Semi-Classical Quantization of Periodic Solutions for Imaginary Time-Dependent Hartree-Fock Theory to Describe Spontaneous Fission

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Introduction:
TDHF & Spontaneous Fission

Our research purpose

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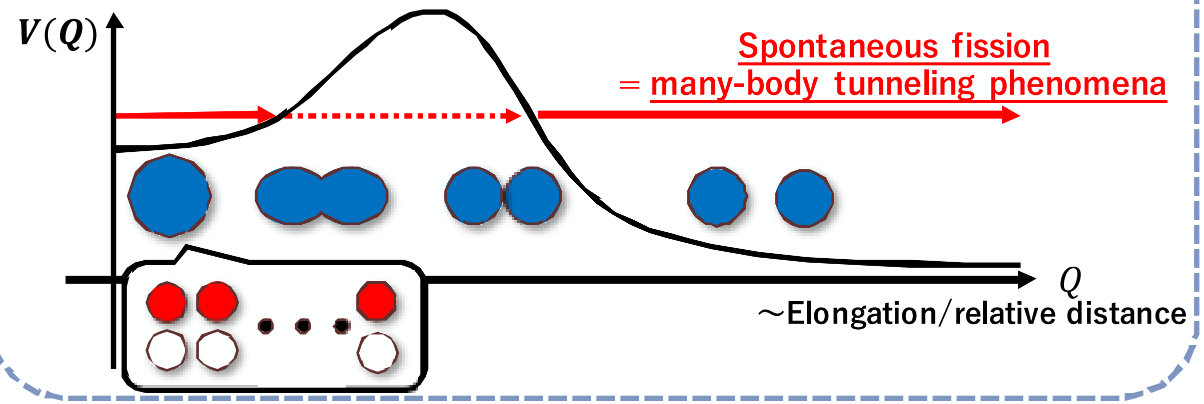
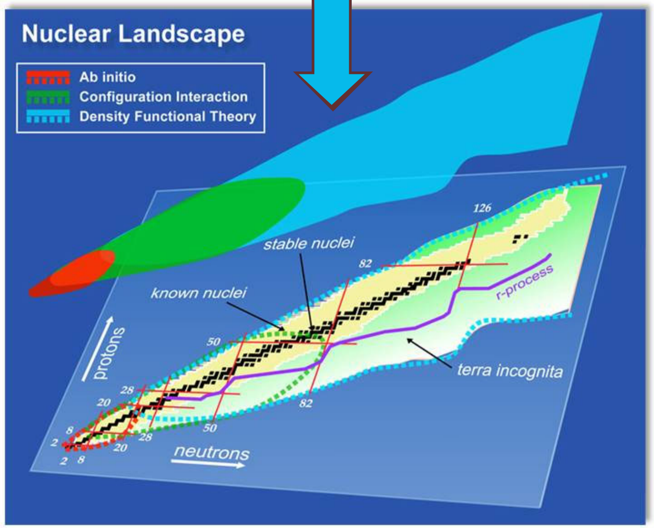
To describe spontaneous nuclear fission
by a microscopic mean-field approach

Potential energy surface by LDM



Spontaneous fission (SF) occurs in heavy nuclei.

TDHF is possible to calculate heavy nuclei.



H Nam *et al* 2012 *J. Phys.: Conf. Ser.* **402** 012033

Time-Dependent Hartree-Fock (TDHF) theory

- Nucleons create a mean field
- Nucleons move independently in the mean field
- Self-consistent equation for single particle wave function

$$i\hbar\partial_t\psi_k(t) = -\frac{\hbar^2}{2m}\nabla^2\psi_k(t) + \frac{\delta\mathcal{V}}{\delta\psi_k^*(t)}$$

Why TDHF is classical theory?

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TDHF equation can be derived using **a path integral formalism**.

S. Levit Phys. Rev. C **21**, 1594 (1980)
S. Levit, J. W. Negele, and Z. Paltiel, Phys. Rev. C **21**, 1603 (1980)

Many-body Quantum Mechanics

$$H = \sum_{\alpha\beta} T_{\alpha\beta}\hat{a}^\dagger_\alpha \hat{a}_\beta + \frac{1}{2}\sum_{\alpha\beta\gamma\delta} v_{\alpha\beta\gamma\delta}\hat{a}^\dagger_\alpha \hat{a}^\dagger_\beta \hat{a}_\delta \hat{a}_\gamma$$

$$U(t_f,t_i) = T\exp\left[-\frac{i}{2}\int_{t_i}^{t_f} dt \sum_{\alpha\beta\gamma\delta} \hat{\rho}_{\alpha\gamma}(t) V_{\alpha\beta\gamma\delta}\hat{\rho}_{\beta\delta}(t)\right]$$

⇩ **Hubbard-Stratonovich transformation**

Path Integral representation

$$U(t_f,t_i) = \int \mathcal{D}[\sigma]\exp\left[\frac{i}{2}\int_{t_i}^{t_f} dt\{\sigma(t)v\sigma(t)\}U_I^\sigma(t_f,t_i)\right]$$

$$U_I^\sigma(t_f,t_i) \equiv T\exp\left[-i\int_{t_i}^{t_f} dt\{\sigma(t)v\hat{\rho}(t)\}\right]$$

※The HS transformation is also used in the Monte Carlo shell model.
T. Otsuka et al., Prog. Part. Nucl. Phys. 47 (2001) 319-400

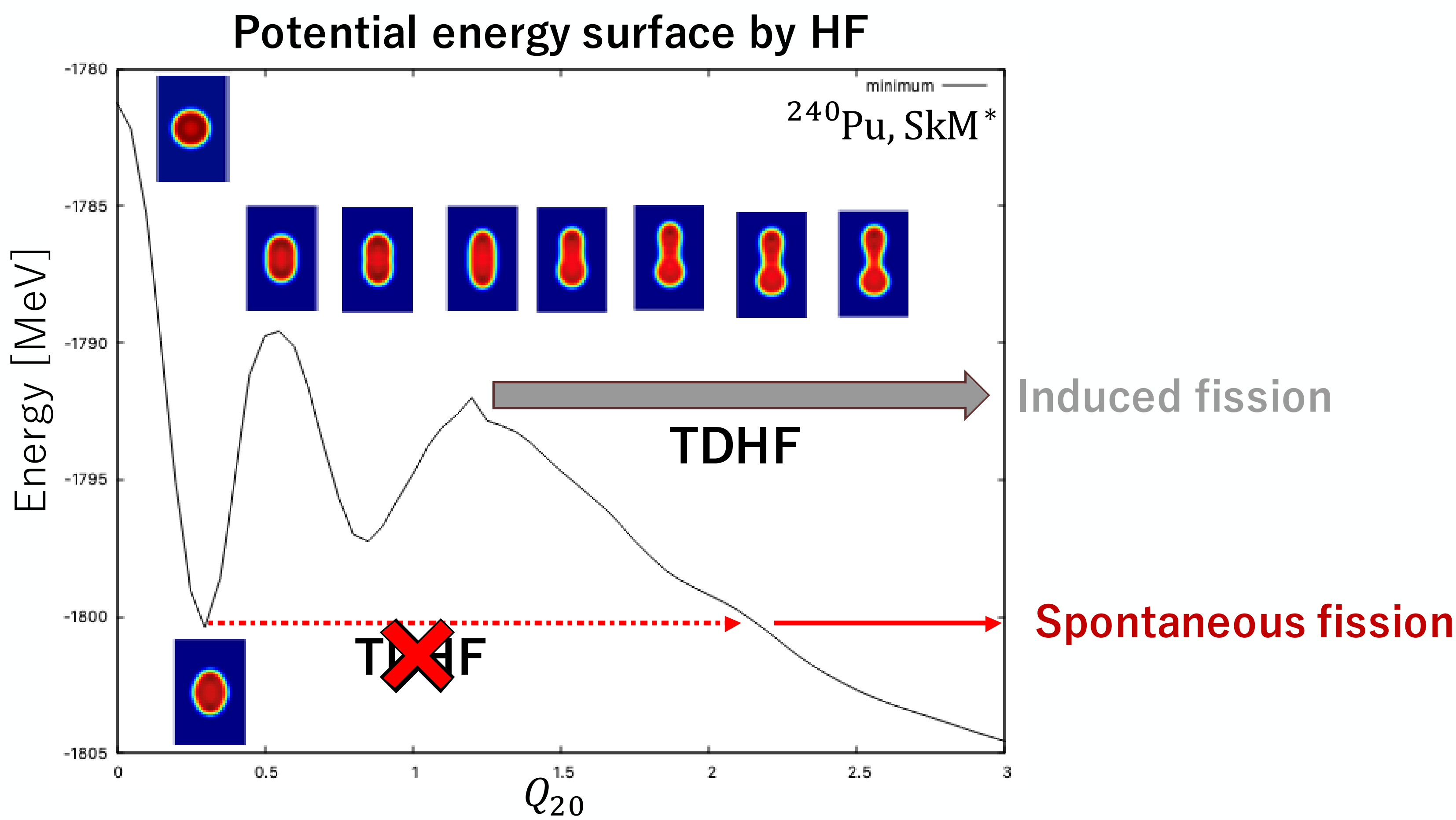
⇩ **Stationary phase approximation ($\delta S = 0$)**

TDHF equation

$$i\hbar\partial_t\psi_k(t) = -\frac{\hbar^2}{2m}\nabla^2\psi_k(t) + \frac{\delta\mathcal{V}}{\delta\psi_k^*(t)}$$

TDHF is classical theory

TDHF cannot describe SF



TDHF can describe microscopically the dynamics of a single particle, but a collective motion is classically.

We must quantize TDHF to describe SF.

Previous study of SF

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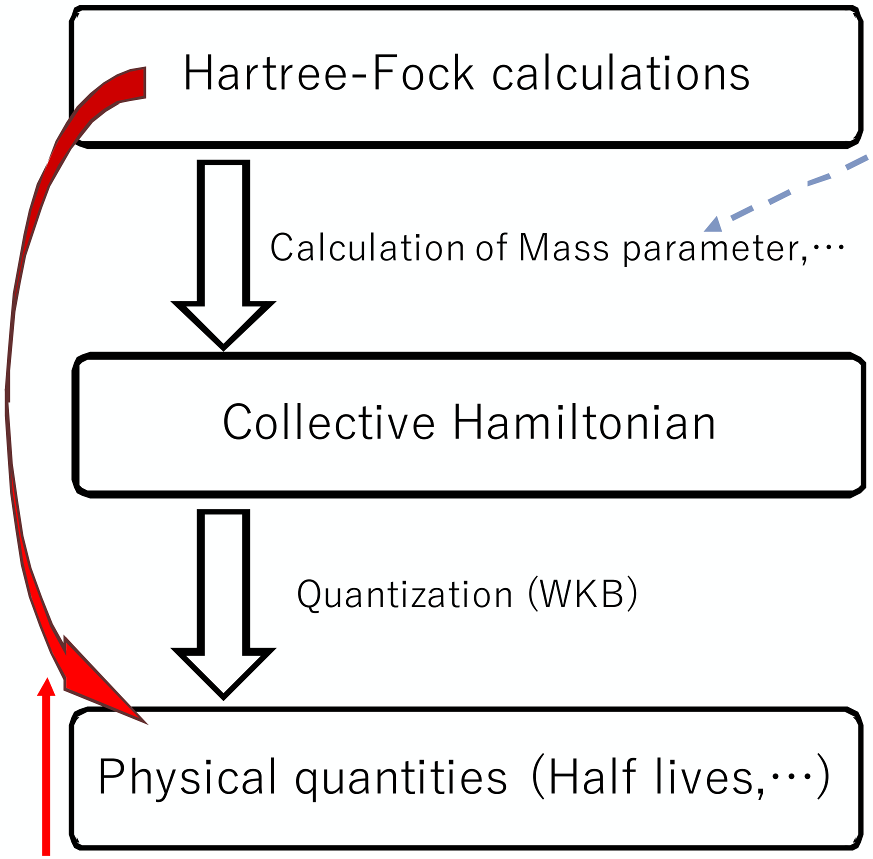
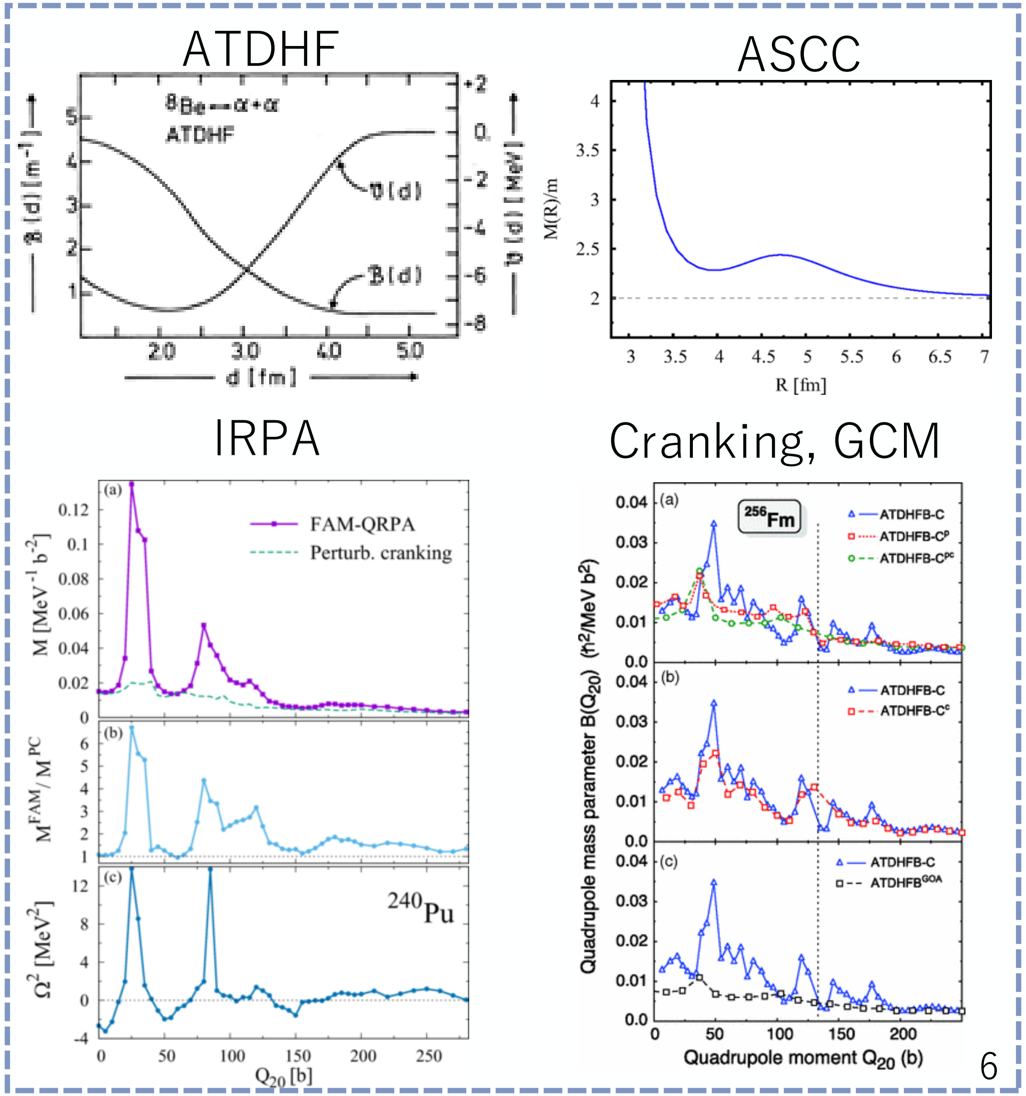
P. –G. Reinhard, J. Maruhn, K. Goeke, Phys. Rev. Lett. **44**, 1740 (1980)
A. Baran et al. , Phys. Rev. C **84**, 054321 (2011)
A. K. Wen, T. Nakatsukasa, Phys. Rev. C **94**, 054618 (2016)
B. K. Washiyama, N. Hinohara, T. Nakatsukasa, Phys. Rev. C **103**, 014306 (2021)



The calculations what we want to do.

⇨ **Imaginary time evolution (analogy of instanton)**



Introduction:
Imaginary TDHF

Imaginary TDHF

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TDHF

$$i\hbar\partial_t\psi_k(t) = -\frac{\hbar^2}{2m}\nabla^2\psi_k(t) + \frac{\delta\mathcal{V}}{\delta\psi_k^*(t)}$$

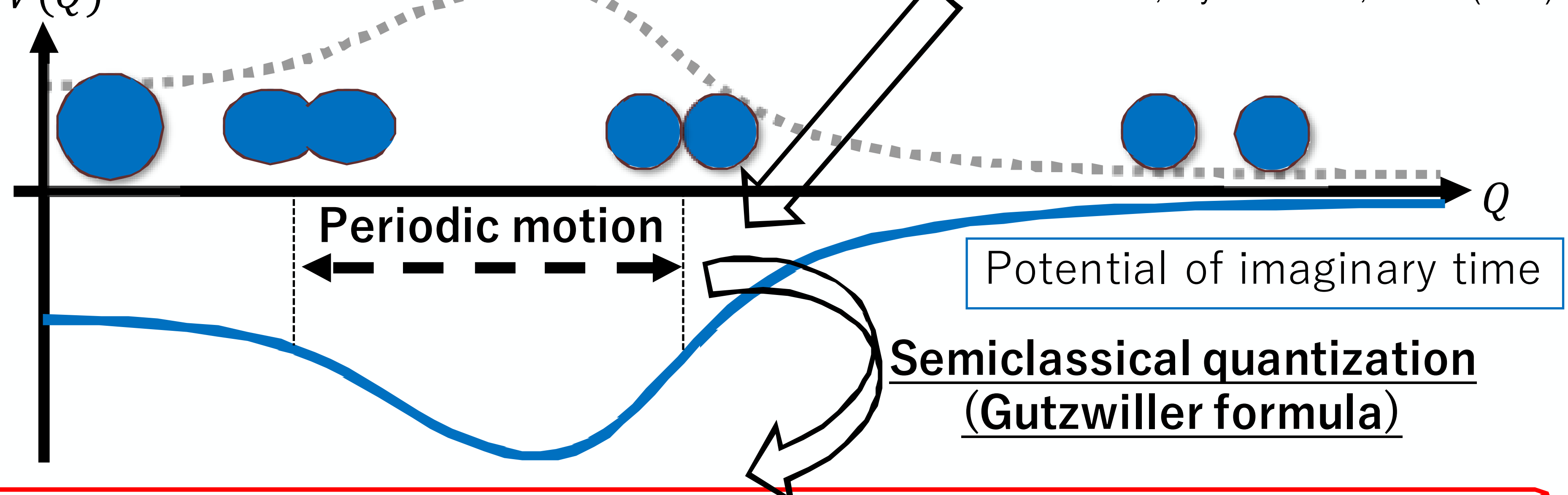
$t \rightarrow -i\tau$

Imaginary TDHF (ITDHF)

$$-\hbar\partial_\tau\psi_k(\tau) = -\frac{\hbar^2}{2m}\nabla^2\psi_k(\tau) + \frac{\delta\mathcal{V}}{\delta\psi_k(-\tau)}$$

$$\psi_k(T/2) = e^{-\alpha_k}\psi_k(-T/2)$$

J. Skalski, Phys. Rev. C **77**, 064610 (2008)



Fission half-life

$$T_{1/2} \approx \exp\left[-\frac{S}{\hbar}\right], \quad S = \hbar\int_{-T/2}^{T/2} d\tau \sum_k \left\langle \psi_k(-\tau) \middle| \frac{\partial\psi_k(\tau)}{\partial\tau} \right\rangle$$

ITDHF was proposed in the 1980s, but there has been little progress since then.

Semi-classical quantization

S. Levit, J. W. Negele, and Z. Paltiel, Phys. Rev. C **21**, 1603 (1980)

Gutzwiller formula

$$G(E) \equiv i\int_0^\infty dT e^{iET}\,\mathrm{tr}\,U(T,0) = \sum_\nu \frac{1}{E_\nu - E}$$

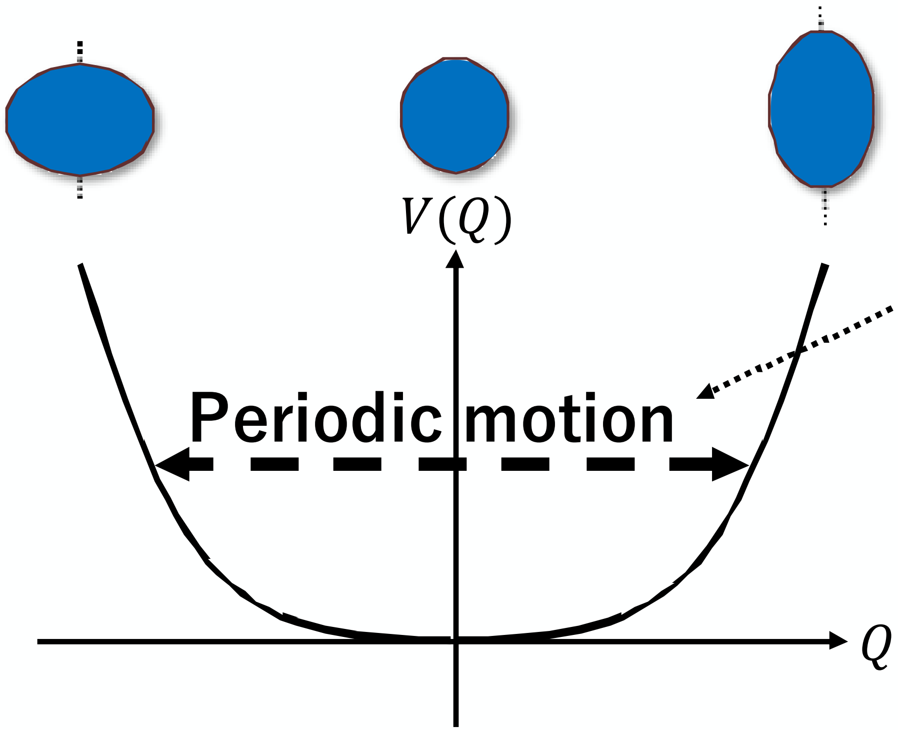
Periodic trajectory Propagator ($\mathrm{tr}\,U(T,0)$)

Quantum theory's Energy (E_ν)

①Describing by periodic TDHF

$$i\hbar\partial_t\psi_k(t) = -\frac{\hbar^2}{2m}\nabla^2\psi_k(t) + \frac{\delta\mathcal{V}}{\delta\psi_k^*(t)}$$

$$\psi_k(T/2) = e^{-i\alpha_k}\psi_k(-T/2)$$

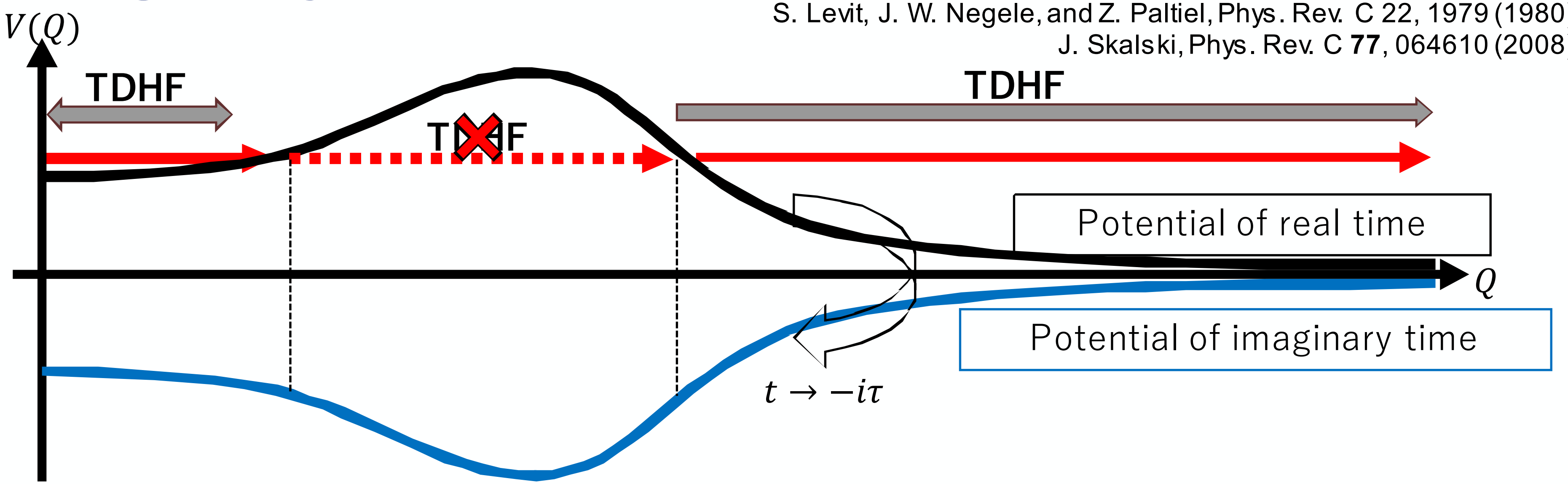


②calculating $G(E)$, the poles give the energy in quantum theory.

However, conventional TDHF cannot penetrate barriers, so the periodic TDHF corresponding to SF cannot be obtained.

Imaginary time evolution

S. Levit, J. W. Negele, and Z. Paltiel, Phys. Rev. C 22, 1979 (1980)
J. Skalski, Phys. Rev. C **77**, 064610 (2008)



Real time TDHF

$$i\hbar\partial_t\psi_k(t) = -\frac{\hbar^2}{2m}\nabla^2\psi_k(t) + \frac{\delta\mathcal{V}}{\delta\psi_k^*(t)}$$

$$\psi_k = \sqrt{\rho_k}e^{-i\chi_k} \qquad \psi_k^* = \sqrt{\rho_k}e^{i\chi_k}$$

$$\mathcal{H} = \frac{1}{2m}\sum_{k=1}^{A}\int\rho_k(\nabla\chi_k)^2dx + \mathcal{V}(\rho)$$

$$\mathcal{V}(\rho) = \frac{1}{8m}\sum_{k=1}^{A}\int\frac{(\nabla\rho_k)^2}{\rho_k}dx + \frac{1}{2}\sum_{k,j=1}^{A}\int\rho_k V\rho_j dxdx'$$

$t \to -i\tau$

Imaginary time TDHF

$$-\hbar\partial_\tau\psi_k(\tau) = -\frac{\hbar^2}{2m}\nabla^2\psi_k(\tau) + \frac{\delta\mathcal{V}}{\delta\psi_k(-\tau)}$$

$$\psi_k = \sqrt{\rho_k}e^{-\chi_k} \qquad \psi_k^* = \sqrt{\rho_k}e^{\chi_k}$$

$$\mathcal{H} = -\frac{1}{2m}\sum_{k=1}^{A}\int\rho_k(\nabla\chi_k)^2dx + \mathcal{V}(\rho)$$

Sign has inverted !

$$\mathcal{V}(\rho) = \frac{1}{8m}\sum_{k=1}^{A}\int\frac{(\nabla\rho_k)^2}{\rho_k}dx + \frac{1}{2}\sum_{k,j=1}^{A}\int\rho_k V\rho_j dxdx'$$

Imaginary TDHF

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TDHF

$$i\hbar\partial_t\psi_k(t) = -\frac{\hbar^2}{2m}\nabla^2\psi_k(t) + \frac{\delta\mathcal{V}}{\delta\psi_k^*(t)}$$

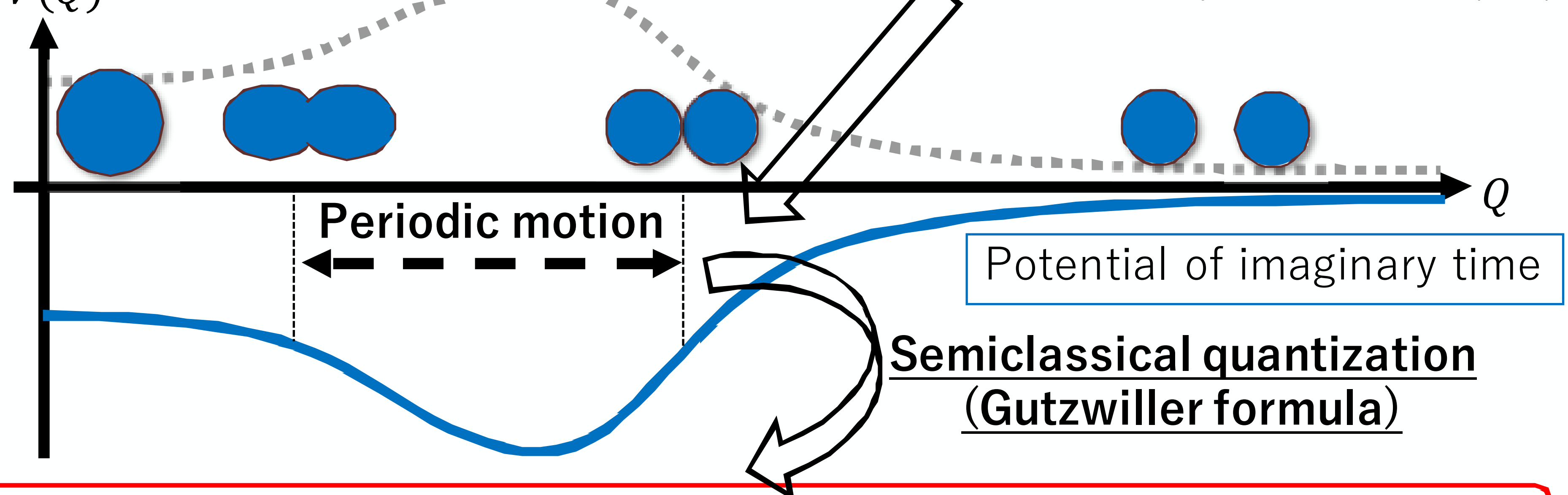
$t \to -i\tau$

Imaginary TDHF (ITDHF)

$$-\hbar\partial_\tau\psi_k(\tau) = -\frac{\hbar^2}{2m}\nabla^2\psi_k(\tau) + \frac{\delta\mathcal{V}}{\delta\psi_k(-\tau)}$$

$$\psi_k(T/2) = e^{-\alpha_k}\psi_k(-T/2)$$

J. Skalski, Phys. Rev. C **77**, 064610 (2008)



Fission half-life $T_{1/2} \approx \exp\left[-\frac{S}{\hbar}\right], \quad S = \hbar\int_{-T/2}^{T/2} d\tau \sum_k \left\langle \psi_k(-\tau) \middle| \frac{\partial\psi_k(\tau)}{\partial\tau} \right\rangle$

ITDHF was proposed in the 1980s, but there has been little progress since then.

Comparison of TDHF and ITDHF

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Conventional TDHF

$$i\hbar\partial_t\psi_k(t) = -\frac{\hbar^2}{2m}\nabla^2\psi_k(t) + \frac{\delta\mathcal{V}}{\delta\psi_k^*(t)}$$

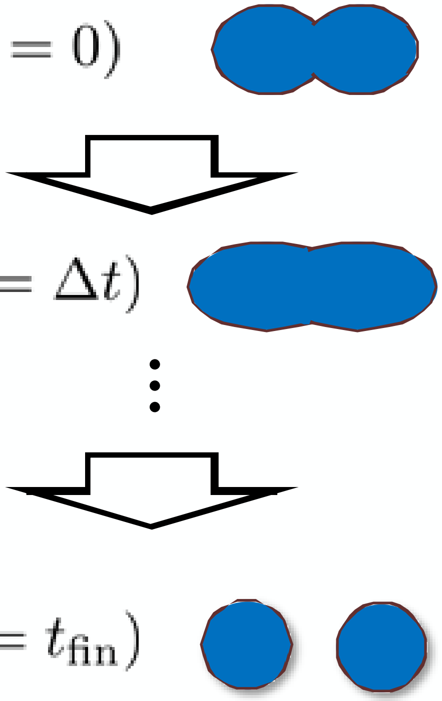
Prepare initial condition
(solution of static HF eq)

$\psi_k(t=0)$

$\psi_k(t=\Delta t)$

⋮

$\psi_k(t=t_{\text{fin}})$

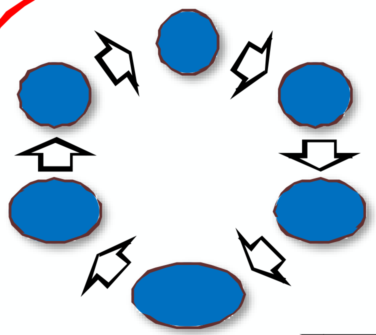


This is "initial value problem".

ITDHF

$$-\hbar\partial_\tau\psi_k(\tau) = -\frac{\hbar^2}{2m}\nabla^2\psi_k(\tau) + \frac{\delta\mathcal{V}}{\delta\psi_k(-\tau)}$$

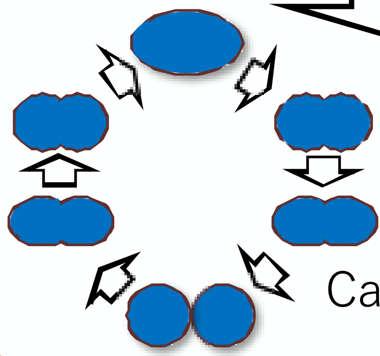
$$\psi_k(T/2) = e^{-\alpha_k}\psi_k(-T/2)$$



Prepare solutions
from $-\frac{T}{2}$ to $\frac{T}{2}$
that satisfy B.C
$\psi_k^{(0)}(T/2),\cdots,\psi_k^{(0)}(-T/2)$

calculate
from $-\frac{T}{2}$ to $\frac{T}{2}$

⋮



$\psi_k(T/2),\cdots,\psi_k(-T/2)$

Calculate until converge

This is space+time dimensional "boundary value problem".

Numerical Calculations of 1D ITDHF eq

① Time evolution operator
② Diagonalizing (1+1)D matrix
③ Imaginary time method for (1+1)D matrix

System of Numerical Calculations

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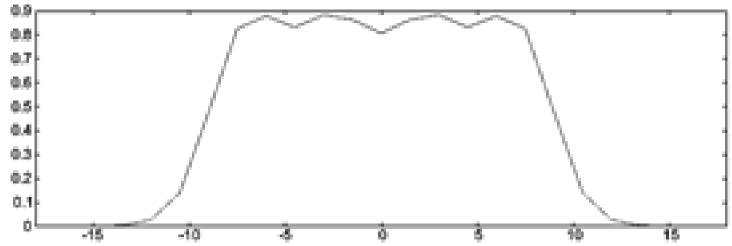
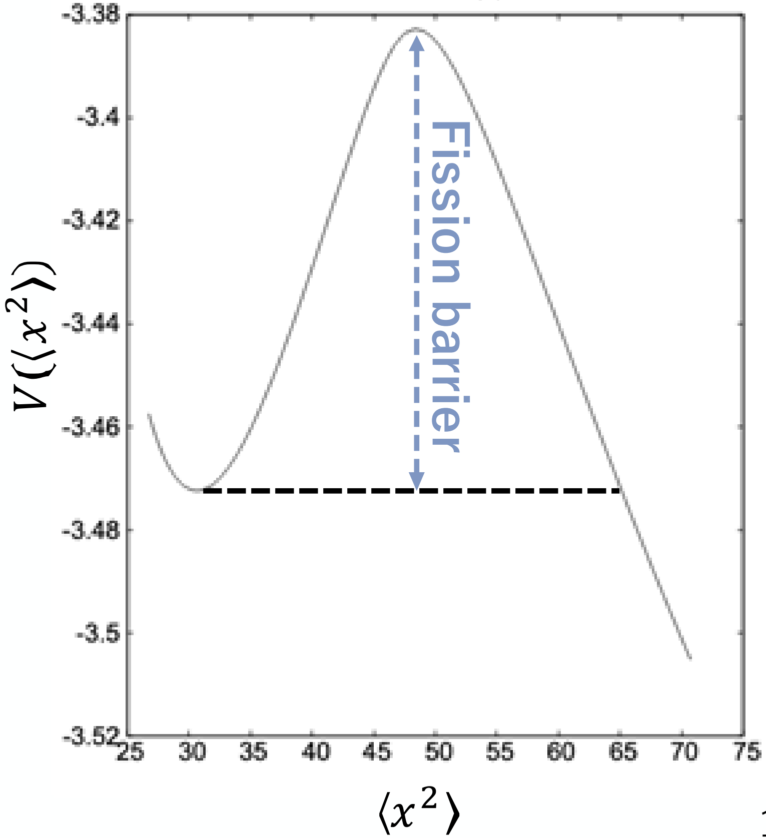
For simplicity, we assume that

- One-dimentional space
- 16 particle system
- Spin-Isospin degeneracy
- No Fock terms





Hamiltonian density of our system

$$\mathcal{H}[\psi(x,-\tau),\psi(x,\tau)] = -M\sum_\alpha \psi_\alpha(x,-\tau)\left(\frac{\partial^2}{\partial x^2}\right)\psi_\alpha(x,\tau)$$
$$+\frac{1}{2}\int dx'\rho(x,\tau)V(x-x')\rho(x',\tau)$$
$$+\frac{1}{3}V_3\rho^3(x,\tau)$$

Three body force **repulsive**

$$V(x) = \frac{V_1}{\sqrt{\pi}\gamma_1}e^{-x^2/\gamma_1^2} + \frac{V_2}{\sqrt{\pi}\gamma_2}e^{-x^2/\gamma_2^2}$$

attractive **repulsive**

Two body force

$$\rho(x,\tau) = M\sum_\alpha \psi_\alpha(x,-\tau)\psi_\alpha(x,\tau)$$

S. Levit, J. W. Negele, and Z. Paltiel, Phys. Rev. C **22**, 1979

Discretization of Time

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For numerical caluculation, the time variable is deiscritized.

G.Puddu , J. W. Negele, Phys. Rev. C **35**, 1007

$$-\frac{\partial \psi_\beta(x,\tau)}{\partial \tau} = \left(-\frac{\partial^2}{\partial x^2} + \int V(x-x')\rho(x',\tau)dx' + V_3\rho^2(x,\tau) + V_\lambda(x)\right)\psi_\beta(x,\tau)$$

$$= h[\rho]\psi_\beta(x,\tau)$$

Constraints to avoid uniform solutions

$$V_\lambda(x) = \lambda\left[\int_{-\frac{1}{2}}^{\frac{1}{2}} d\eta \int x'^2\rho(x',\eta)dx' - x_0^2\right]x^2$$

B.C. $\psi_\beta(x,\frac{T}{2}) = e^{-\lambda_\beta}\psi_\beta(x,-\frac{T}{2})$

Discretization of time $\tau_0, \tau_1, \dots \tau_{N_\tau}$

Number of meshes: 32, Mesh width: 3.5

$$\psi_\beta(x_i,\tau_{k+1}) = \exp\left(-h\left[\frac{\rho_{k+1}+\rho_k}{2}\right]\Delta\tau\right)\psi_\beta(x_i,\tau_k)$$

Time evolution operator

Boundary condition $\psi_\beta(x_i,\tau_{N_\tau}) = e^{-\alpha_\beta}\psi_\beta(x_i,\tau_1)$

Computational Flow ①

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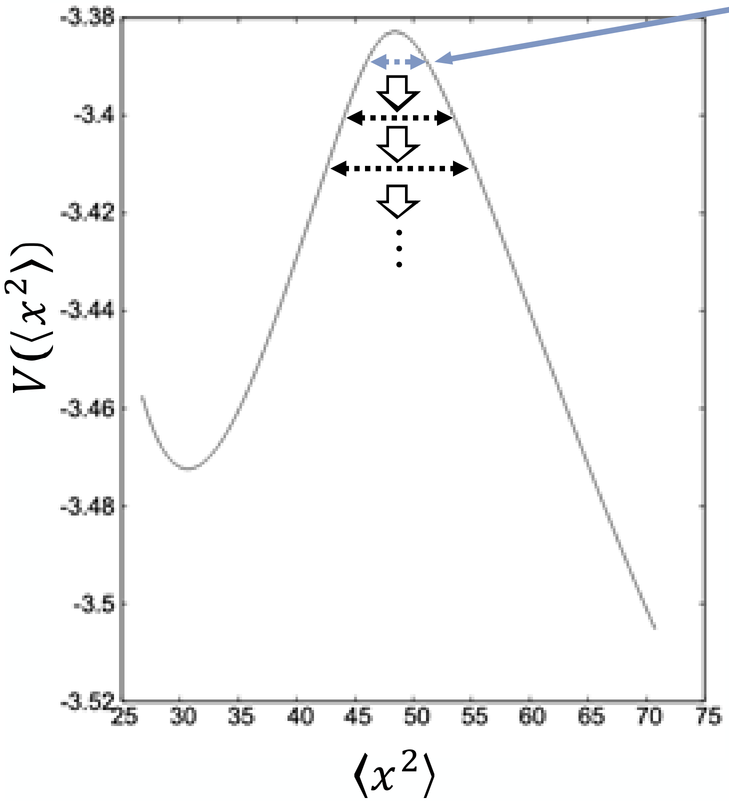
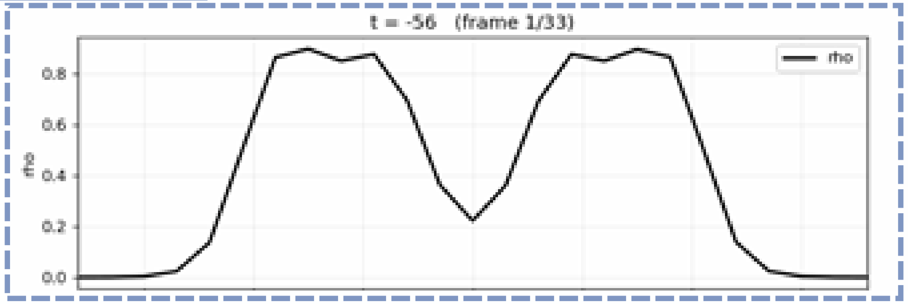
G.Puddu , J. W. Negele, Phys. Rev. C **35**, 1007



Initial pass



infinitesimal dilatation mode of saddle point density

Imaginary Time evolving

Imaginary Time evolution operator

$$\psi_\beta(x_i, \tau_{k+1}) = \exp\left(-h\left[\frac{\rho(x_i, k+1) + \rho(x_i, k)}{2}\right]\Delta\tau\right)\psi_\beta(x_i, \tau_k)$$

※Orthogonalize at each step to avoid divergence.

Update the density

$$\rho_{\text{new}}(x, \tau) = (1 - K)\rho_{\text{old}}(x, \tau) + KM\sum_{\alpha=1}^{4}\psi_\alpha(x, -\tau)\psi_\alpha(x, \tau)$$

If $\rho_{\text{new}}(x, \tau) = \rho_{\text{old}}(x, \tau)$, calcuration stop

The existence of V_λ means that ρ_{new} is not ρ_{old} generally

$$V_\lambda(x) = \lambda\left[\int_{-\frac{1}{2}}^{\frac{1}{2}} d\eta \int x'^2 \rho(x', \eta)dx' - x_0^2\right]x^2$$

Constraints to avoid uniform solutions

Computational Flow ①

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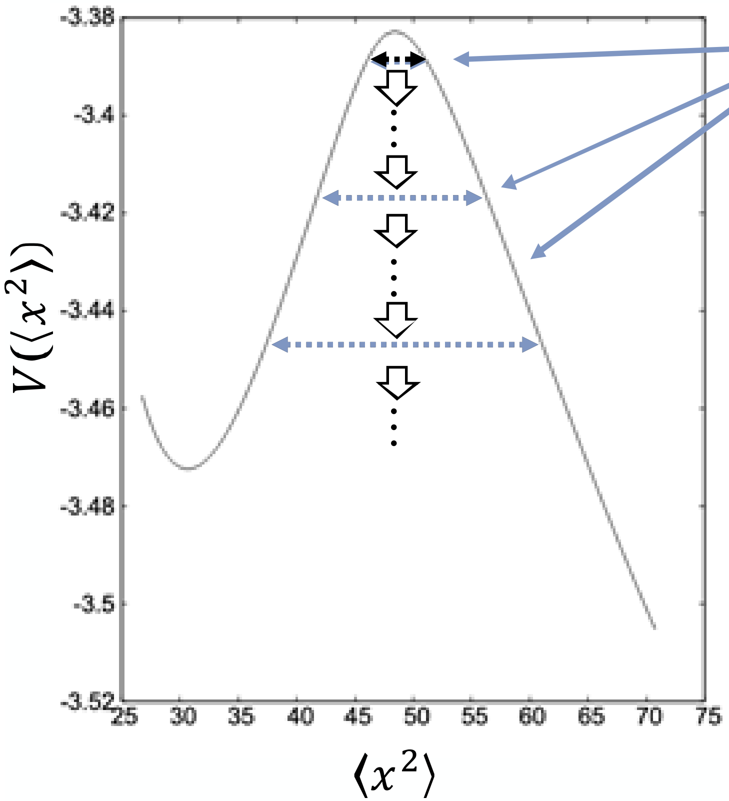
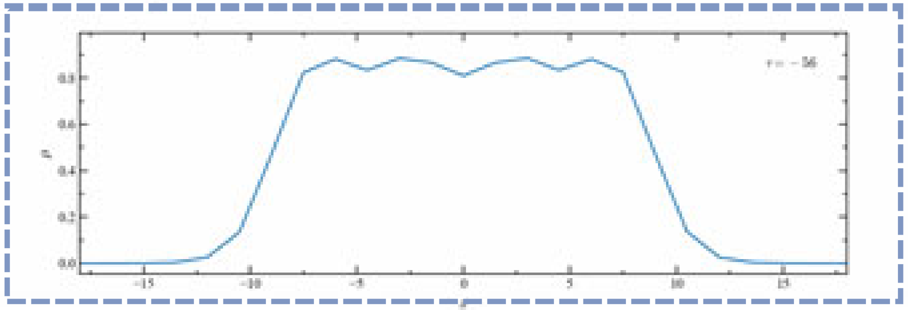
G.Puddu , J. W. Negele, Phys. Rev. C **35**, 1007



100 iterations

Initial phase



infinitesimal dilatation mode of saddle point density

Imaginary Time evolving

From initial time $-\frac{T}{2}$ to final time $\frac{T}{2}$

$$\psi_\beta(x_i, \tau_{k+1}) = \exp\left(-h\left[\frac{\rho(x_i, k+1) + \rho(x_i, k)}{2}\right]\Delta\tau\right)\psi_\beta(x_i, \tau_k)$$

※Orthogonalize at each step to avoid divergence.

