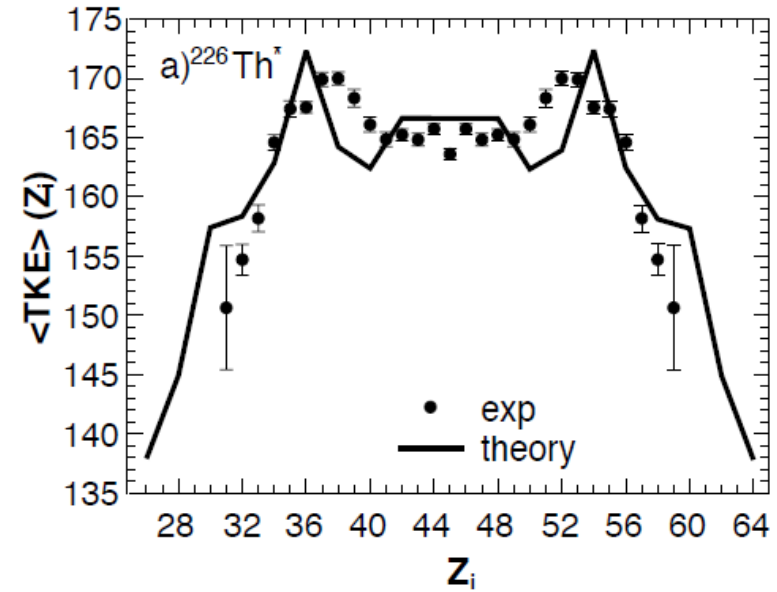
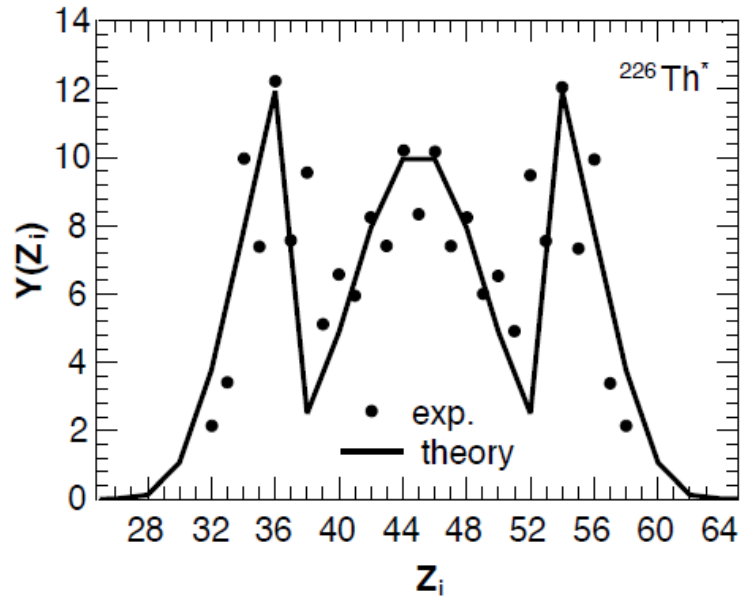
$$\Delta_\nu = \left(E_i^* - \sum_{k=1}^{\nu} B_k \right) / T_i$$

$$P_L(E_L^*) \sim \rho_L(E_L^*) \rho_H(E^* - E_L^*)$$

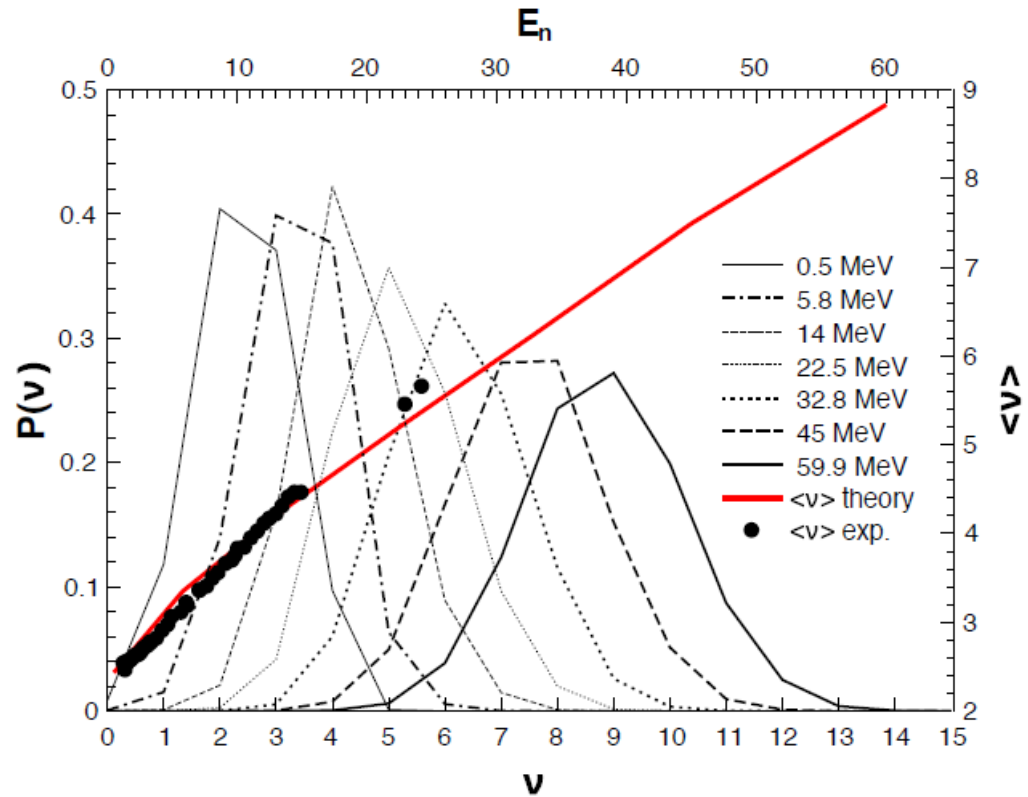
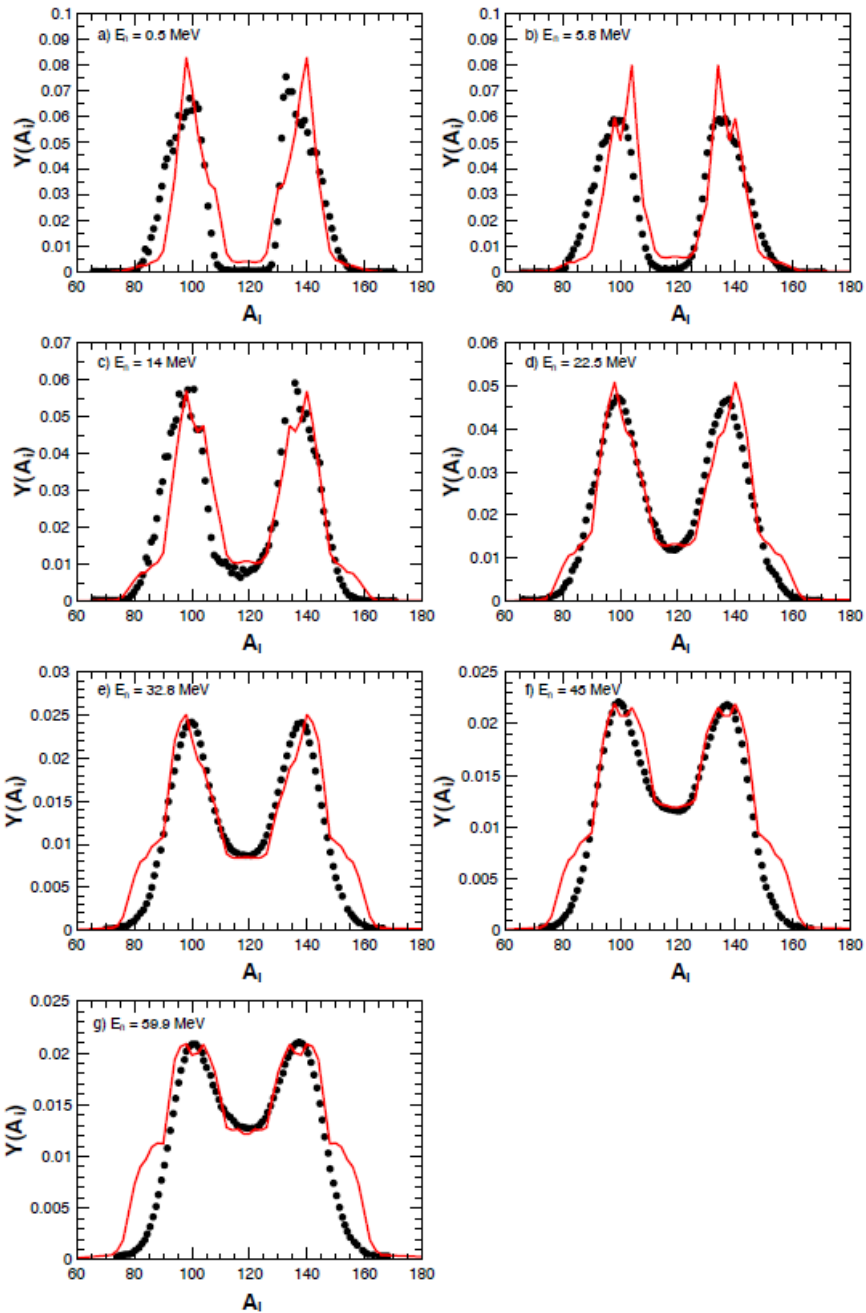
$$\rho_i(E_i^*) \sim \exp[2(a_i E_i^*)^{1/2}]$$