Update the density

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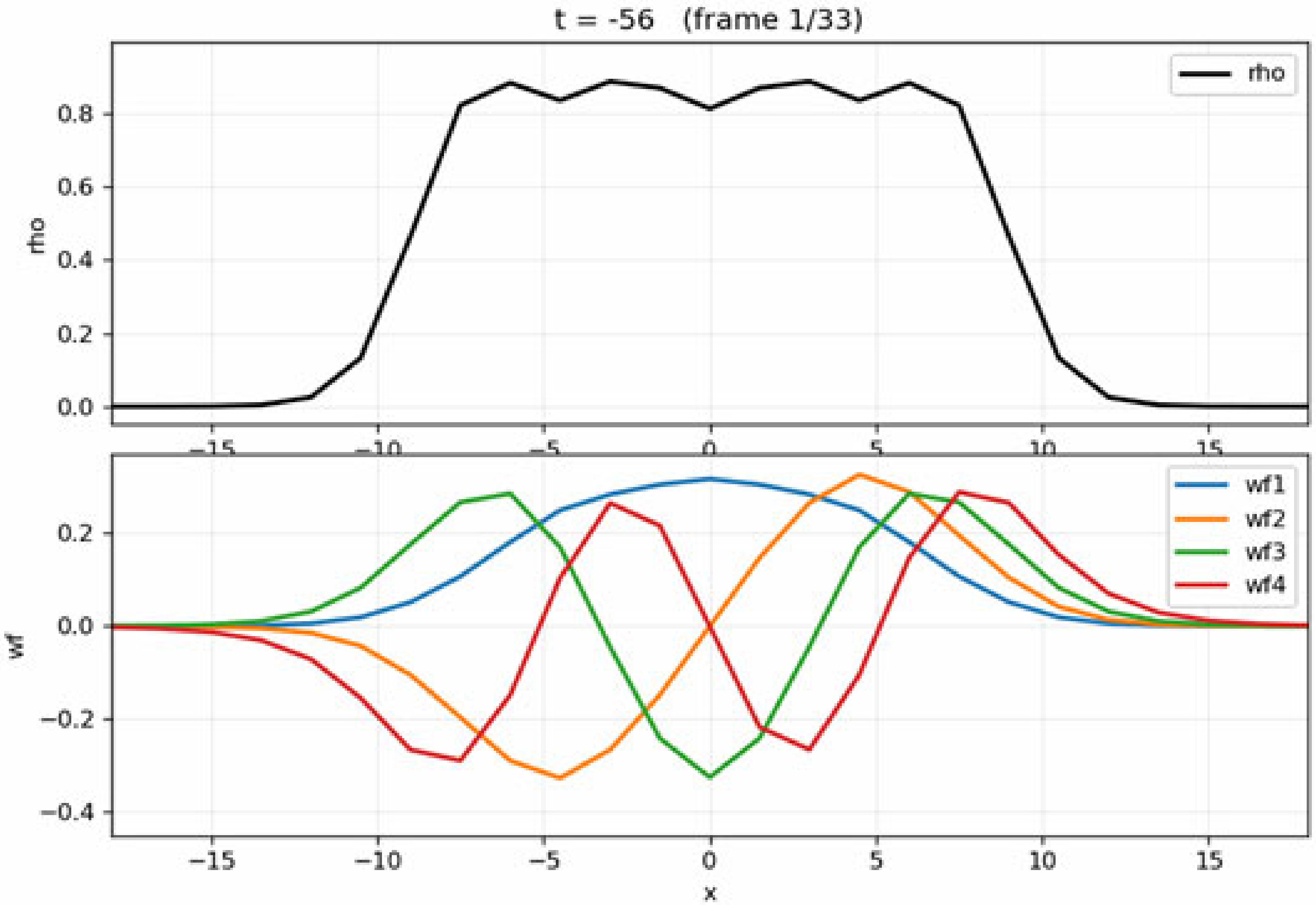
The existence of V_λ means that ρ_{new} is not ρ_{old} generally

$$V_\lambda(x) = \lambda\left[\int_{-\frac{1}{2}}^{\frac{1}{2}} d\eta \int x'^2 \rho(x', \eta)dx' - x_0^2\right] x^2$$

Constraints to avoid uniform solutions

Results of Iterative Method



It seems like fission and periodic motion.

Transformation of ITDHF

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Imaginary TDHF (ITDHF)

$$-\hbar\partial_\tau\psi_k(\tau) = -\frac{\hbar^2}{2m}\nabla^2\psi_k(\tau) + \frac{\delta\mathcal{V}}{\delta\psi_k(-\tau)}$$

$$\|$$

$$\left(\partial_\tau + \hat{h}(x,\tau)\right)\psi_k(x,\tau) = 0$$

B.C. $\psi_k(T/2) = e^{-\alpha_k}\psi_k(-T/2)$

Can be solved by iterative method

$$\phi_\beta(x_i,\tau_{k+1}) = \exp\left(-h\left[\frac{\rho_{k+1}+\rho_k}{2}\right]\Delta\tau\right)\phi_\beta(x_i,\tau_k)$$

Unitary transformation in Imaginary time

$$\phi_k(x,\tau) = e^{-\lambda_k(\tau+T/2)}\psi_k(x,\tau)$$

$$\lambda_k = \alpha_k/T$$

$$\left(\partial_\tau + \hat{h}(x,\tau)\right)\phi_k(x,\tau) = \lambda_k\phi_k(x,\tau)$$

B.C. $\phi_k(x,T/2) = \phi_k(x,-T/2)$

Eigenvalue equation

$N_x \times N_\tau$

$N_x \times N_\tau$

$$\left[\left(\partial_\tau + \hat{h}(x,\tau)\right)\right]\left(\phi_k(x,\tau)\right) = \lambda_k\left(\phi_k(x,\tau)\right)$$

Computational Flow ②

(transformed) ITDHF eq

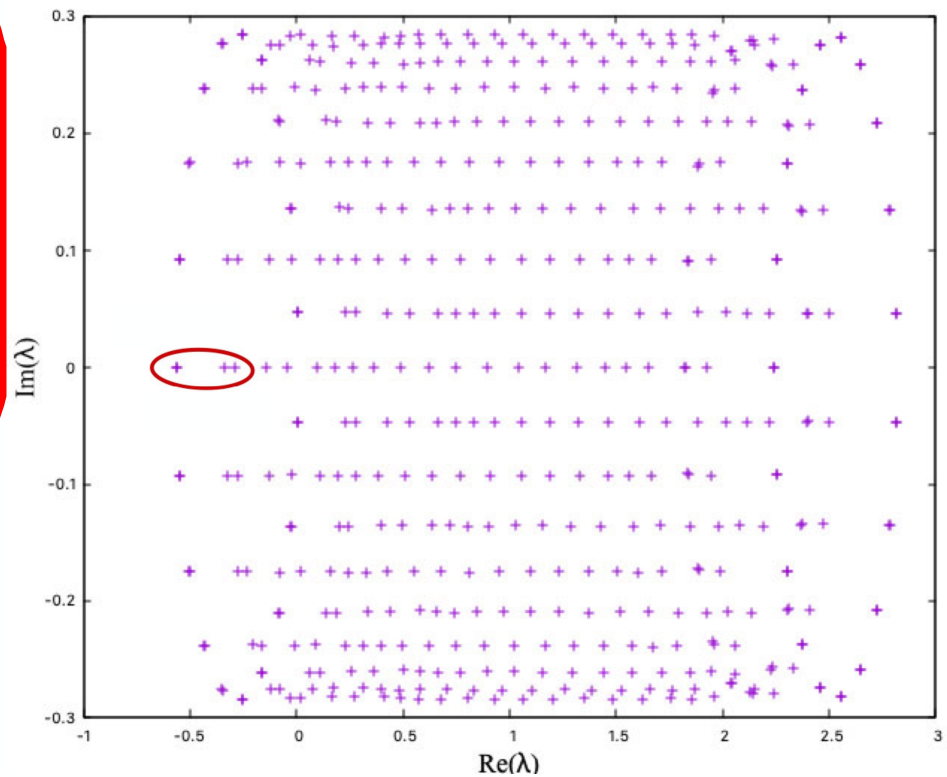
$$\left(\partial_\tau + \hat{h}(x,\tau)\right)\phi_k(x,\tau) = \lambda_k \phi_k(x,\tau)$$

B.C. $\phi_k(x, T/2) = \phi_k(x, -T/2)$

Diagonalization

$\{\phi_k(x,\tau), \lambda_k\}$

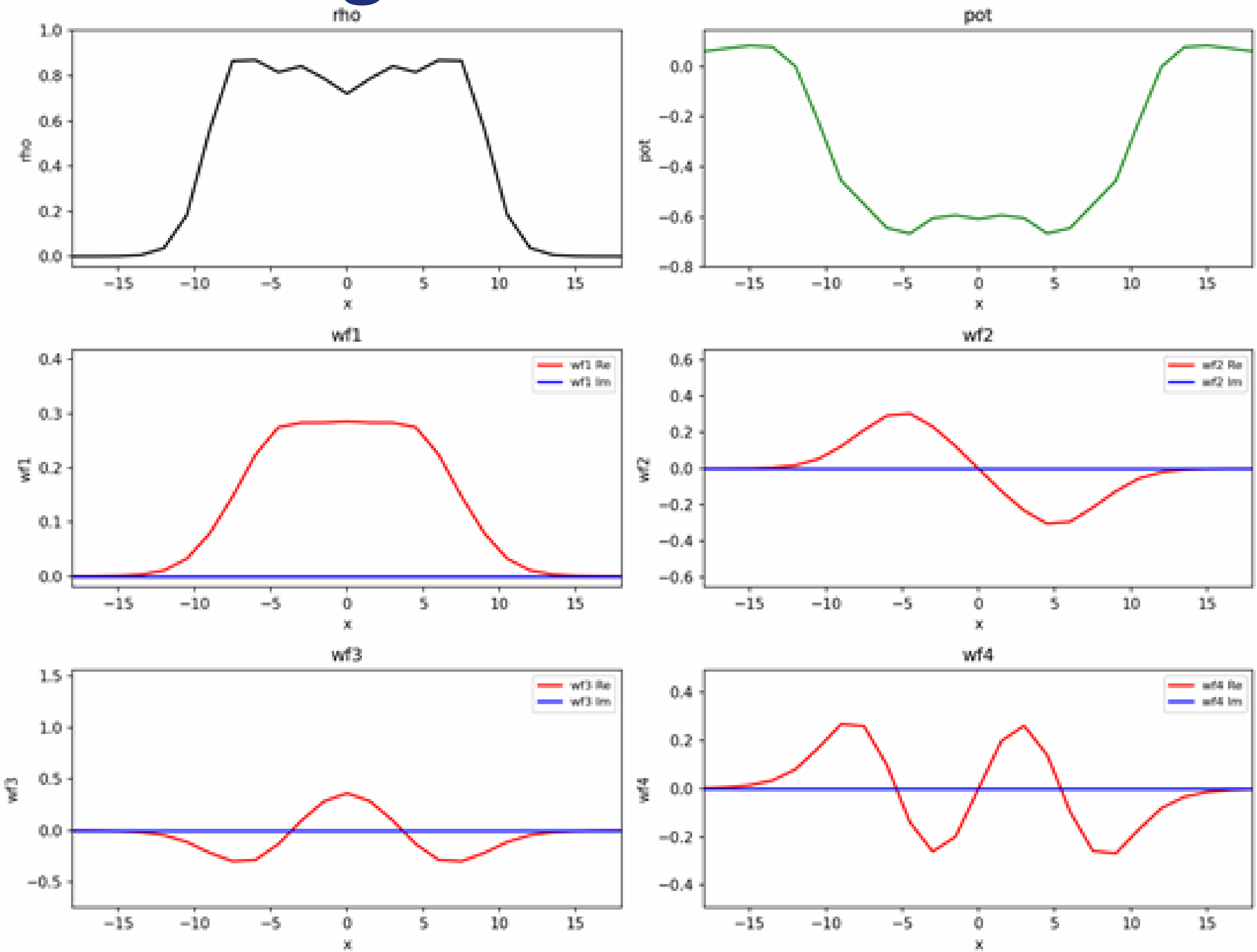
# of eigenvalue = $N_x \times N_\tau$



Pick up the four smallest real eigenvalues in order.

$$\rho(x,\tau) = M \sum_{k=1}^{4} \phi_k(x,-\tau)\phi_k(x,\tau)$$

Results of Diag Method



- It appears that this solution's behavior differs from that of the previous solution. (e.g. wf1 and wf3's norm)

Computational Flow ③

(transformed) ITDHF eq

$$\left(\partial_\tau + \hat{h}(x,\tau)\right)\phi_k(x,\tau) = \lambda_k \phi_k(x,\tau)$$

B.C. $\phi_k(x, T/2) = \phi_k(x, -T/2)$

$$\phi_k^{\mathrm{new}} = \phi_k^{\mathrm{old}} - \left(\partial_\tau + \hat{h}\right)\phi_k^{\mathrm{old}}\Delta t$$

Orthonormalization ϕ_k^{new}

$$\rho(x,\tau) = M\sum_{k=1}^{4}\phi_k(x,-\tau)\phi_k(x,\tau)$$

This method is analogy of imaginary time method for static HF calculation.

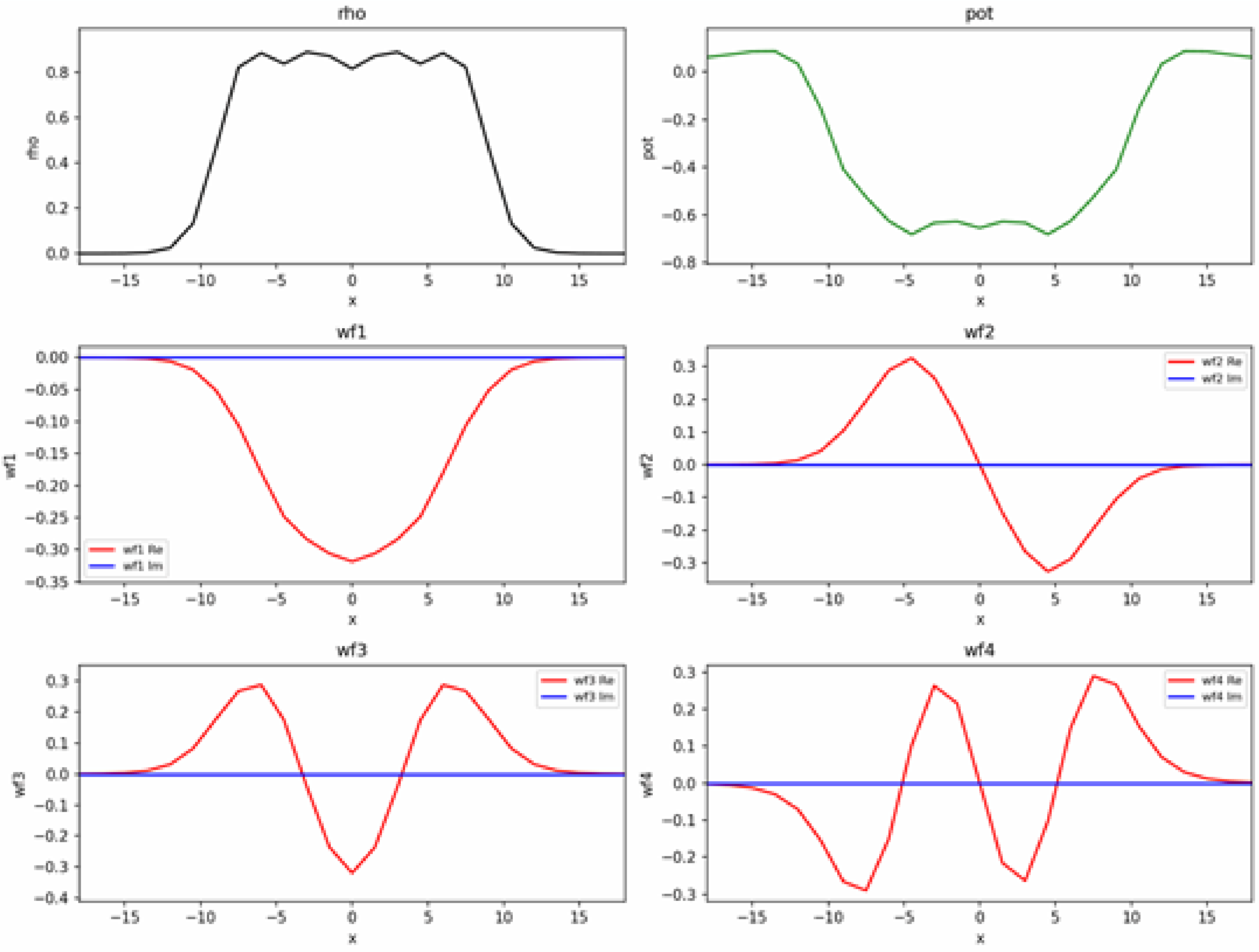
$N_x \times N_\tau$

$N_x \times N_\tau$

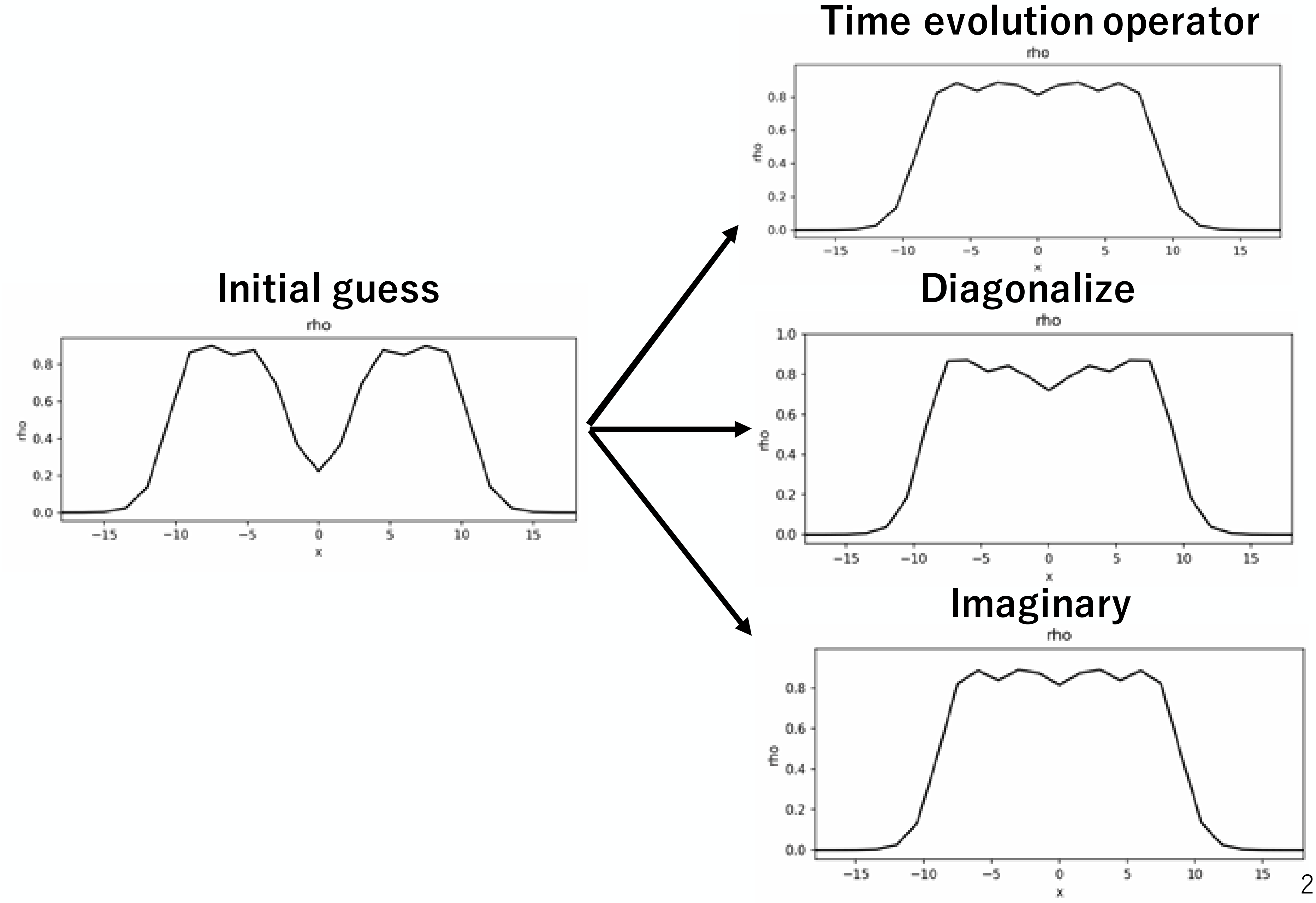
$$\left[\left(\partial_\tau + \hat{h}(x,\tau)\right)\right]\left(\phi_k(x,\tau)\right)$$

1+1-dimensional imaginary time method

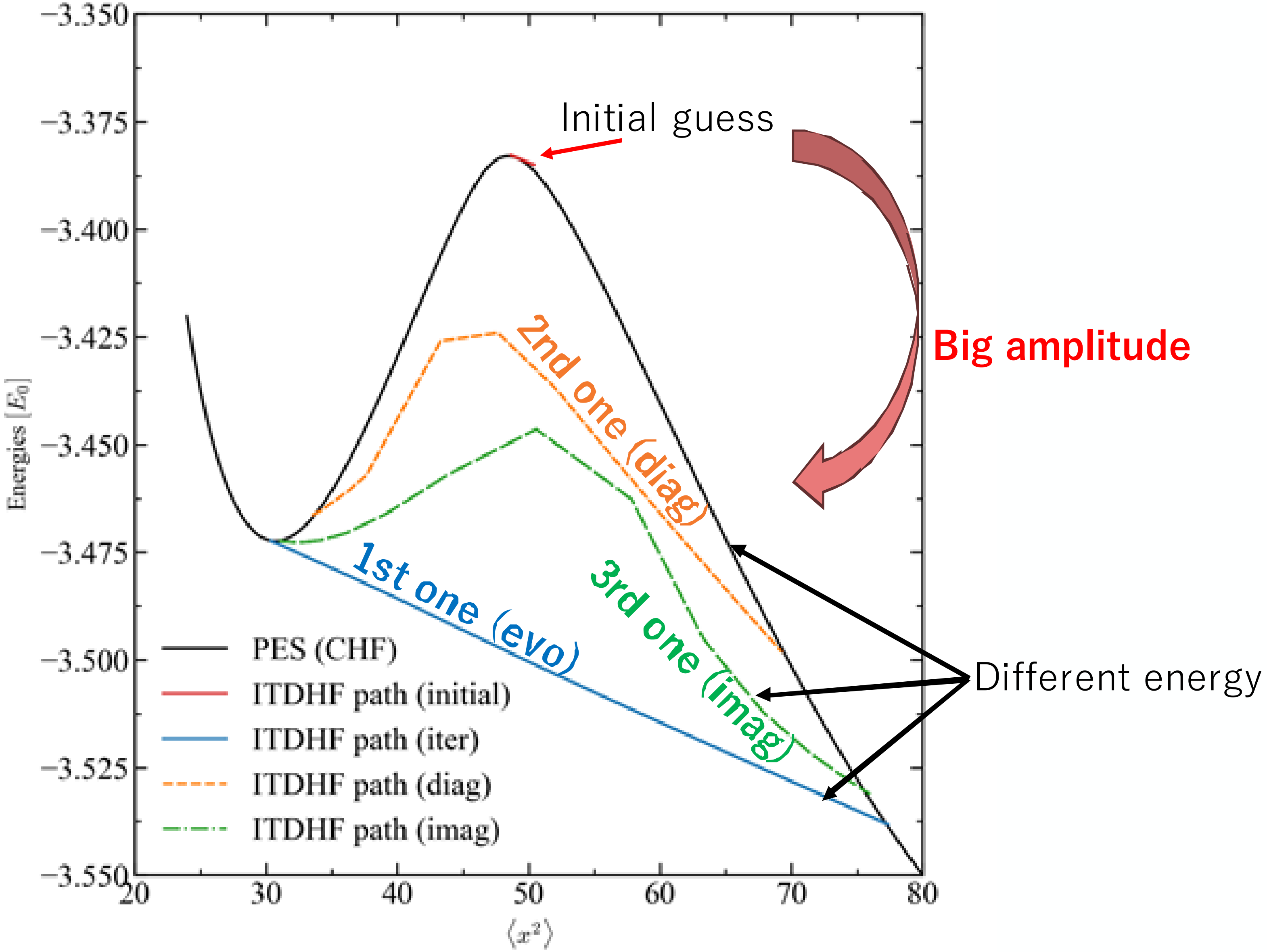
Results of Imag Method



Each ITDHF Path’s Energy



Each ITDHF Path's Energy

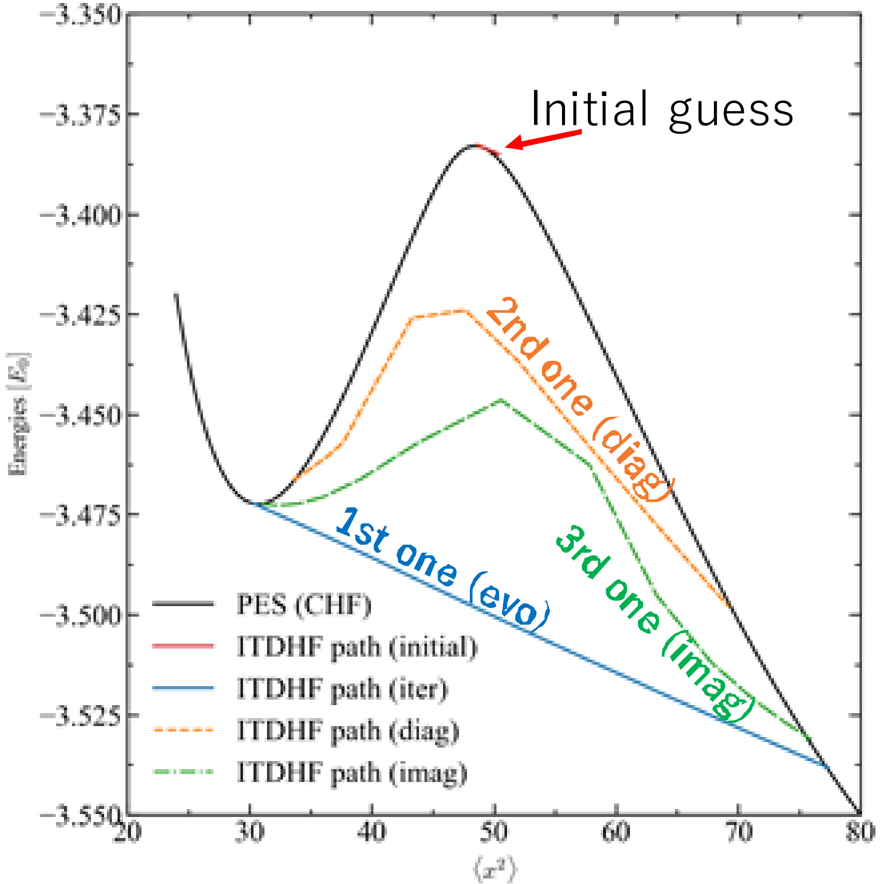


All paths are not conserving energy.

Discussion

In a converged path, it appears that energy is NOT conserved.

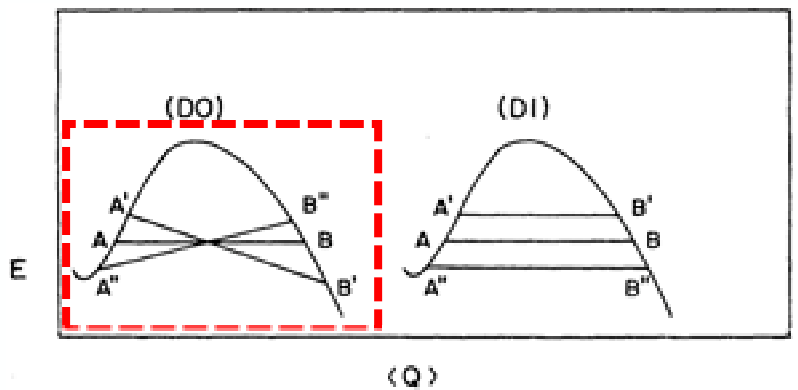
Potential energy surface



Analytically, we can prove that energy is conserved in imaginary time.

$$\frac{\partial \mathcal{H}}{\partial \tau} = \hbar \sum_i \left(-\frac{\delta \mathcal{H}}{\delta \phi_i(\tau)} \frac{\delta \mathcal{H}}{\delta \phi_i^*(-\tau)} + \frac{\delta \mathcal{H}}{\delta \phi_i^*(-\tau)} \frac{\delta \mathcal{H}}{\delta \phi_i(\tau)} \right) = 0$$

However, numerically, orthogonalizing each time step could change the energy. Furthermore, if we do not orthogonalize each time step, the calculation diverges.



Previous research has suggested that this is because the infinitesimal dilatation mode used for the initial conditions contains **an Random Phase Approximation mode.**

G.Puddu , J. W. Negele, Phys. Rev. C **35**, 1007

Summary & Future Work

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Summary

- Our research purpose is to describe spontaneous fission from nucleon degrees of freedom.
- We quantized TDHF by periodic Imaginary TDHF (ITDHF).
- ITDHF equation can be interpreted as eigenvalue equation in space and time dimension (Conventional TDHF calculation is initial value problem.)
- We calculated simple 1D system's ITDHF equation by three method. (time evolution operator calculation, diagonalize calculation and "imaginary method" calculation)

Future Work

- Investigate initial guess dependency for converged path.
- Calculate the half-life in the 1D system and compare it with the half-lives calculated using other theories.

Results

Discussion & Conclusion

Discussion

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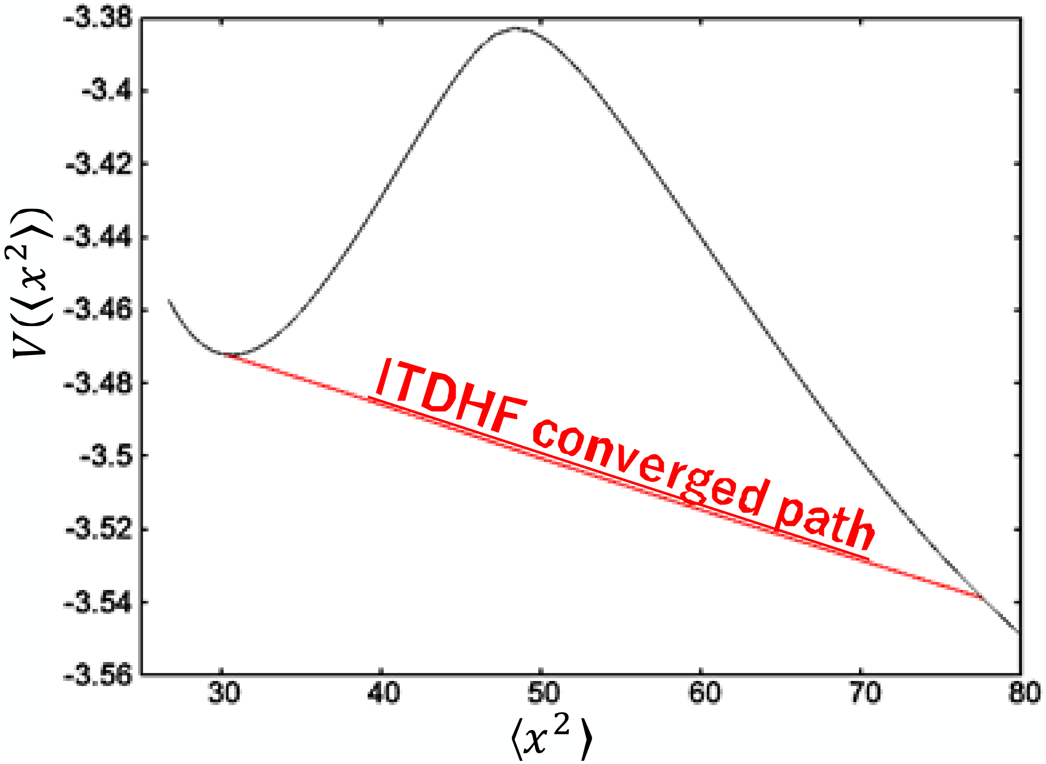
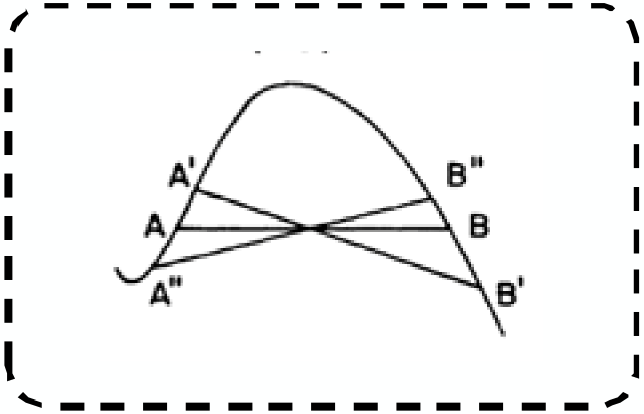
In a converged path, it appears that energy is not conserved.

Potential energy surface



In original paper, the energy also may not conserved.



However, we still do not fully understand the original paper…

Mathematically, we can prove that energy is conserved in imaginary time.

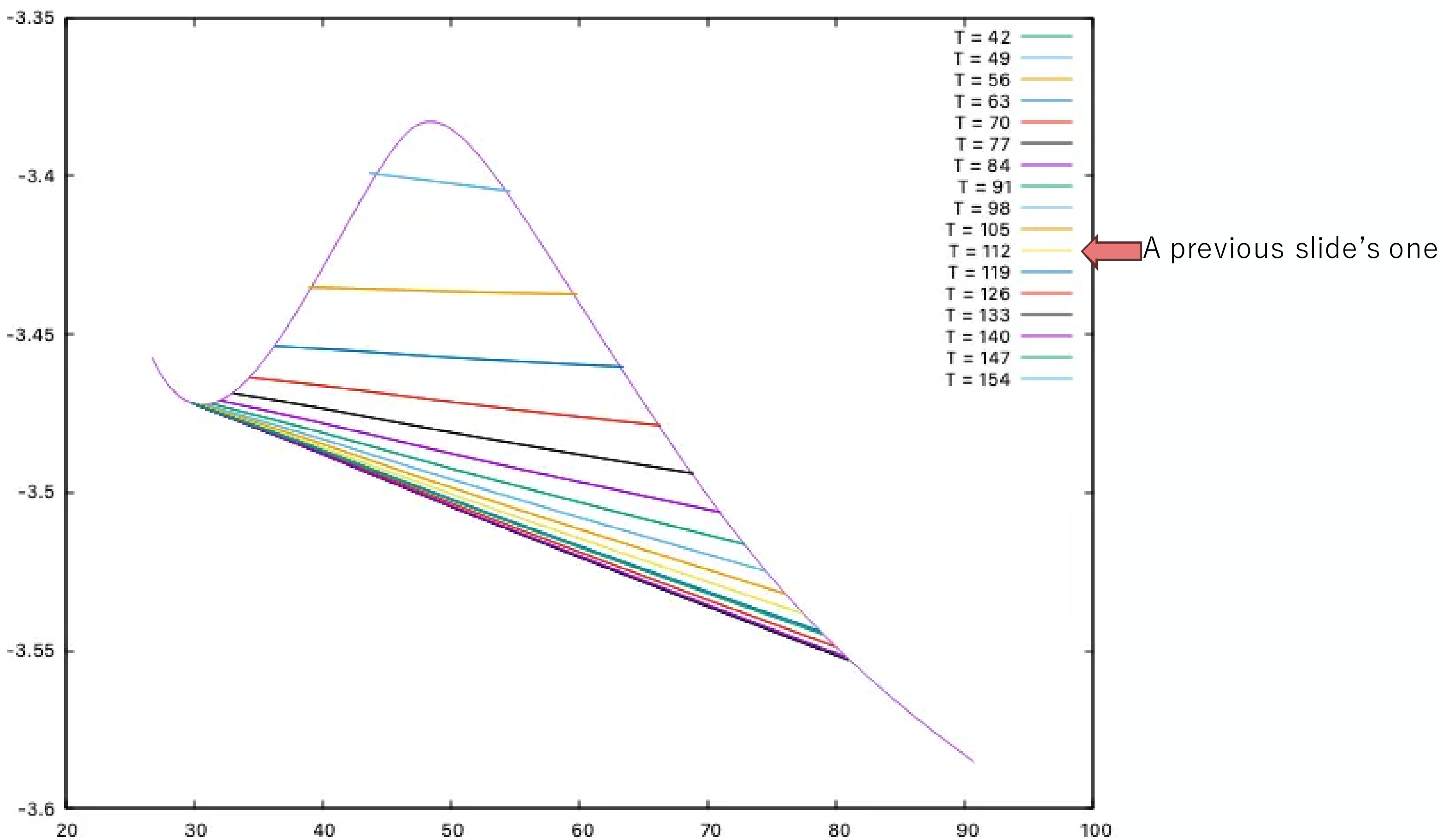
$$\frac{\partial \mathcal{H}}{\partial \tau} = \hbar \sum_i \left(-\frac{\delta \mathcal{H}}{\delta \phi_i(\tau)} \frac{\delta \mathcal{H}}{\delta \phi_i^*(-\tau)} + \frac{\delta \mathcal{H}}{\delta \phi_i^*(-\tau)} \frac{\delta \mathcal{H}}{\delta \phi_i(\tau)} \right) = 0$$

However, numerically, orthogonalizing each time step could change the energy. Furthermore, if we do not orthogonalize each time step, the calculation diverges.

Discussion

The time period T is the free parameter we can choose. If T is change, the converged path is also change.

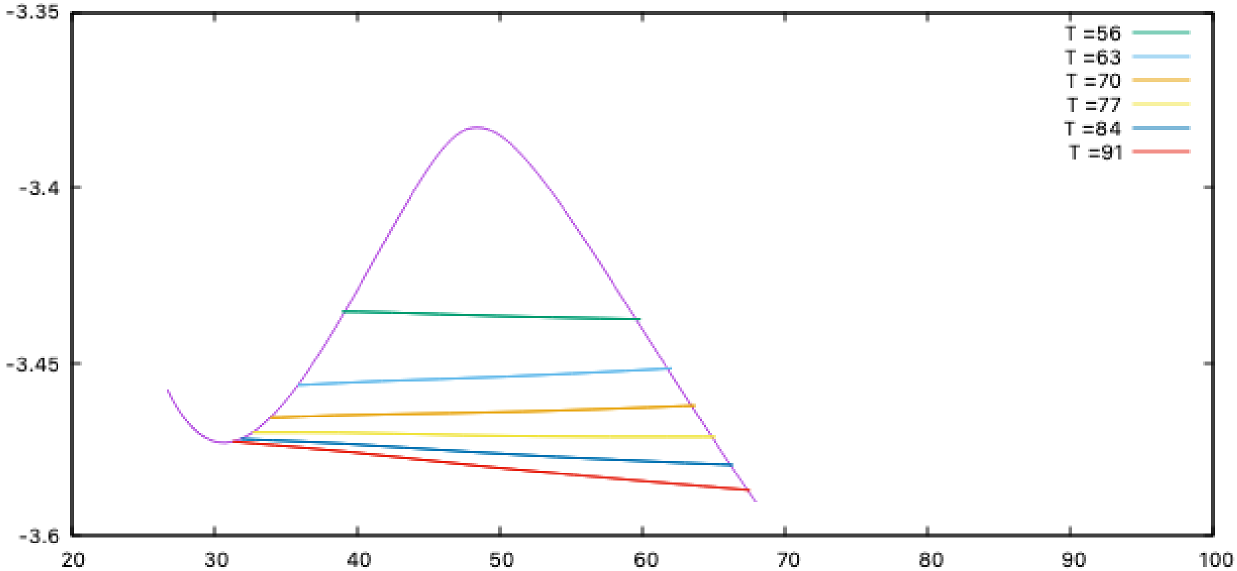
Boundary condition $\phi_\beta(x, \frac{T}{2}) = e^{-\lambda_\beta}\phi_\beta(x, -\frac{T}{2})$



However, simply changing T could not obtain the path of energy conservation and through the ground states.

Discussion

Even if we pick up the energy conserved path ($T = 56$) as a initial path and extend T, near the ground states, the energy is not conserved.



Furthermore, it differs from direct and extended calculations.
It may show that there is initial value dependency…?

Even with a constraint, the initial guess must be close in some sense to a periodic solution, or else iteration will evolve it into an uninteresting solution, usually a static Hartree-Fock solution. An adequate strategy for finding eigenstates of large-amplitude collective motion is first to solve the RPA equations for infinitesimal vibrations. Starting with the time-dependent wave functions for a single mode, a series of sequential self-consistent calculations may be performed gradually increasing the period from the RPA value $T_0 = 2\pi/\omega$. By continuity, the ini-

J. W. Negele, Rev. Mod. Phys. **54**, 913 (1982)

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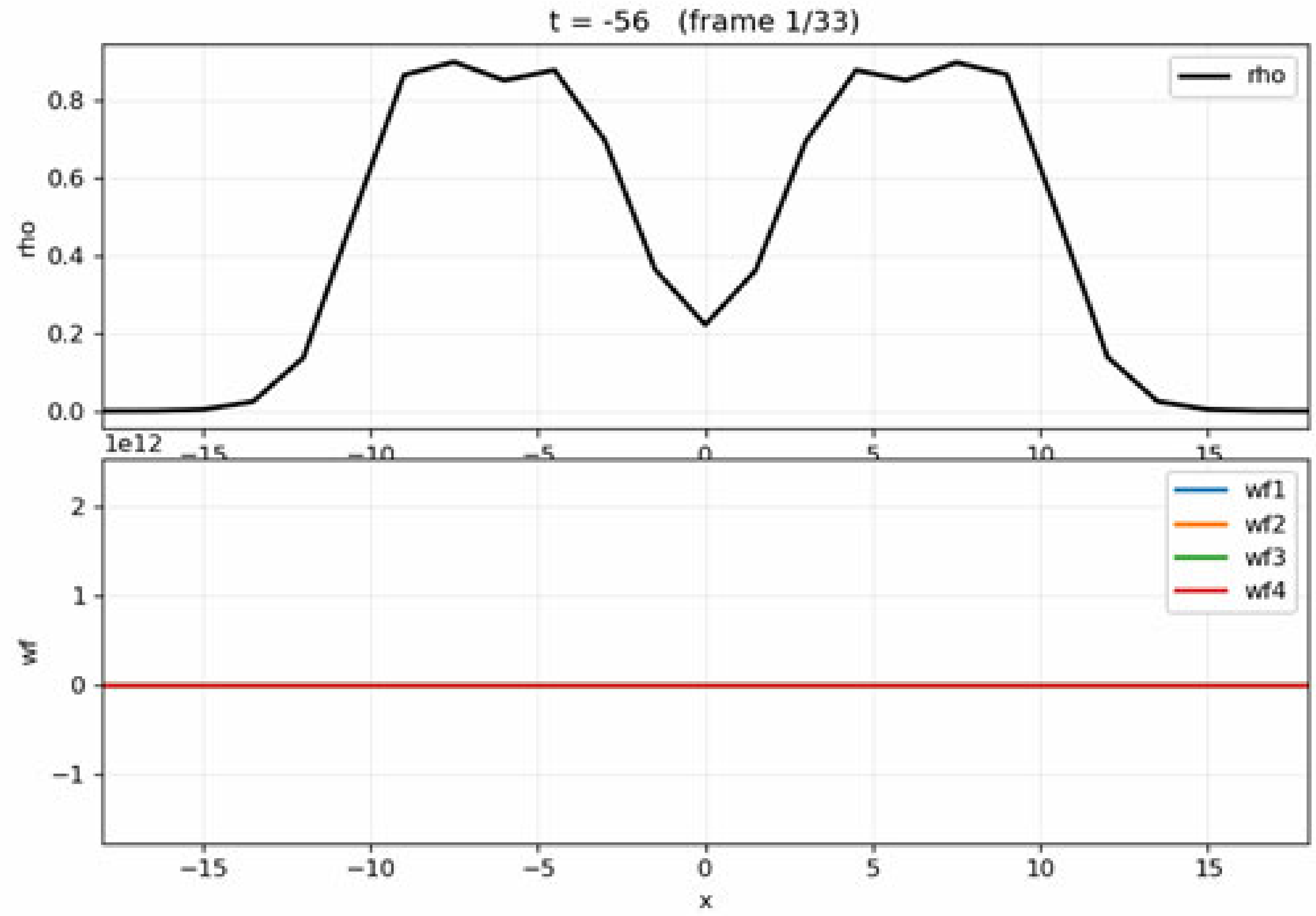
Back up