$$P_L(\nu) = \frac{\int_0^{E^*} dE_L^* P_\nu^L(E_L^*) \rho_L(E_L^*) \rho_H(E^* - E_L^*)}{\int_0^{E^*} dE_L^* \rho_L(E_L^*) \rho_H(E^* - E_L^*)}$$

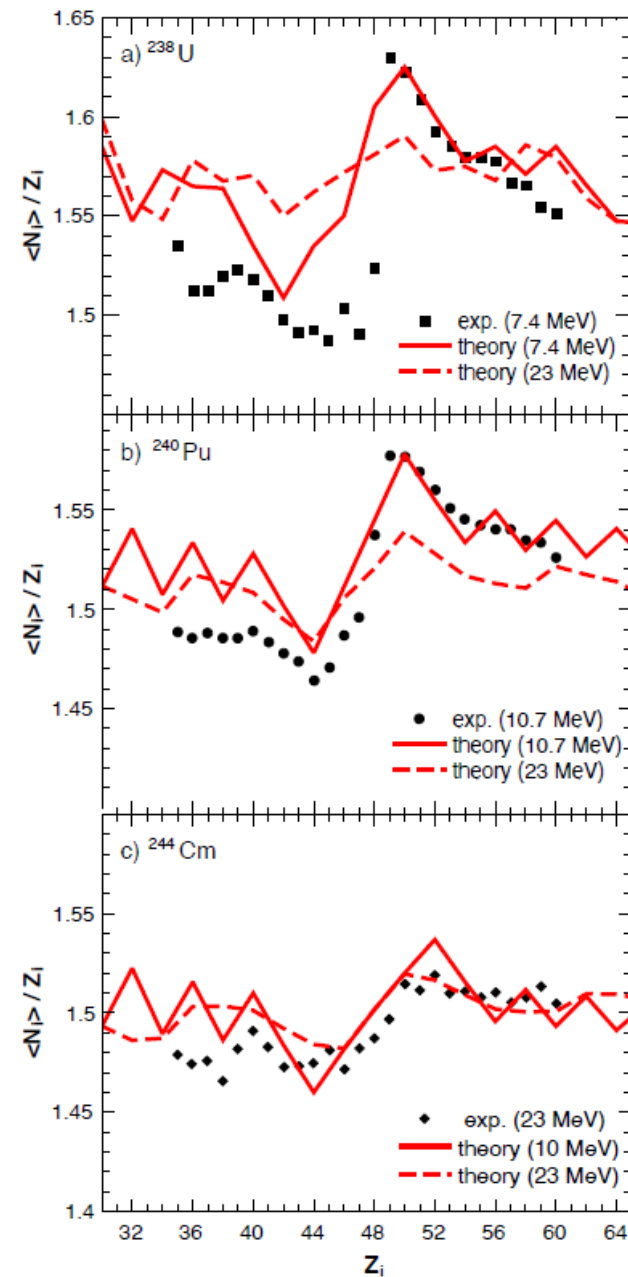
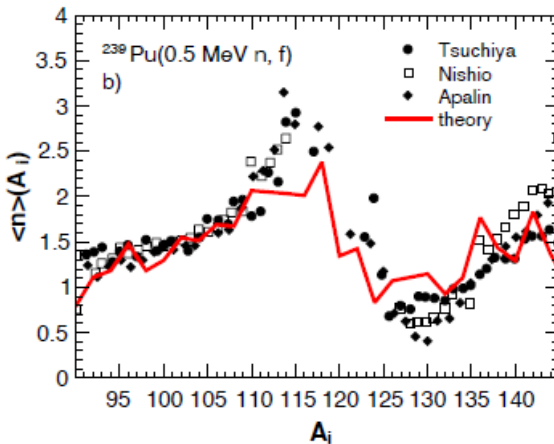
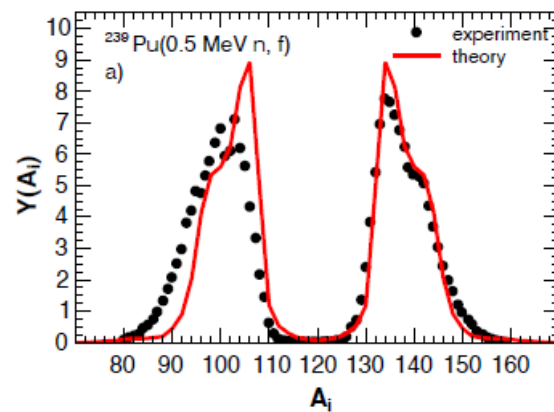
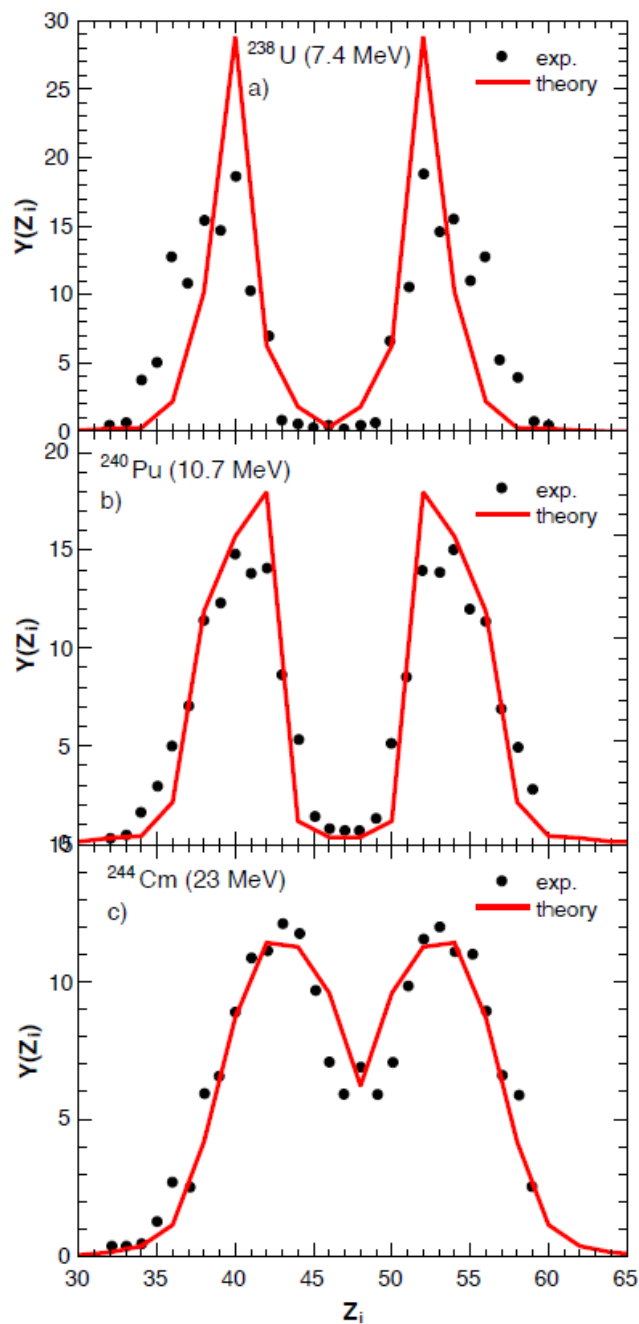
Results - thorium isotopes