Back up



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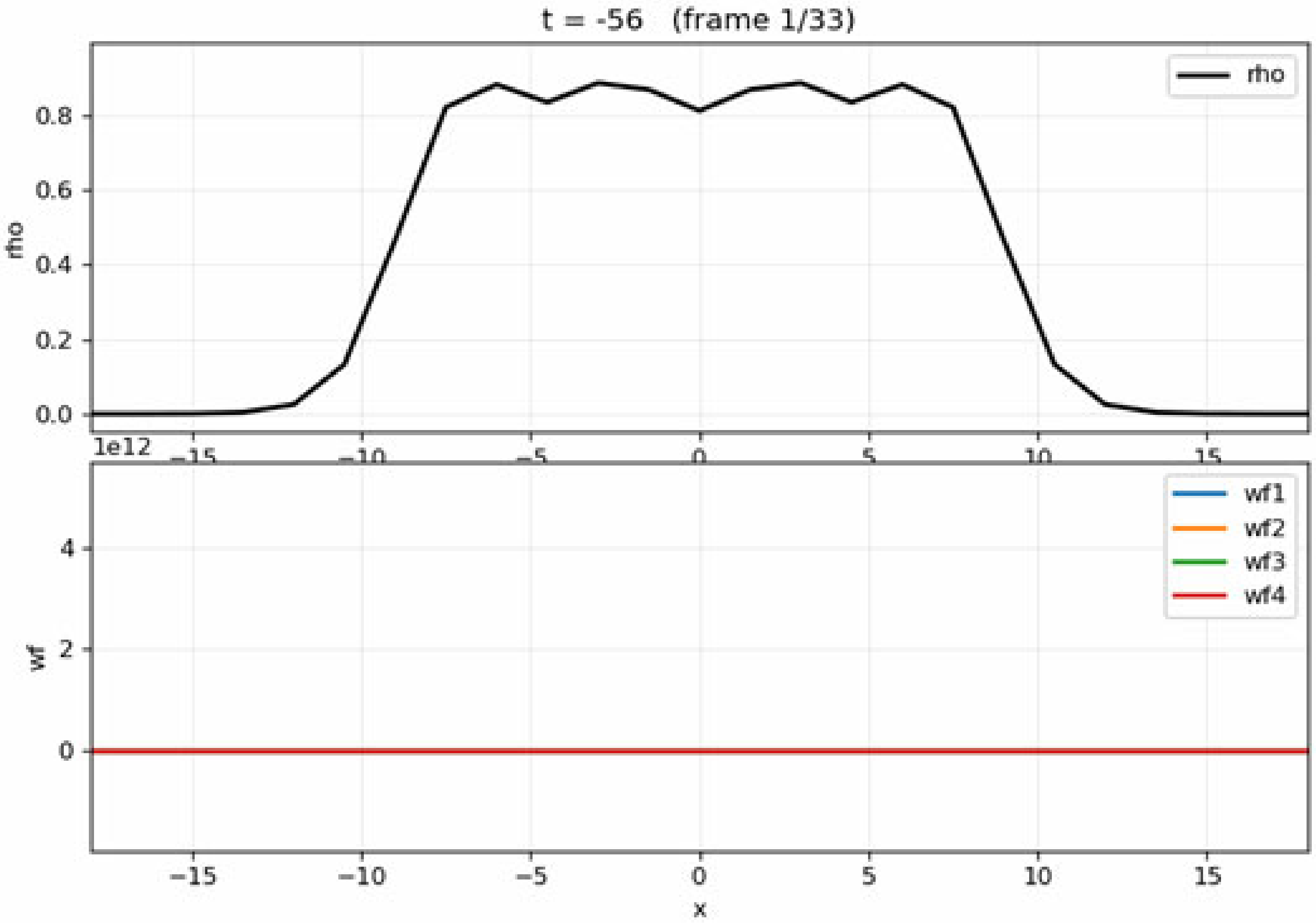
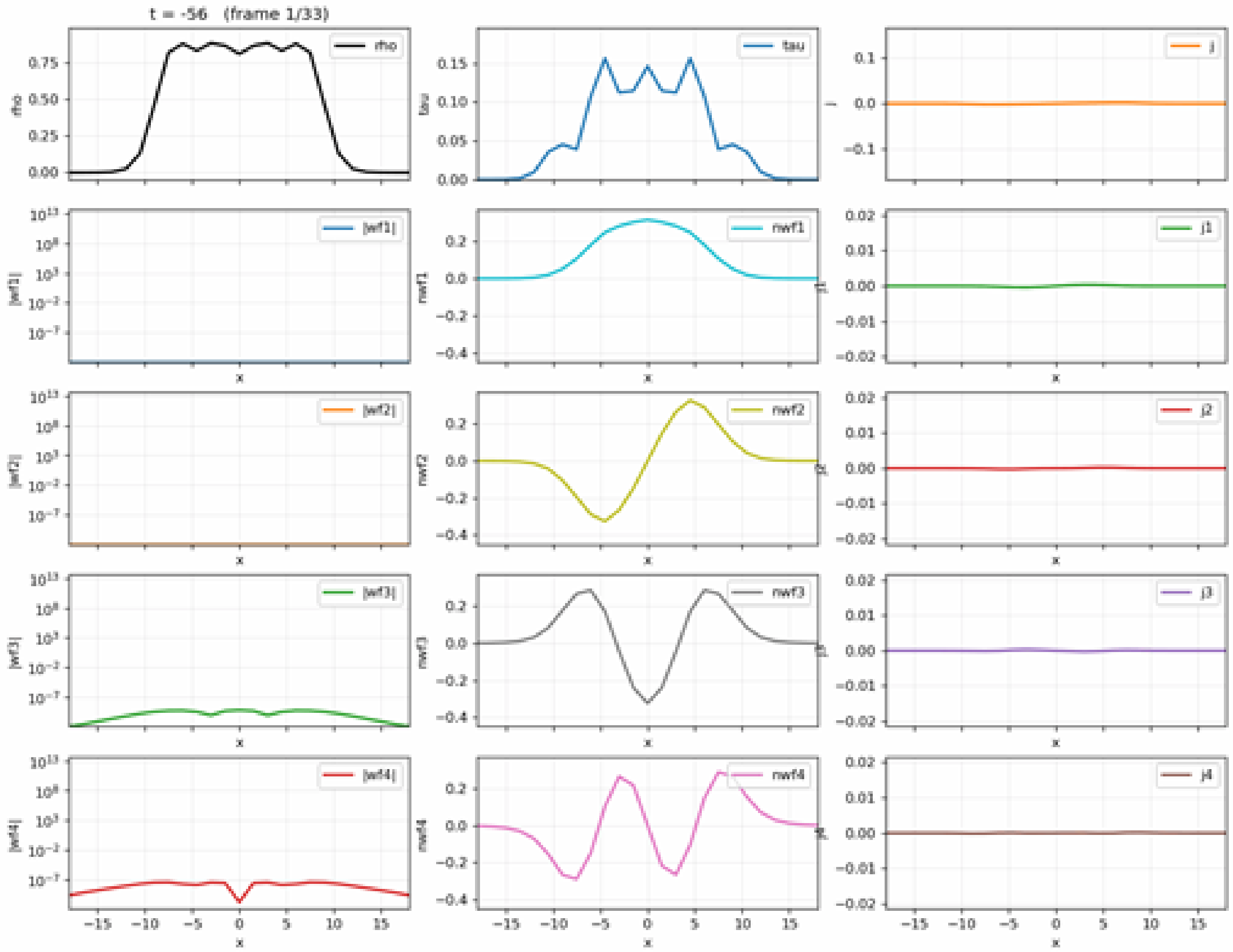
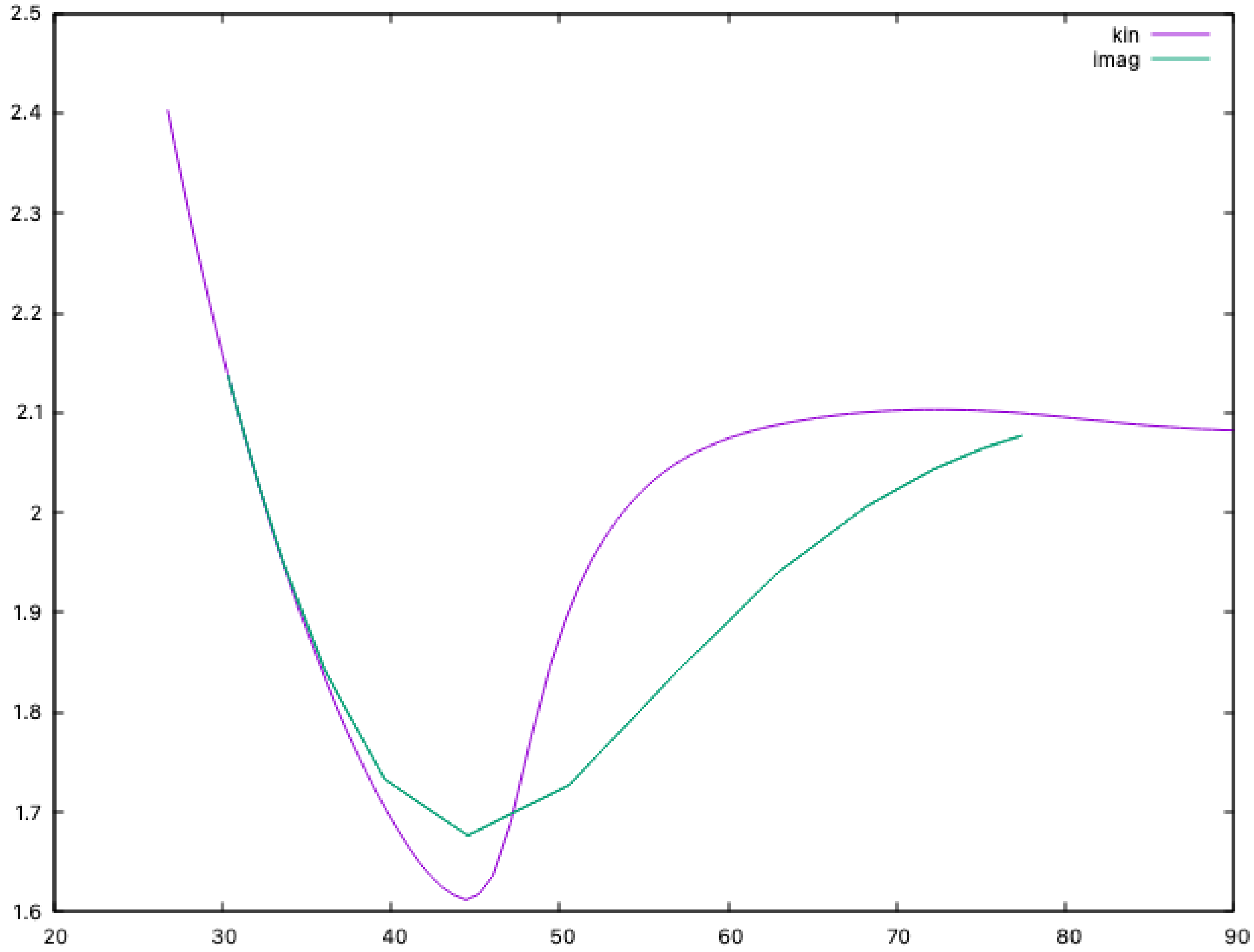


t = -56 (frame 1/33)
rho
0.8
0.6
0.4
0.2
0.0
1e12
wf1
wf2
wf3
wf4
wf
4
2
0
−15
−10
−5
0
5
10
15
x

Back up


t = -56 (frame 1/33)
rho
tau
j
|wf1|
nwf1
j1
|wf2|
nwf2
j2
|wf3|
nwf3
j3
|wf4|
nwf4
j4
x

Back up



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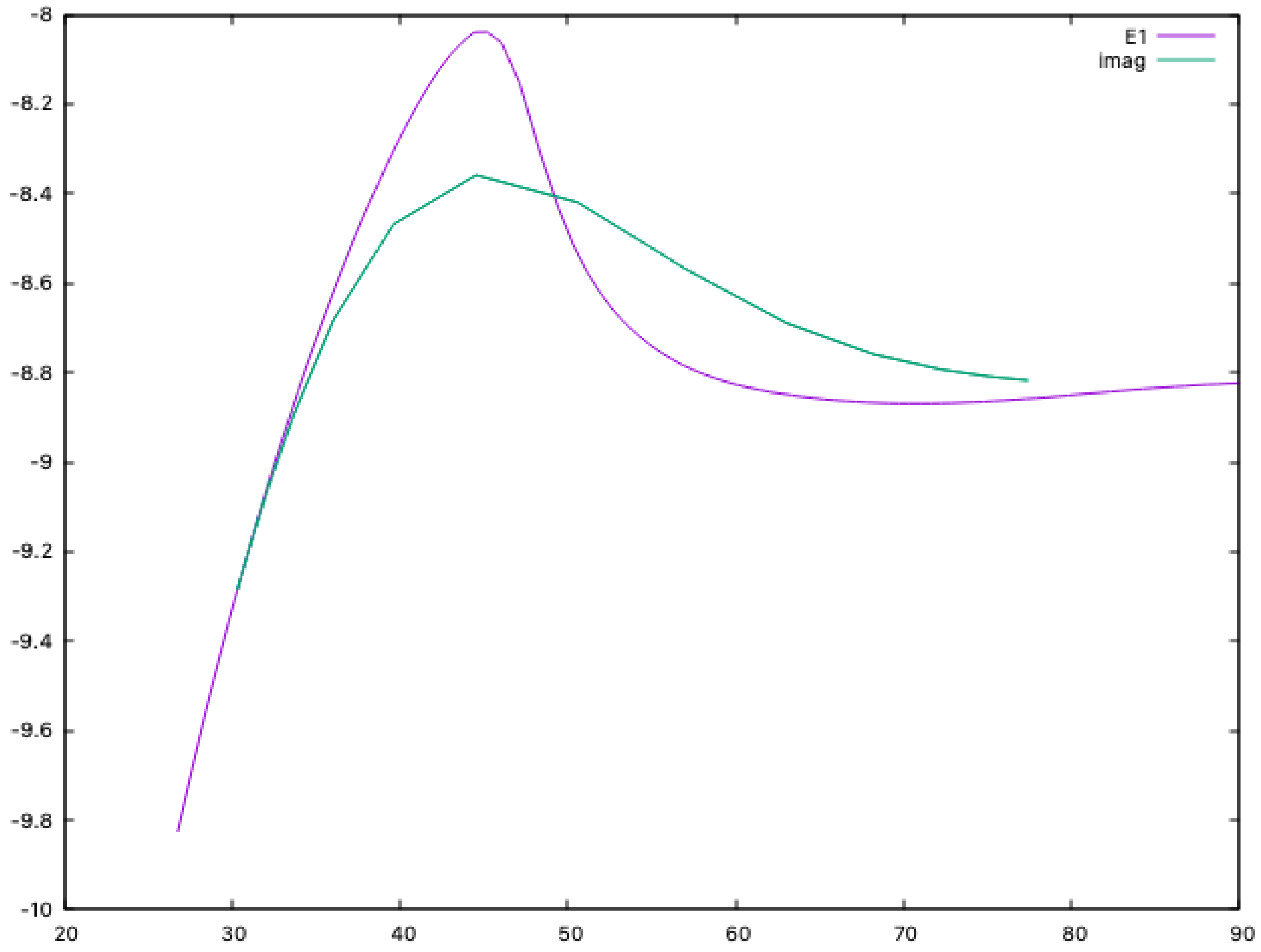

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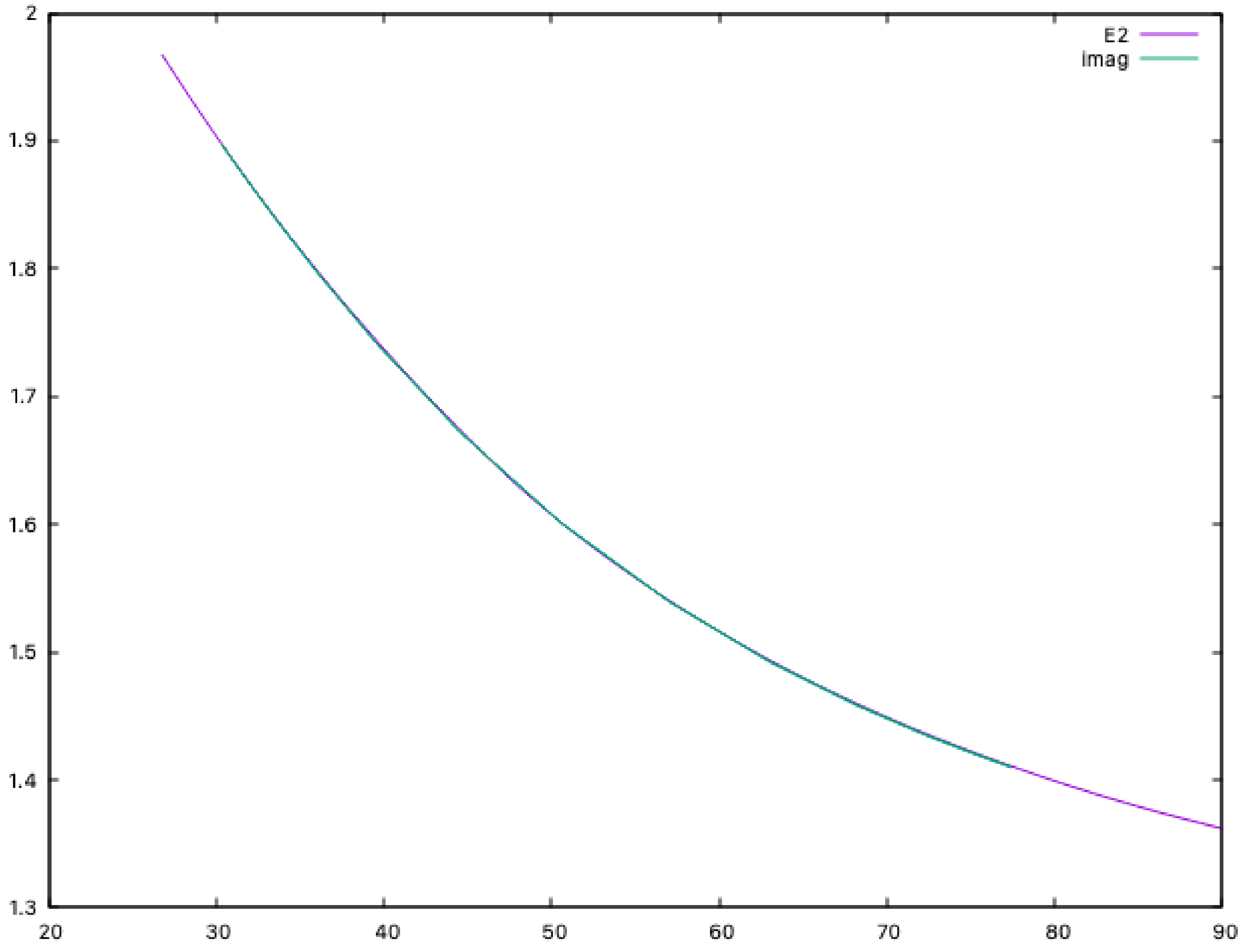

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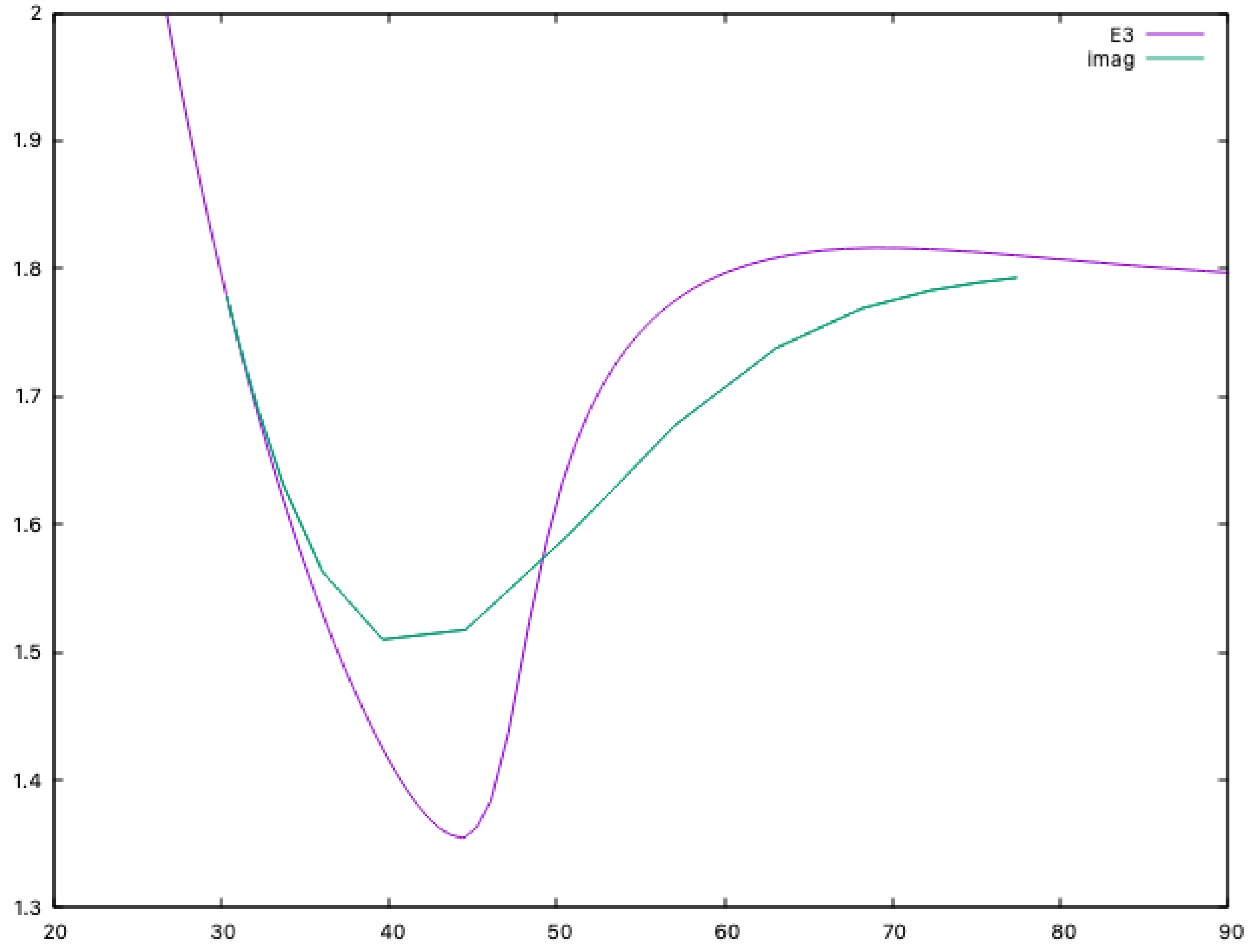

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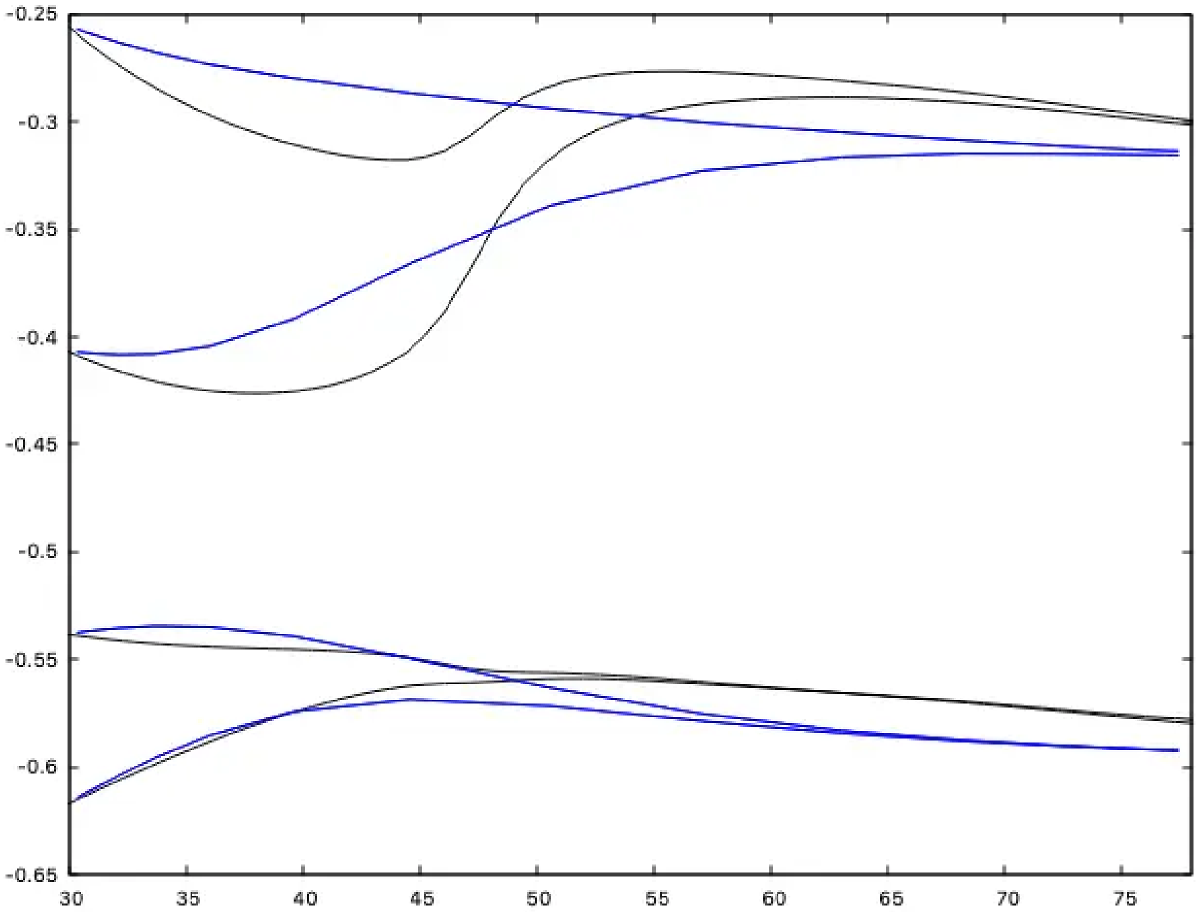
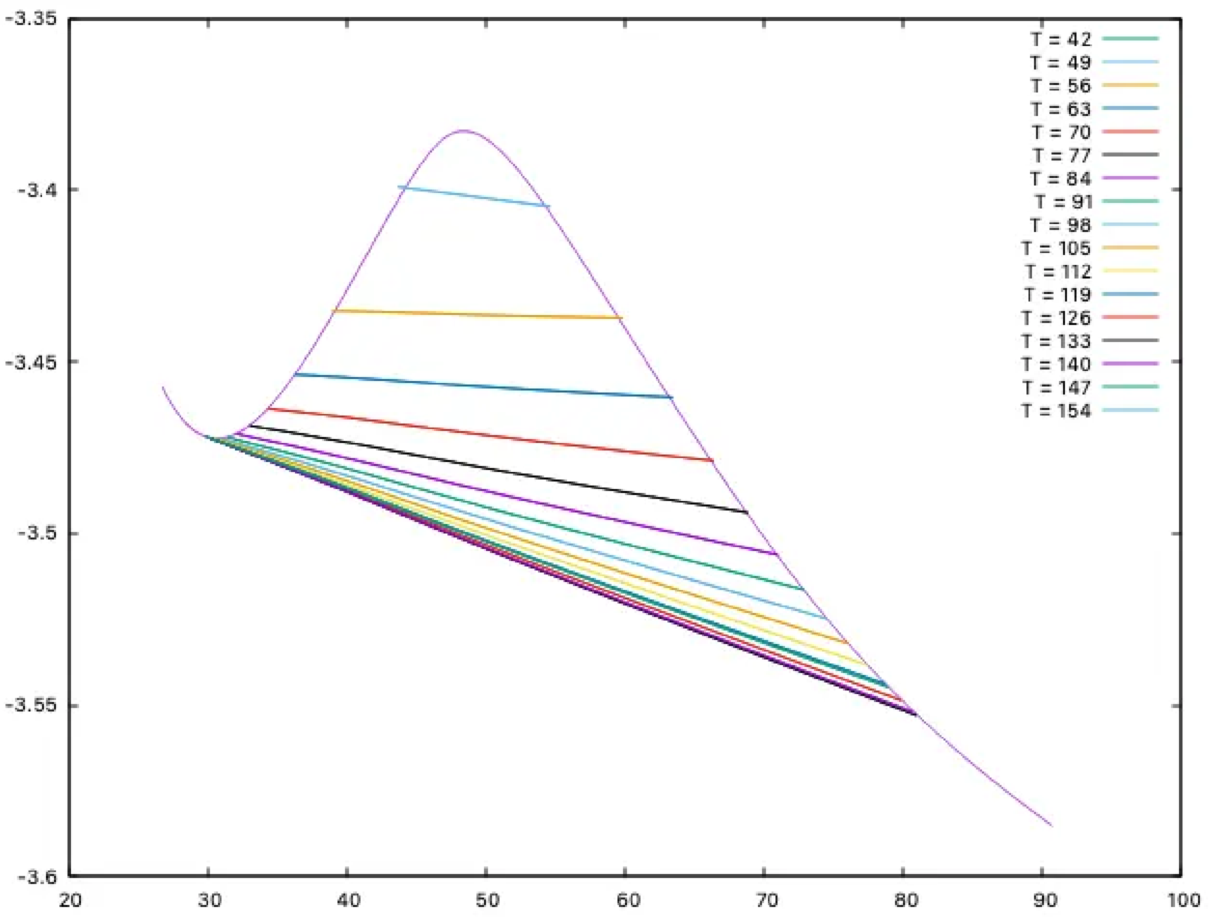
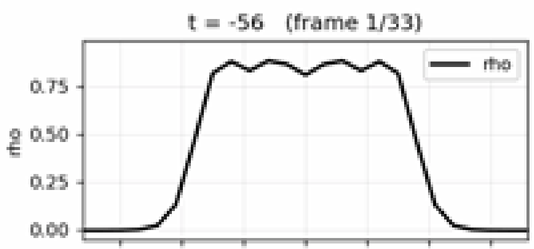
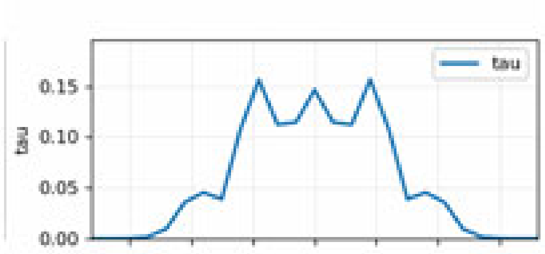


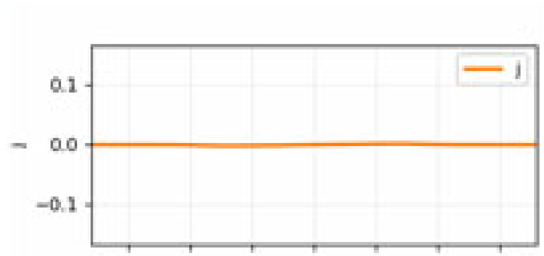
Back up


-3.35
-3.4
-3.45
-3.5
-3.55
-3.6
20
30
40
50
60
70
80
90
100
T = 42
T = 49
T = 56
T = 63
T = 70
T = 77
T = 84
T = 91
T = 98
T = 105
T = 112
T = 119
T = 126
T = 133
T = 140
T = 147
T = 154

Discussion①







自発核分裂のモデルとしての1次元虚時間TDHF方程式の周期解の計算

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東京科学大学, M2

指導教員: 関澤 一之

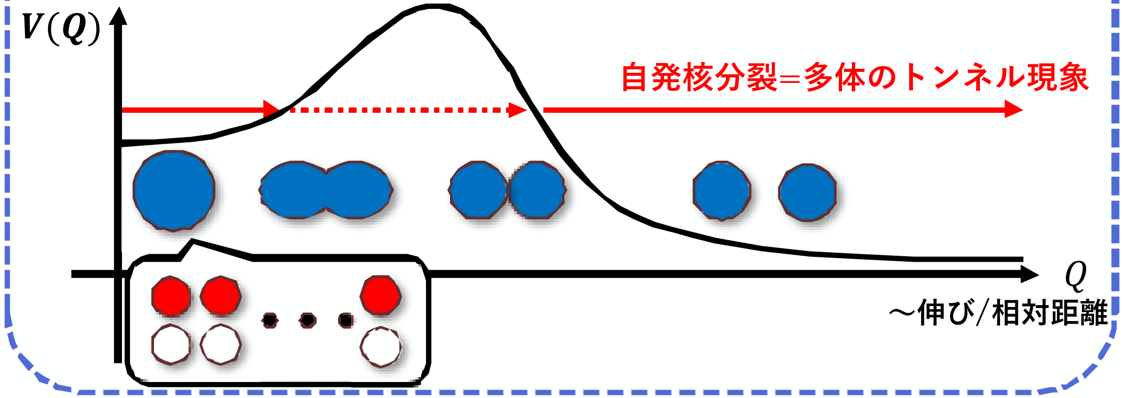
日本物理学会 2026年春季大会@ オンライン・2026/3/25
公演番号: 25aU1-2

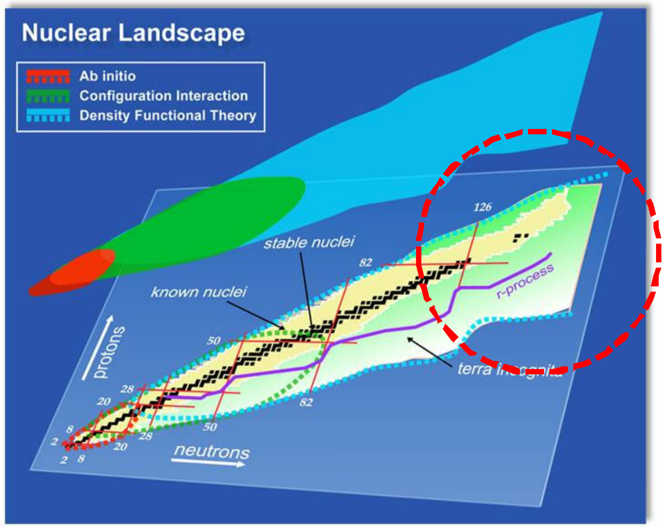
Introduction

研究の目的

自発核分裂の微視的平均場理論による記述

Potential energy surface by LDM





H Nam *et al* 2012 *J. Phys.: Conf. Ser.* **402** 012033

Time-Dependent Hartree-Fock (TDHF) 理論

$$S = \int \langle \Psi(t')|(i\hbar\partial_t - \hat{H})|\Psi(t')\rangle \, dt'$$

$$\Downarrow \quad \frac{\delta S}{\delta \psi_k^*(t)} = 0$$

$$i\hbar\partial_t\psi_k(t) = -\frac{\hbar^2}{2m}\nabla^2\psi_k(t) + \frac{\delta \mathcal{V}}{\delta \psi_k^*(t)}$$

- 原子核のダイナミクスを記述
- 核子を自由度とする
- 重い核でも計算可能

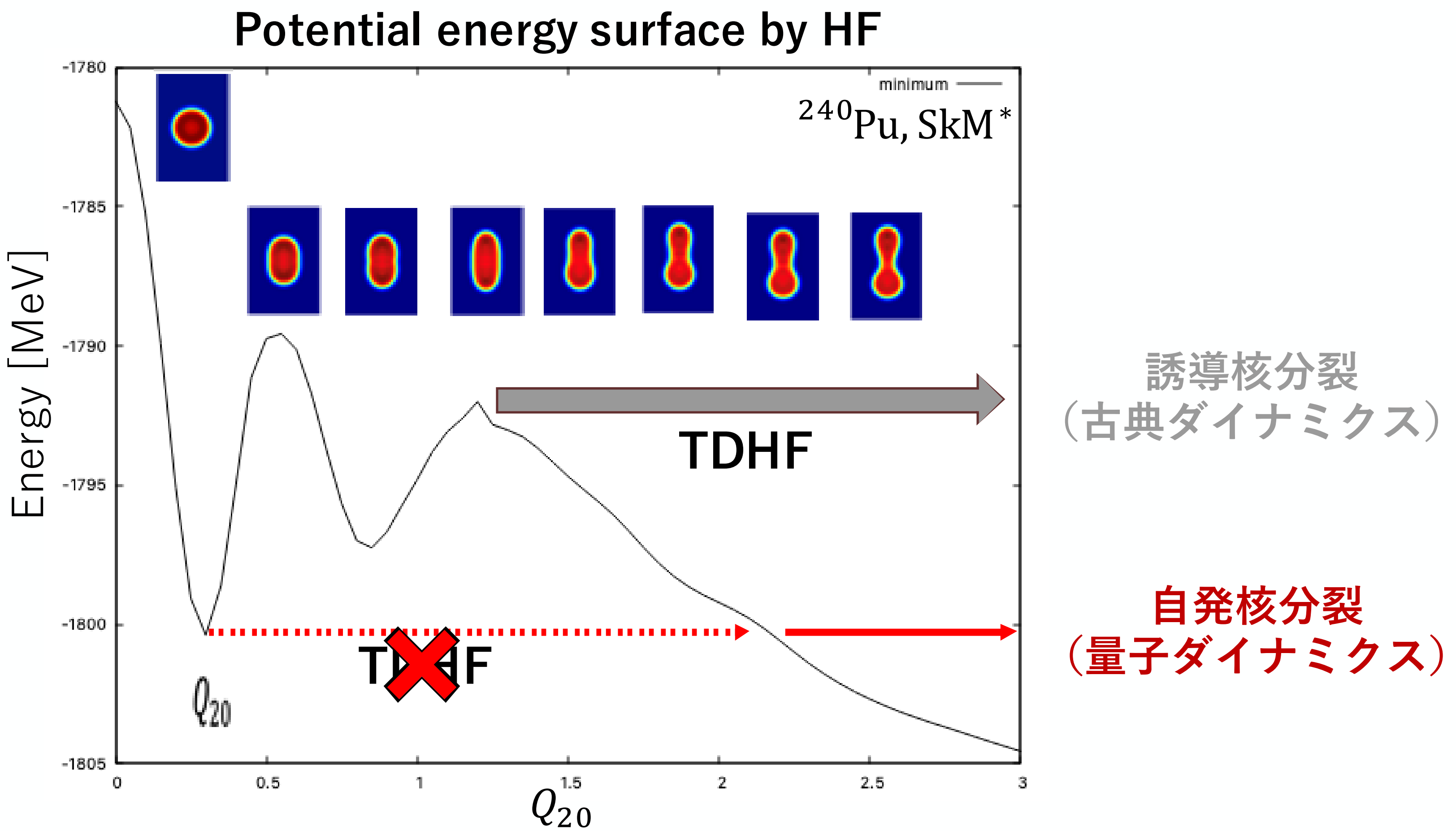
理論的に自発核分裂を記述できれば,
超重核の予言が可能に

⇓

Fission recycling, r-processに寄与

TDHFの量子化

TDHF理論の問題



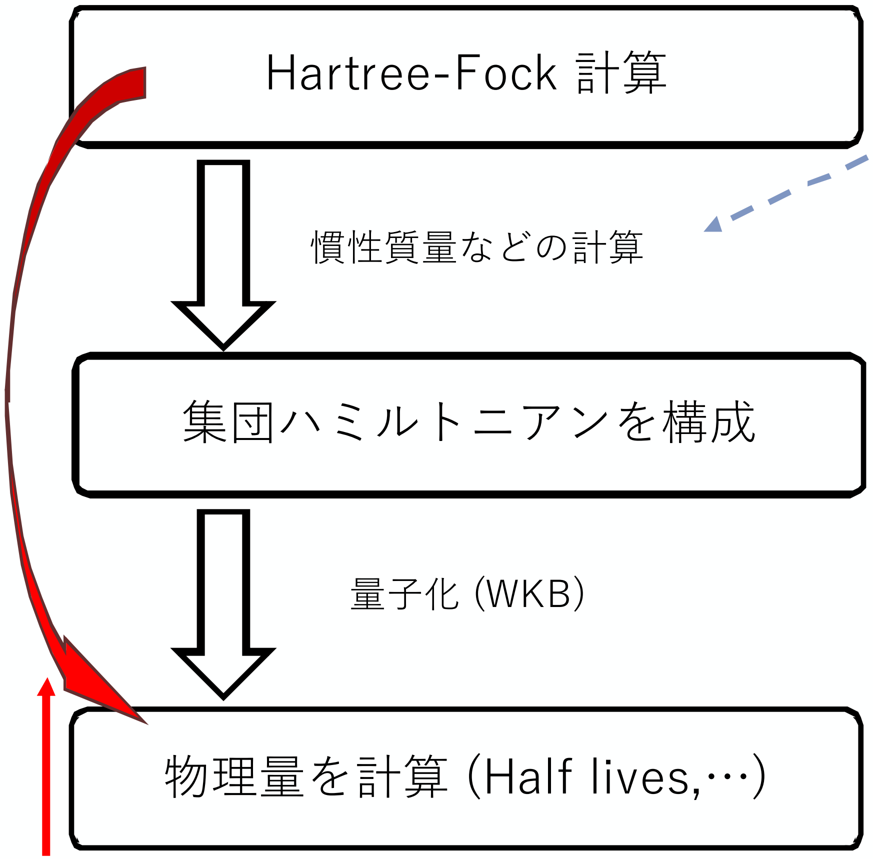
TDHFは **1粒子のダイナミクス**は微視的に記述できるが,
<u>**集団運動については古典的にしか記述できない.**</u>

自発核分裂の記述には<u>TDHFの量子化</u>が必要！

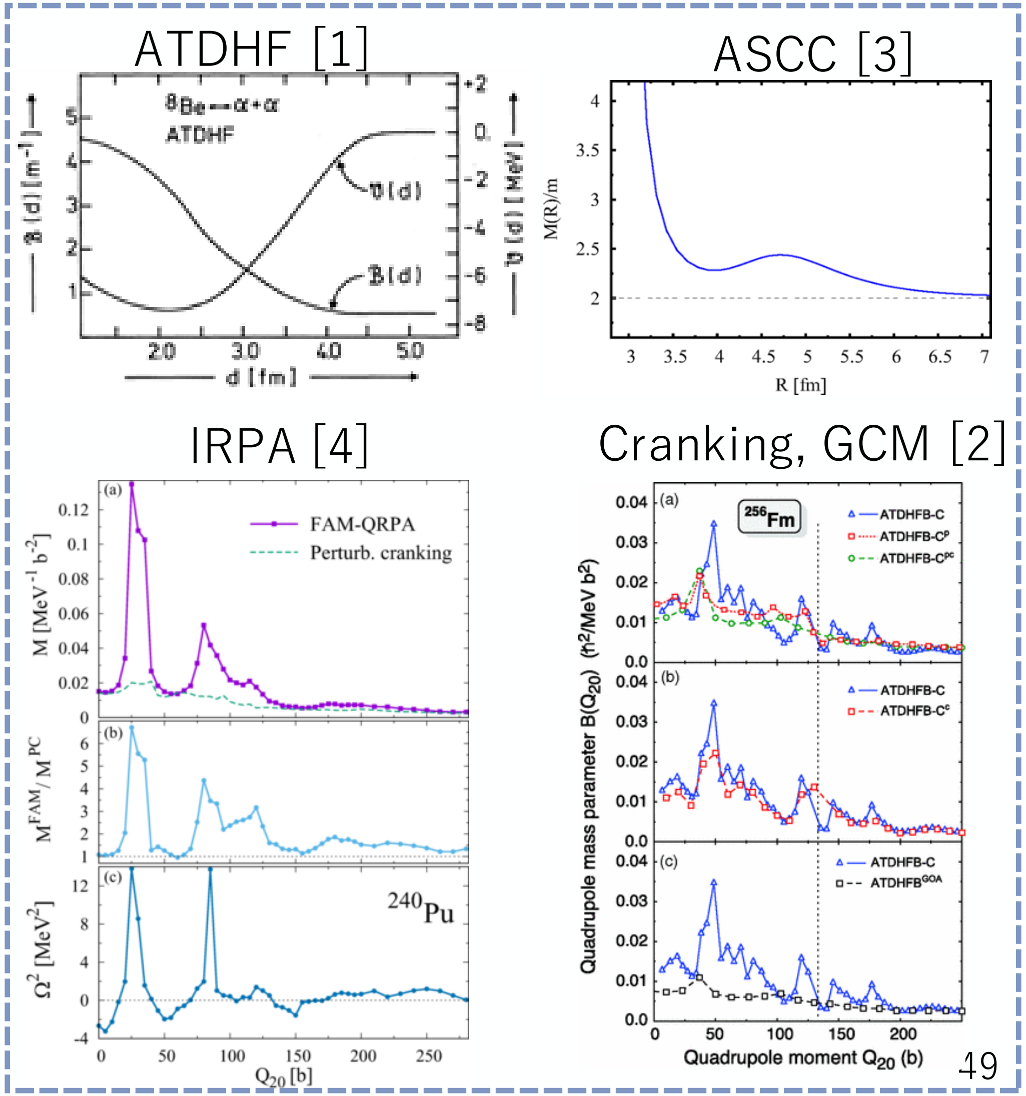
先行研究

Institute of SCIENCE TOKYO

[1] P. –G. Reinhard, J. Maruhn, K. Goeke, Phys. Rev. Lett. **44**, 1740 (1980)
[2] A. Baran et al. , Phys. Rev. C **84**, 054321 (2011)
[3] K. Wen, T. Nakatsukasa, Phys. Rev. C **94**, 054618 (2016)
[4] K. Washiyama, N. Hinohara, T. Nakatsukasa, Phys. Rev. C **103**, 014306 (2021)



集団ハミルトニアンを経ずに直接計算したい



ITDHFを用いた自発核分裂の記述

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TDHF

$$i\hbar\partial_t\psi_k(t) = -\frac{\hbar^2}{2m}\nabla^2\psi_k(t) + \frac{\delta\mathcal{V}}{\delta\psi_k^*(t)}$$

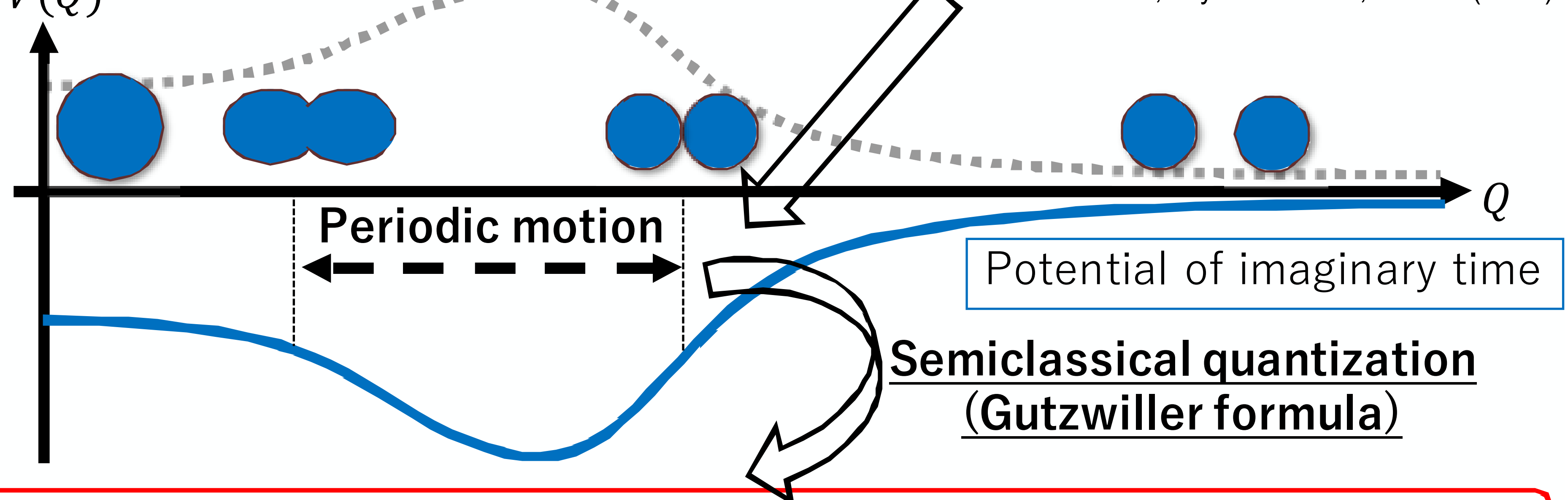
$t \rightarrow -i\tau$

Imaginary TDHF (ITDHF)

$$-\hbar\partial_\tau\psi_k(\tau) = -\frac{\hbar^2}{2m}\nabla^2\psi_k(\tau) + \frac{\delta\mathcal{V}}{\delta\psi_k(-\tau)}$$

$$\psi_k(T/2) = e^{-\alpha_k}\psi_k(-T/2)$$

J. Skalski, Phys. Rev. C **77**, 064610 (2008)



Fission half-life

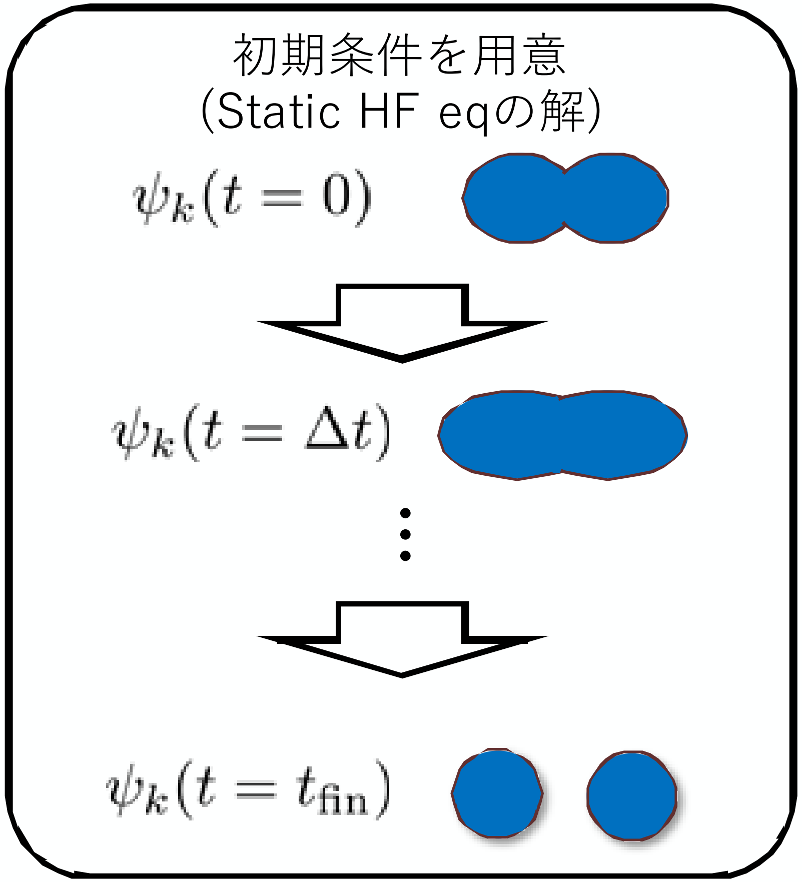
$$T_{1/2} \approx \exp\left[-\frac{S}{\hbar}\right], \quad S = \hbar\int_{-T/2}^{T/2} d\tau \sum_k \left\langle \psi_k(-\tau) \middle| \frac{\partial\psi_k(\tau)}{\partial\tau} \right\rangle$$

ITDHFは1980年代に提案されたが，それ以降の進展は少ない．

TDHFとITDHFの違い

Conventional TDHF

$$i\hbar\partial_t\psi_k(t) = -\frac{\hbar^2}{2m}\nabla^2\psi_k(t) + \frac{\delta\mathcal{V}}{\delta\psi_k^*(t)}$$

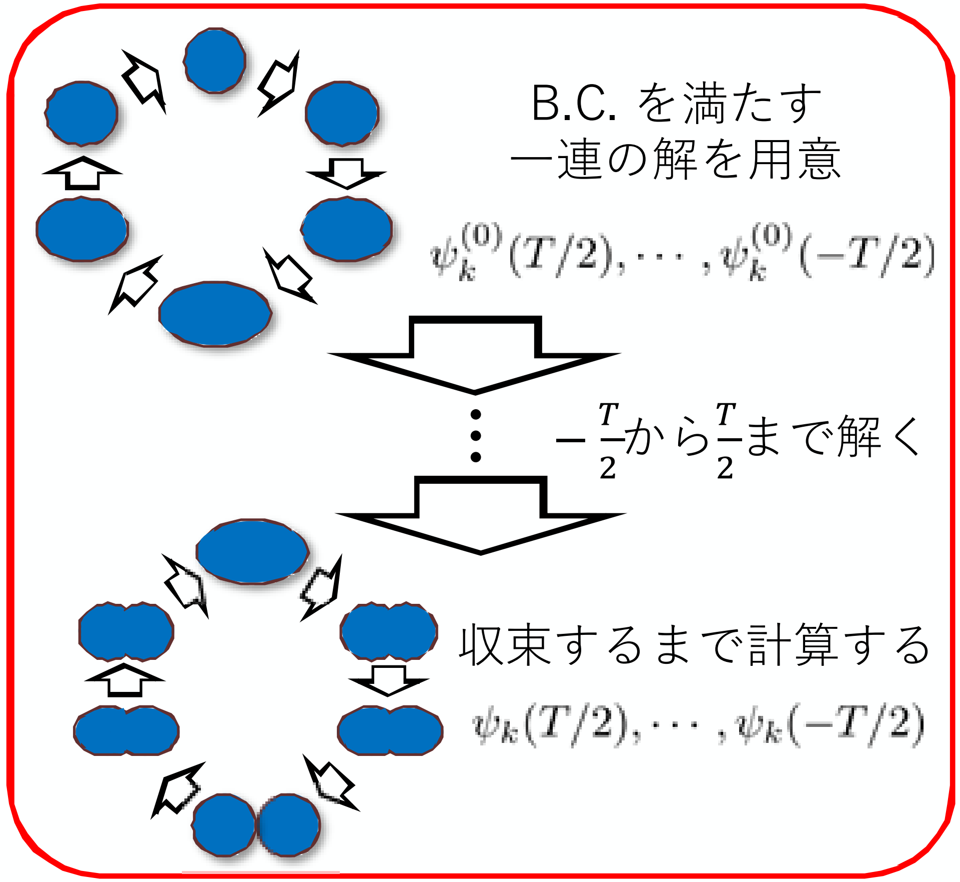


$t=0$で解いたのち
実時間発展を行う"初期値問題"

ITDHF

$$-\hbar\partial_\tau\psi_k(\tau) = -\frac{\hbar^2}{2m}\nabla^2\psi_k(\tau) + \frac{\delta\mathcal{V}}{\delta\psi_k(-\tau)}$$

$$\psi_k(T/2) = e^{-\alpha_k}\psi_k(-T/2)$$



周期境界条件を満たすために，空間と時間を同時に解く"1+1次元の境界値問題"

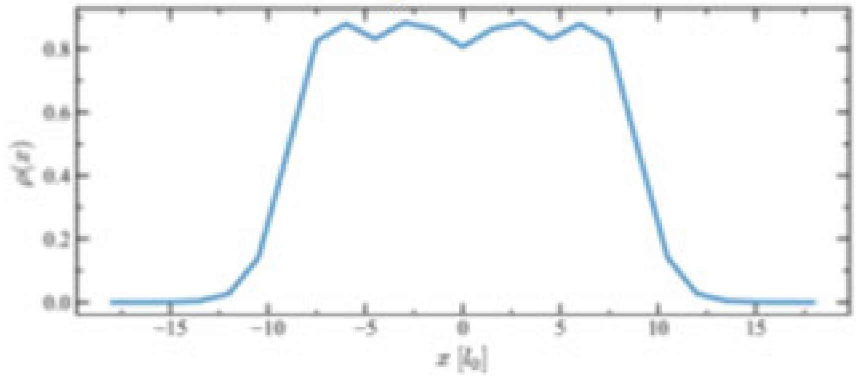
1次元ITDHFの数値計算

系の静的詳細

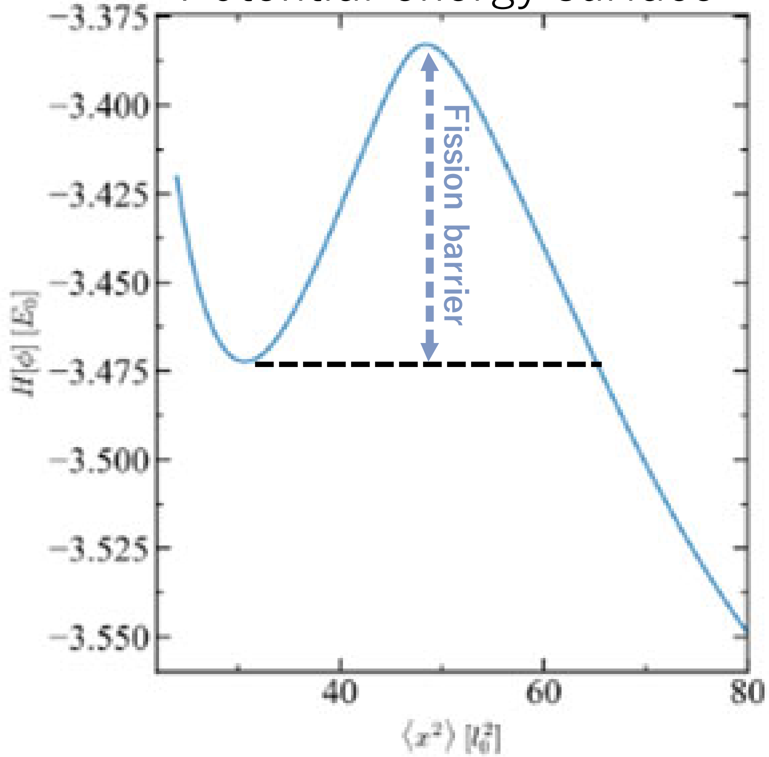
簡単のため，以下の条件を仮定した

- One-dimensional space
- 16-particle system
- Spin-Isospin degeneracy
- No Fock terms

Density distribution



Potential energy surface



Energy density of our system

$$\mathcal{H}[\phi(x,-\tau),\phi(x,\tau)] = -M\sum_{\alpha}\phi_{\alpha}(x,-\tau)\left(\frac{\partial^2}{\partial x^2}\right)\phi_{\alpha}(x,\tau) + \frac{1}{2}\int dx'\rho(x,\tau)V(x-x')\rho(x',\tau) + \frac{1}{3}V_3\rho^3(x,\tau)$$

Three body force: repulsive

$$V(x) = \frac{V_1}{\sqrt{\pi}\gamma_1}e^{-x^2/\gamma_1^2} + \frac{V_2}{\sqrt{\pi}\gamma_2}e^{-x^2/\gamma_2^2}$$

attractive　repulsive

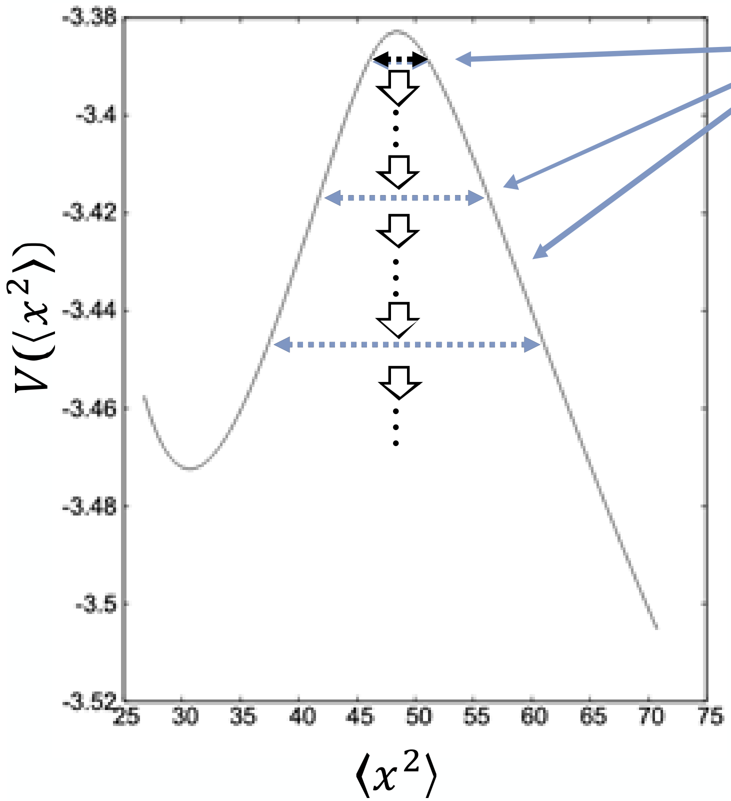
Two body force

$$\rho(x,\tau) = M\sum_{\alpha}\phi_{\alpha}(x,-\tau)\phi_{\alpha}(x,\tau)$$

S. Levit, J. W. Negele, and Z. Paltiel, Phys. Rev. C **22**, 1979

計算の流れ

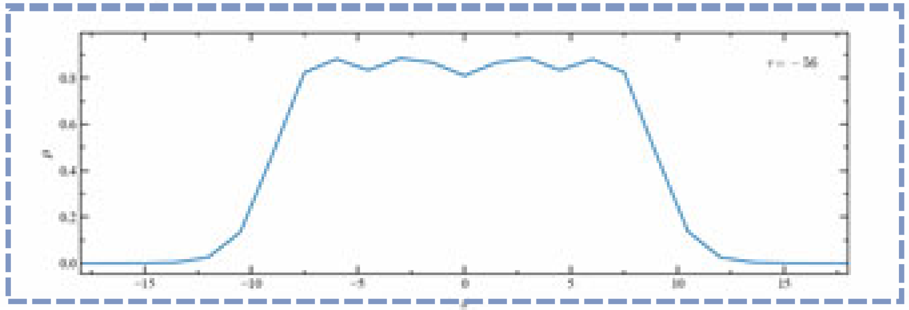
G.Puddu , J. W. Negele, Phys. Rev. C **35**, 1007



時間的に一様な解を避けるための拘束

$$V_\lambda(x) = \lambda \left[\int_{-\frac{1}{2}}^{\frac{1}{2}} d\eta \int x'^2 \rho(x', \eta) dx' - x_0^2 \right] x^2$$

100 iterations
initial phase



infinitesimal dilatation mode of saddle point density

Imaginary Time evolving

From initial time $-\frac{T}{2}$ to final time $\frac{T}{2}$

$$-\hbar \partial_\tau \psi_k(\tau) = -\frac{\hbar^2}{2m} \nabla^2 \psi_k(\tau) + \frac{\delta \mathcal{V}}{\delta \psi_k(-\tau)}$$

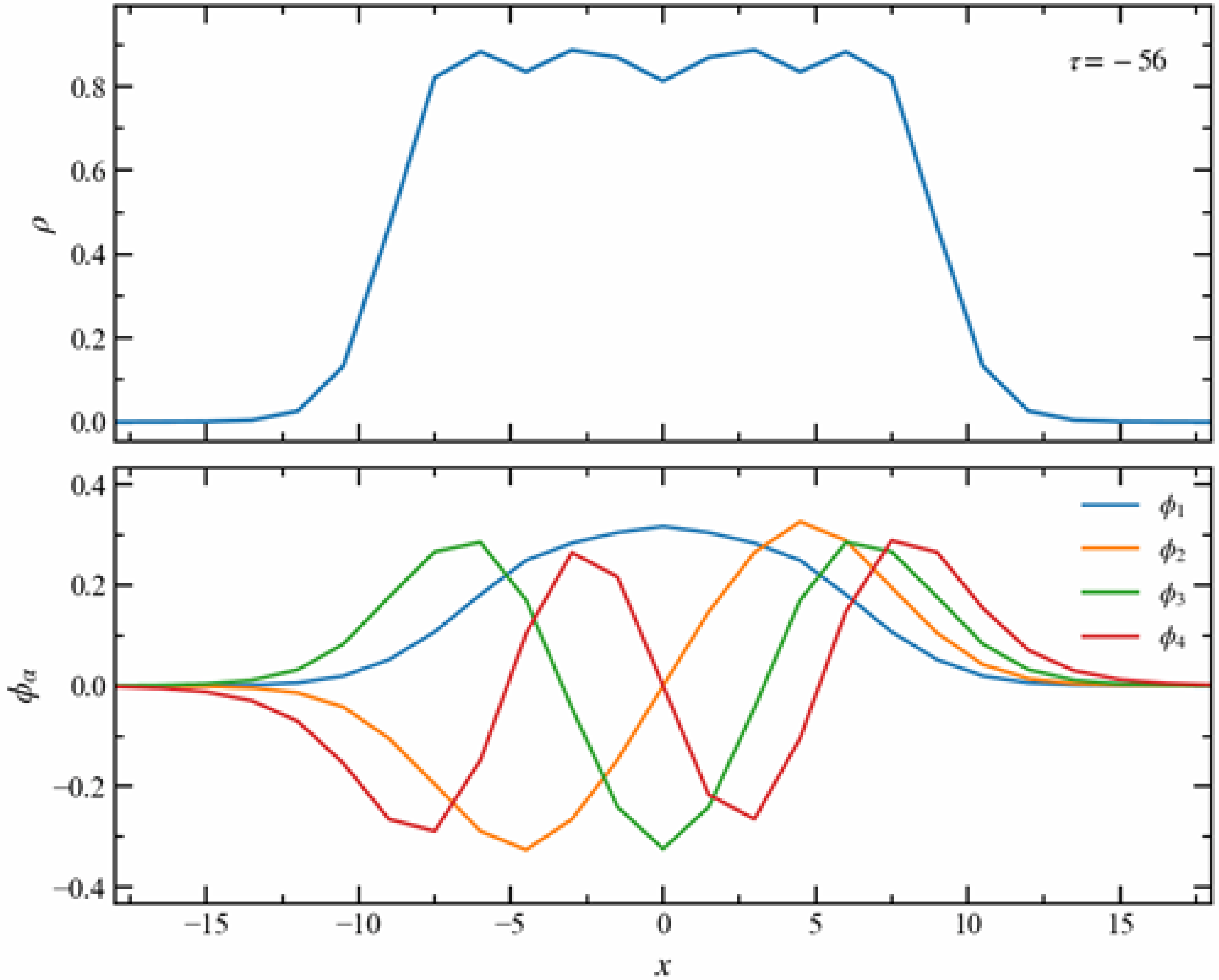
※発散を避けるため，各時間stepで直交化を行う

Update the density

$$\rho_{\rm new}(x,\tau) = (1-K)\rho_{\rm old}(x,\tau) + KM \sum_{\alpha=1}^{4} \phi_\alpha(x,-\tau)\phi_\alpha(x,\tau)$$

If $\rho_{\rm new}(x,\tau) = \rho_{\rm old}(x,\tau)$, calcuration stop

結果

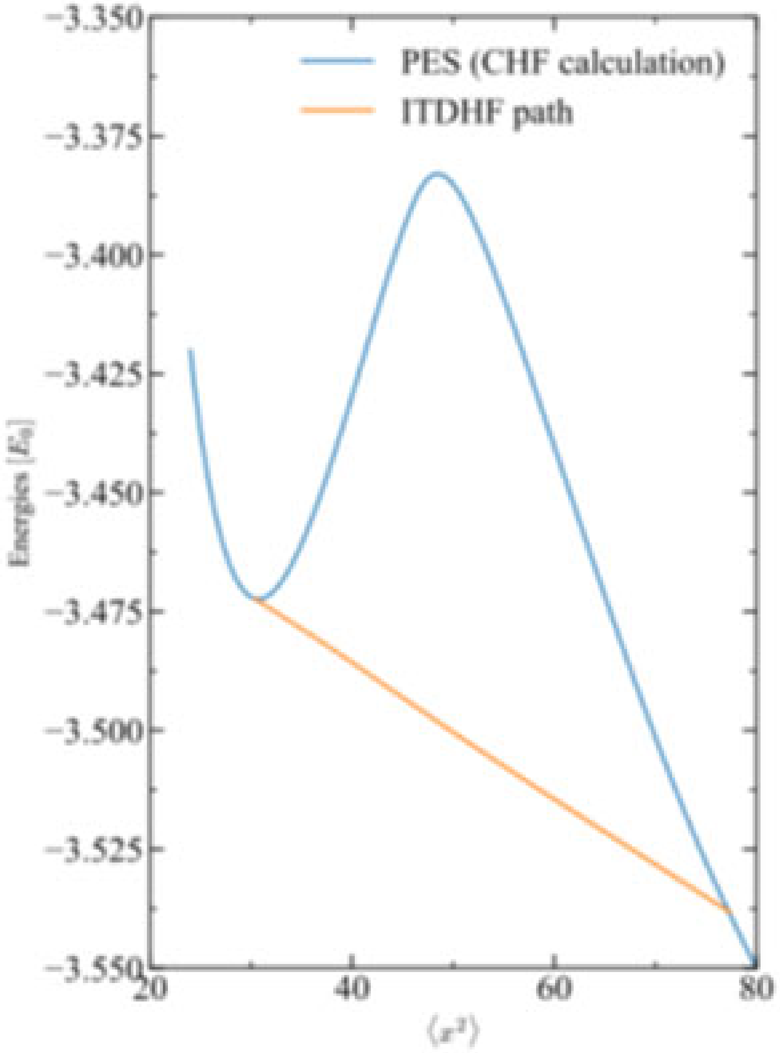


周期運動かつ分裂をしているようにみえる

エネルギーの非保存

得られた周期解は，エネルギーが保存していない…

Potential energy surface

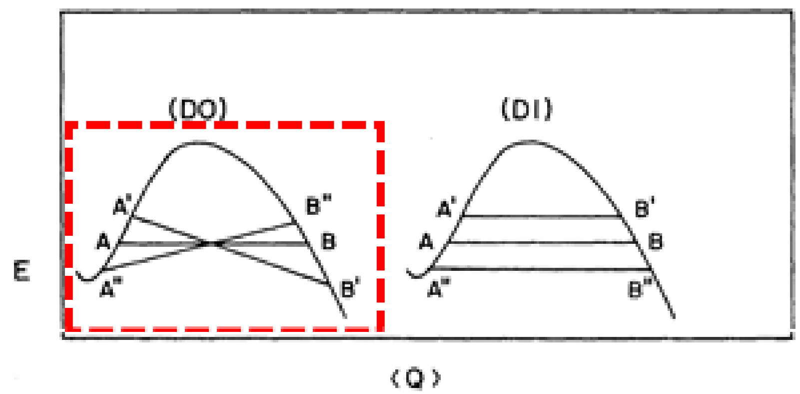


解析的にはITDHF方程式はエネルギーを保存する

$$\frac{\partial \mathcal{H}}{\partial \tau} = \hbar \sum_i \left(-\frac{\delta \mathcal{H}}{\delta \phi_i(\tau)} \frac{\delta \mathcal{H}}{\delta \phi_i^*(-\tau)} + \frac{\delta \mathcal{H}}{\delta \phi_i^*(-\tau)} \frac{\delta \mathcal{H}}{\delta \phi_i(\tau)} \right) = 0$$

しかしながら，数値的には各時間stepで直交化しているのでエネルギーの非保存が起こる．

先行研究では，エネルギーの非保存は初期値によって起こされると報告されている．



初期条件が特定のRPAモードのとき，エネルギーの非保存が起こる

G.Puddu , J. W. Negele, Phys. Rev. C **35**, 1007

まとめと今後の展望

まとめ

- 我々の研究は，自発核分裂を核子自由度から微視的に記述することを目的としている．
- Time-Dependent Hartree-Fock（TDHF）理論は微視的な平均場理論であるが，集団運動については古典的であるため，自発核分裂は直接記述できない．
- 自発核分裂をTDHFに基づいて記述するためには，その量子化を行う必要がある．
- 量子化されたTDHFの1つである，Imaginary TDHFに注目し，提案された1980年代に行われている数値計算を行い，結果を再現するとともにその数値計算手法を理解した．

今後の展望

- エネルギーの非保存問題を解決するために，Ramdom Phase Approximation（RPA）モードを初期条件として用意し，1次元系のITDHF計算を行う．
- より現実的な系 （3次元系など）で計算を行う．

Imaginary time evolution

Real time TDHF

$$i\hbar\partial_t\psi_k(t) = -\frac{\hbar^2}{2m}\nabla^2\psi_k(t) + \frac{\delta\mathcal{V}}{\delta\psi_k^*(t)}$$

$$\psi_k = \sqrt{\rho_k}e^{-i\chi_k} \qquad \psi_k^* = \sqrt{\rho_k}e^{i\chi_k}$$

$$\mathcal{H} = \frac{1}{2m}\sum_{k=1}^{A}\int \rho_k(\nabla\chi_k)^2 dx + \mathcal{V}(\rho)$$

$$\mathcal{V}(\rho) = \frac{1}{8m}\sum_{k=1}^{A}\int\frac{(\nabla\rho_k)^2}{\rho_k}dx + \frac{1}{2}\sum_{k,j=1}^{A}\int \rho_k V \rho_j dx dx'$$

$t \rightarrow -i\tau$

Imaginary time TDHF

$$-\hbar\partial_\tau\psi_k(\tau) = -\frac{\hbar^2}{2m}\nabla^2\psi_k(\tau) + \frac{\delta\mathcal{V}}{\delta\psi_k(-\tau)}$$

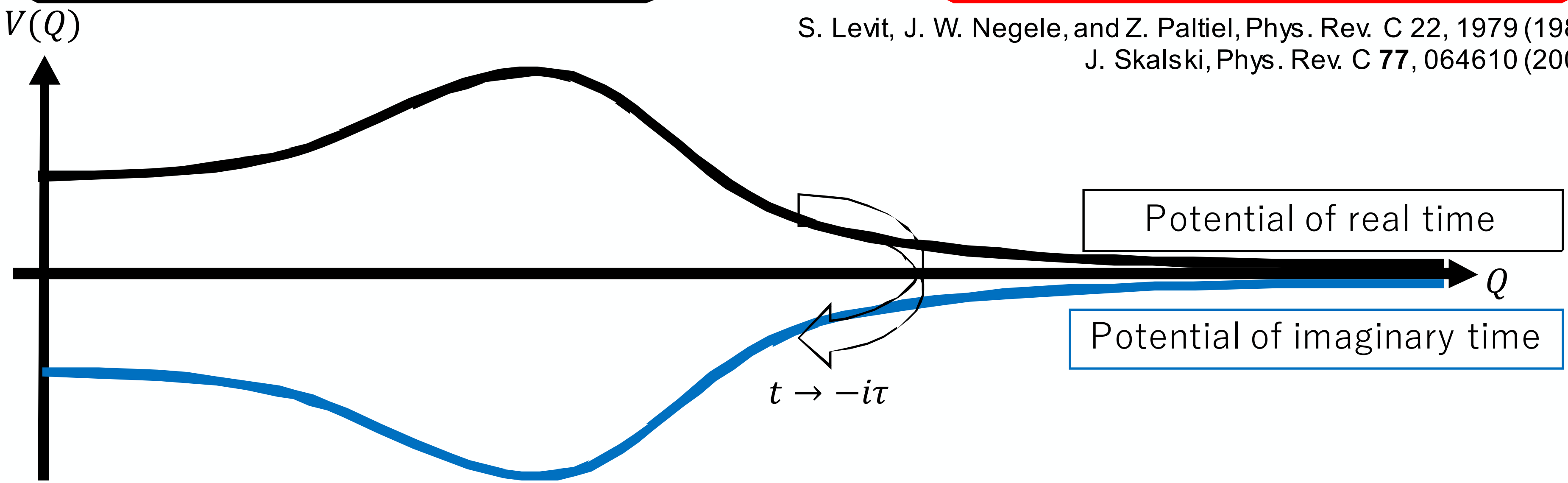
$$\psi_k = \sqrt{\rho_k}e^{-\chi_k} \qquad \psi_k^* = \sqrt{\rho_k}e^{\chi_k}$$

$$\mathcal{H} = -\frac{1}{2m}\sum_{k=1}^{A}\int \rho_k(\nabla\chi_k)^2 dx + \mathcal{V}(\rho)$$

符号が反転している!

$$\mathcal{V}(\rho) = \frac{1}{8m}\sum_{k=1}^{A}\int\frac{(\nabla\rho_k)^2}{\rho_k}dx + \frac{1}{2}\sum_{k,j=1}^{A}\int \rho_k V \rho_j dx dx'$$

S. Levit, J. W. Negele, and Z. Paltiel, Phys. Rev. C 22, 1979 (1980)
J. Skalski, Phys. Rev. C **77**, 064610 (2008)



ただ時間を反転させただけでは物理的意味は無い。

半古典量子化

S. Levit, J. W. Negele, and Z. Paltiel, Phys. Rev. C **21**, 1603 (1980)

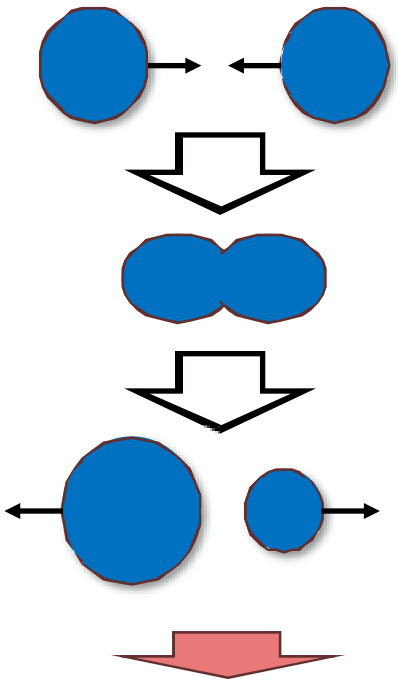
Gutzwiller formula

$$G(E) \equiv i \int_0^\infty dT e^{iET} \operatorname{tr} U(T,0) = \sum_\nu \frac{1}{E_\nu - E}$$

Periodic trajectory Propagator → $\operatorname{tr} U(T,0)$

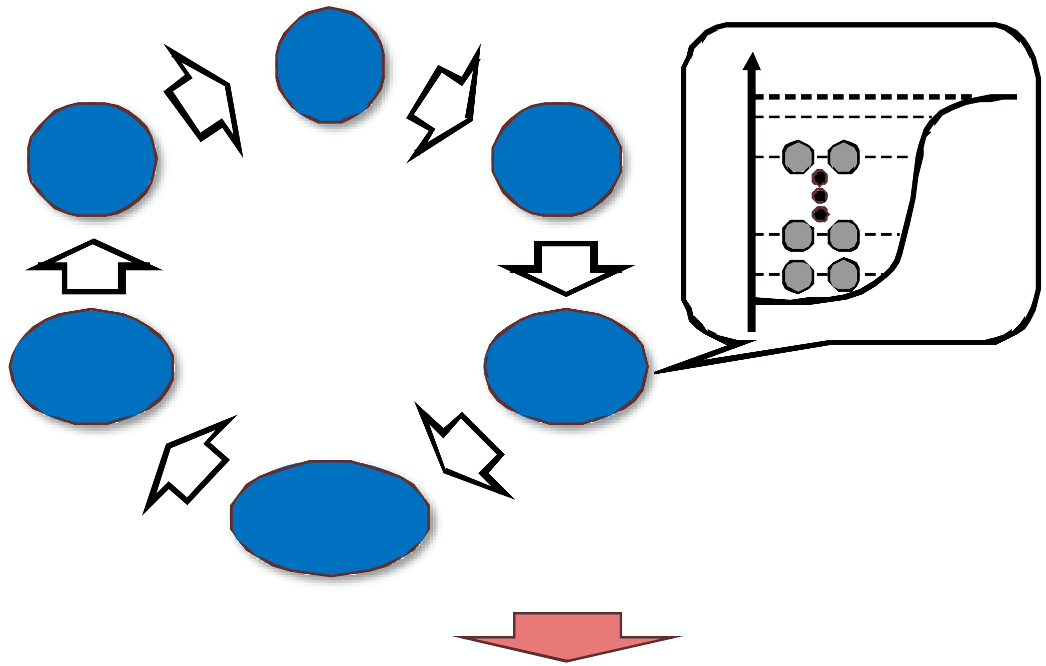
theory's Energy → E_ν

NON-periodic trajectory in TDHF



量子化できない

Periodic trajectory in TDHF



量子化できる

ITDHFを用いた自発核分裂の記述

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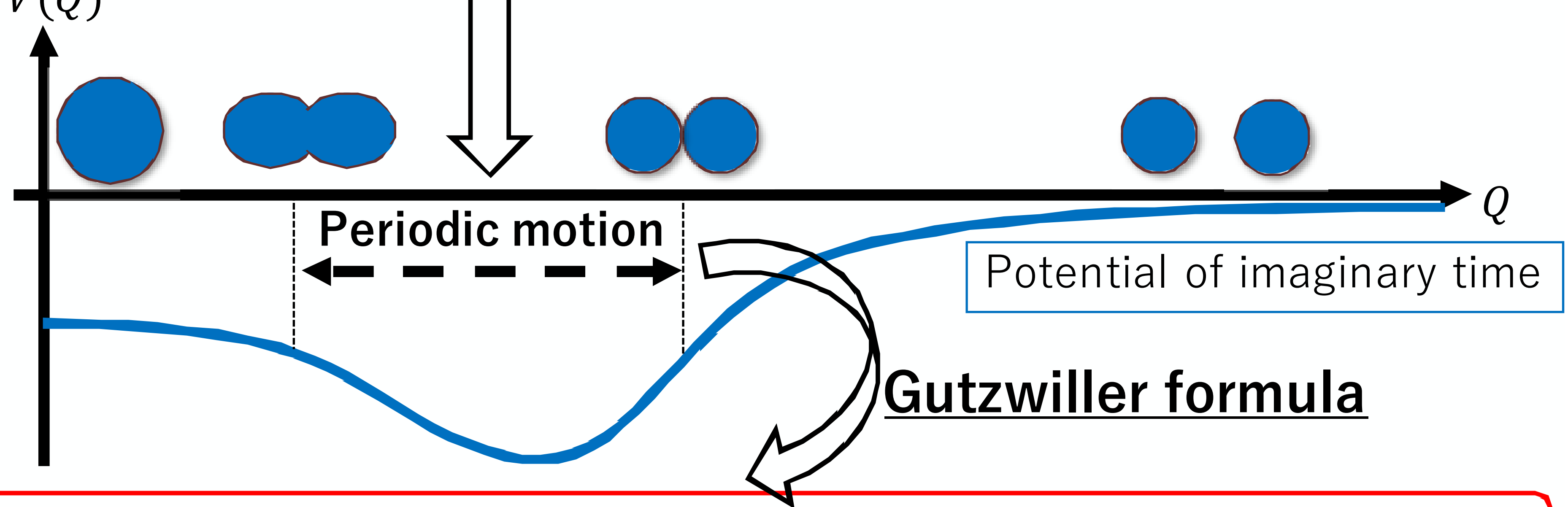
Periodic TDHF ↓ **Imaginary time evolution** ↓

$$-\hbar\partial_\tau\psi_k(\tau) = -\frac{\hbar^2}{2m}\nabla^2\psi_k(\tau) + \frac{\delta\mathcal{V}}{\delta\psi_k(-\tau)} \qquad \psi_k(T/2) = e^{-\alpha_k}\psi_k(-T/2)$$

J. Skalski, Phys. Rev. C **77**, 064610 (2008)



Fission half-life $\quad T_{1/2} \approx \exp\left[-\frac{S}{\hbar}\right], \quad S = \hbar\int_{-T/2}^{T/2} d\tau \sum_k \left\langle \phi_k(-\tau) \middle| \frac{\partial\phi_k(\tau)}{\partial\tau} \right\rangle$

ITDHF was proposed in the 1980s, but there has been little progress since then.

Computation of Periodic Solutions of One-Dimensional Imaginary TDHF Equations as a Model for Spontaneous Fission

Kotaro Koga

Institute of Science Tokyo, M2

Supervisor: K.Sekizawa

Sado2025・2025/11/4

Computation of Periodic Solutions of One-Dimensional Imaginary TDHF Equations as a Model for Spontaneous Fission

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DFT meeting @ Hokkaido・2026/1/8

Introduction:
Spontaneous fission and TDHF

Our research purpose

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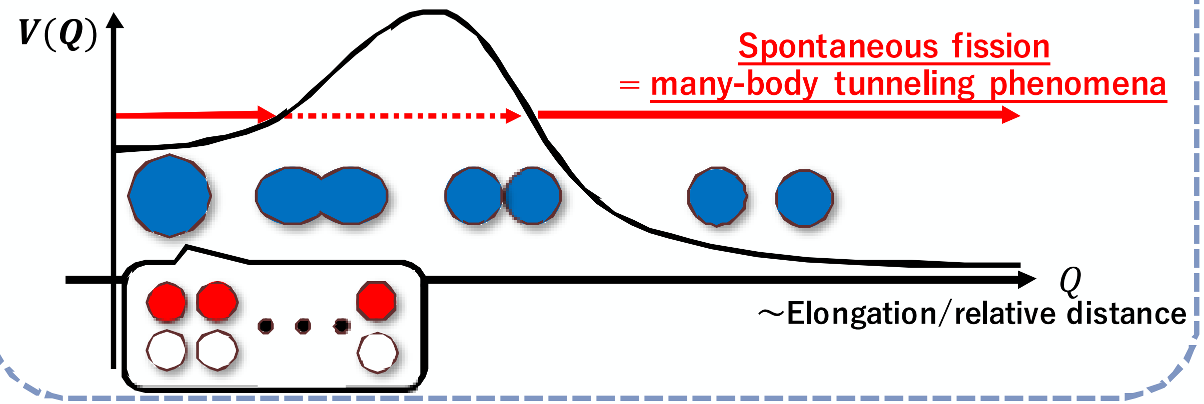
To describe spontaneous nuclear fission by a microscopic mean-field approach

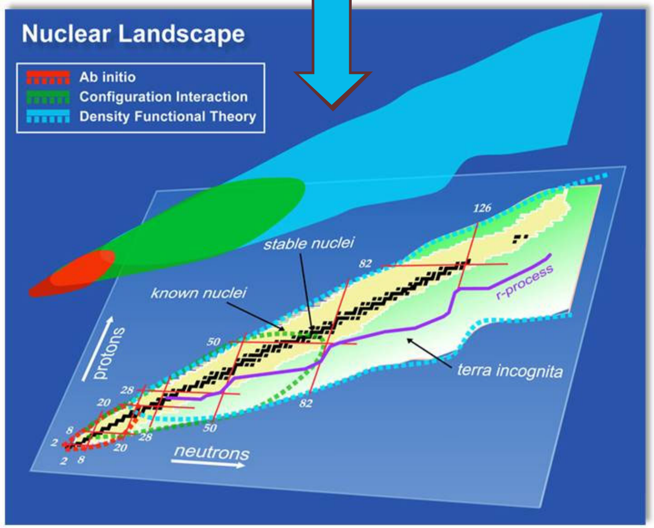
Potential energy surface by LDM



Spontaneous fission (SF) occurs in heavy nuclei.

⇩

TDHF is possible to calculate heavy nuclei.



Time-Dependent Hartree-Fock (TDHF) theory

- Nucleons create a mean field
- Nucleons move independently in the mean field
- Self-consistent equation for single particle wave function

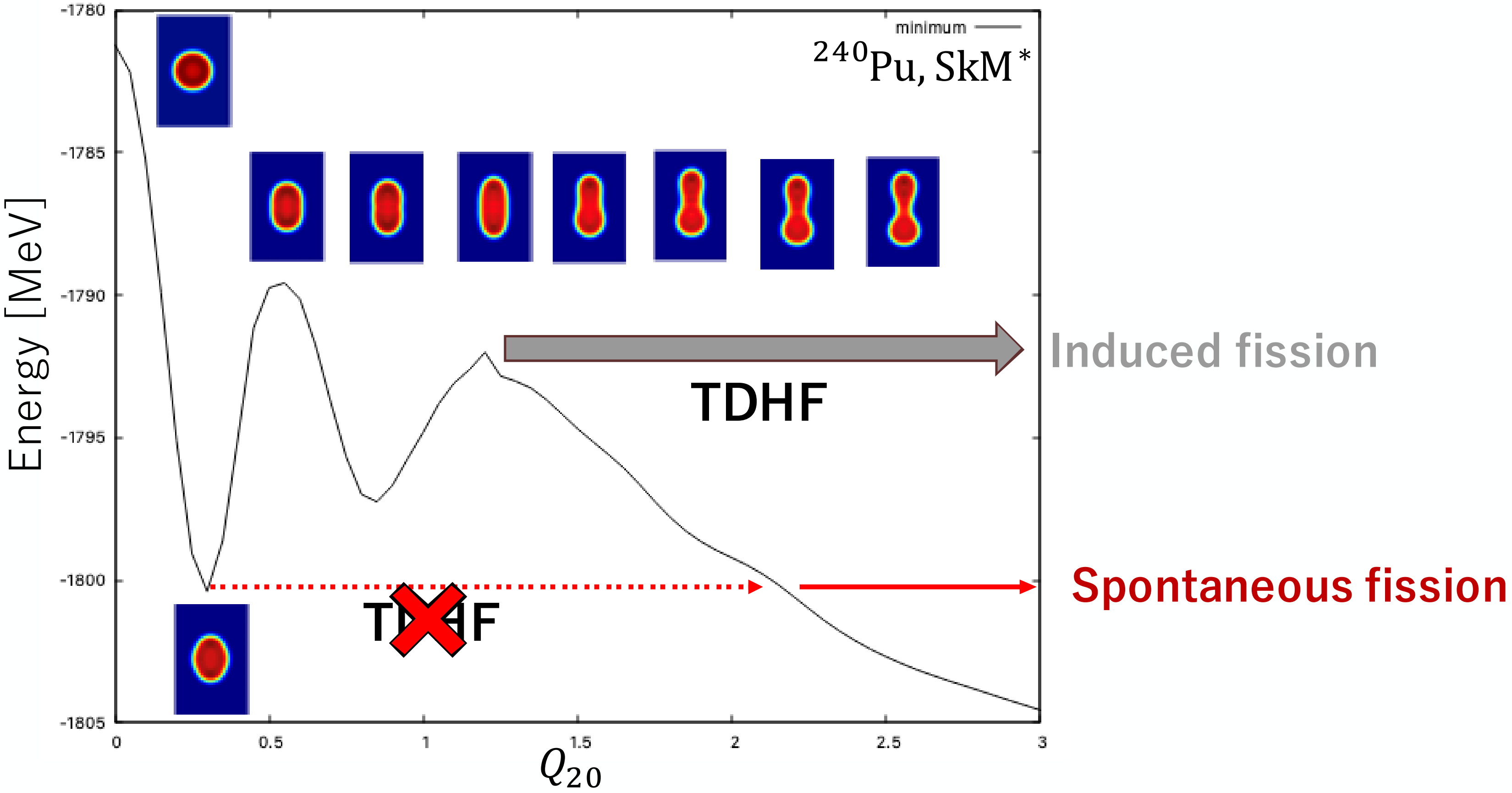
$$i\hbar\partial_t\psi_k(t) = -\frac{\hbar^2}{2m}\nabla^2\psi_k(t) + \frac{\delta\mathcal{V}}{\delta\psi_k^*(t)}$$

H Nam *et al* 2012 *J. Phys.: Conf. Ser.* **402** 012033

TDHF cannot describe SF

There are various theories that describe SF in microscopically, although almost all of them **<u>cannot reproduce the experimental values.</u>**

Potential energy surface by HF



TDHF can describe microscopically the dynamics of **<u>a single particle</u>**, but **<u>a collective motion is classically.</u>**

We must <u>quantize TDHF</u> to describe SF.

Previous study of SF

P. –G. Reinhard, J. Maruhn, K. Goeke, Phys. Rev. Lett. **44**, 1740 (1980)
A. Baran et al. , Phys. Rev. C **84**, 054321 (2011)
A. K. Wen, T. Nakatsukasa, Phys. Rev. C **94**, 054618 (2016)
B. K. Washiyama, N. Hinohara, T. Nakatsukasa, Phys. Rev. C **103**, 014306 (2021)

Hartree-Fock calculations

Calculation of Mass parameter,⋯

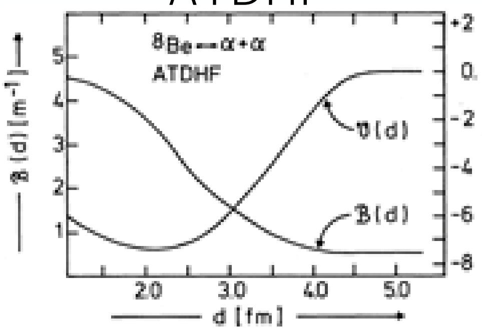
Collective Hamiltonian

Quantization (WKB)

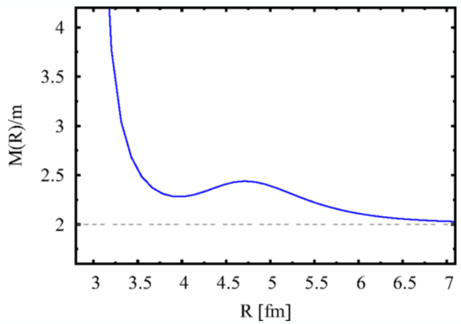
Physical quantities (Half lives,⋯)

The calculations what we want to do.

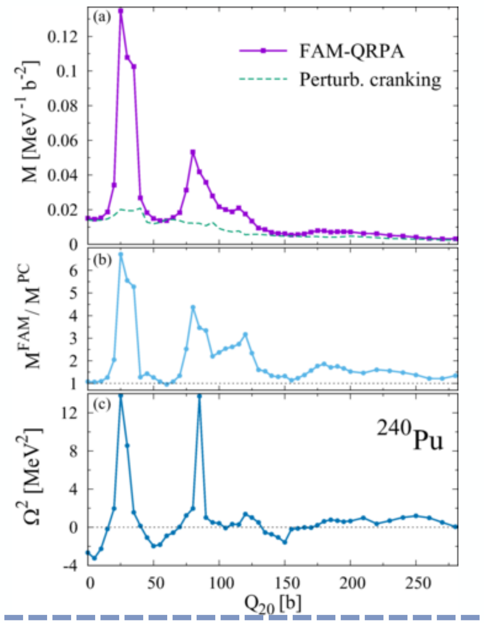
ATDHF



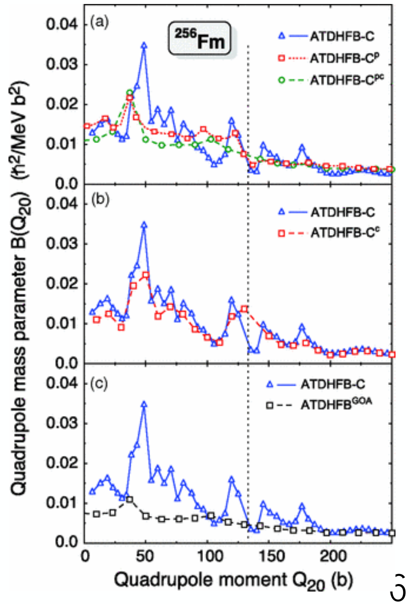
ASCC



IRPA



Cranking, GCM



Why TDHF is classical theory?

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Conventionally, TDHF is derived using the variational principle, but now, we derive it using **a path integral formalism**.