What about excitation energy of CN? $^{238}\text{U}(n,f)$



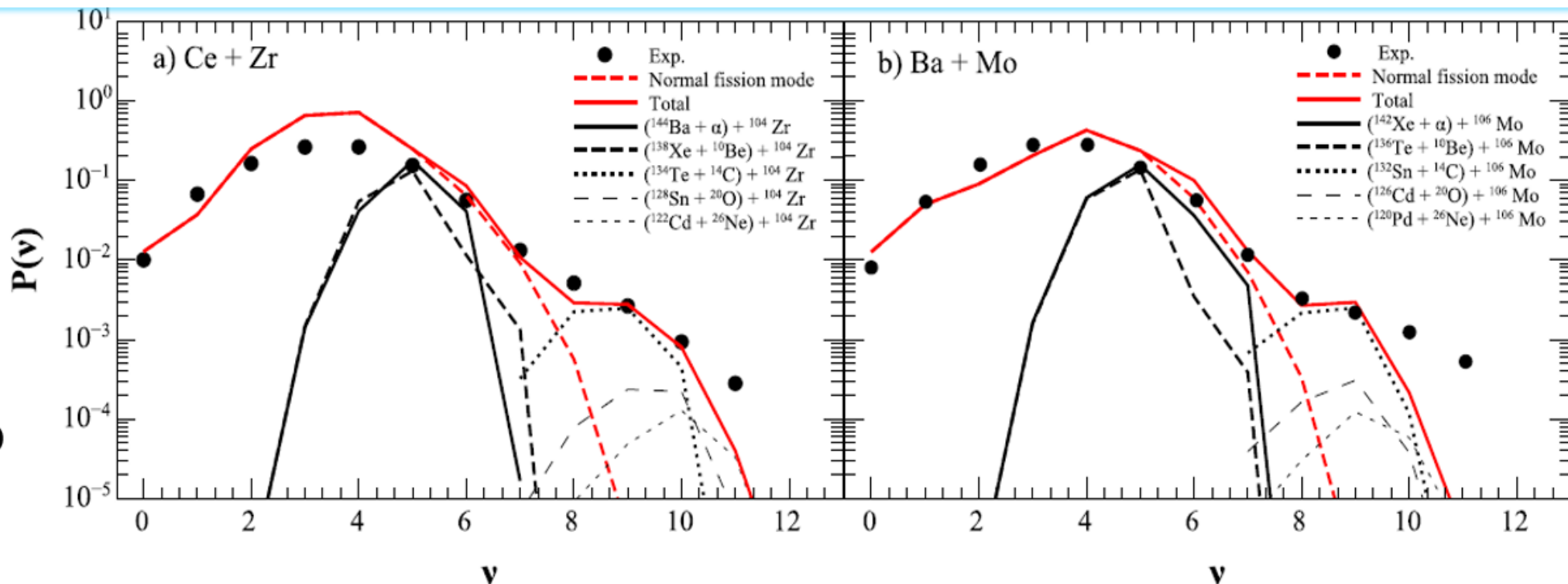
Results - $\langle N/Z \rangle$ ratios in primary fragments



Manifestation of ternary clusterization in binary spontaneous fission of ^{252}Cf [PLB 864, 139444, (2025)]

H. Pasca, G.G. Adamian, N.V. Antonenko

Probability $P(\nu)$ of emitting exactly ν neutrons from the pairs Ce+Zr (a) and Ba+Mo (b) resulting from SF of ^{252}Cf .



Exp.

G.M. Ter-Akopian,

J. Hamilton,

Yu. Ts. Oganessian

et al.