S. Levit Phys. Rev. C **21**, 1594 (1980)
S. Levit, J. W. Negele, and Z. Paltiel, Phys. Rev. C **21**, 1603 (1980)

Many-body Quantum Mechanics

$$H = \sum_{\alpha\beta} T_{\alpha\beta}\hat{a}^\dagger_\alpha \hat{a}_\beta + \frac{1}{2}\sum_{\alpha\beta\gamma\delta} v_{\alpha\beta\gamma\delta}\hat{a}^\dagger_\alpha \hat{a}^\dagger_\beta \hat{a}_\delta \hat{a}_\gamma$$

$$U(t_f, t_i) = T\exp\left[-\frac{i}{2}\int_{t_i}^{t_f} dt \sum_{\alpha\beta\gamma\delta} \hat{\rho}_{\alpha\gamma}(t) V_{\alpha\beta\gamma\delta}\hat{\rho}_{\beta\delta}(t)\right]$$

⇩ **Hubbard-Stratonovich transformation**

Path Integral representation

$$U(t_f, t_i) = \int \mathcal{D}[\sigma]\exp\left[\frac{i}{2}\int_{t_i}^{t_f} dt\{\sigma(t)v\sigma(t)\}U_I^\sigma(t_f, t_i)\right]$$

$$U_I^\sigma(t_f, t_i) \equiv T\exp\left[-i\int_{t_i}^{t_f} dt\{\sigma(t)v\hat{\rho}(t)\}\right]$$

※The HS transformation is also used in the Monte Carlo shell model.
T. Otsuka et al., Prog. Part. Nucl. Phys. 47 (2001) 319-400

⇩ **Stationary phase approximation ($\delta S = 0$)**

TDHF equation

$$i\hbar\partial_t\psi_k(t) = -\frac{\hbar^2}{2m}\nabla^2\psi_k(t) + \frac{\delta\mathcal{V}}{\delta\psi_k^*(t)}$$

TDHF is classical theory

Quantization of TDHF

Semi-classical quantization

S. Levit, J. W. Negele, and Z. Paltiel, Phys. Rev. C **21**, 1603 (1980)

Gutzwiller formula

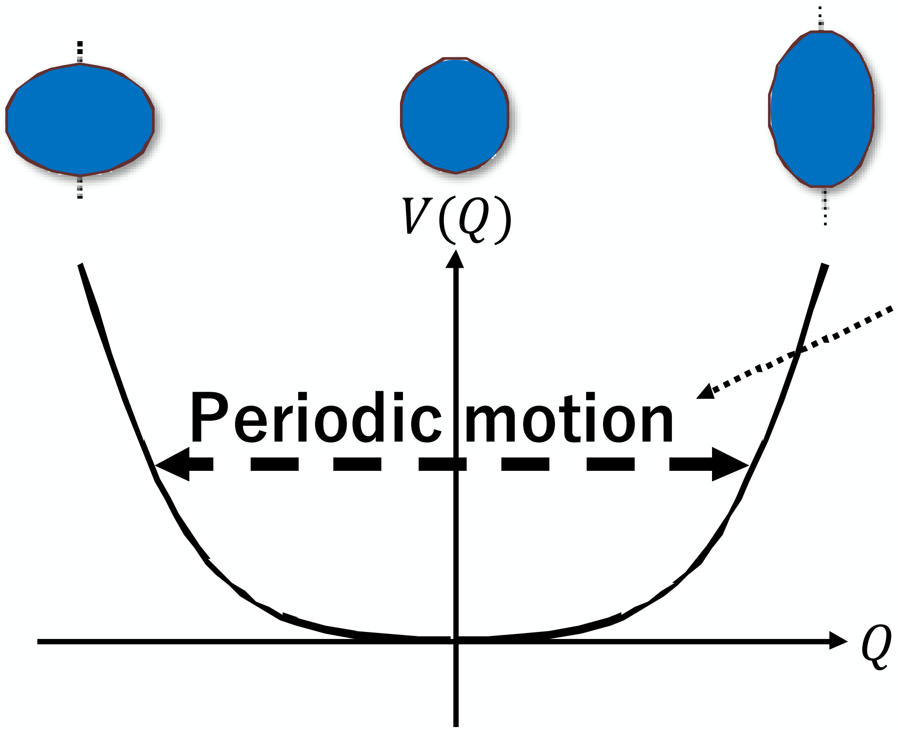
$$G(E) \equiv i\int_0^{\infty} dT e^{iET}\,\mathrm{tr}\,U(T,0) = \sum_{\nu} \frac{1}{E_{\nu} - E}$$

Periodic trajectory Propagator — Quantum theory's Energy

①Describing by periodic TDHF

$$i\hbar\partial_t\psi_k(t) = -\frac{\hbar^2}{2m}\nabla^2\psi_k(t) + \frac{\delta\mathcal{V}}{\delta\psi_k^*(t)}$$

$$\psi_k(T/2) = e^{-i\alpha_k}\psi_k(-T/2)$$

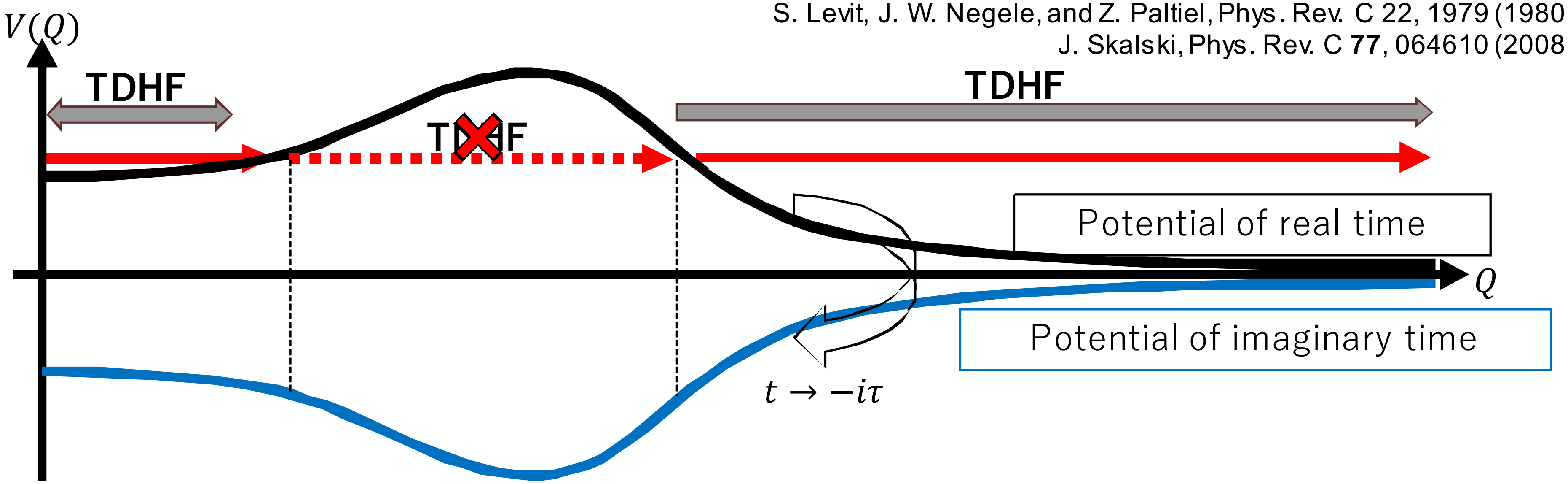


②calculating $G(E)$, the poles give the energy in quantum theory.

However, conventional TDHF cannot penetrate barriers,
so the periodic TDHF corresponding to SF cannot be obtained.

Imaginary time evolution

S. Levit, J. W. Negele, and Z. Paltiel, Phys. Rev. C 22, 1979 (1980)
J. Skalski, Phys. Rev. C **77**, 064610 (2008)



Real time TDHF

$$i\hbar\partial_t\psi_k(t) = -\frac{\hbar^2}{2m}\nabla^2\psi_k(t) + \frac{\delta\mathcal{V}}{\delta\psi_k^*(t)}$$

$$\psi_k = \sqrt{\rho_k}e^{-i\chi_k} \qquad \psi_k^* = \sqrt{\rho_k}e^{i\chi_k}$$

$$\mathcal{H} = \frac{1}{2m}\sum_{k=1}^{A}\int \rho_k(\nabla\chi_k)^2 dx + \mathcal{V}(\rho)$$

$$\mathcal{V}(\rho) = \frac{1}{8m}\sum_{k=1}^{A}\int\frac{(\nabla\rho_k)^2}{\rho_k}dx + \frac{1}{2}\sum_{k,j=1}^{A}\int \rho_k V \rho_j dx dx'$$

$t \rightarrow -i\tau$

Imaginary time TDHF

$$-\hbar\partial_\tau\psi_k(\tau) = -\frac{\hbar^2}{2m}\nabla^2\psi_k(\tau) + \frac{\delta\mathcal{V}}{\delta\psi_k(-\tau)}$$

$$\psi_k = \sqrt{\rho_k}e^{-\chi_k} \qquad \psi_k^* = \sqrt{\rho_k}e^{\chi_k}$$

$$\mathcal{H} = -\frac{1}{2m}\sum_{k=1}^{A}\int \rho_k(\nabla\chi_k)^2 dx + \mathcal{V}(\rho)$$

Sign has inverted !

$$\mathcal{V}(\rho) = \frac{1}{8m}\sum_{k=1}^{A}\int\frac{(\nabla\rho_k)^2}{\rho_k}dx + \frac{1}{2}\sum_{k,j=1}^{A}\int \rho_k V \rho_j dx dx'$$

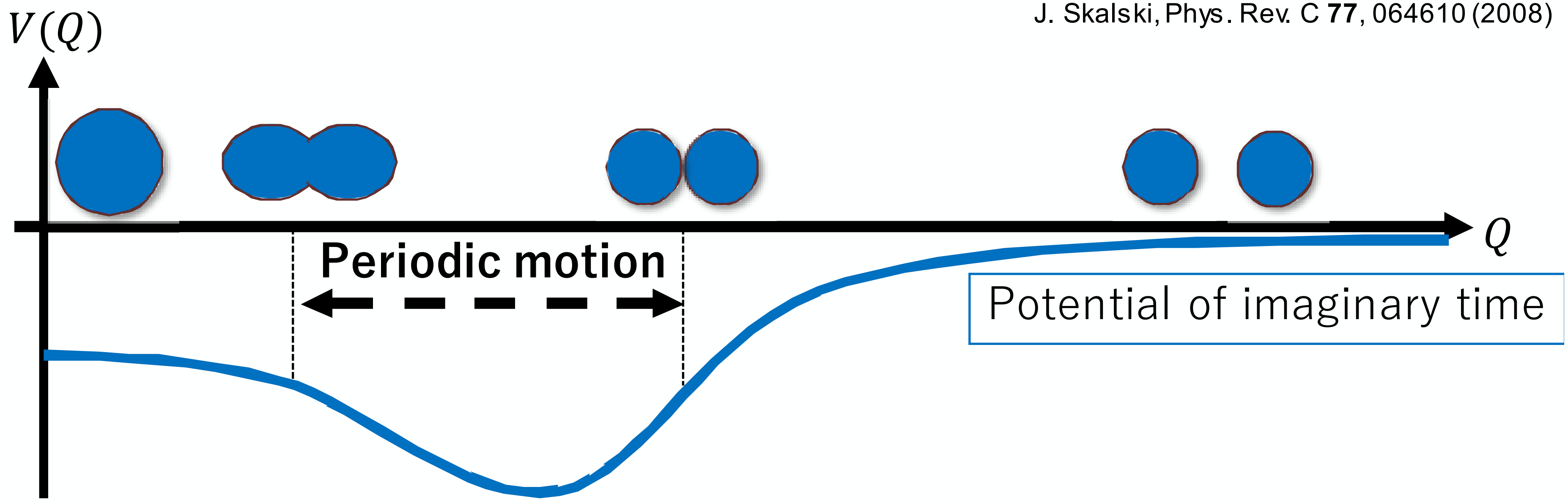
Description of SF using ITDHF

Periodic TDHF ↓ **Imaginary time evolution** ↓ **Gutzwiller formula** ↓

$$-\hbar\partial_\tau\psi_k(\tau) = -\frac{\hbar^2}{2m}\nabla^2\psi_k(\tau) + \frac{\delta\mathcal{V}}{\delta\psi_k(-\tau)} \qquad \psi_k(T/2) = e^{-\alpha_k}\psi_k(-T/2)$$

$$\exp\left[-\frac{S}{\hbar}\right], \quad S = \hbar\int_{-T/2}^{T/2} d\tau \sum_k \left\langle \phi_k(-\tau) \middle| \frac{\partial\phi_k(\tau)}{\partial\tau} \right\rangle.$$

J. Skalski, Phys. Rev. C **77**, 064610 (2008)



ITDHF was proposed in the 1980s, but there has been little progress since then.

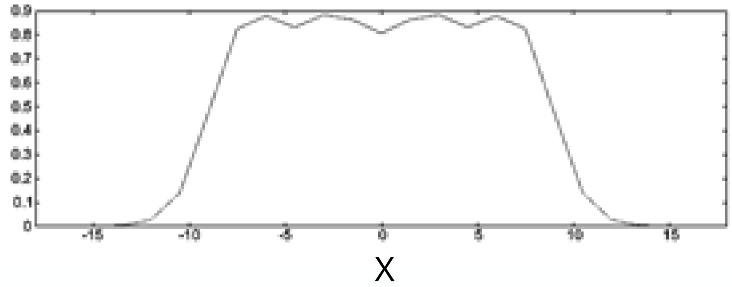
Setup for Numerical Calculations

System

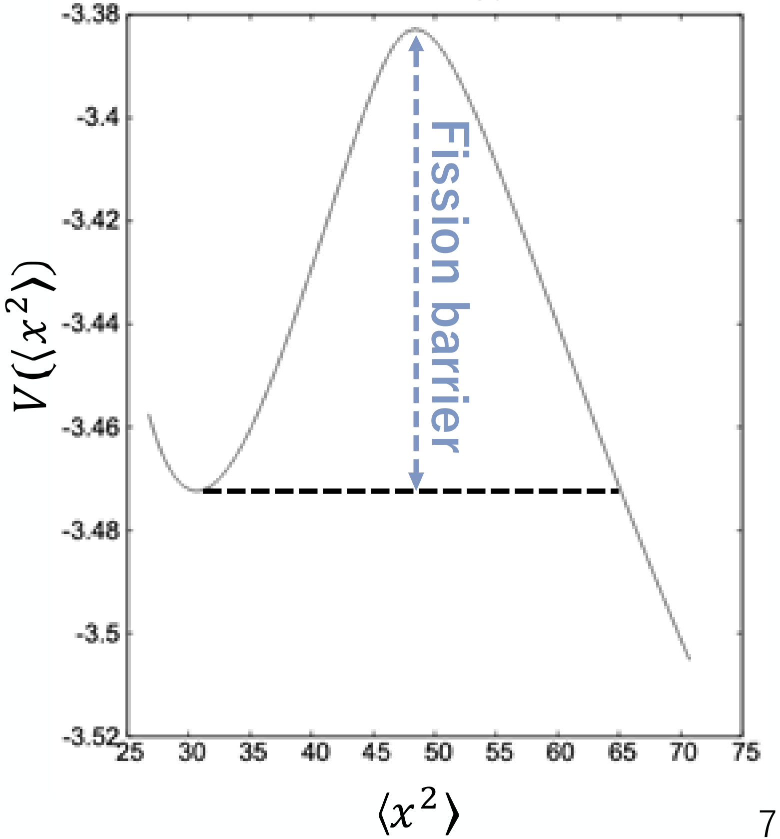
For simplicity, we assume that

- One-dimentional space
- 16 particle system
- Spin-Isospin degeneracy
- No Fock terms

Density



Potential energy surface



Hamiltonian density of our system

$$\mathcal{H}[\phi(x,-\tau),\phi(x,\tau)] = -M\sum_{\alpha}\phi_{\alpha}(x,-\tau)\left(\frac{\partial^2}{\partial x^2}\right)\phi_{\alpha}(x,\tau) + \frac{1}{2}\int dx'\rho(x,\tau)V(x-x')\rho(x',\tau) + \frac{1}{3}V_3\rho^3(x,\tau)$$

Three body force — **repulsive**

$$V(x) = \frac{V_1}{\sqrt{\pi}\gamma_1}e^{-x^2/\gamma_1^2} + \frac{V_2}{\sqrt{\pi}\gamma_2}e^{-x^2/\gamma_2^2}$$

attractive **repulsive**

Two body force

$$\rho(x,\tau) = M\sum_{\alpha}\phi_{\alpha}(x,-\tau)\phi_{\alpha}(x,\tau)$$

S. Levit, J. W. Negele, and Z. Paltiel, Phys. Rev. C **22**, 1979

Discretization of Time

For numerical caluculation, the time variable is deiscritized.

G.Puddu , J. W. Negele, Phys. Rev. C **35**, 1007

$$-\frac{\partial \phi_\beta(x,\tau)}{\partial \tau} = \left(-\frac{\partial^2}{\partial x^2} + \int V(x-x')\rho(x',\tau)dx' + V_3\rho^2(x,\tau) + V_\lambda(x)\right)\phi_\beta(x,\tau)$$

$$= h[\rho]\phi_\beta(x,\tau)$$

Constraints to avoid uniform solutions

$$V_\lambda(x) = \lambda\left[\int_{-\frac{1}{2}}^{\frac{1}{2}} d\eta \int x'^2\rho(x',\eta)dx' - x_0^2\right]x^2$$

B.C. $\phi_\beta(x,\frac{T}{2}) = e^{-\lambda_\beta}\phi_\beta(x,-\frac{T}{2})$

Discretization of time $\tau_0, \tau_1, \ldots \tau_{N_\tau}$

Number of meshes: 32, Mesh width: 3.5

$$\phi_\beta(x_i,\tau_{k+1}) = \exp\left(-h\left[\frac{\rho_{k+1}+\rho_k}{2}\right]\Delta\tau\right)\phi_\beta(x_i,\tau_k)$$

Time evolution operator

Eigenvalue problem $\phi_\beta(x_i,\tau_{N_\tau}) = e^{-\lambda_\beta}\phi_\beta(x_i,\tau_1)$

Computational Flow

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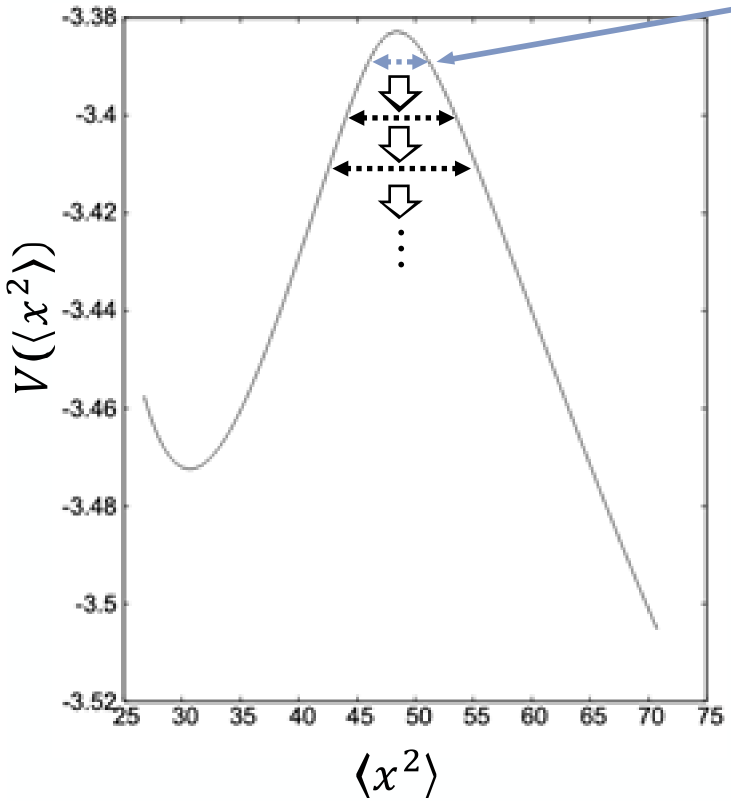
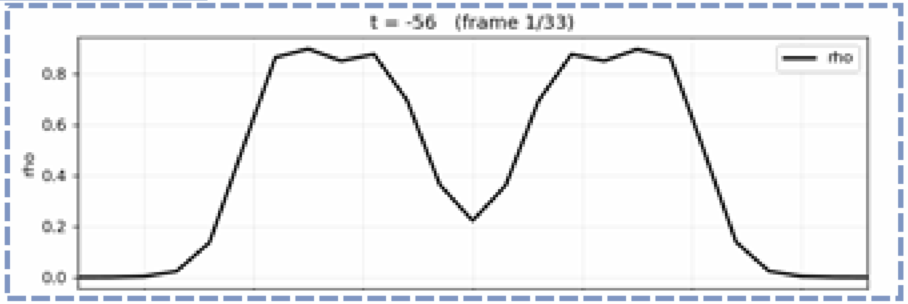
G.Puddu , J. W. Negele, Phys. Rev. C **35**, 1007



Initial pass



infinitesimal dilatation mode of saddle point density

Imaginary Time evolving

Imaginary Time evolution operator

$$\phi_\beta(x_i, \tau_{k+1}) = \exp\left(-h\left[\frac{\rho(x_i, k+1) + \rho(x_i, k)}{2}\right]\Delta\tau\right)\phi_\beta(x_i, \tau_k)$$

※Orthogonalize at each step to avoid divergence.

Update the density

$$\rho_{\rm new}(x,\tau) = (1-K)\rho_{\rm old}(x,\tau) + KM\sum_{\alpha=1}^{4}\phi_\alpha(x,-\tau)\phi_\alpha(x,\tau)$$

If $\rho_{\rm new}(x,\tau) = \rho_{\rm old}(x,\tau)$, calcuration stop

The existence of V_λ means that $\rho_{\rm new}$ is not $\rho_{\rm old}$ generally

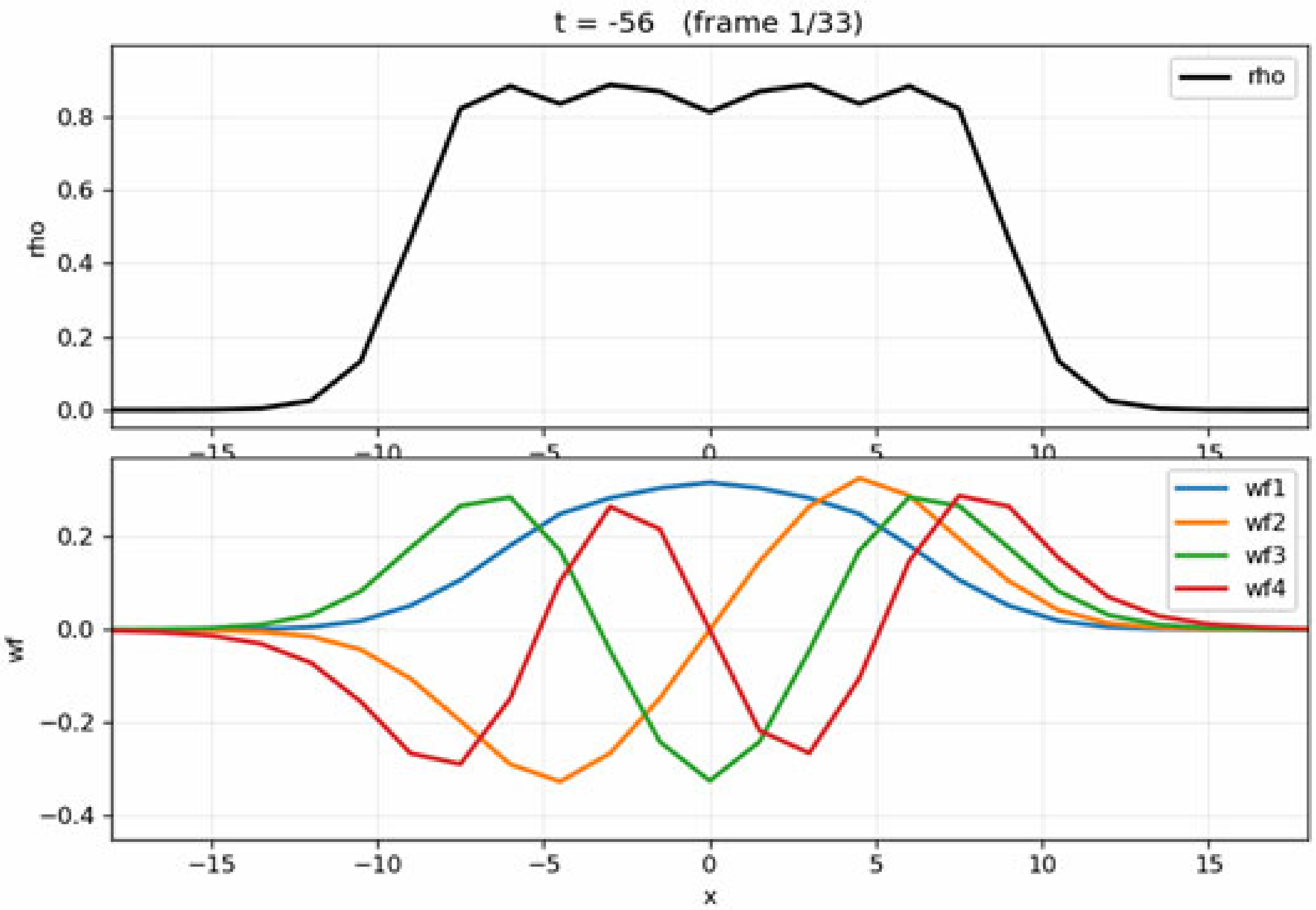
$$V_\lambda(x) = \lambda\left[\int_{-\frac{1}{2}}^{\frac{1}{2}} d\eta \int x'^2 \rho(x', \eta) dx' - x_0^2\right] x^2$$

Constraints to avoid uniform solutions

Results

Results

After ~1000 iterations, the density converged



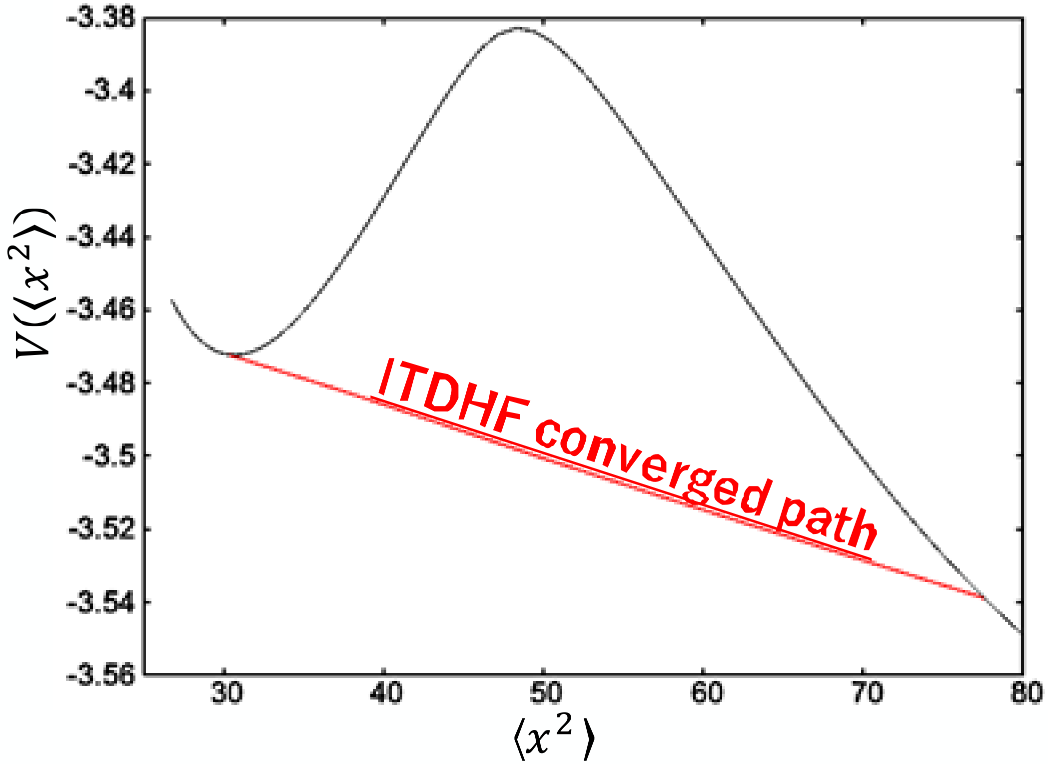
It seems like fission and periodic motion.

Discussion & Conclusion

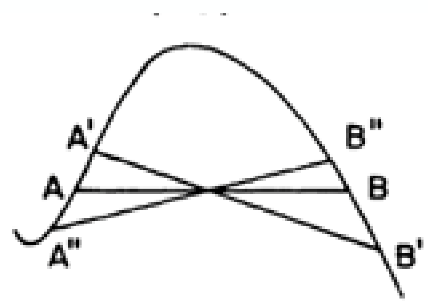
Discussion

In a converged path, it appears that energy is not conserved.

Potential energy surface



In original paper, the energy also may not conserved.



However, we still do not fully understand the original paper…

Mathematically, we can prove that energy is conserved in imaginary time.

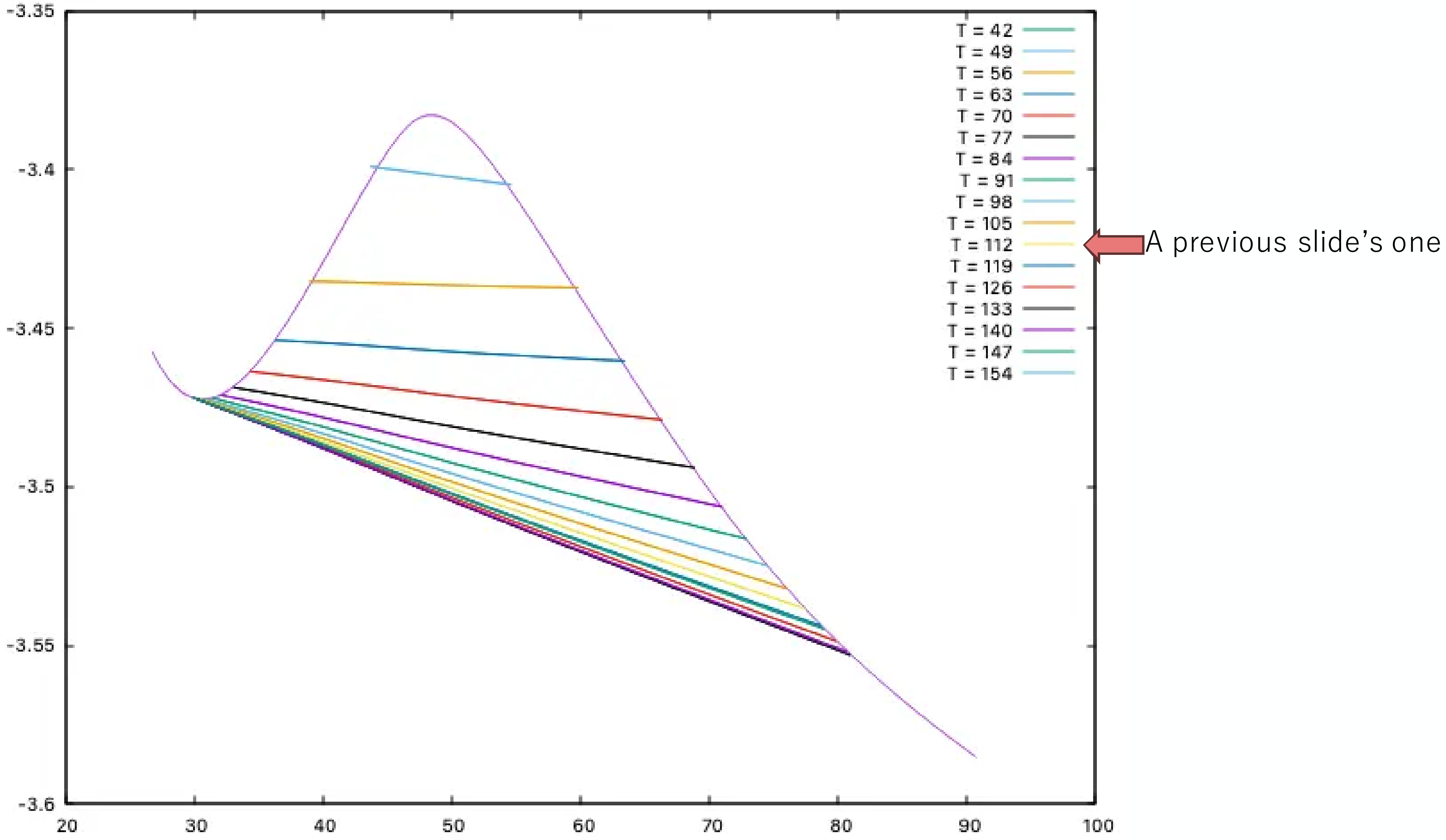
$$\frac{\partial \mathcal{H}}{\partial \tau} = \hbar \sum_i \left(-\frac{\delta \mathcal{H}}{\delta \phi_i(\tau)} \frac{\delta \mathcal{H}}{\delta \phi_i^*(-\tau)} + \frac{\delta \mathcal{H}}{\delta \phi_i^*(-\tau)} \frac{\delta \mathcal{H}}{\delta \phi_i(\tau)} \right) = 0$$

However, numerically, orthogonalizing each time step could change the energy.
Furthermore, if we do not orthogonalize each time step, the calculation diverges.

Discussion

The time period T is the free parameter we can choose. If T is change, the converged path is also change.

Boundary condition $\phi_\beta(x, \frac{T}{2}) = e^{-\lambda_\beta}\phi_\beta(x, -\frac{T}{2})$



However, simply changing T could not obtain the path of energy conservation and through the ground states.

Discussion

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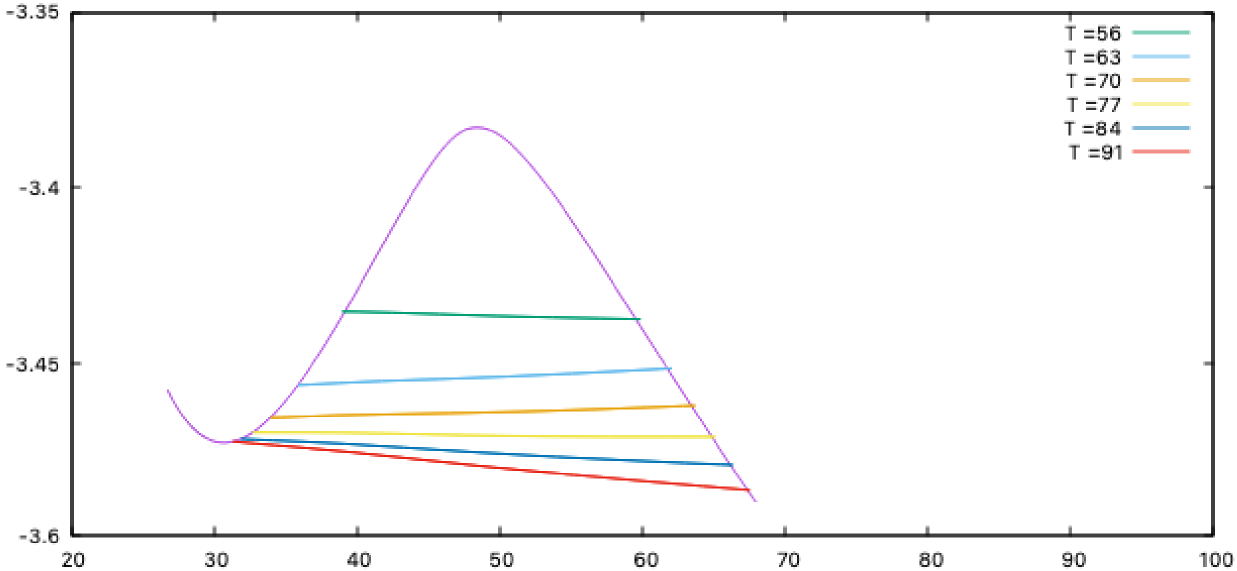
Even if we pick up the energy conserved path ($T = 56$) as a initial path and extend T, near the ground states, the energy is not conserved.



Furthermore, it differs from direct and extended calculations.
It may show that there is initial value dependency…?

Even with a constraint, the initial guess must be close in some sense to a periodic solution, or else iteration will evolve it into an uninteresting solution, usually a static Hartree-Fock solution. An adequate strategy for finding eigenstates of large-amplitude collective motion is first to solve the RPA equations for infinitesimal vibrations. Starting with the time-dependent wave functions for a single mode, a series of sequential self-consistent calculations may be performed gradually increasing the period from the RPA value $T_0 = 2\pi/\omega$. By continuity, the ini-

J. W. Negele, Rev. Mod. Phys. **54**, 913 (1982)

Summary & Future Work

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Summary

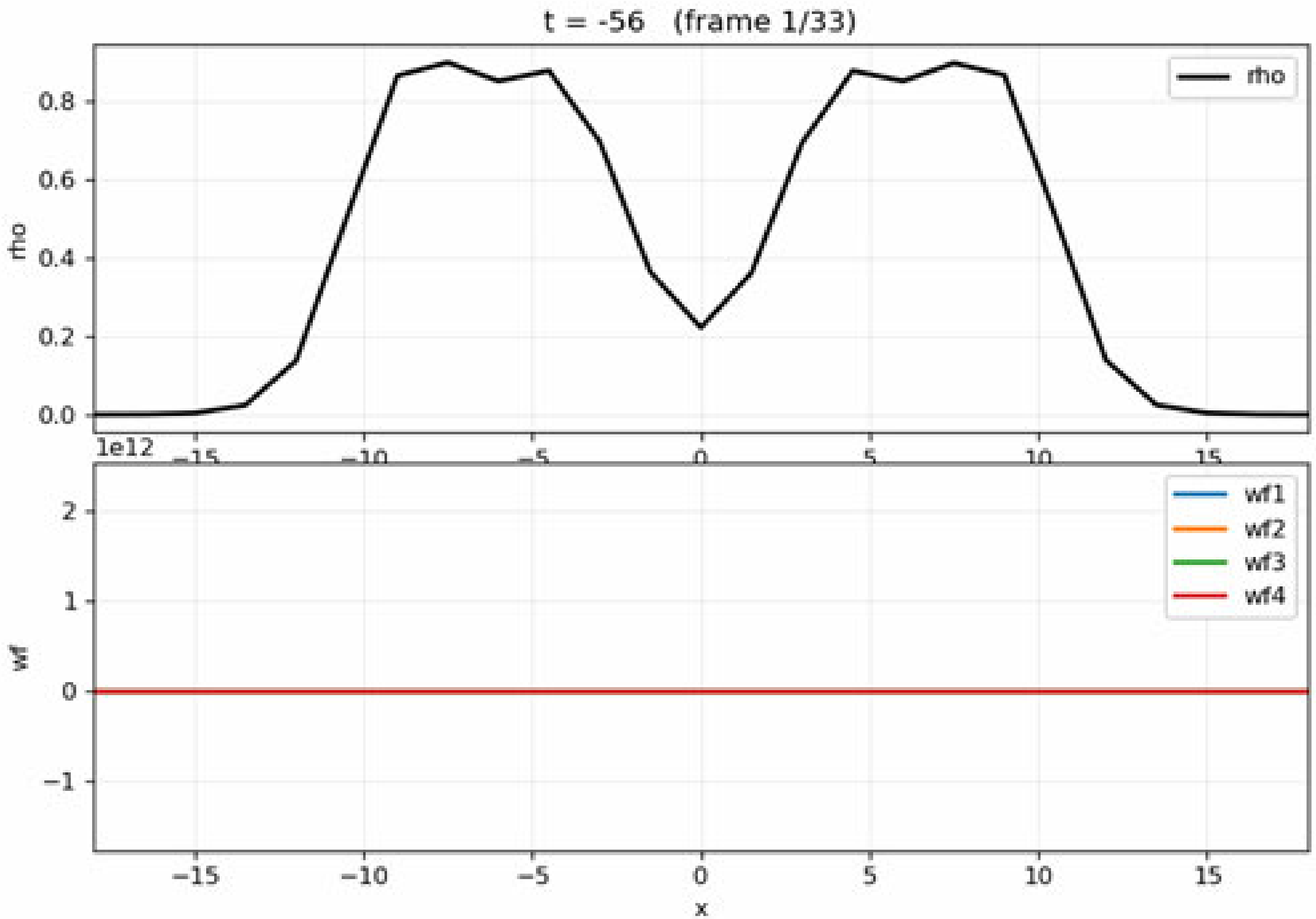
- Our research purpose is to describe spontaneous fission from nucleon degrees of freedom.
- Time-Dependent Hartree-Fock（TDHF）method is known as a method for microscopically describing the dynamics of nuclei, however, since mean-field motion is classical, quantization of TDHF is necessary to describe spontaneous fission.
- We quantized TDHF by periodic Imaginary TDHF and calculated simple 1D systems.

Future Work

- Investigate why energy is not conserved.
- Calculate the half-life in the 1D system and compare it with the half-lives calculated using other theories.

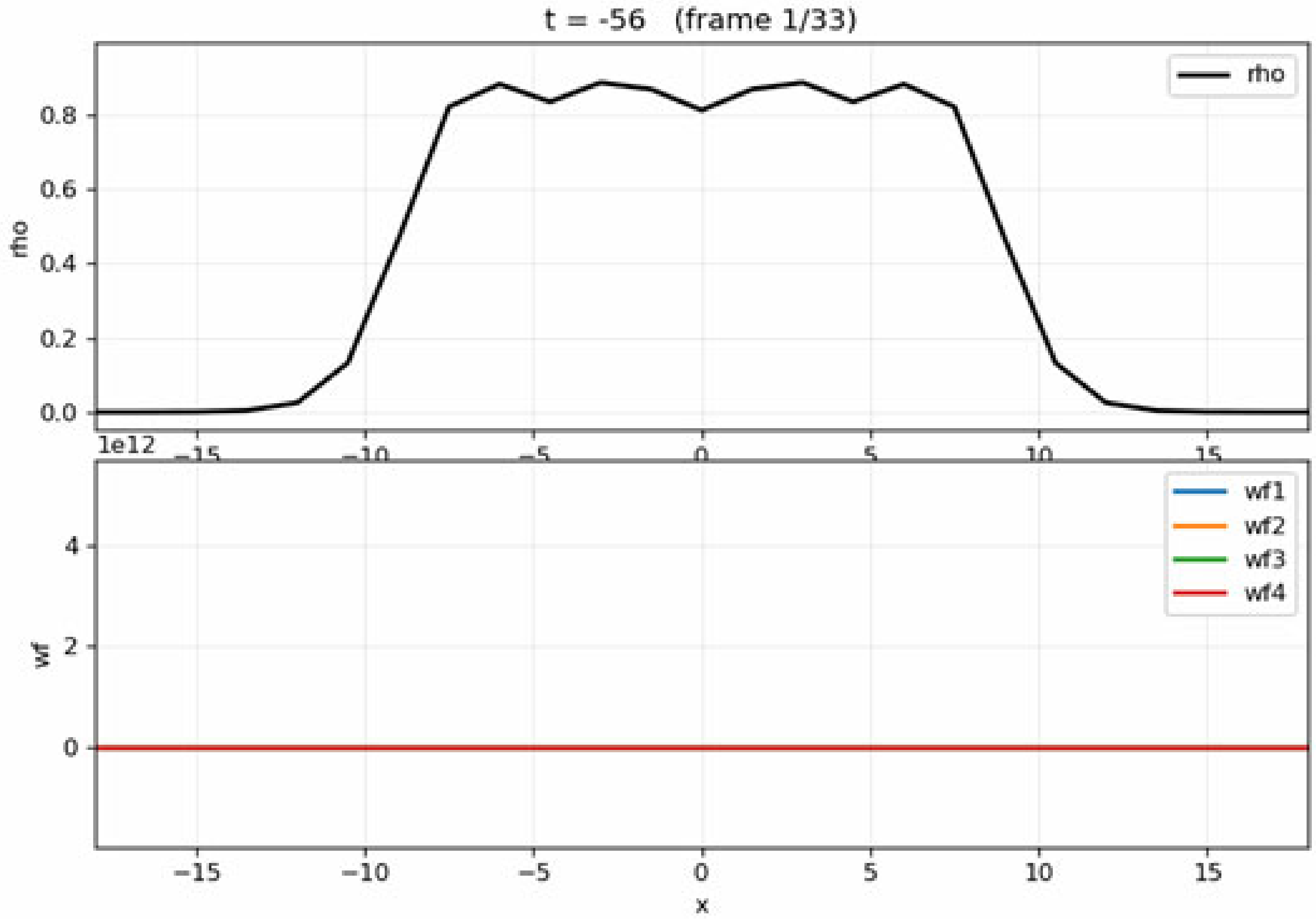
Back up

Back up



t = -56 (frame 1/33)
rho
0.8
0.6
0.4
0.2
0.0
1e12
wf1
wf2
wf3
wf4
wf
2
1
0
−1
−15
−10
−5
0
5
10
15
x

Back up



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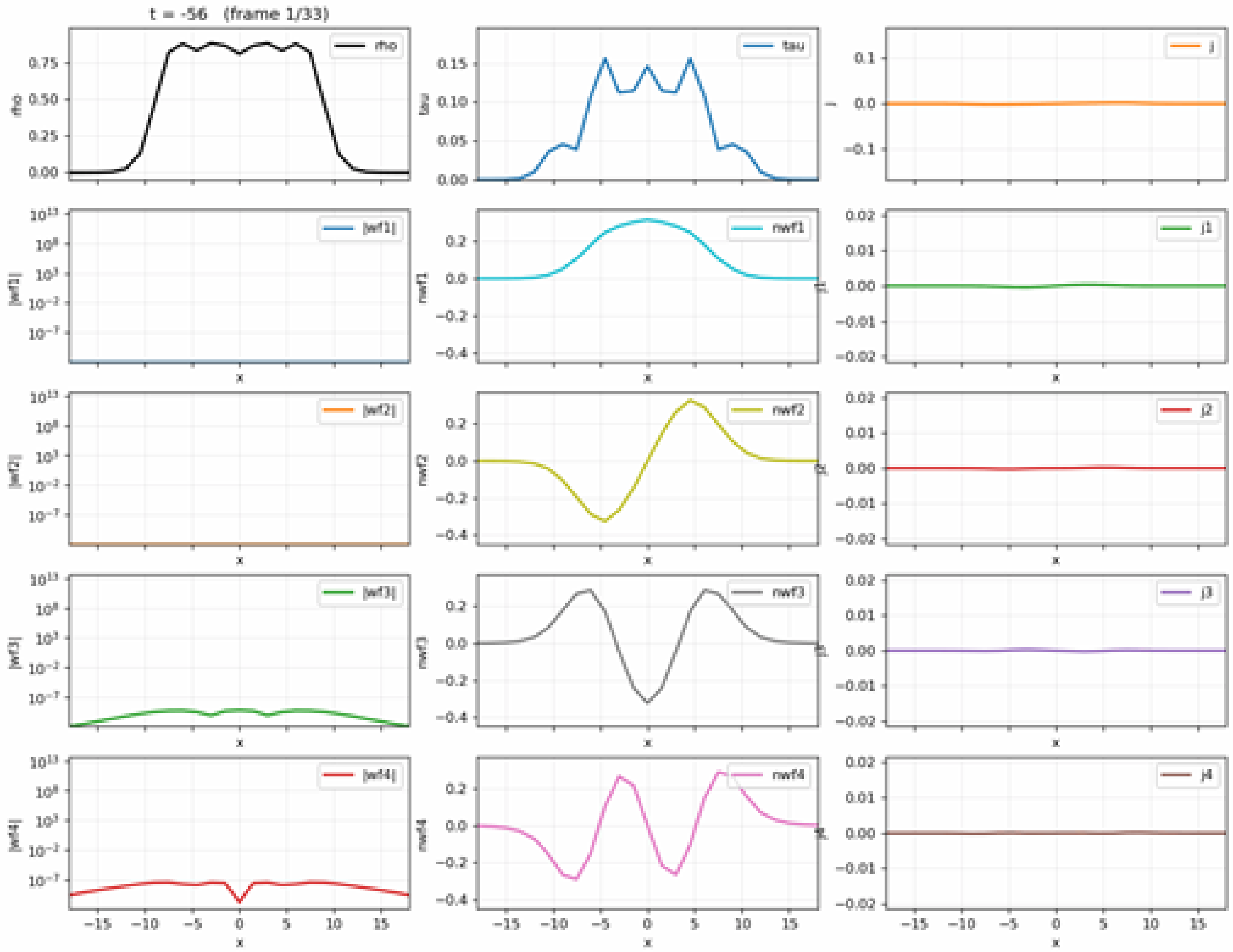

t = -56 (frame 1/33)
rho
tau
j
|wf1|
nwf1
j1
|wf2|
nwf2
j2
|wf3|
nwf3
j3
|wf4|
nwf4
j4
x