PRL 77(96)32,

PRC 55(97)1196,

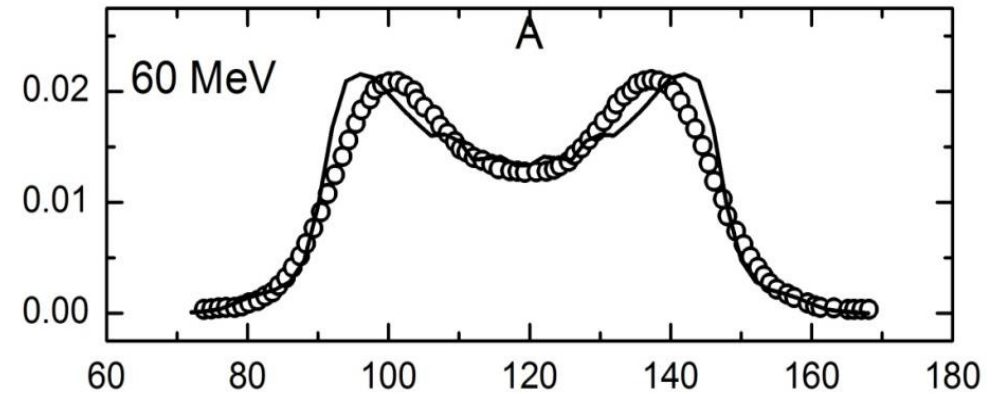
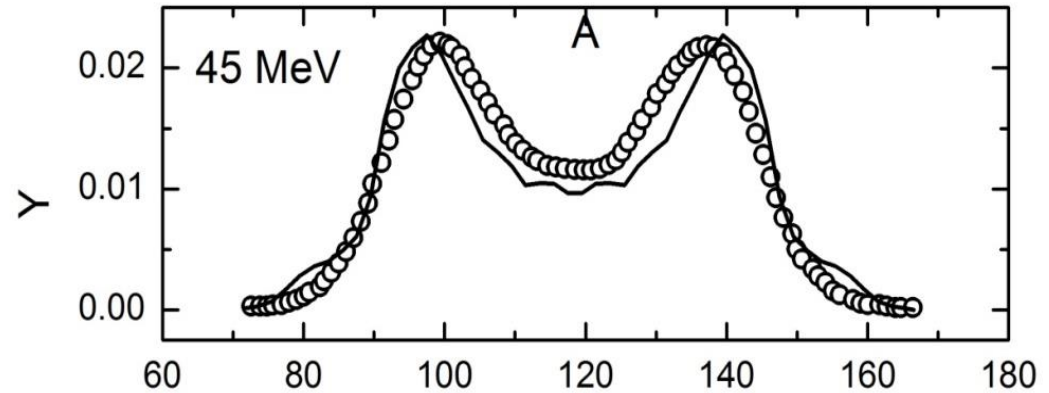