Back up

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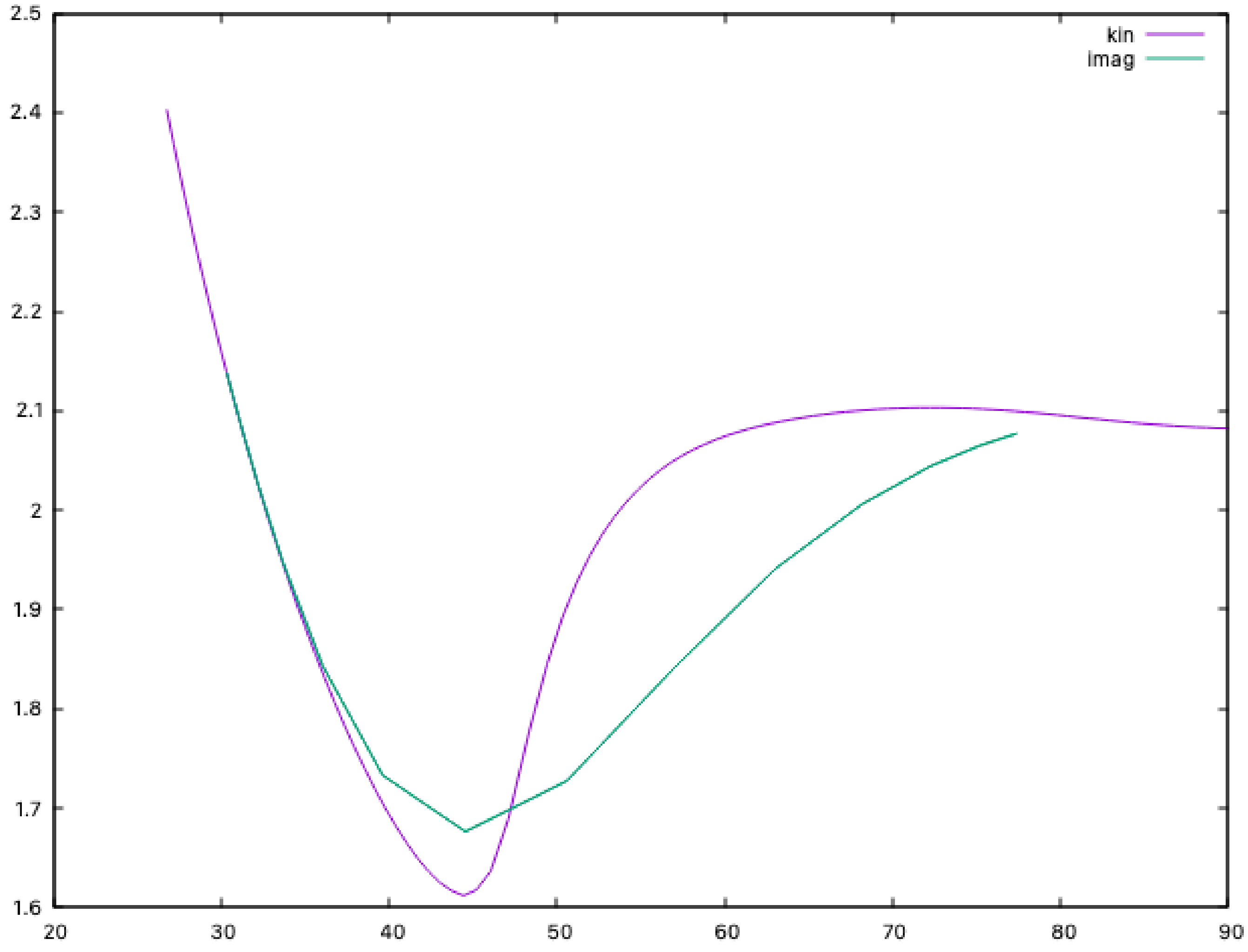

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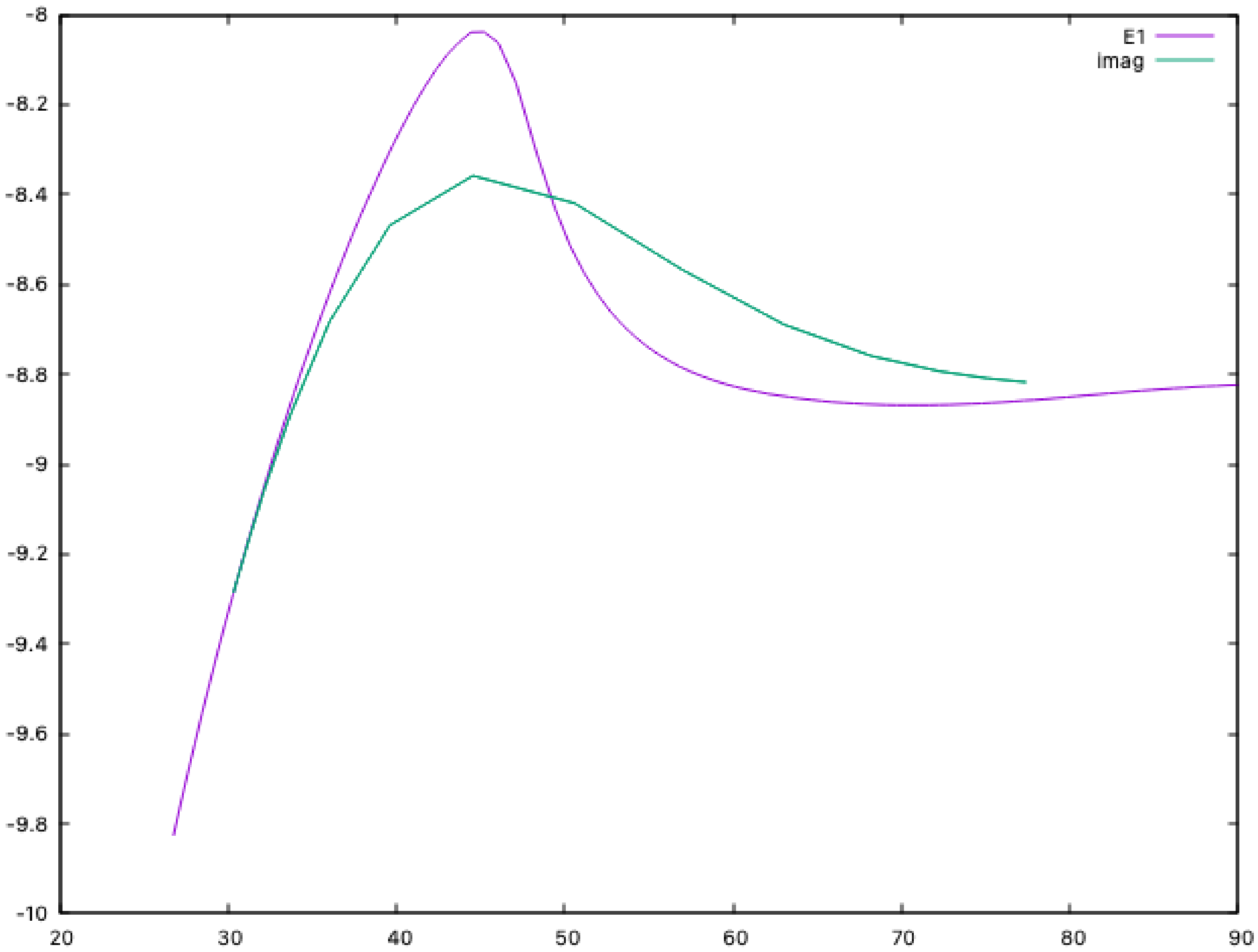

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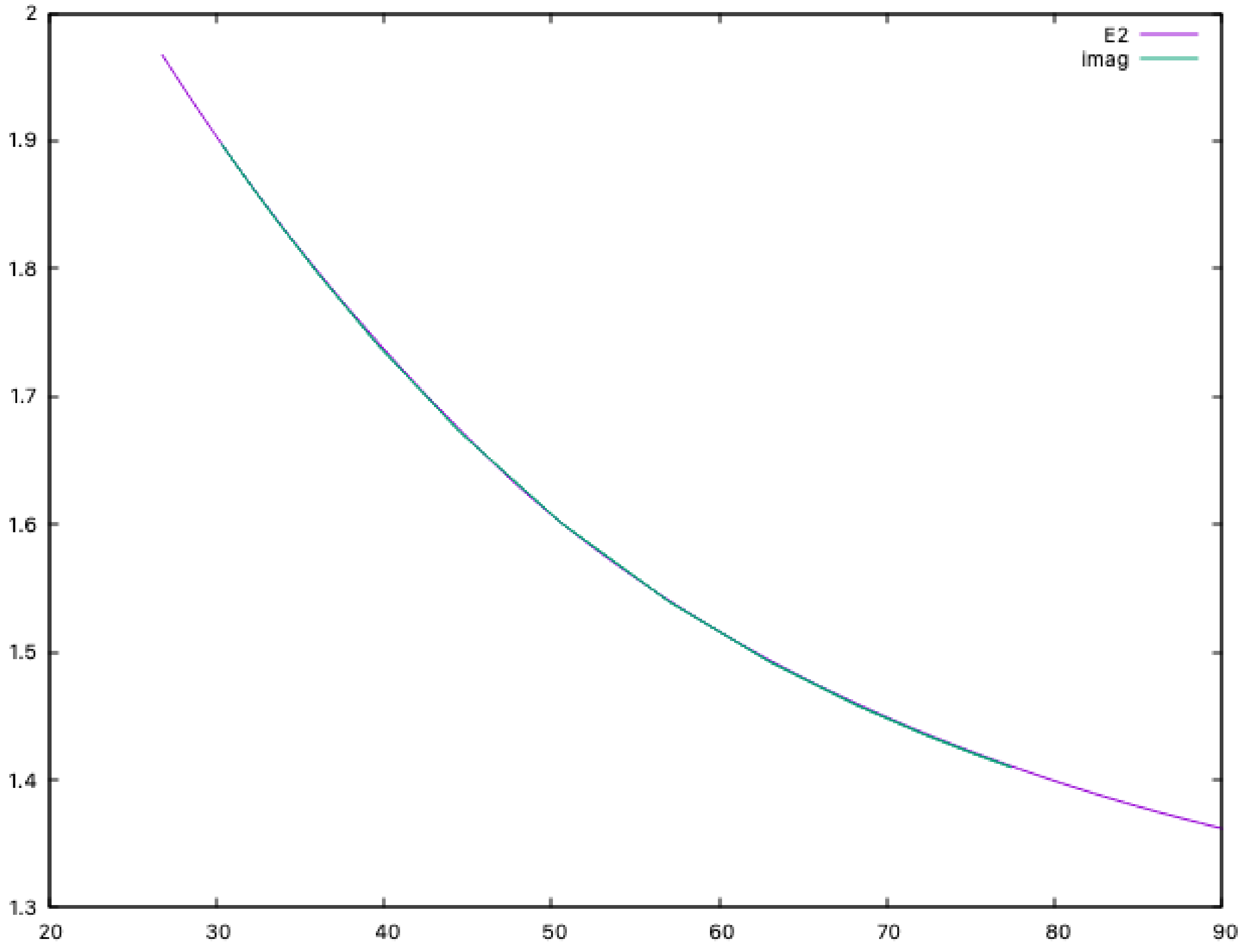

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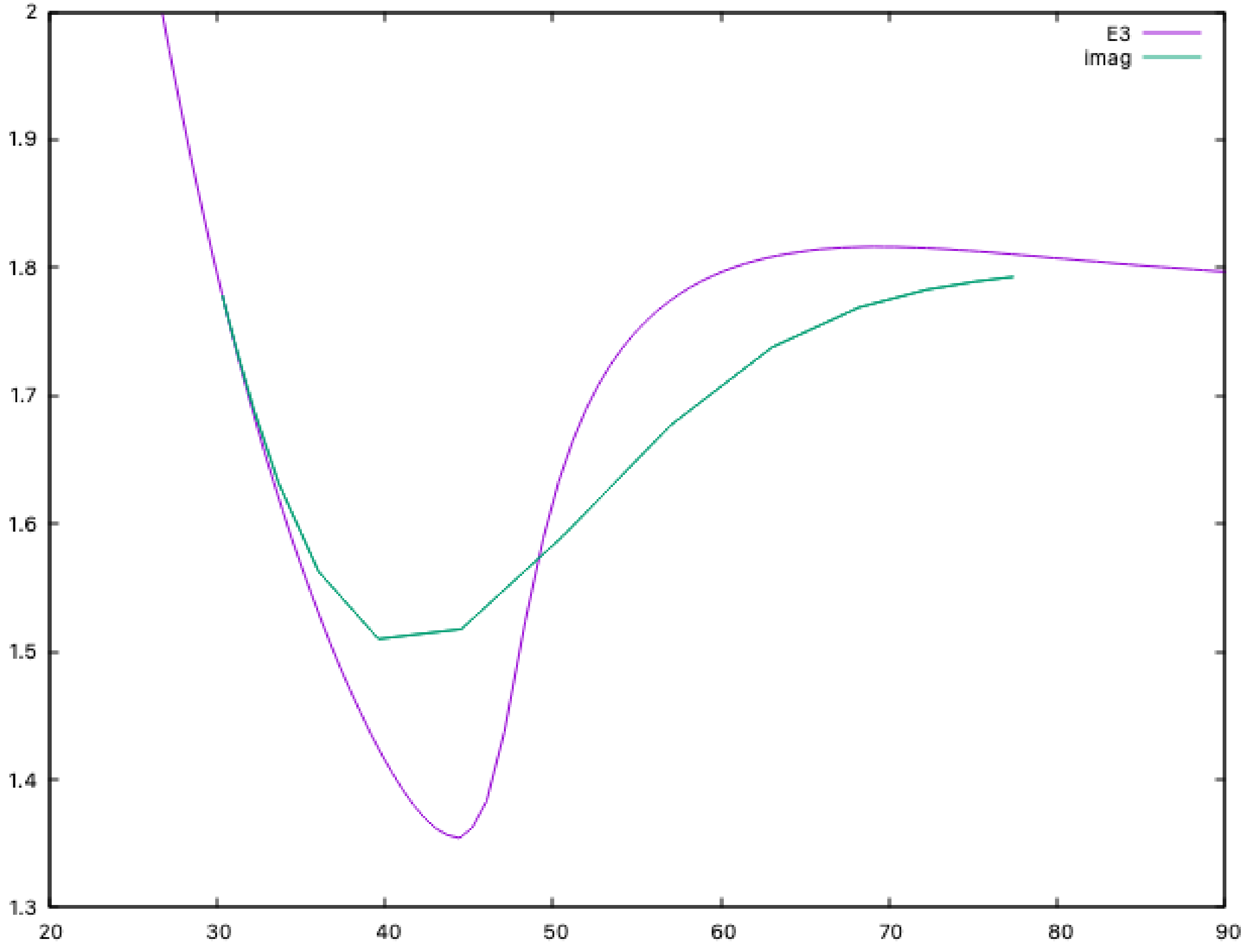

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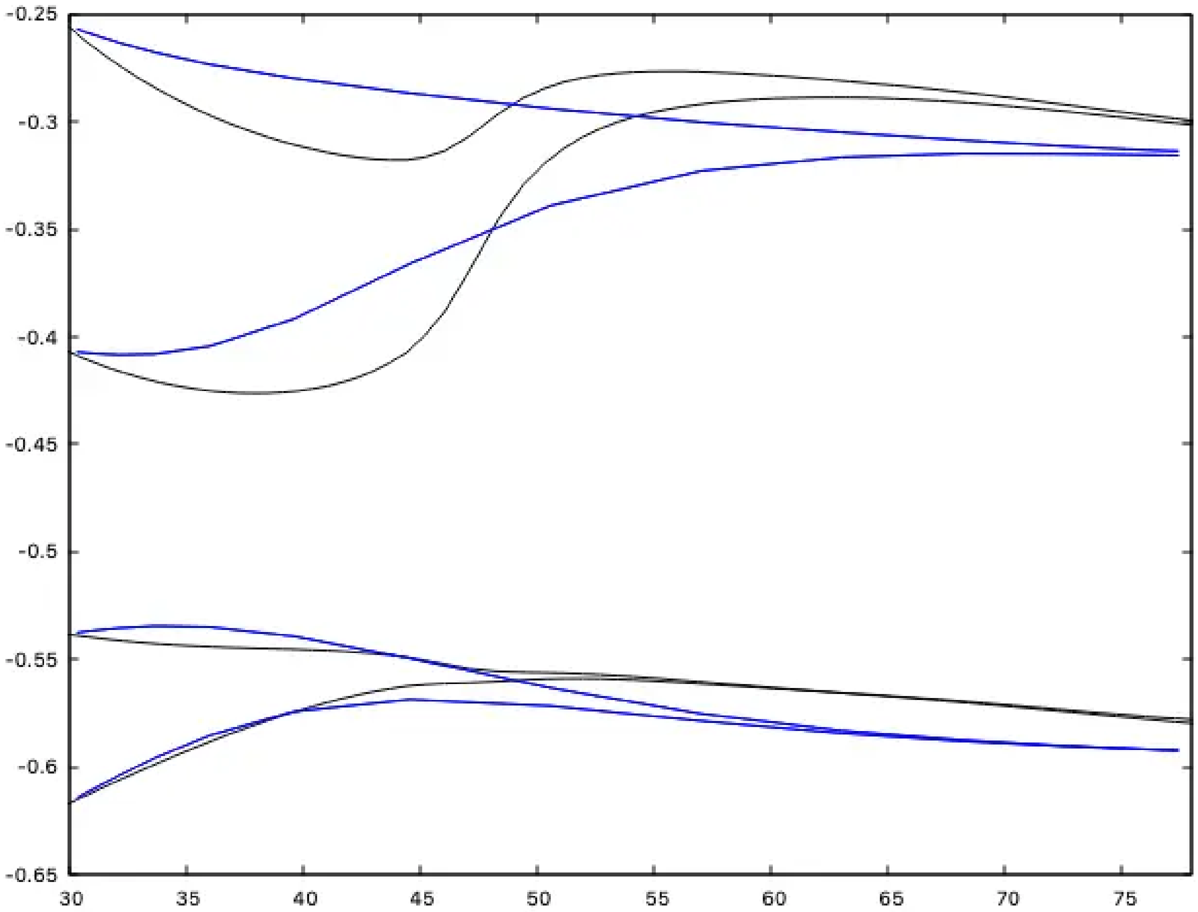

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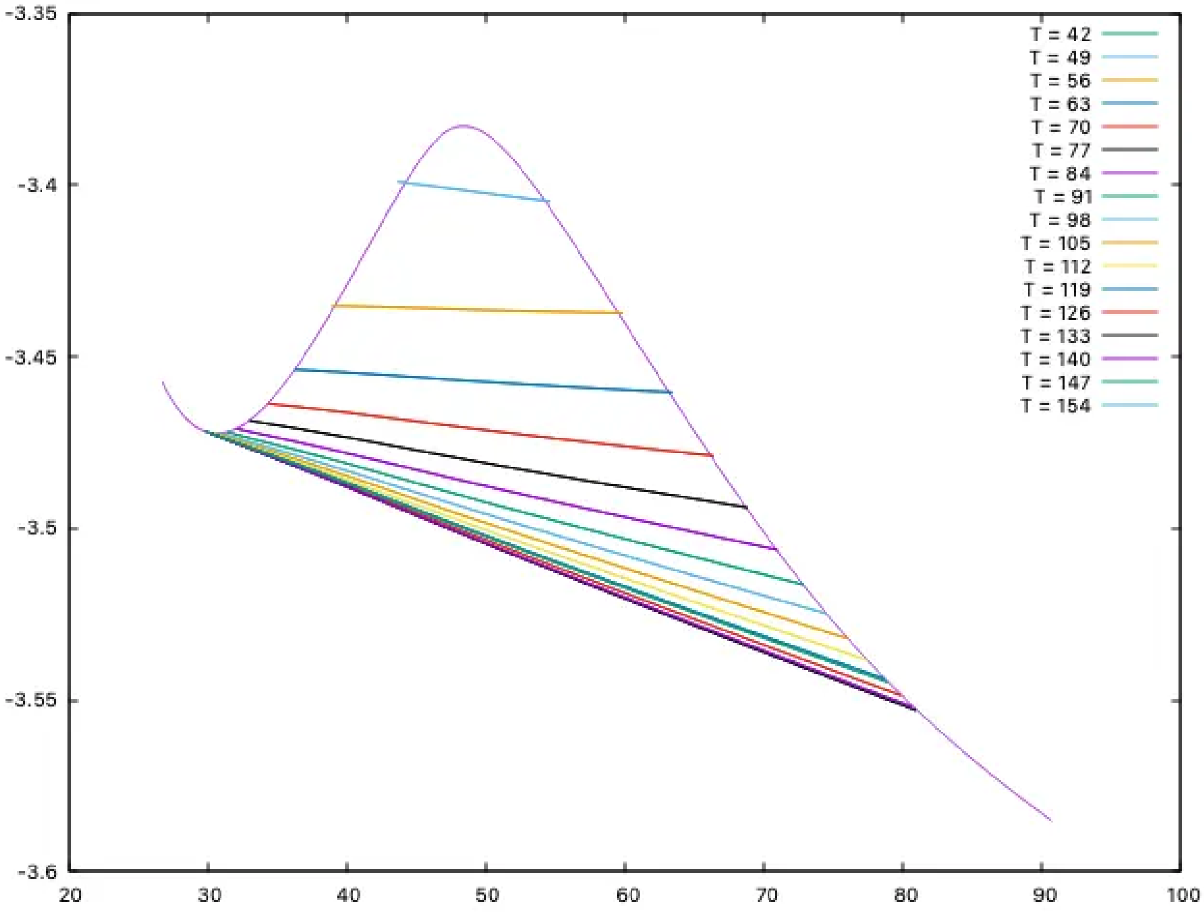
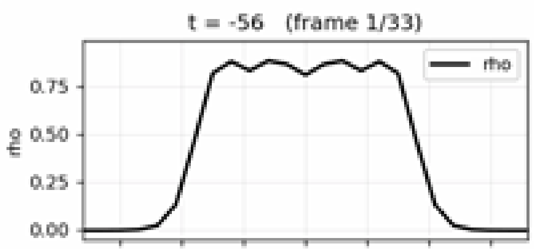
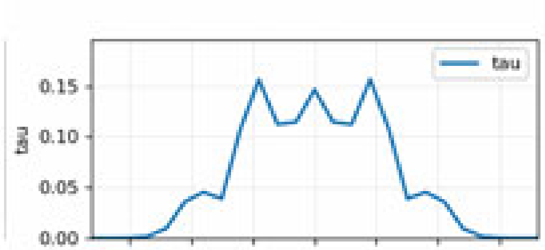
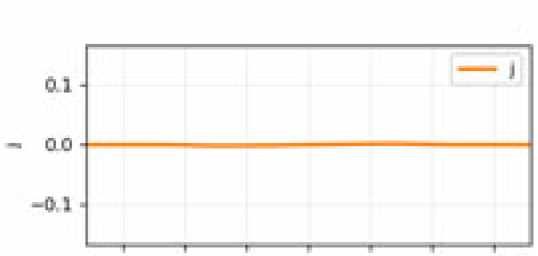


-3.35
-3.4
-3.45
-3.5
-3.55
-3.6
20
30
40
50
60
70
80
90
100
T = 42
T = 49
T = 56
T = 63
T = 70
T = 77
T = 84
T = 91
T = 98
T = 105
T = 112
T = 119
T = 126
T = 133
T = 140
T = 147
T = 154

Discussion①







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Institute of Science Tokyo, M2

Supervisor: K.Sekizawa

Nuclear and Hadron Physics Laboratories Joint Workshop 2025・2025/12/8

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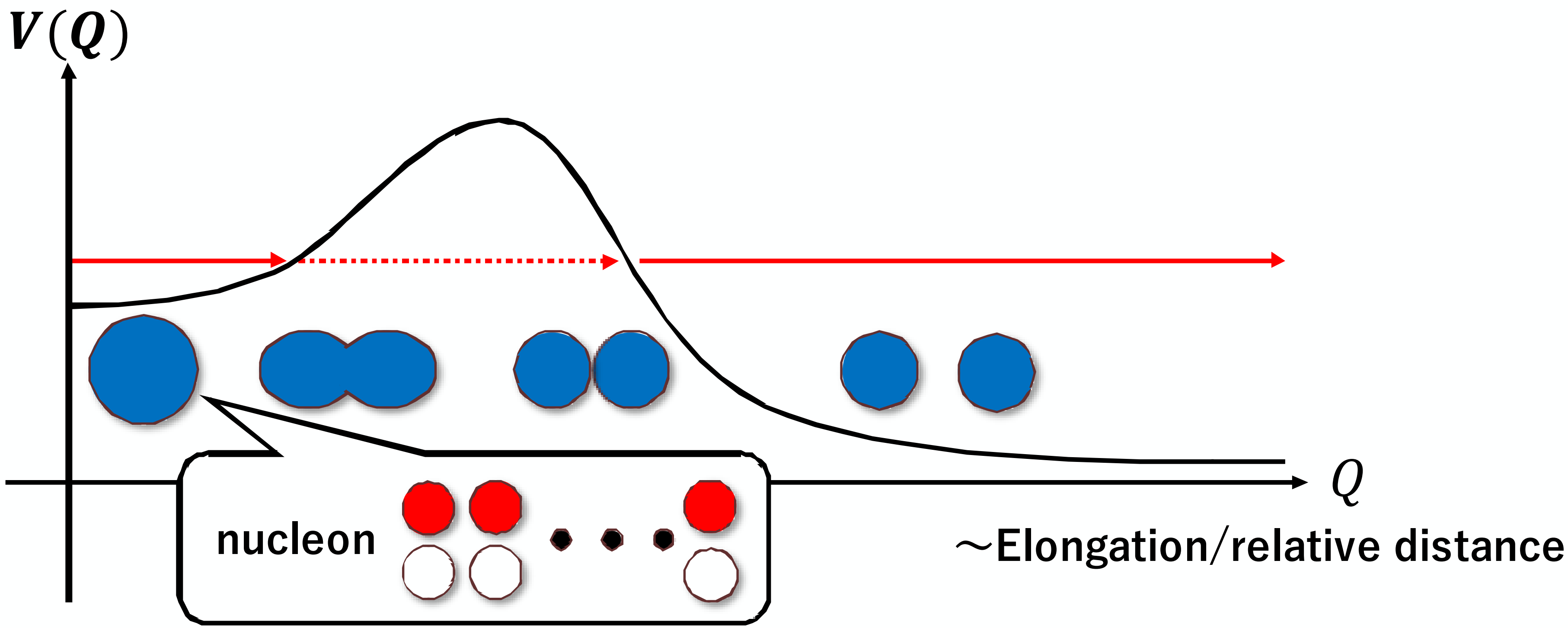
Problem of TDHF

Our research purpose

To describe spontaneous nuclear fission by a microscopic mean-field approach



Spontaneous fission is caused by **many-body tunneling effect.**

Our research purpose

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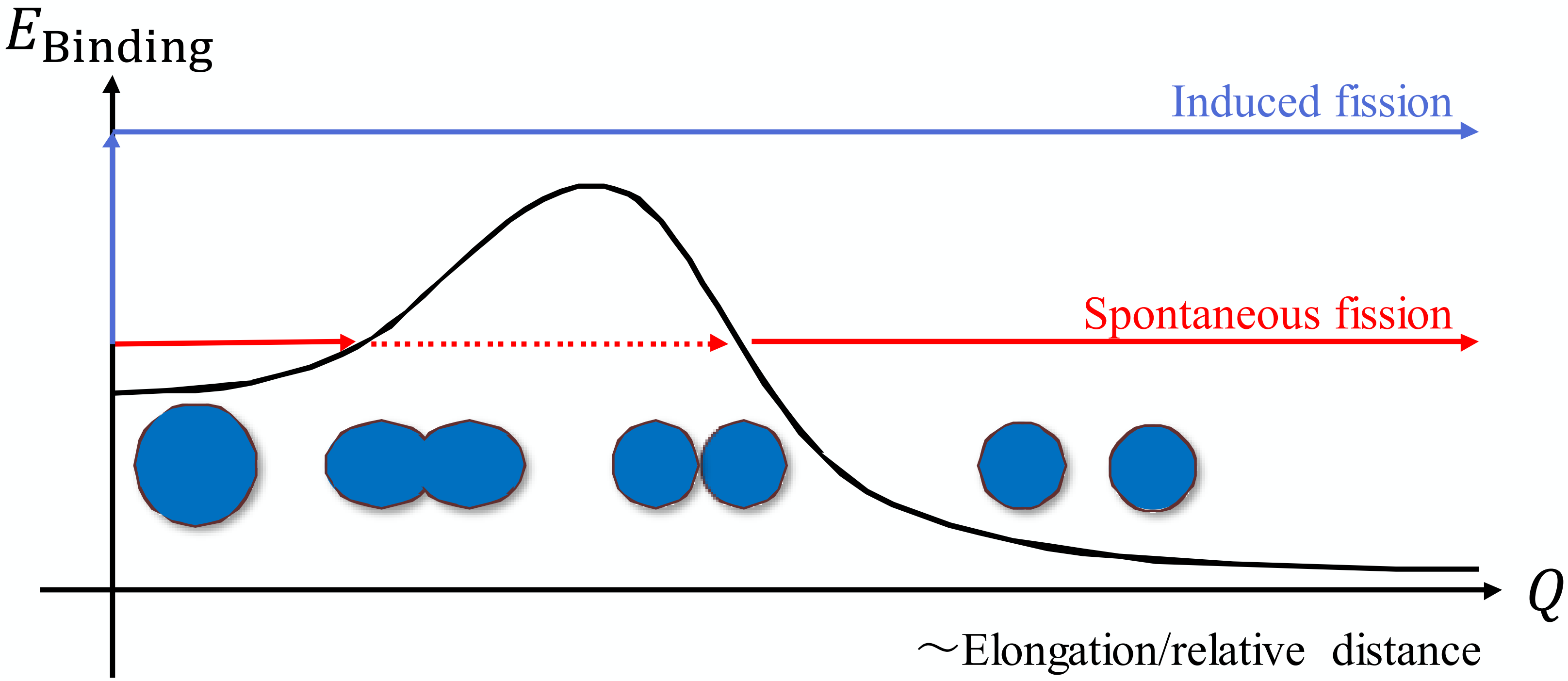
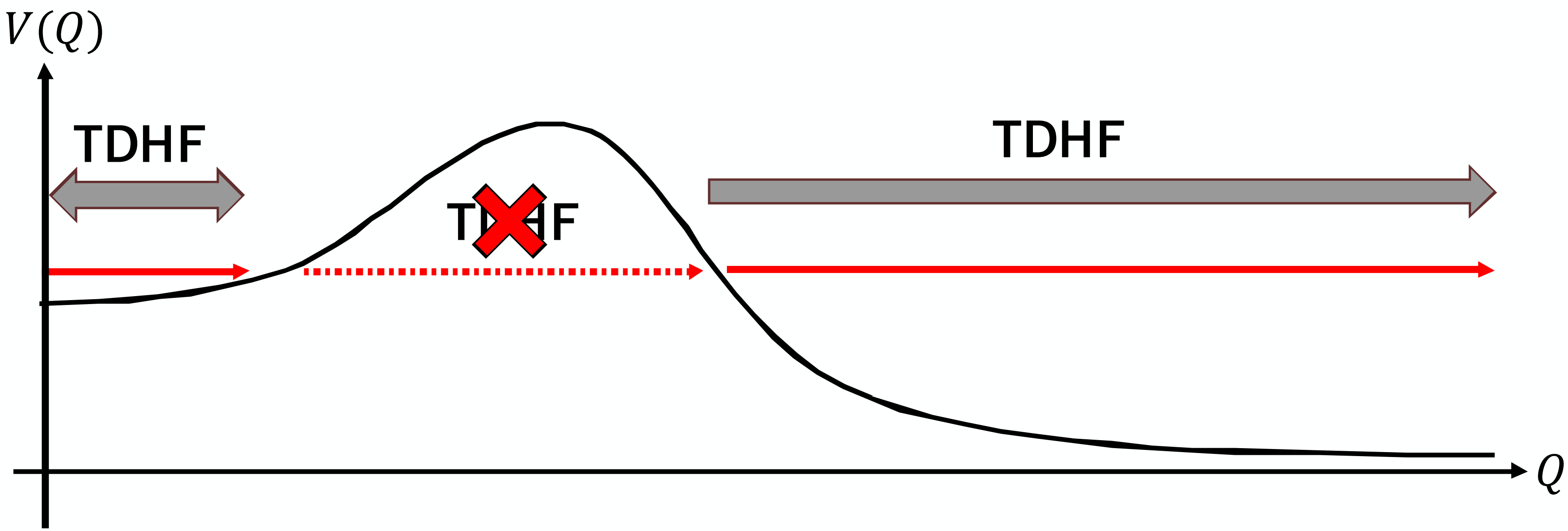
To describe spontaneous nuclear fission by a microscopic mean-field approach



Spontaneous fission is caused by **many-body tunneling effect.**

TDHF cannot describe SF

Time-Dependent Hartree-Fock (TDHF) can describe microscopically the dynamics of **a single particle**, but…



A motion of mean field (collective motion) is classically, so it cannot describe spontaneous fission (SF).

We should quantize TDHF to describe SF.

Quantization of TDHF

Derivation TDHF

Conventionally, TDHF is derived using the variational principle, but now, we derive it using **a path integral formalism**.

S. Levit Phys. Rev. C **21**, 1594 (1980)
S. Levit, J. W. Negele, and Z. Paltiel, Phys. Rev. C **21**, 1603 (1980)

Many-body Quantum Mechanics

$$H = \sum_{\alpha\beta} T_{\alpha\beta}\hat{a}^\dagger_\alpha \hat{a}_\beta + \frac{1}{2}\sum_{\alpha\beta\gamma\delta} v_{\alpha\beta\gamma\delta}\hat{a}^\dagger_\alpha \hat{a}^\dagger_\beta \hat{a}_\delta \hat{a}_\gamma$$

$$U(t_f, t_i) = T\exp\left[-\frac{i}{2}\int_{t_i}^{t_f} dt \sum_{\alpha\beta\gamma\delta}\hat{\rho}_{\alpha\gamma}(t) V_{\alpha\beta\gamma\delta}\hat{\rho}_{\beta\delta}(t)\right]$$

⇩ **Hubbard-Stratonovich transformation**

Path Integral representation

$$U(t_f, t_i) = \int \mathcal{D}[\sigma]\exp\left[\frac{i}{2}\int_{t_i}^{t_f} dt\{\sigma(t) v \sigma(t)\} U_I^\sigma(t_f, t_i)\right]$$

$$U_I^\sigma(t_f, t_i) \equiv T\exp\left[-i\int_{t_i}^{t_f} dt\{\sigma(t) v \hat{\rho}(t)\}\right]$$

⇩ **Stationary phase approximation ($\delta S = 0$)**

TDHF equation

$$i\hbar\partial_t \psi_k(t) = -\frac{\hbar^2}{2m}\nabla^2\psi_k(t) + \frac{\delta \mathcal{V}}{\delta\psi_k^*(t)}$$

TDHF is classical theory

Semi-classical quantization

S. Levit, J. W. Negele, and Z. Paltiel, Phys. Rev. C **21**, 1603 (1980)

Gutzwiller formula

$$G(E) \equiv i \int_0^\infty dT e^{iET} \, \mathrm{tr}\, U(T,0) = \sum_\nu \frac{1}{E_\nu - E}$$

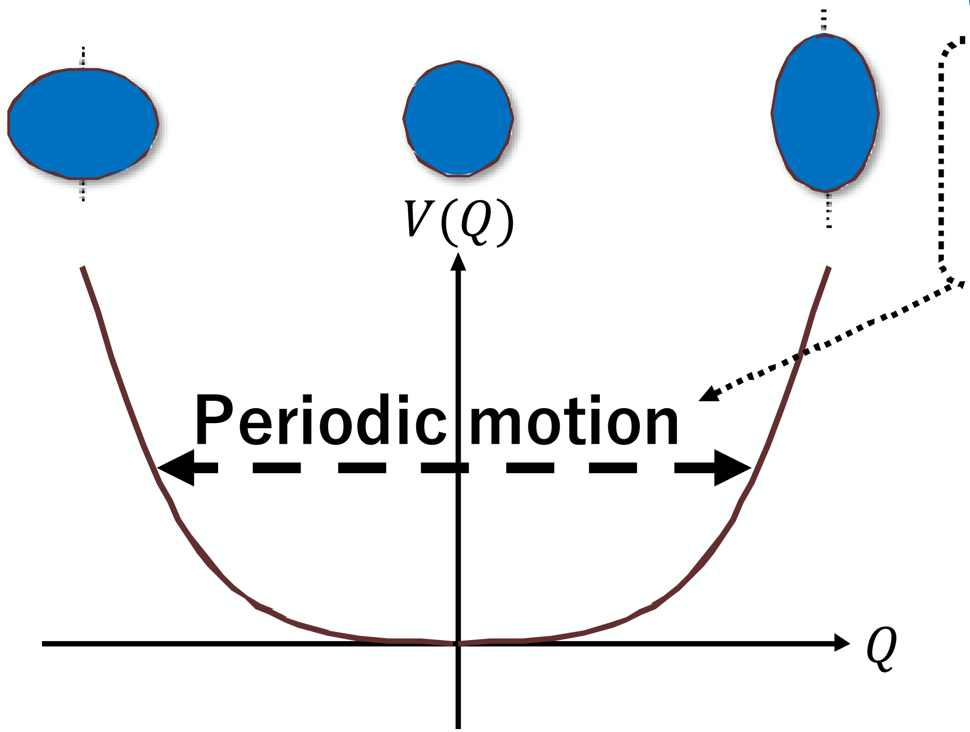
Periodic trajectory Propagator

Quantum theory's Energy

①Describing by periodic TDHF

$$i\hbar \partial_t \psi_k(t) = -\frac{\hbar^2}{2m}\nabla^2 \psi_k(t) + \frac{\delta \mathcal{V}}{\delta \psi_k^*(t)}$$

$$\psi_k(T/2) = e^{-i\alpha_k}\psi_k(-T/2)$$

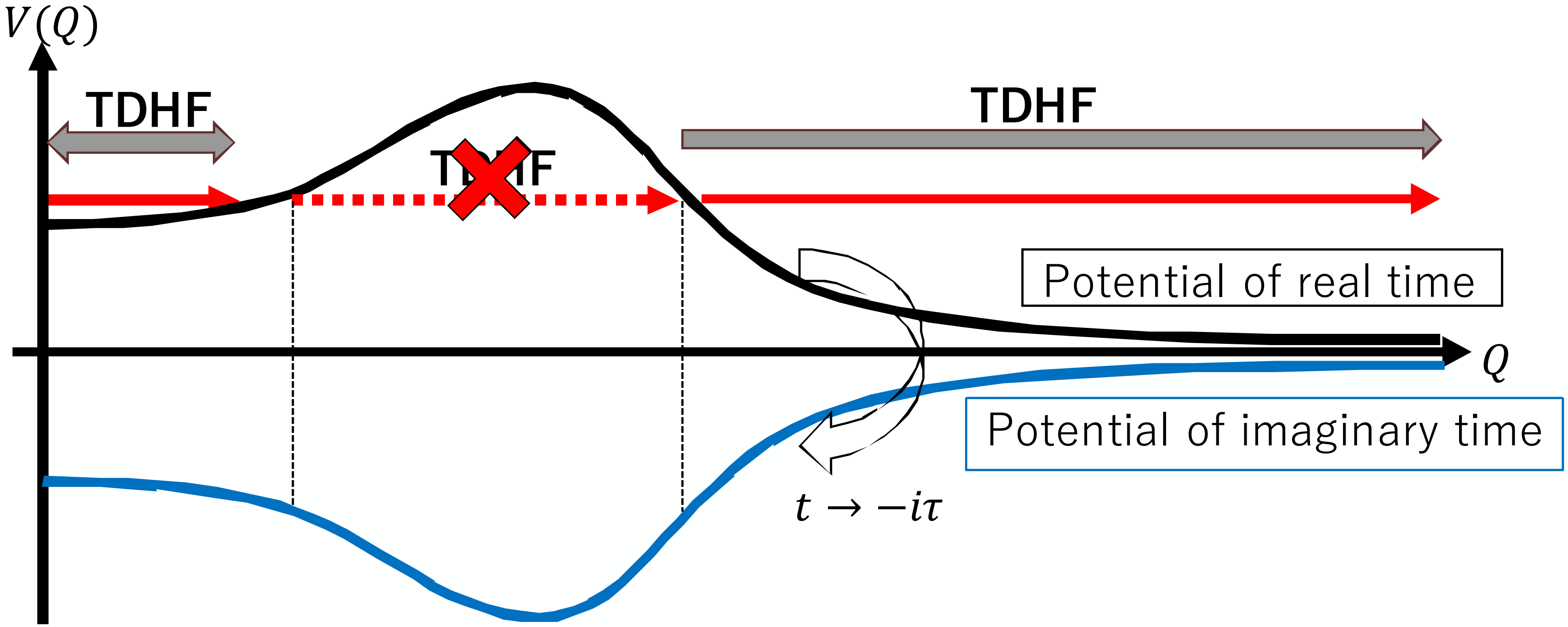


②calculating $G(E)$, the poles give the energy in quantum theory.