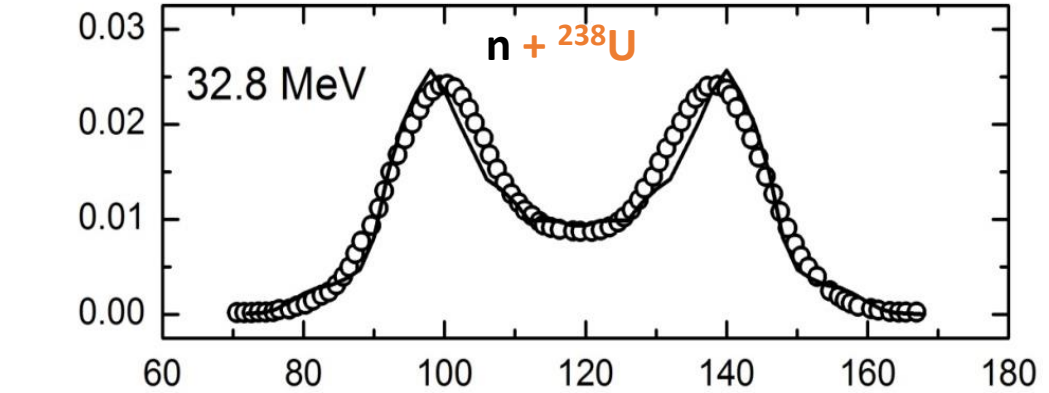
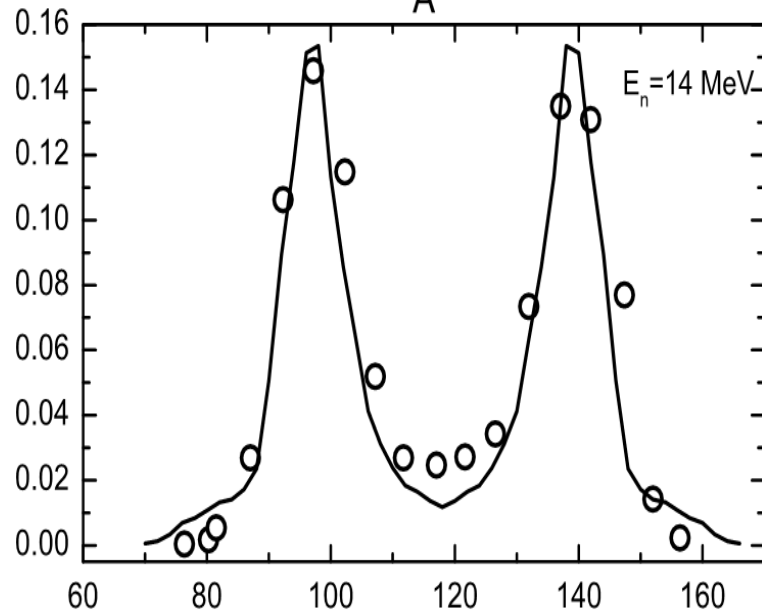
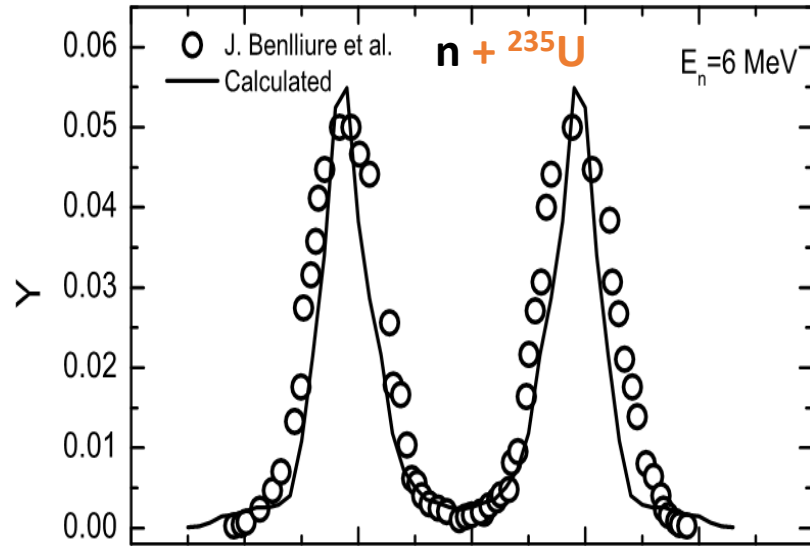
PRC 101(20)034610