Imaginary time evolution

As an analogy of the one particle QM,

J. Skalski, Phys. Rev. C **77**, 064610 (2008)



Real time TDHF

$$i\hbar\partial_t\psi_k(t) = -\frac{\hbar^2}{2m}\nabla^2\psi_k(t) + \frac{\delta\mathcal{V}}{\delta\psi_k^*(t)}$$

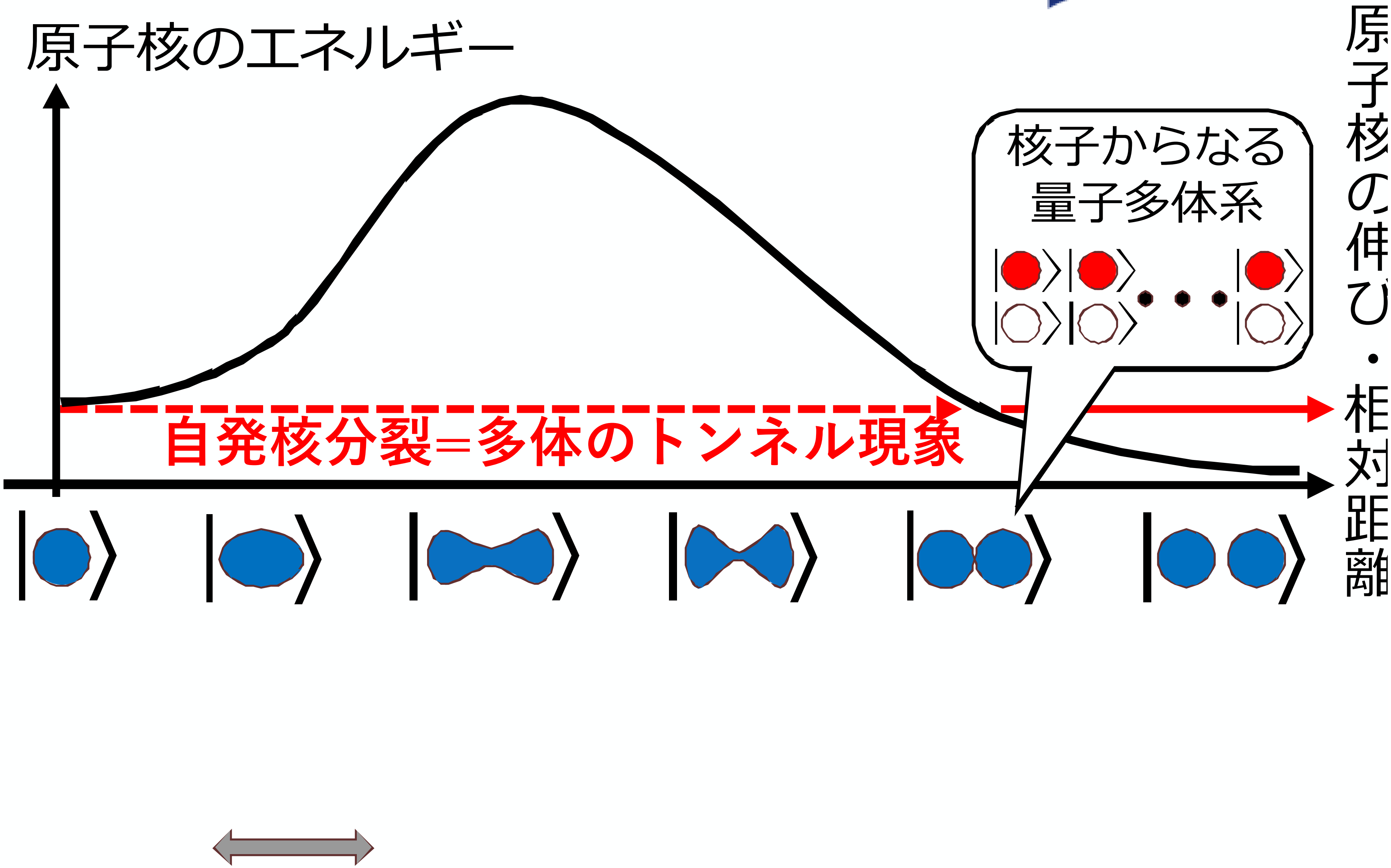
$$\langle A(t)\rangle = \sum_k \int dx\,\psi_k^*(x,t)\hat{A}\psi_k(x,t)$$

$t \to -i\tau$

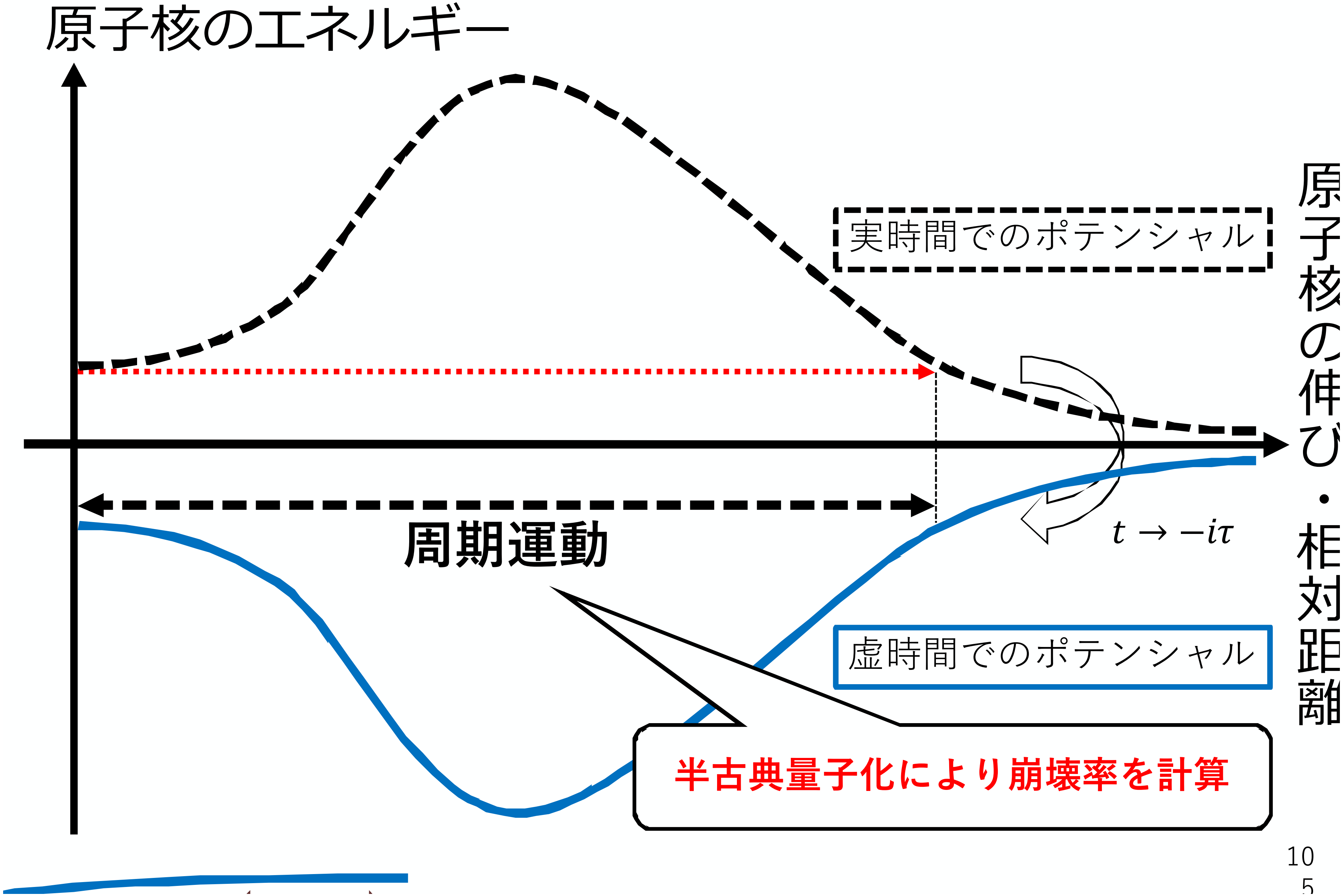
Imaginary time TDHF

$$-\hbar\partial_\tau\psi_k(\tau) = -\frac{\hbar^2}{2m}\nabla^2\psi_k(\tau) + \frac{\delta\mathcal{V}}{\delta\psi_k(-\tau)}$$

$$\langle A(\tau)\rangle = \sum_k \int dx\,\psi_k(x,-\tau)\hat{A}\psi_k(x,\tau)$$



Potential of real time


原子核のエネルギー
実時間でのポテンシャル
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周期運動
虚時間でのポテンシャル
半古典量子化により崩壊率を計算

Description of SF using ITDHF

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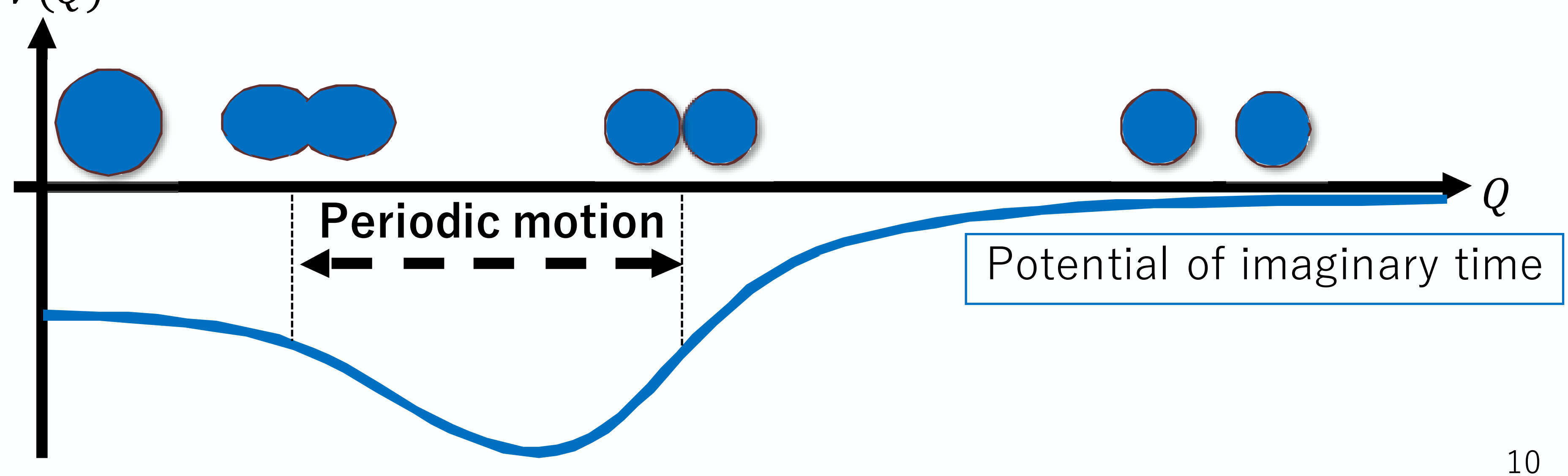
Periodic TDHF → **Imaginary TDHF** → **Gutzwiller formula**

$$-\hbar\partial_\tau\psi_k(\tau) = -\frac{\hbar^2}{2m}\nabla^2\psi_k(\tau) + \frac{\delta\mathcal{V}}{\delta\psi_k(-\tau)} \qquad \psi_k(T/2) = e^{-\alpha_k}\psi_k(-T/2)$$

$$\exp\left[-\frac{S}{\hbar}\right], \quad S = \hbar\int_{-T/2}^{T/2} d\tau \sum_k \left\langle \phi_k(-\tau) \middle| \frac{\partial\phi_k(\tau)}{\partial\tau} \right\rangle .$$

J. Skalski, Phys. Rev. C **77**, 064610 (2008)



Setup for Numerical Calculations

System

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EDF of our system

$$\mathcal{H}[\phi(x,-\tau),\phi(x,\tau)] = -M\sum_\alpha \phi_\alpha(x,-\tau)\left(\frac{\partial^2}{\partial x^2}\right)\phi_\alpha(x,\tau) + \frac{1}{2}\int dx'\rho(x,\tau)V(x-x')\rho(x',\tau) + \frac{1}{3}V_3\rho^3(x,\tau)$$

Three body force

$$V(x) = \frac{V_1}{\sqrt{\pi}\gamma_1}e^{-x^2/\gamma_1^2} + \frac{V_2}{\sqrt{\pi}\gamma_2}e^{-x^2/\gamma_2^2} \qquad \rho(x,\tau) = M\sum_\alpha \phi_\alpha(x,-\tau)\phi_\alpha(x,\tau)$$

Two body force

S. Levit, J. W. Negele, and Z. Paltiel, Phys. Rev. C **22**, 1979

parameter	value	
M	4	← Spin-Isospin degeneracy
γ_1	2	
γ_2	10	
V_1	-1.489	← Attractive
V_2	0.40	← Repulsive (Coulomb like)
V_3	0.5	

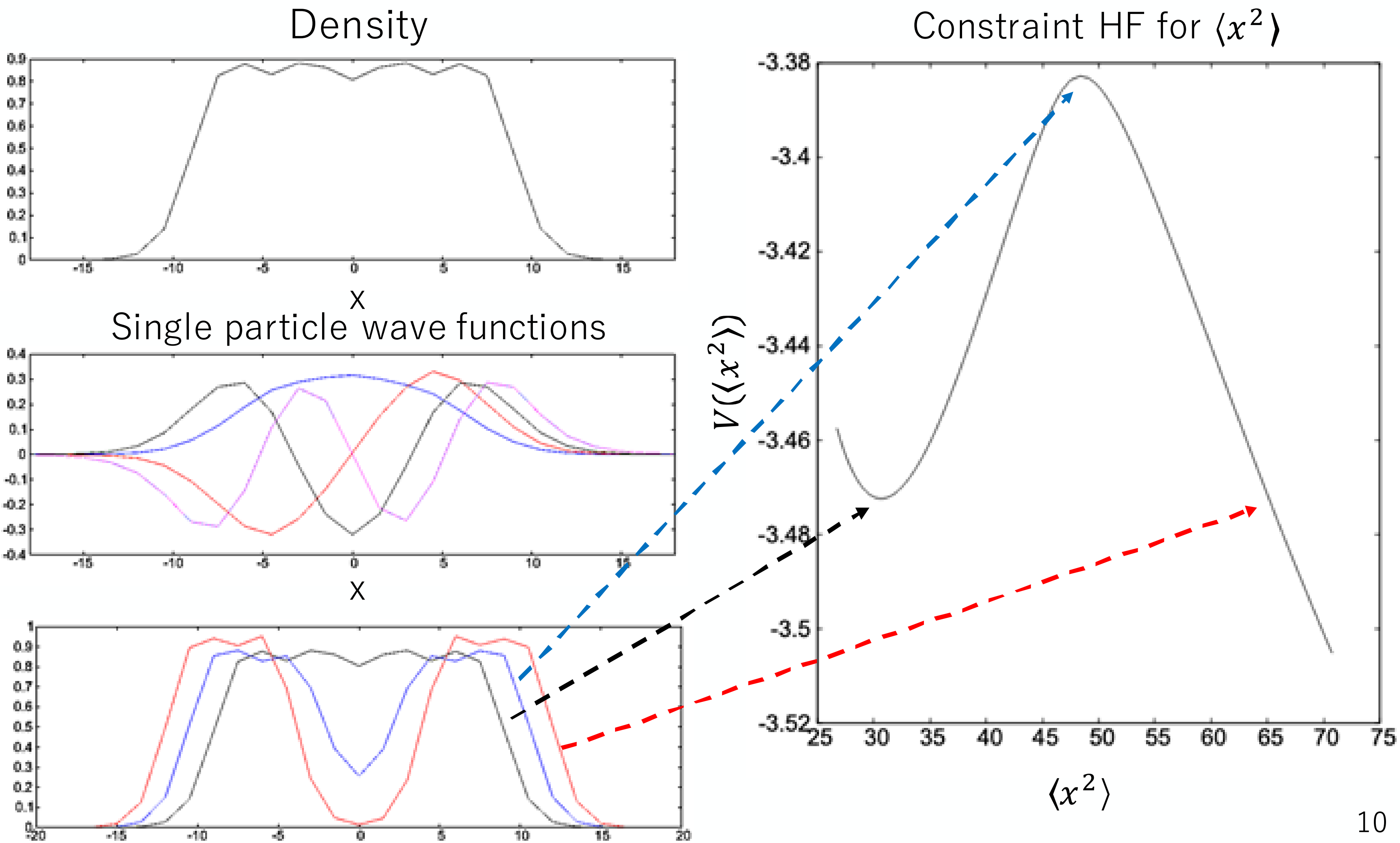
For simplicity,

- One-dimentional space
- 16 particle system
- No Fock terms

Static solutions

Discretization of space $x_0, x_1, \dots x_{N_x}$

Number of meshes: 24, Mesh width: 1.5



Discretization of Time

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G.Puddu , J. W. Negele, Phys. Rev. C **35**, 1007

$$-\frac{\partial\phi_\beta(x,\tau)}{\partial\tau} = \left(-\frac{\partial^2}{\partial x^2} + \int V(x-x')\rho(x',\tau)dx' + V_3\rho^2(x,\tau) + V_\lambda(x)\right)\phi_\beta(x,\tau)$$

$$= h[\rho]\phi_\beta(x,\tau)$$

Constraints to avoid uniform solutions

$$V_\lambda(x) = \lambda\left[\int_{-\frac{1}{2}}^{\frac{1}{2}} d\eta \int x'^2\rho(x',\eta)dx' - x_0^2\right]x^2$$

B.C. $\phi_\beta(x,\frac{T}{2}) = e^{-\lambda_\beta}\phi_\beta(x,-\frac{T}{2})$

Discretization of time $\tau_0, \tau_1, \dots \tau_{N_\tau}$

Number of meshes: 32, Mesh width: 3.5

$$\phi_\beta(x_i,\tau_{k+1}) = \exp\left(-h\left[\frac{\rho_{k+1}+\rho_k}{2}\right]\Delta\tau\right)\phi_\beta(x_i,\tau_k) \equiv U(\tau_{k+1},\tau_k)\phi_\beta(x_i,\tau_k)$$

Time evolution operator

Eigenvalue problem $U(\tau_N,\tau_1)\phi_\beta(x_i,\tau_1) = e^{-\lambda_\beta}\phi_\beta(x_i,\tau_1)$

Computational Flow

G.Puddu , J. W. Negele, Phys. Rev. C **35**, 1007

1. As an initial path, preparing $\rho(x_i, \tau_j)$ and $\phi_k(x_i, \tau_0)$
2. Time evolving $\phi_1(x_i, \tau_0)$ to $\phi_1(x_i, \tau_N)$

$$\phi_1(x_i, \tau_0) \rightarrow \phi_1(x_i, \tau_1) \rightarrow \phi_1(x_i, \tau_2) \rightarrow \cdots \rightarrow \phi_1(x_i, \tau_N)$$

$$\phi_1(x_i, \tau_{k+1}) = \exp\left(-h\left[\frac{\rho_{k+1}+\rho_k}{2}\right]\Delta\tau\right)\phi_1(x_i, \tau_k)$$

3. Normalizing $\phi_1(x_i, \tau_j)$

$$\int dx \phi_1(x, \tau)\phi_1(x, -\tau) = 1$$

4. Time evolving and orthogonalizing $\phi_{2,3,4}(x_i, \tau_0)$ to $\phi_{2,3,4}(x_i, \tau_N)$

$$\phi_{2,3,4}(x_i, \tau_{k+1}) = \exp\left(-h\left[\frac{\rho_{k+1}+\rho_k}{2}\right]\Delta\tau\right)\phi_{2,3,4}(x_i, \tau_k)$$

$$\int dx \phi_{2,3,4}(x, \tau)\phi_1(x, -\tau) = 0$$

※Orthogonalize at each step

5. Normalizing $\phi_{2,3,4}(x_i, \tau_j)$
6. Calculating new density $\rho(x_i, \tau_j)$

$$\rho(x, \tau) = M\sum_{\alpha=1}^{4}\phi_\alpha(x, -\tau)\phi_\alpha(x, \tau)$$

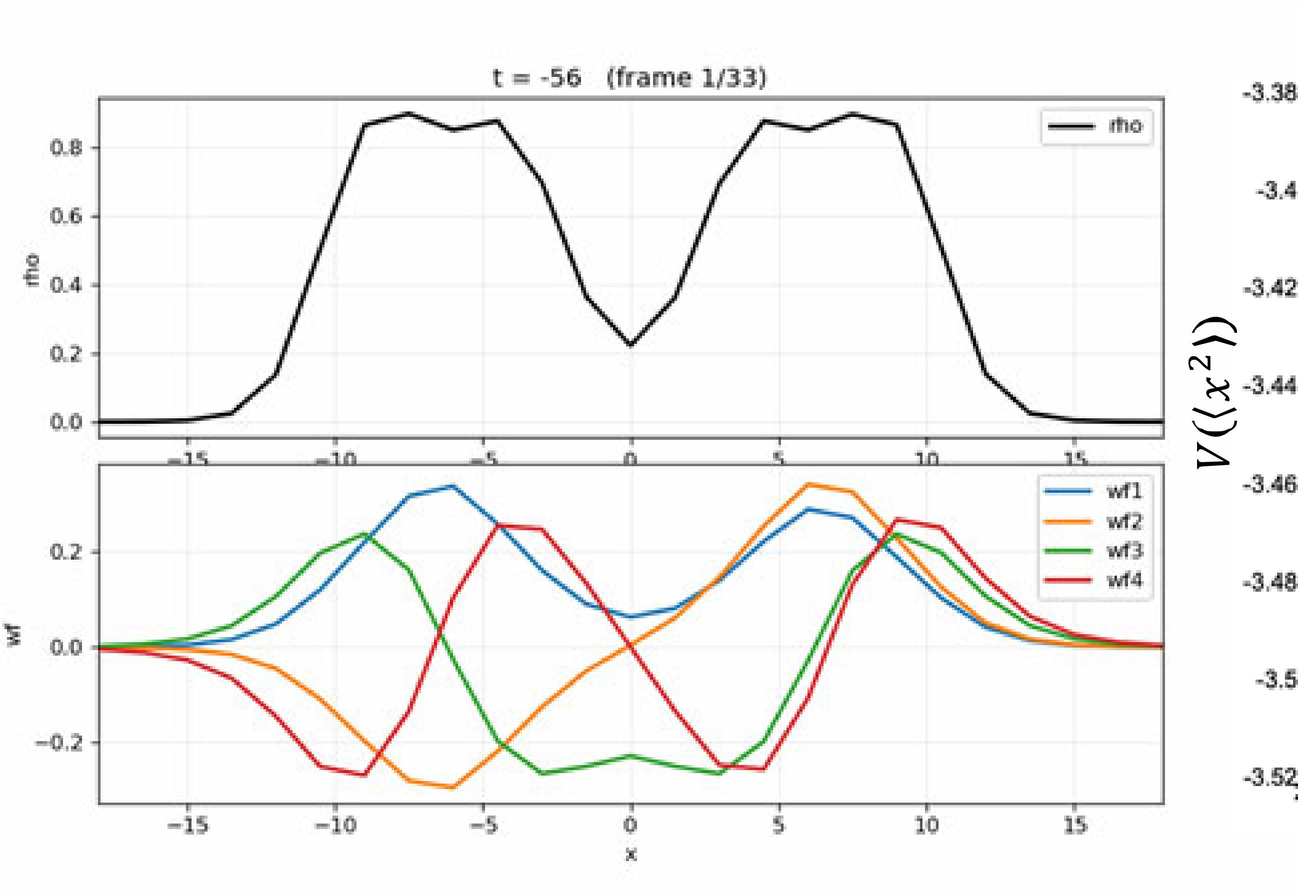
7. Return to step 2 until convergence

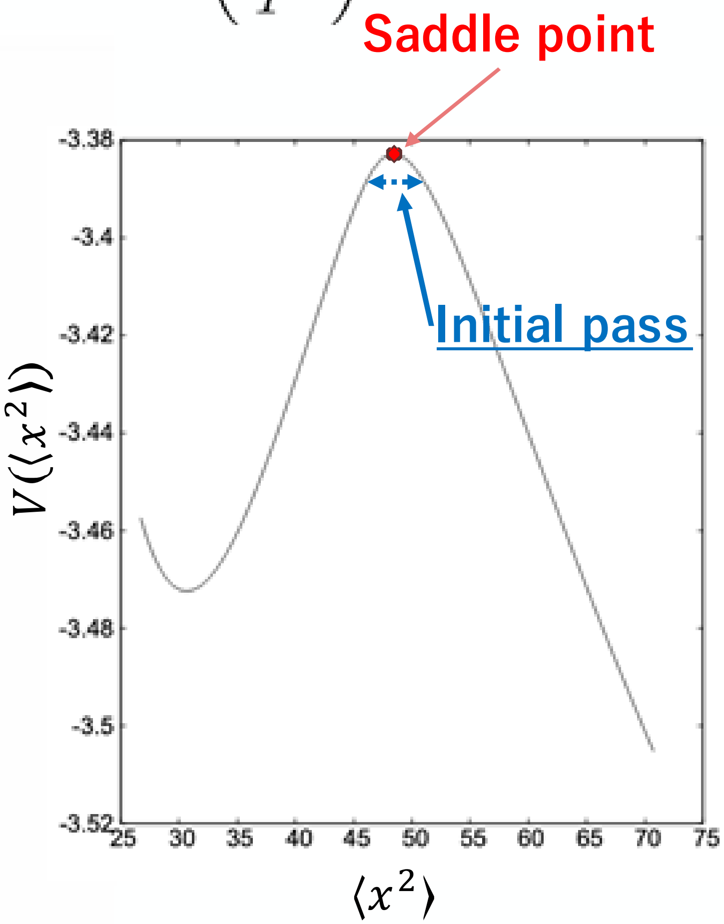
Initial pass

G.Puddu , J. W. Negele, Phys. Rev. C **35**, 1007

An initial pass is infinitesimal dilatation mode of saddle point density

$$\rho_{\rm ini}(x,\tau) = \rho_{\rm sad}(x) + \epsilon\left(\rho_{\rm sad}(x) + x\frac{\partial}{\partial x}\rho_{\rm sad}(x)\right)\cos\left(\frac{2\pi}{T}\tau\right)$$





Results

Results

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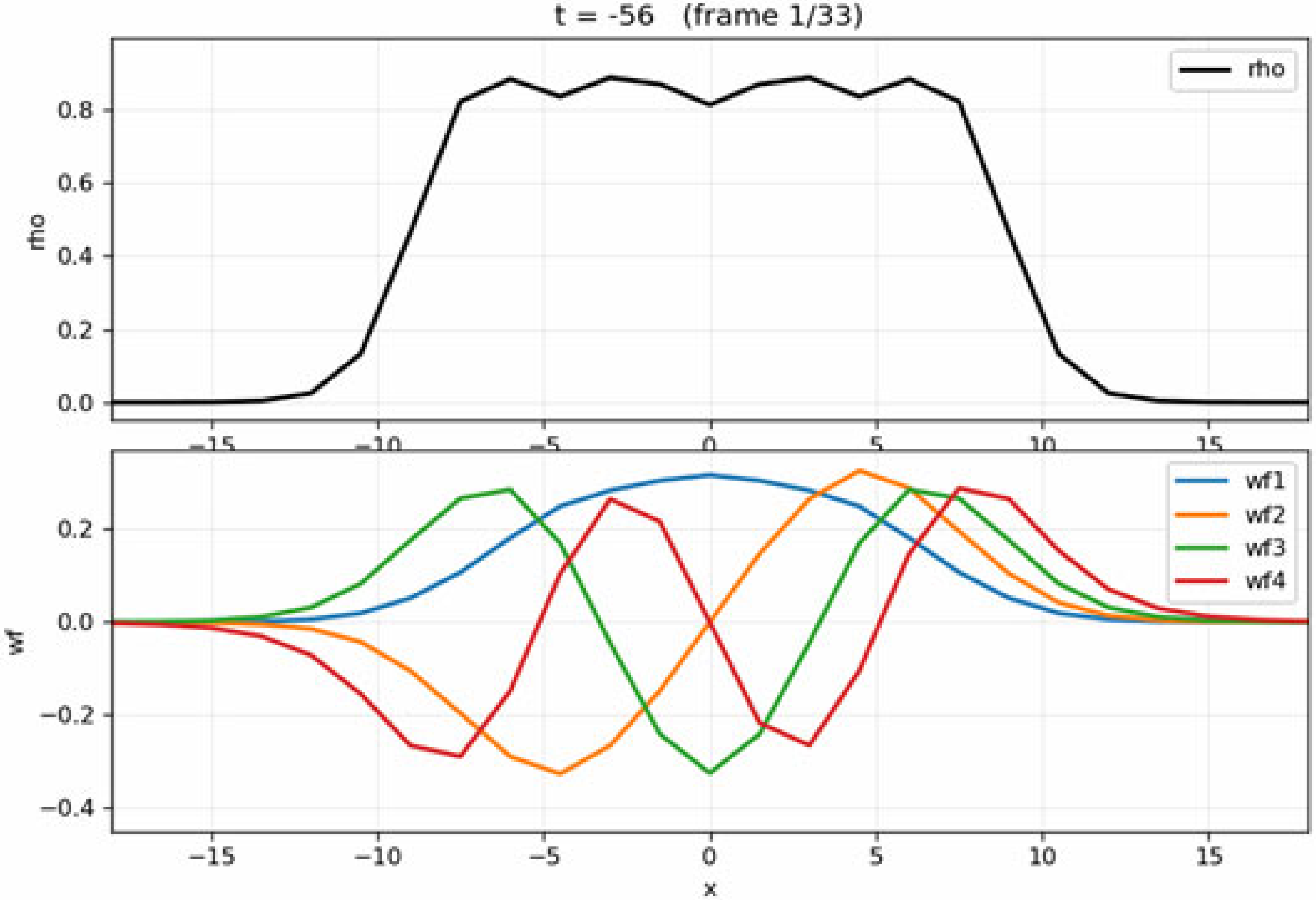
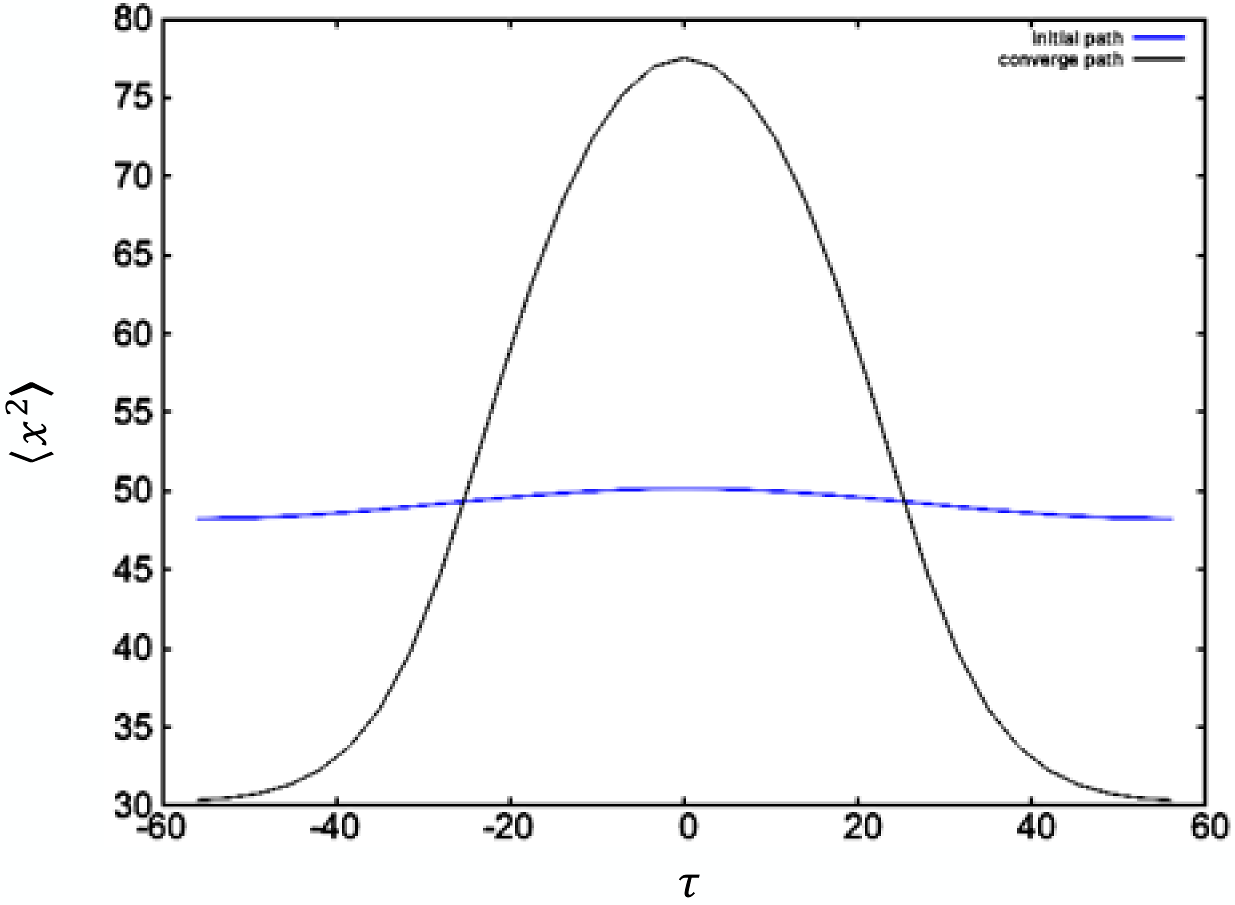
After ~1000 iterations, the density converged to the following:



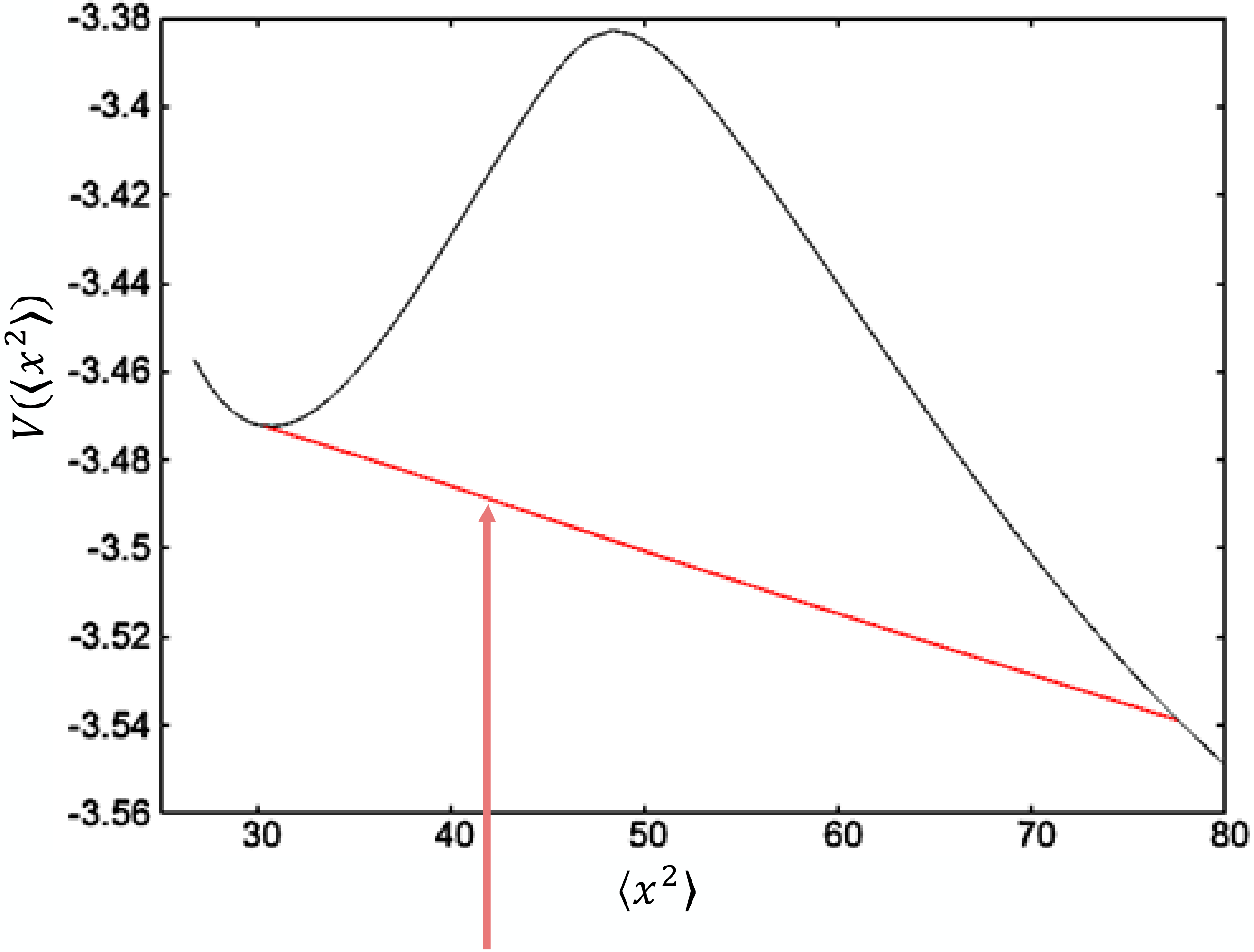
Discussion & Conclusion

Discussion① τ-$\langle x^2 \rangle$



The amplitude of $\langle x^2 \rangle$ is very small in initial path, but it **become large in converged path**.

Discussion② $\langle x^2 \rangle$-V



Energy is not conserved???

Conclusion & Future Work

Conclusion

- Our research purpose is to describe spontaneous fission from nucleon degrees of freedom.
- Time-Dependent Hartree-Fock (TDHF) method is known as a method for microscopically describing the dynamics of nuclei, however, since mean-field motion is classical, quantization of TDHF is necessary to describe spontaneous fission.
- We quantized TDHF by periodic Imaginary TDHF and calculated simple 1D systems.

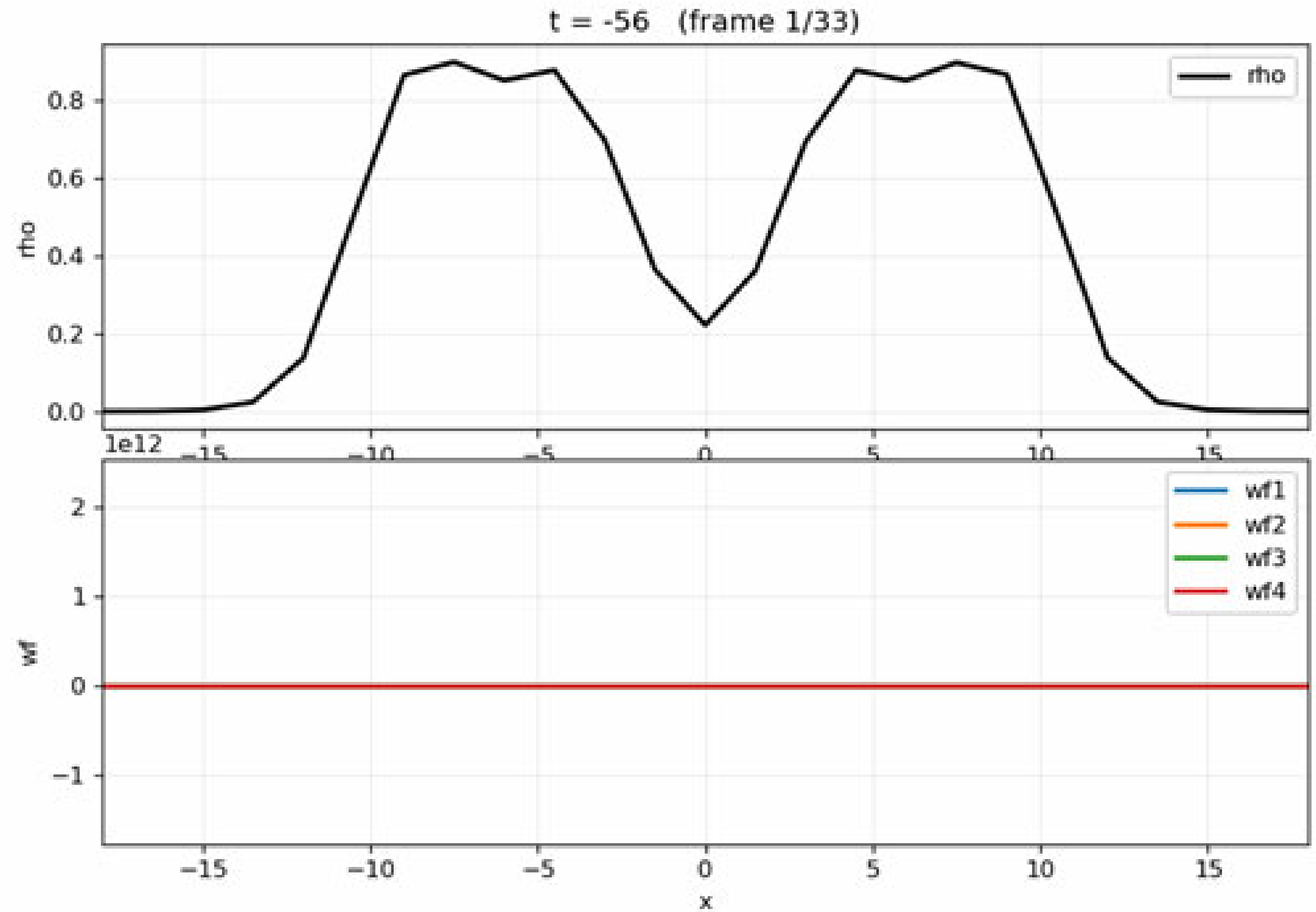
Future Work

- Investigate why energy is not conserved.
- Calculate a half-life in the 1D system.
- Investigate the region where convergence occurs by varying the parameters.

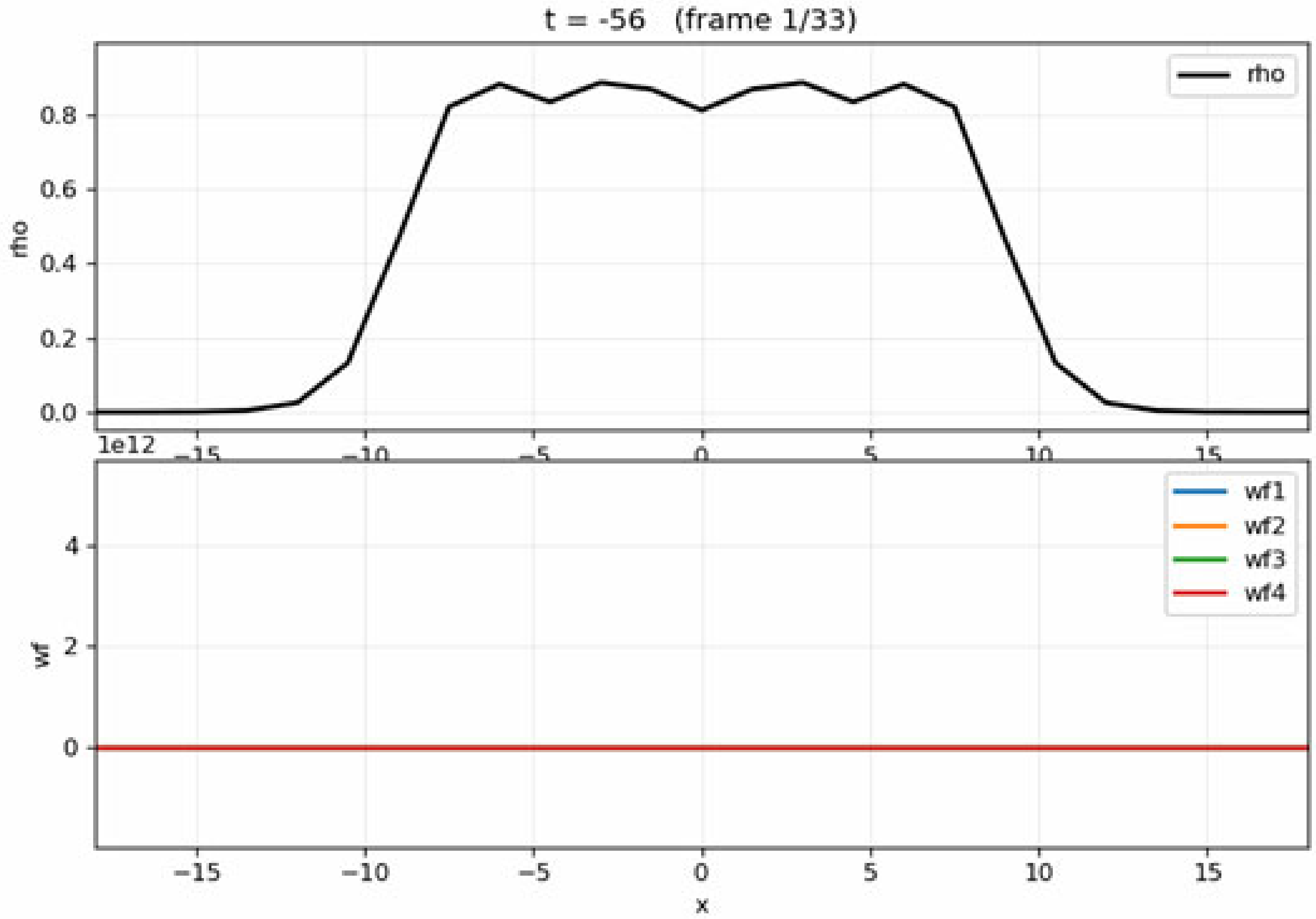
Future Future Work

- Develop 3D periodic imaginary TDHF using Skyrme interactions.

Back up



t = -56 (frame 1/33)
rho
0.8
0.6
0.4
0.2
0.0
1e12
2
1
0
−1
wf
wf1
wf2
wf3
wf4
−15
−10
−5
0
5
10
15
x



t = -56 (frame 1/33)
rho
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0.6
0.4
0.2
0.0
1e12
wf1
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wf3
wf4
wf
4
2
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−15
−10
−5
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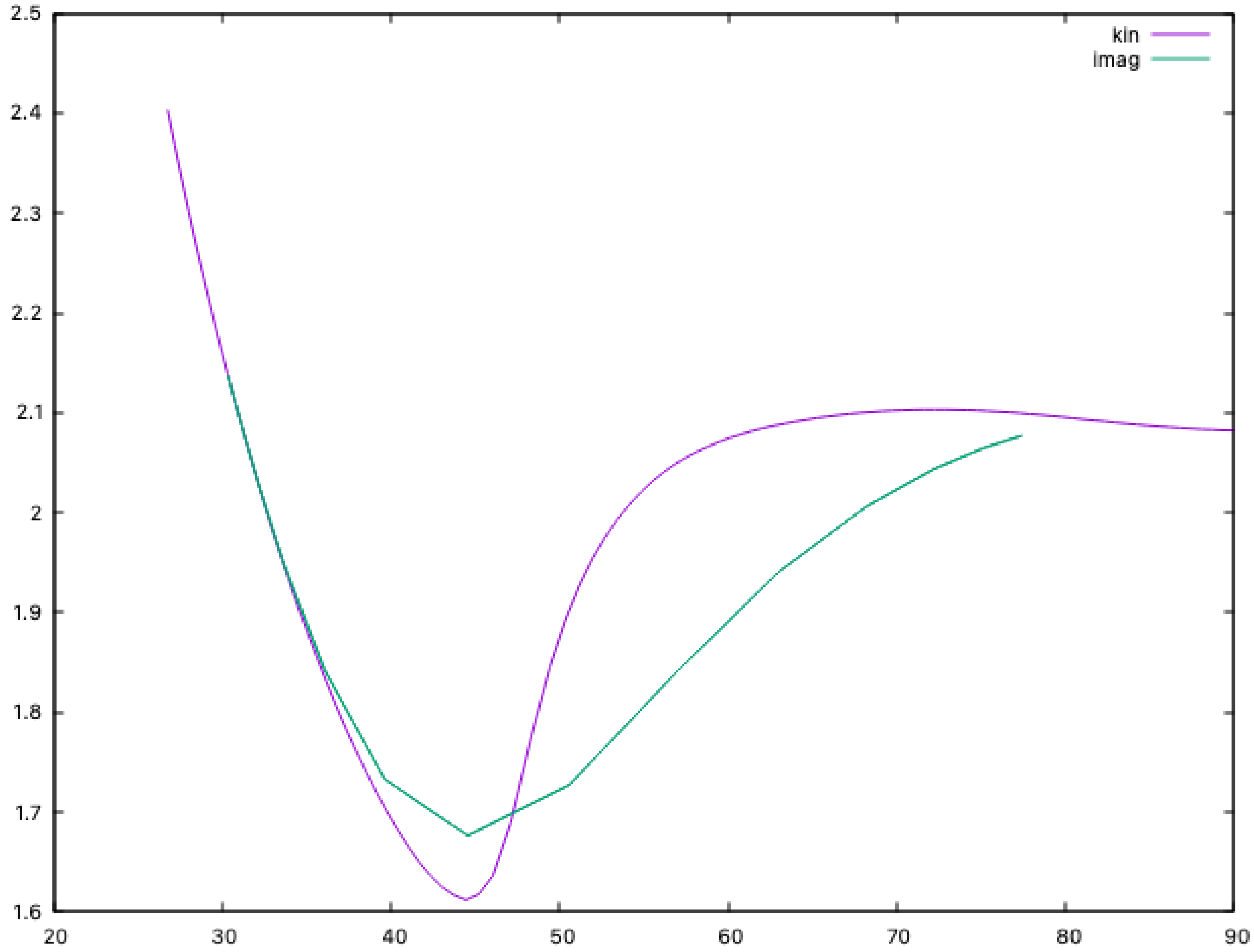
x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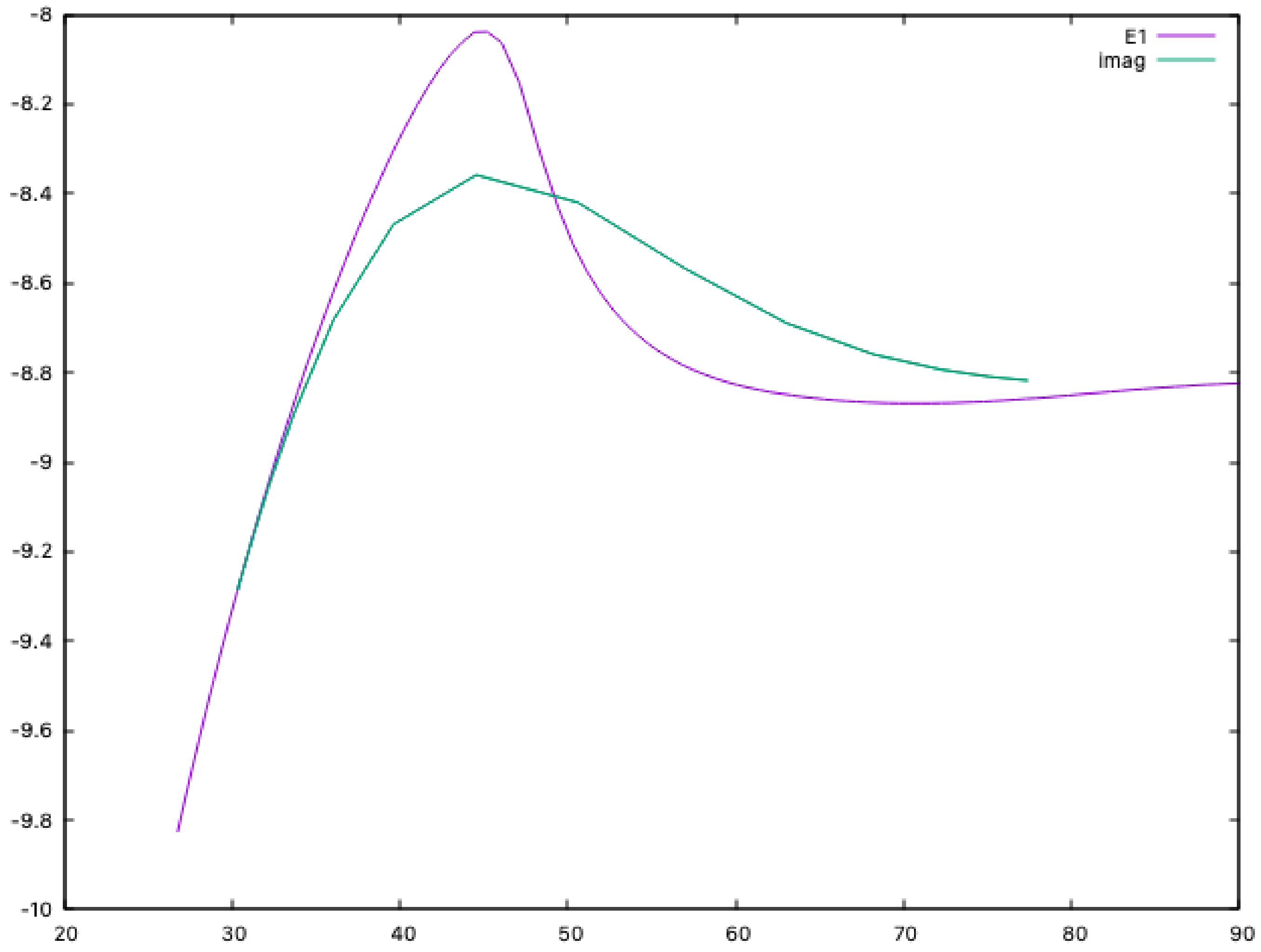