Suggesting the formation of the trinuclear systems (C, O, Ne between two fragments) at scission and their binary decay into a light fragment and a dinuclear system with subsequent fusion of the nuclei of the dinuclear system and the formation of a hyperexcited heavy fragment, one can describe the 8–11 neutron evaporation channels for Ce+Zr and Ba+Mo in the SF of ^{252}Cf .

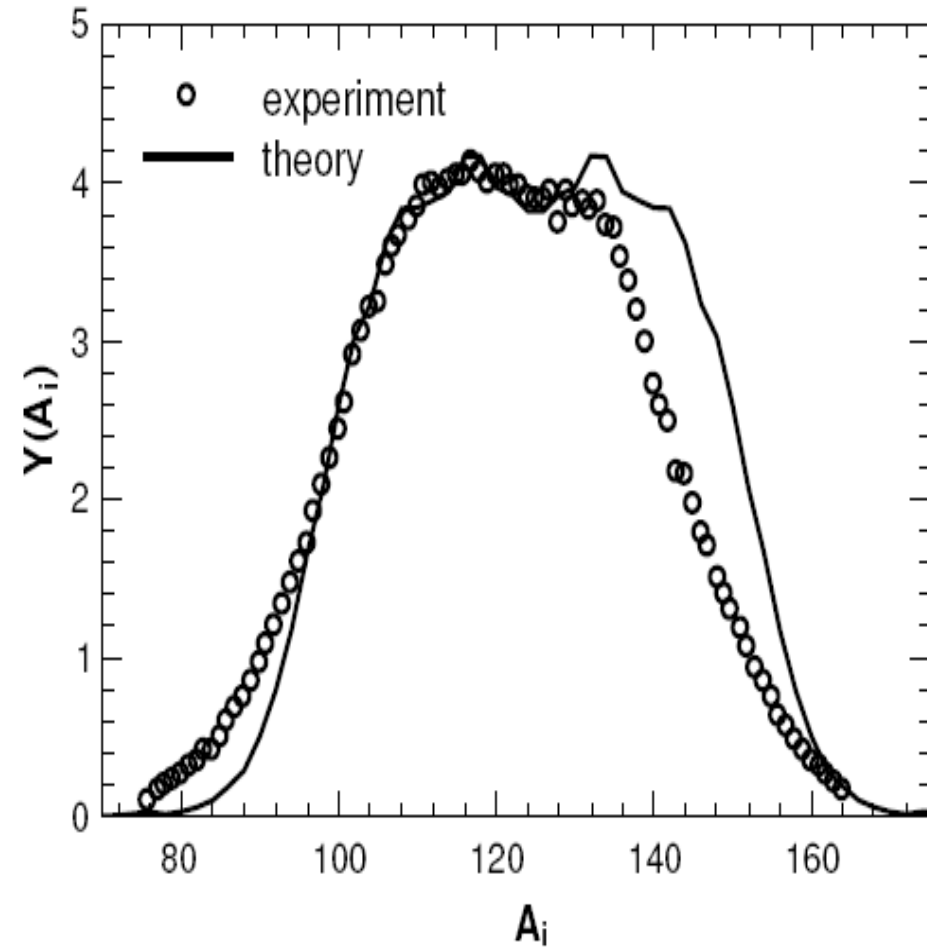
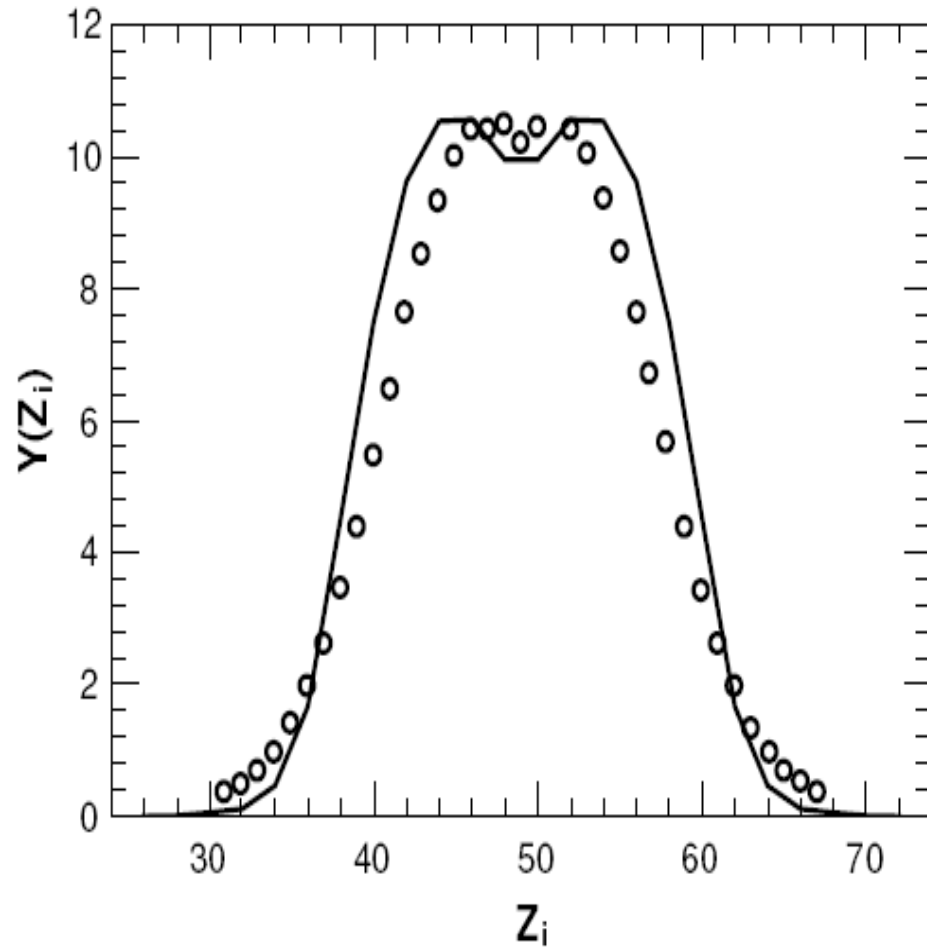
This abnormal (ternary cluster) mode is new type of bimodal fission with “normal” and low TKE for the same charge/mass fragmentation Ba+Mo or Ce+Zr in SF of ^{252}Cf .

Ba+Mo: 1st mode: $\text{TKE}_{\text{exp}} = 189 \pm 1 \text{ MeV}$, $\text{TKE}_{\text{th}} = 192 \text{ MeV}$, 2nd mode: $\text{TKE}_{\text{exp}} = 153 \pm 3 \text{ MeV}$, $\text{TKE}_{\text{th}} = 156 \text{ MeV}$

High excitation energy of fissioning nucleus



^{250}Cf (46 MeV)



Exp.: D. Ramos et al.
PRC 97(2018)054612

Transfer reactions

Transfer reactions

Exper.:K.Hirose et al.
PRL119(2017)222501