E1
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-8
-8.2
-8.4
-8.6
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-9.2
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-9.6
-9.8
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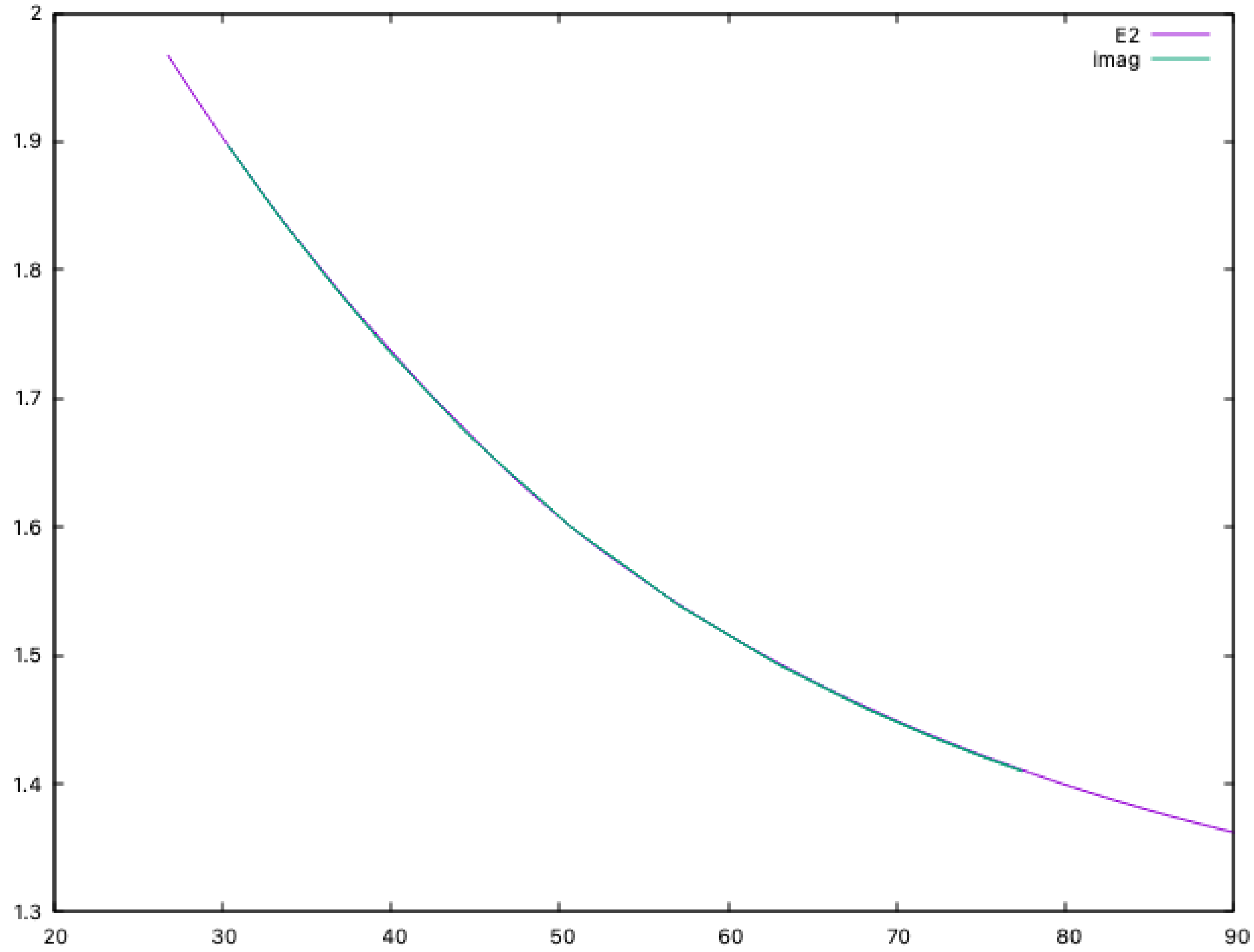
E2
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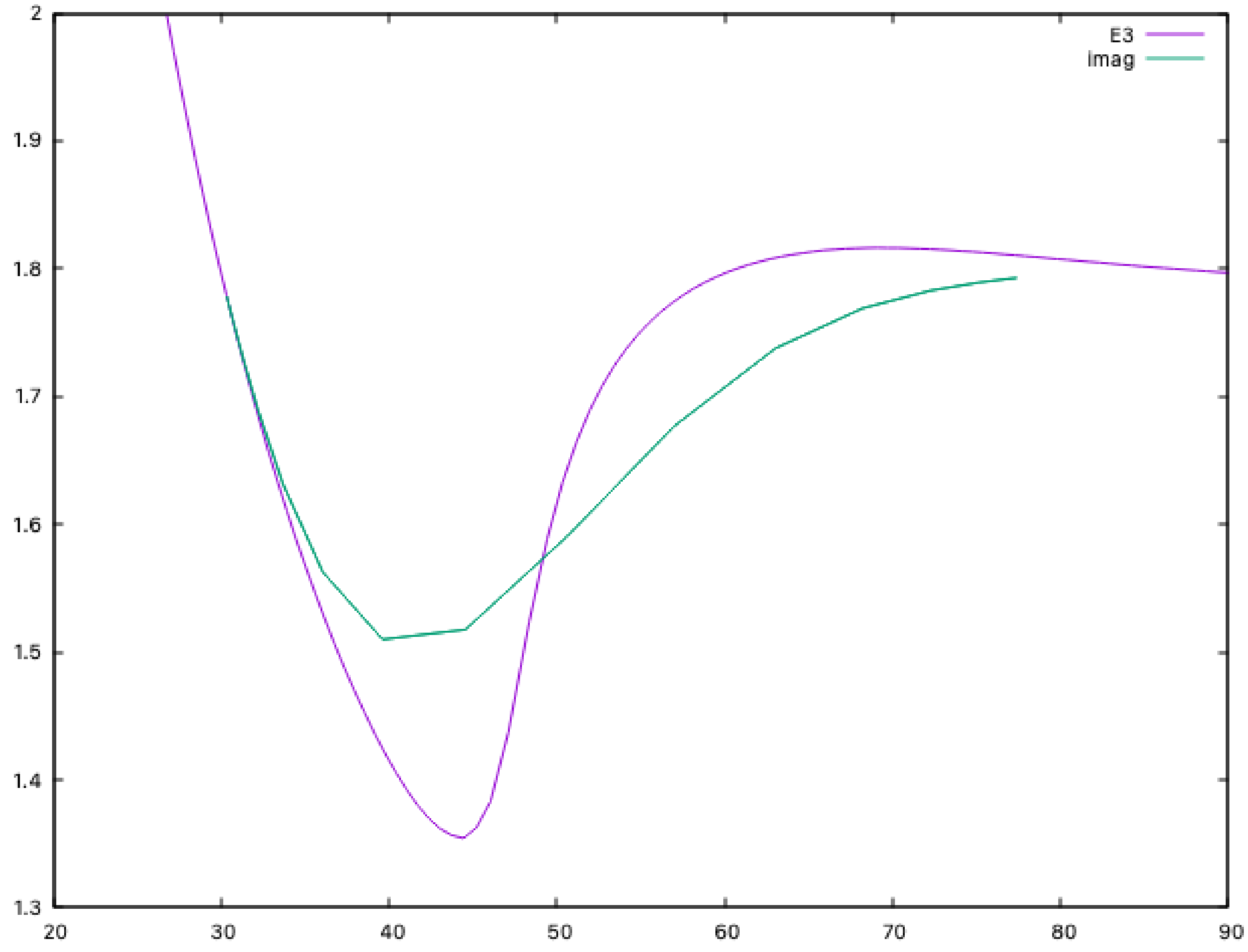
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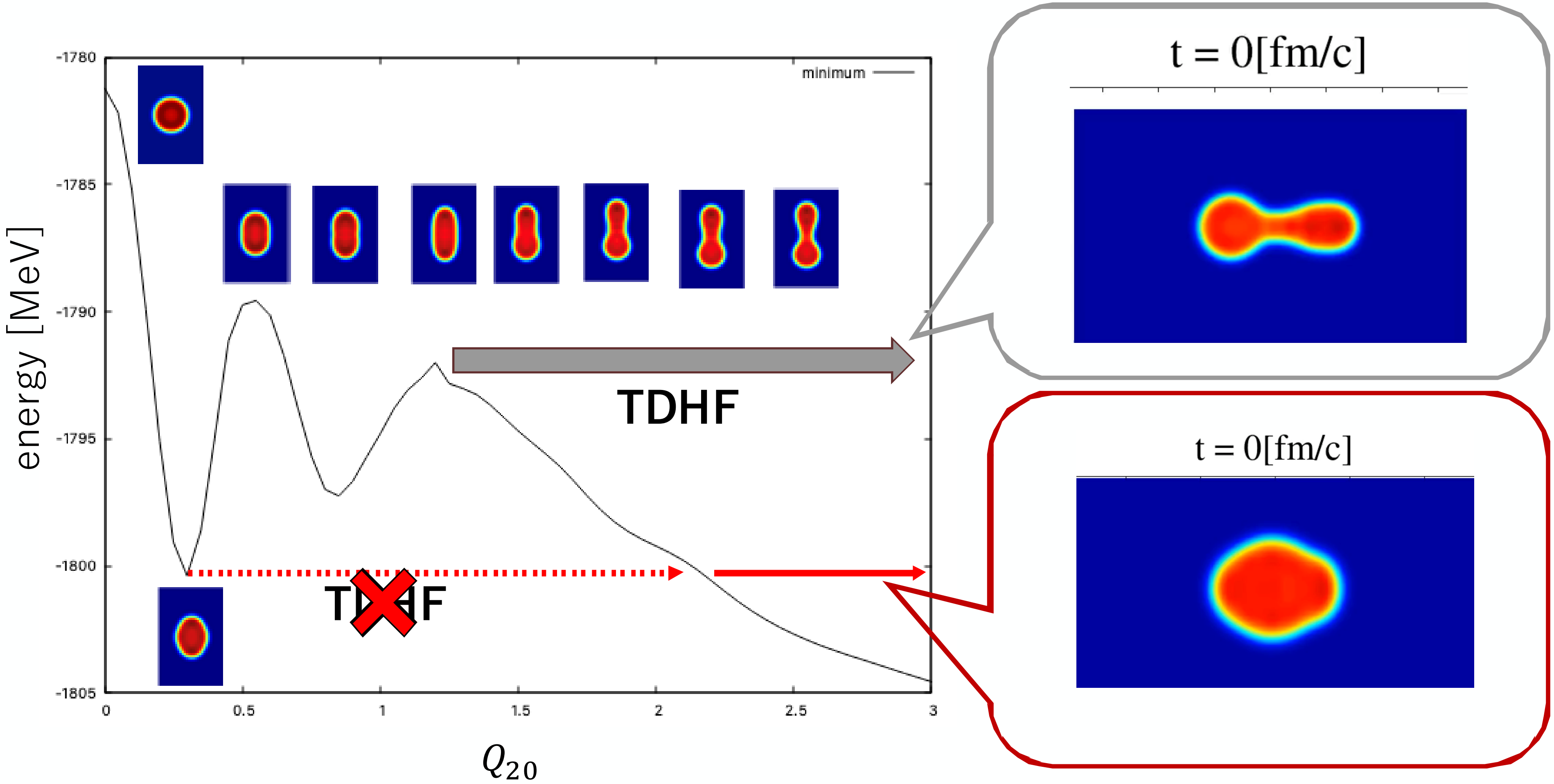
E3
imag

TDHF cannot describe SF

Time-Dependent Hartree-Fock (TDHF) can describe microscopically the dynamics of <u>a single particle</u>, but <u>a motion of mean field (collective motion) is classically</u>

Induced fission



We must <u>quantize TDHF</u> to describe SF.

Previous Studies for Quantized

Various studies have been conducted to describe spontaneous fission microscopically.

- HF+Cranking formula
- HF + local RPA
- ATDHF
- ASCC
- …

Introduction:
Spontaneous fission and TDHF

Our research purpose

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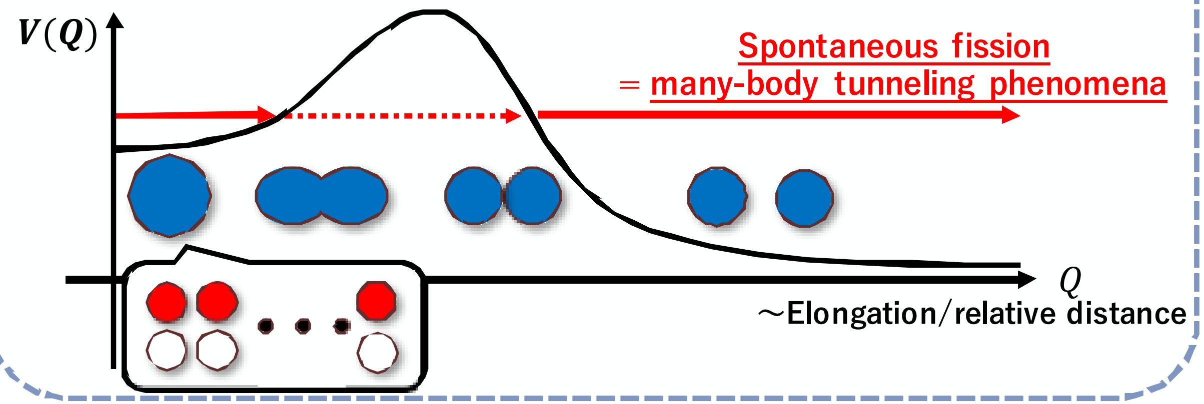
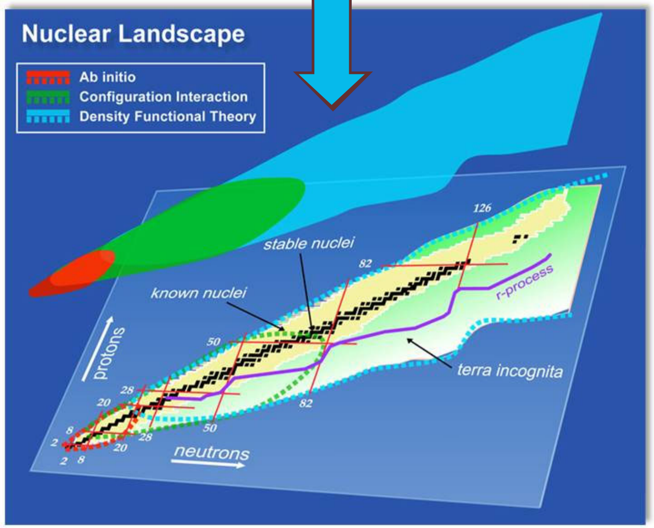
To describe spontaneous nuclear fission by a microscopic mean-field approach

Potential energy surface by LDM



Spontaneous fission (SF) occurs in heavy nuclei.

TDHF is possible to calculate heavy nuclei.



H Nam *et al* 2012 *J. Phys.: Conf. Ser.* **402** 012033

Time-Dependent Hartree-Fock (TDHF) theory

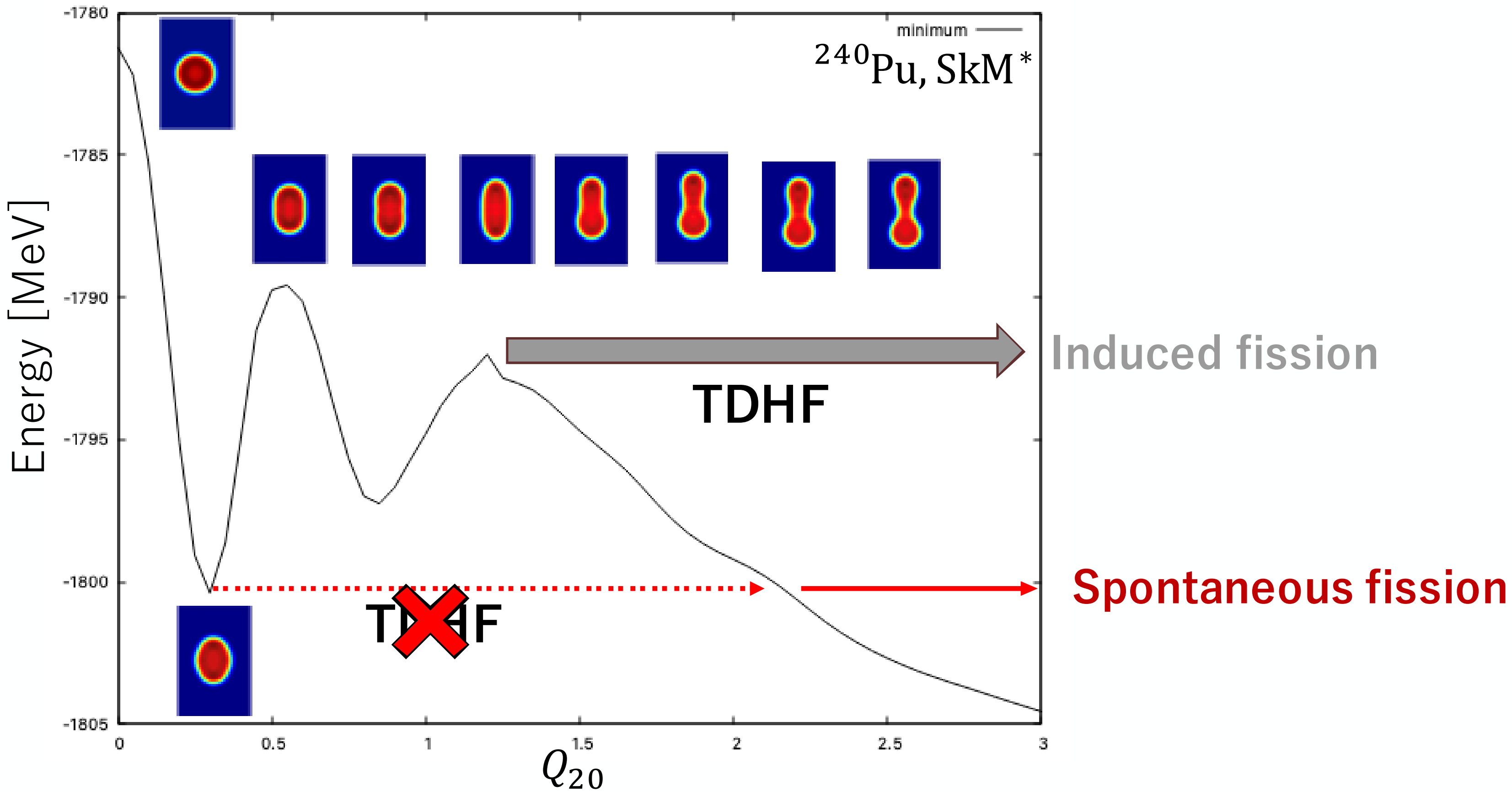
- Nucleons create a mean field
- Nucleons move independently in the mean field
- Self-consistent equation for single particle wave function

$$i\hbar\partial_t\psi_k(t) = -\frac{\hbar^2}{2m}\nabla^2\psi_k(t) + \frac{\delta\mathcal{V}}{\delta\psi_k^*(t)}$$

TDHF cannot describe SF

There are various theories that describe SF in microscopically, although almost all of them **cannot reproduce the experimental values.**

Potential energy surface by HF



TDHF can describe microscopically the dynamics of **a single particle**, but **a motion of mean field (collective motion) is classically.**

We must quantize TDHF to describe SF.

Quantization of TDHF

Why TDHF is classical theory?

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Conventionally, TDHF is derived using the variational principle, but now, we derive it using **a path integral formalism**.

S. Levit Phys. Rev. C **21**, 1594 (1980)
S. Levit, J. W. Negele, and Z. Paltiel, Phys. Rev. C **21**, 1603 (1980)

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$$U(t_f,t_i)=T\exp\left[-\frac{i}{2}\int_{t_i}^{t_f}dt\sum_{\alpha\beta\gamma\delta}\hat{\rho}_{\alpha\gamma}(t)V_{\alpha\beta\gamma\delta}\hat{\rho}_{\beta\delta}(t)\right]$$

⇩ **Hubbard-Stratonovich transformation**

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$$U(t_f,t_i)=\int\mathcal{D}[\sigma]\exp\left[\frac{i}{2}\int_{t_i}^{t_f}dt\{\sigma(t)v\sigma(t)\}U_I^\sigma(t_f,t_i)\right]$$

$$U_I^\sigma(t_f,t_i)\equiv T\exp\left[-i\int_{t_i}^{t_f}dt\{\sigma(t)v\hat{\rho}(t)\}\right]$$

⇩ **Stationary phase approximation ($\delta S=0$)**

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S. Levit, J. W. Negele, and Z. Paltiel, Phys. Rev. C **21**, 1603 (1980)

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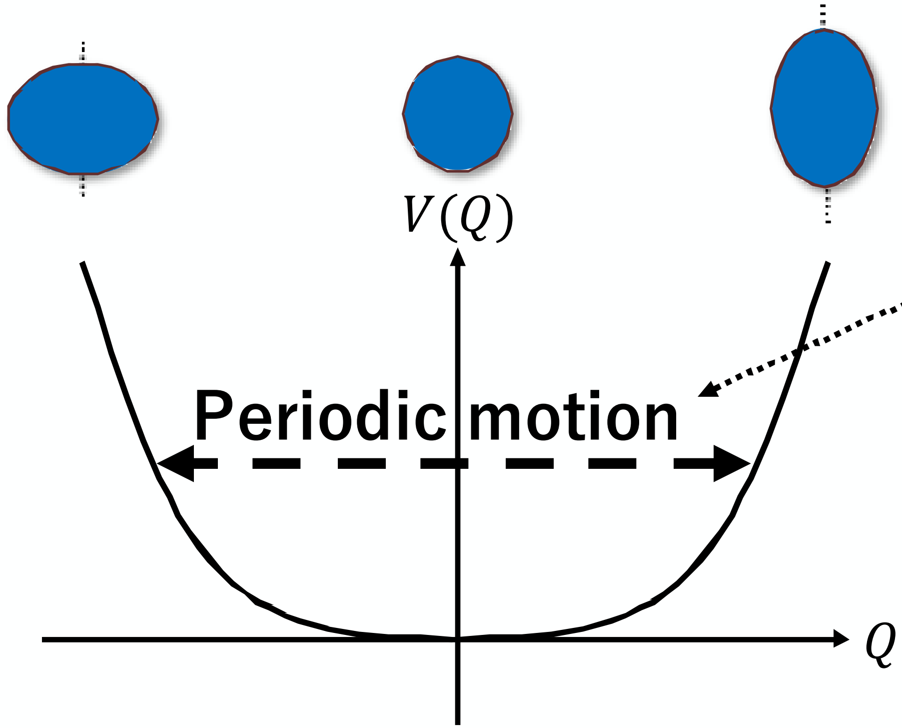
Periodic trajectory Propagator

Quantum theory's Energy



①Describing by periodic TDHF

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Imaginary time evolution

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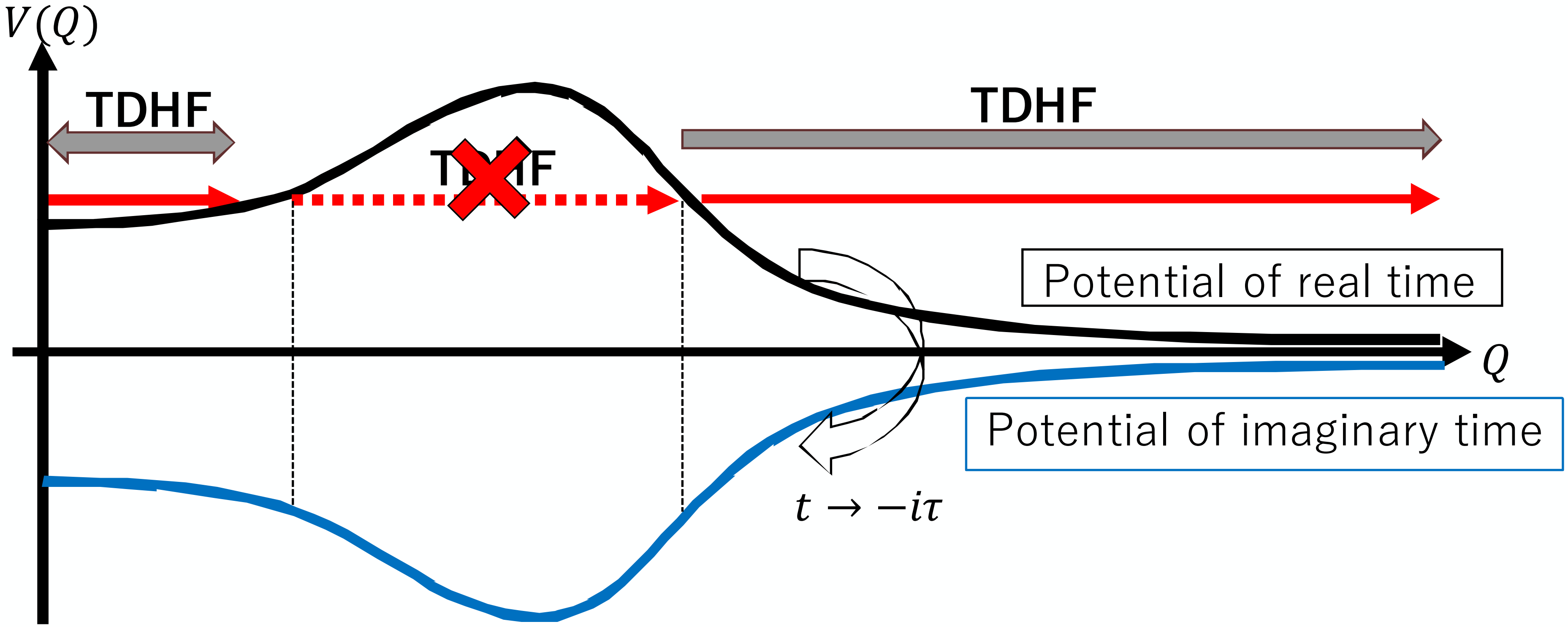
J. Skalski, Phys. Rev. C **77**, 064610 (2008)



Real time TDHF

$$i\hbar\partial_t\psi_k(t) = -\frac{\hbar^2}{2m}\nabla^2\psi_k(t) + \frac{\delta\mathcal{V}}{\delta\psi_k^*(t)}$$

$$\langle A(t)\rangle = \sum_k \int dx\,\psi_k^*(x,t)\hat{A}\psi_k(x,t)$$

$t \to -i\tau$

Imaginary time TDHF

$$-\hbar\partial_\tau\psi_k(\tau) = -\frac{\hbar^2}{2m}\nabla^2\psi_k(\tau) + \frac{\delta\mathcal{V}}{\delta\psi_k(-\tau)}$$

$$\langle A(\tau)\rangle = \sum_k \int dx\,\psi_k(x,-\tau)\hat{A}\psi_k(x,\tau)$$

Description of SF using ITDHF

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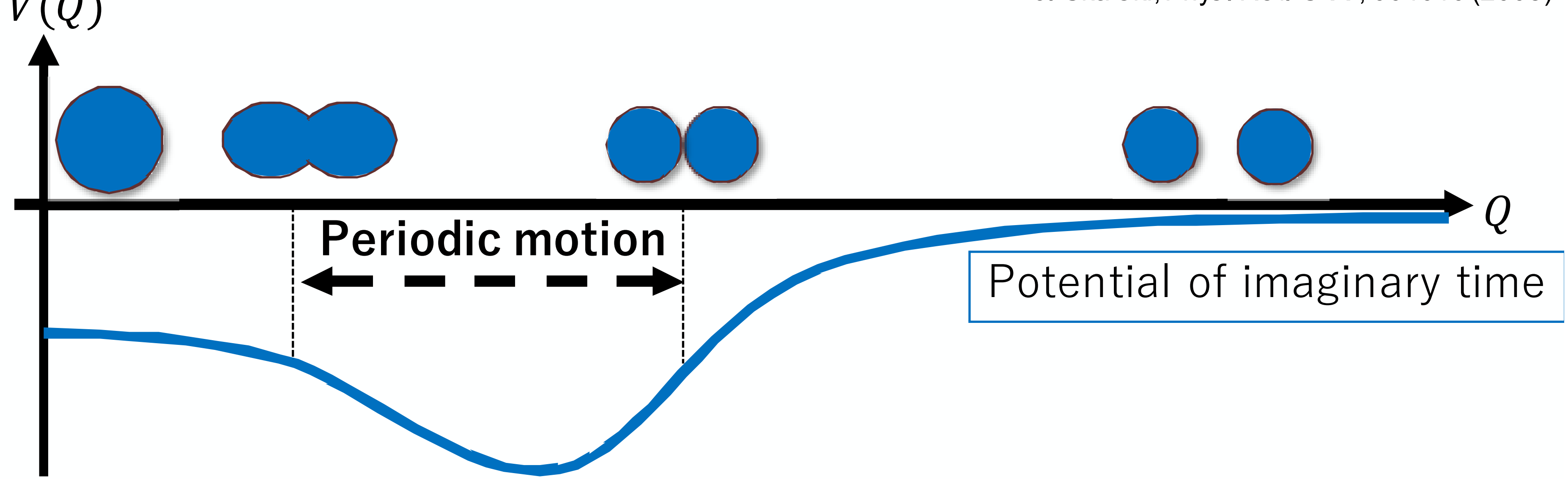
Periodic TDHF | **Imaginary time evolution** | **Gutzwiller formula**

$$-\hbar\partial_\tau \psi_k(\tau) = -\frac{\hbar^2}{2m}\nabla^2\psi_k(\tau) + \frac{\delta \mathcal{V}}{\delta\psi_k(-\tau)} \qquad \psi_k(T/2) = e^{-\alpha_k}\psi_k(-T/2)$$

$$\exp\left[-\frac{S}{\hbar}\right], \quad S = \hbar\int_{-T/2}^{T/2} d\tau \sum_k \left\langle \phi_k(-\tau) \middle| \frac{\partial\phi_k(\tau)}{\partial\tau} \right\rangle .$$

J. Skalski, Phys. Rev. C **77**, 064610 (2008)



ITDHF was proposed in the 1980s, but there has been little progress since then.

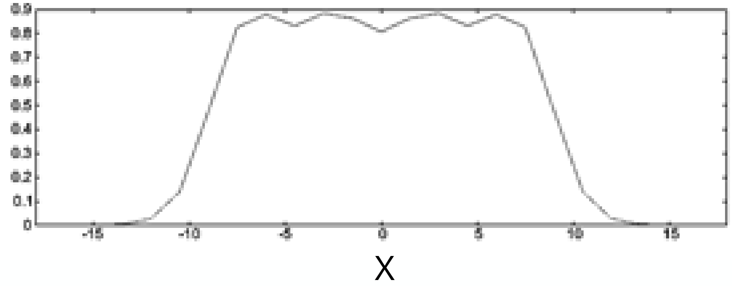
Setup for Numerical Calculations

System

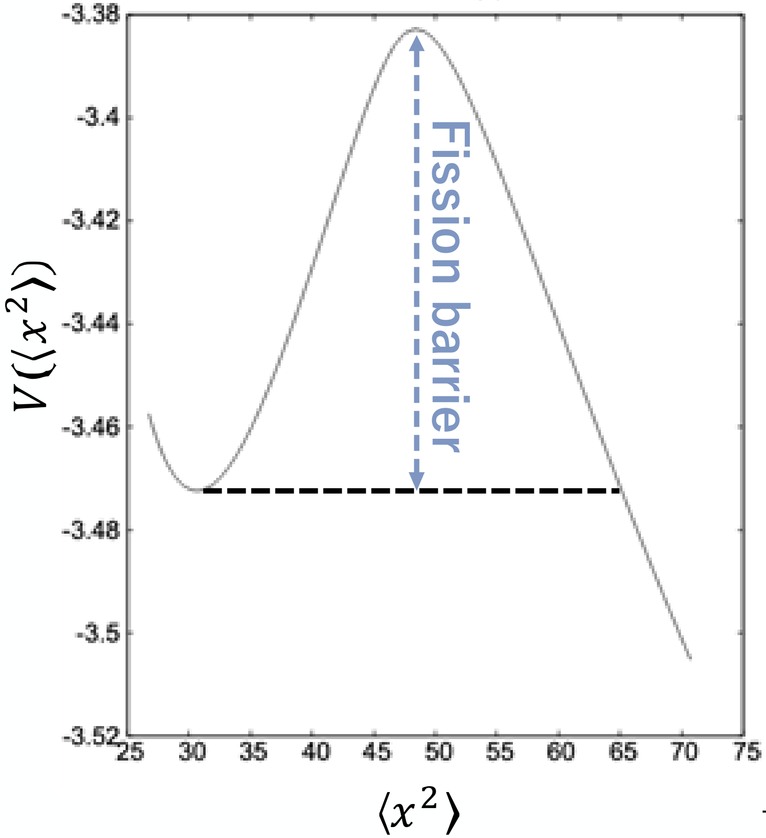
For simplicity, we assume that

- One-dimentional space
- 16 particle system
- Spin-Isospin degeneracy
- No Fock terms

Density



Potential energy surface



Hamiltonian density of our system

$$\mathcal{H}[\phi(x,-\tau),\phi(x,\tau)] = -M\sum_{\alpha}\phi_\alpha(x,-\tau)\left(\frac{\partial^2}{\partial x^2}\right)\phi_\alpha(x,\tau)$$

$$+\frac{1}{2}\int dx'\rho(x,\tau)V(x-x')\rho(x',\tau)$$

$$+\frac{1}{3}V_3\rho^3(x,\tau)$$

Three body force **repulsive**

$$V(x) = \frac{V_1}{\sqrt{\pi}\gamma_1}e^{-x^2/\gamma_1^2} + \frac{V_2}{\sqrt{\pi}\gamma_2}e^{-x^2/\gamma_2^2}$$

attractive **repulsive**

Two body force

$$\rho(x,\tau) = M\sum_{\alpha}\phi_\alpha(x,-\tau)\phi_\alpha(x,\tau)$$

S. Levit, J. W. Negele, and Z. Paltiel, Phys. Rev. C **22**, 1979

Discretization of Time

For numerical caluculation, the time variable is deiscritized.

G.Puddu , J. W. Negele, Phys. Rev. C **35**, 1007

$$-\frac{\partial \phi_\beta(x,\tau)}{\partial \tau} = \left(-\frac{\partial^2}{\partial x^2} + \int V(x-x')\rho(x',\tau)dx' + V_3\rho^2(x,\tau) + V_\lambda(x)\right)\phi_\beta(x,\tau)$$

$$= h[\rho]\phi_\beta(x,\tau)$$

Constraints to avoid uniform solutions

$$V_\lambda(x) = \lambda\left[\int_{-\frac{1}{2}}^{\frac{1}{2}} d\eta \int x'^2\rho(x',\eta)dx' - x_0^2\right]x^2$$

B.C. $\phi_\beta(x,\frac{T}{2}) = e^{-\lambda_\beta}\phi_\beta(x,-\frac{T}{2})$

Discretization of time $\tau_0, \tau_1, \ldots \tau_{N_\tau}$

Number of meshes: 32, Mesh width: 3.5

$$\phi_\beta(x_i,\tau_{k+1}) = \exp\left(-h\left[\frac{\rho_{k+1}+\rho_k}{2}\right]\Delta\tau\right)\phi_\beta(x_i,\tau_k) \equiv U(\tau_{k+1},\tau_k)\phi_\beta(x_i,\tau_k)$$

Time evolution operator

Eigenvalue problem $U(\tau_N,\tau_1)\phi_\beta(x_i,\tau_1) = e^{-\lambda_\beta}\phi_\beta(x_i,\tau_1)$

Computational Flow

G.Puddu , J. W. Negele, Phys. Rev. C **35**, 1007

1. As an initial path, preparing $\rho(x_i, \tau_j)$ and $\phi_k(x_i, \tau_0)$
2. Time evolving $\phi_1(x_i, \tau_0)$ to $\phi_1(x_i, \tau_N)$

$$\phi_1(x_i, \tau_0) \rightarrow \phi_1(x_i, \tau_1) \rightarrow \phi_1(x_i, \tau_2) \rightarrow \cdots \rightarrow \phi_1(x_i, \tau_N)$$

$$\phi_1(x_i, \tau_{k+1}) = \exp\left(-h\left[\frac{\rho_{k+1} + \rho_k}{2}\right]\Delta\tau\right)\phi_1(x_i, \tau_k)$$

3. Normalizing $\phi_1(x_i, \tau_j)$

$$\int dx \phi_1(x, \tau)\phi_1(x, -\tau) = 1$$

4. Time evolving and orthogonalizing $\phi_{2,3,4}(x_i, \tau_0)$ to $\phi_{2,3,4}(x_i, \tau_N)$

$$\phi_{2,3,4}(x_i, \tau_{k+1}) = \exp\left(-h\left[\frac{\rho_{k+1} + \rho_k}{2}\right]\Delta\tau\right)\phi_{2,3,4}(x_i, \tau_k)$$

$$\int dx \phi_{2,3,4}(x, \tau)\phi_1(x, -\tau) = 0$$

※Orthogonalize at each step

5. Normalizing $\phi_{2,3,4}(x_i, \tau_j)$
6. Calculating new density $\rho(x_i, \tau_j)$

$$\rho(x, \tau) = M\sum_{\alpha=1}^{4}\phi_\alpha(x, -\tau)\phi_\alpha(x, \tau)$$

7. Return to step 2 until convergence

Initial pass

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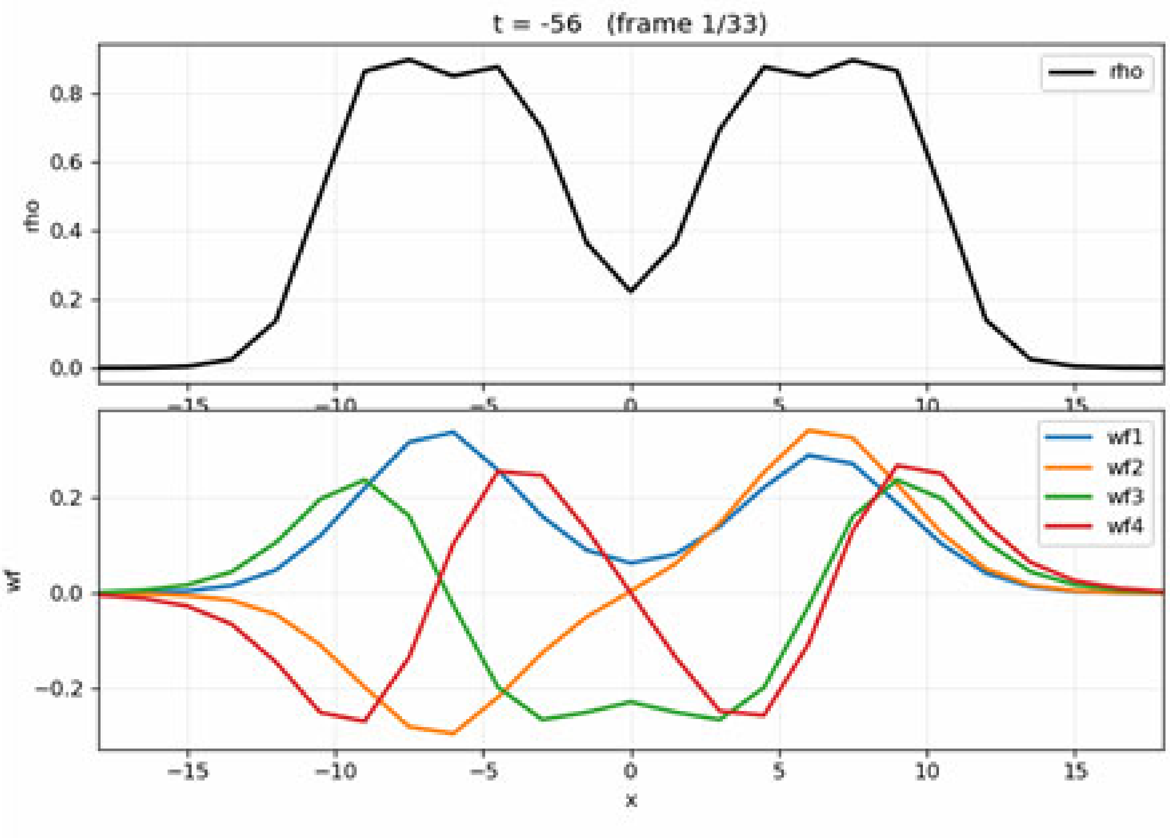
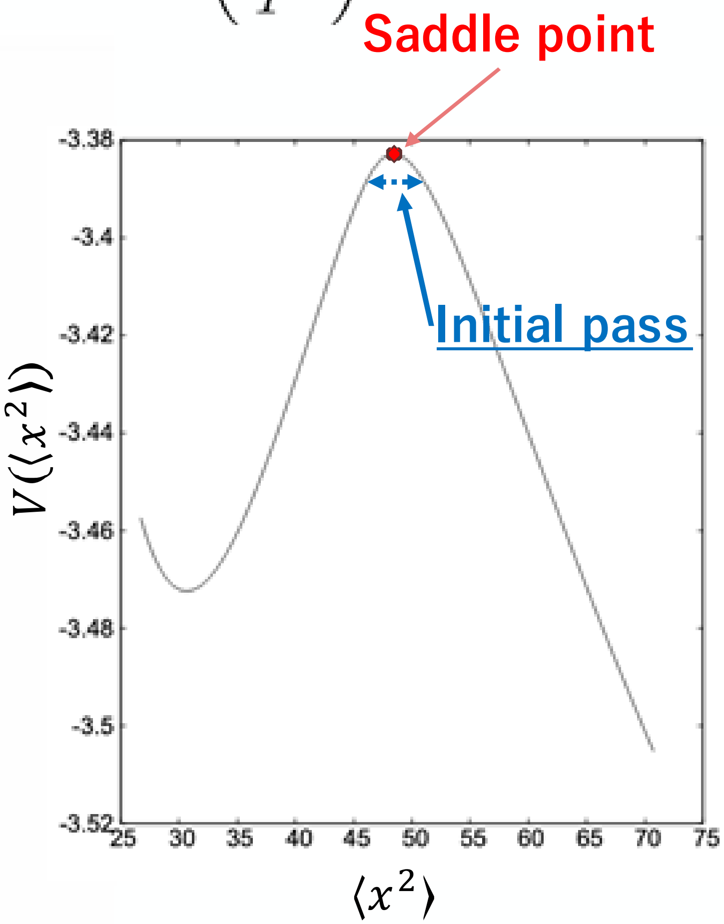
An initial pass is infinitesimal dilatation mode of saddle point density

$$\rho_{\mathrm{ini}}(x,\tau) = \rho_{\mathrm{sad}}(x) + \epsilon\left(\rho_{\mathrm{sad}}(x) + x\frac{\partial}{\partial x}\rho_{\mathrm{sad}}(x)\right)\cos\left(\frac{2\pi}{T}\tau\right)$$

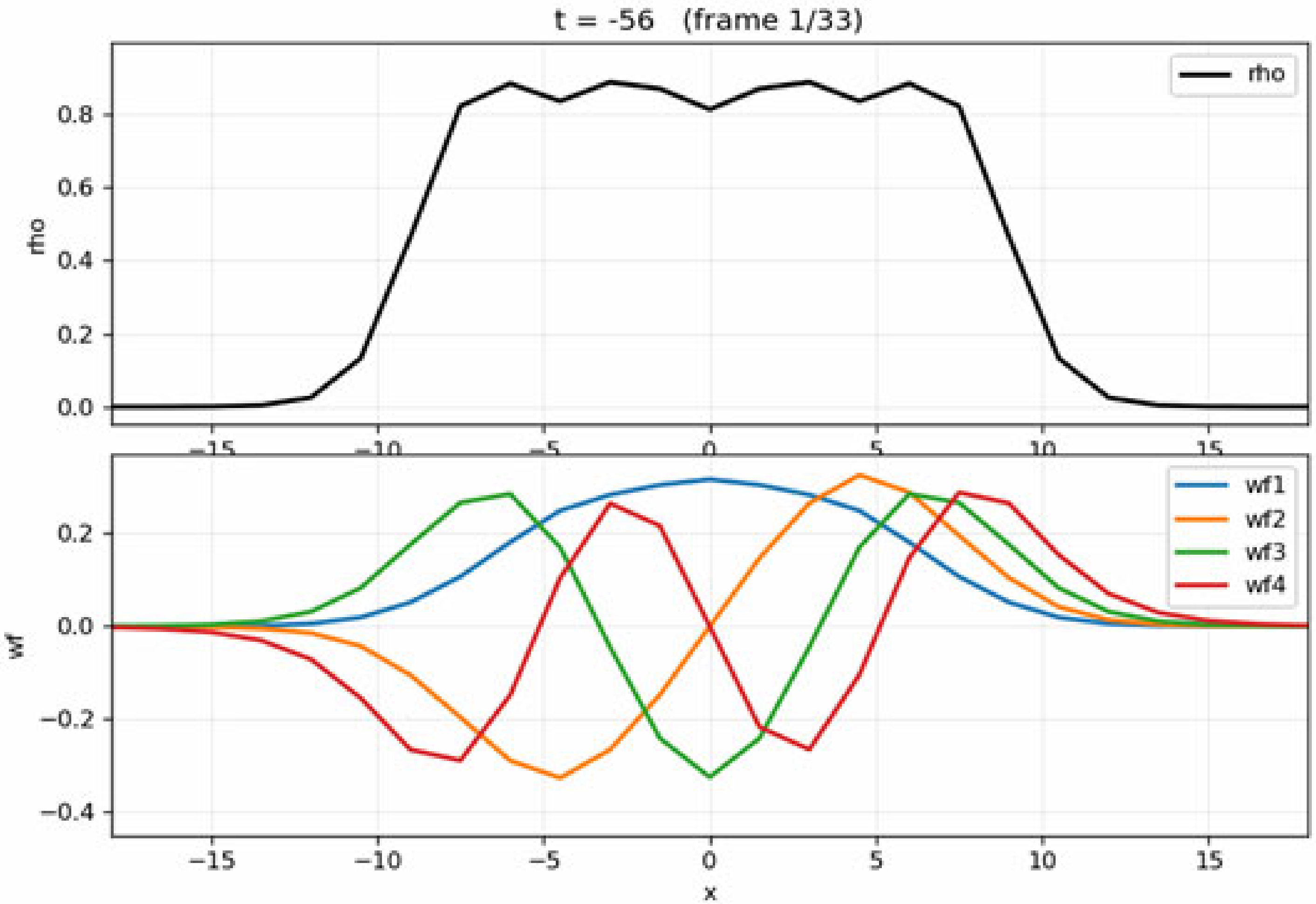




Results

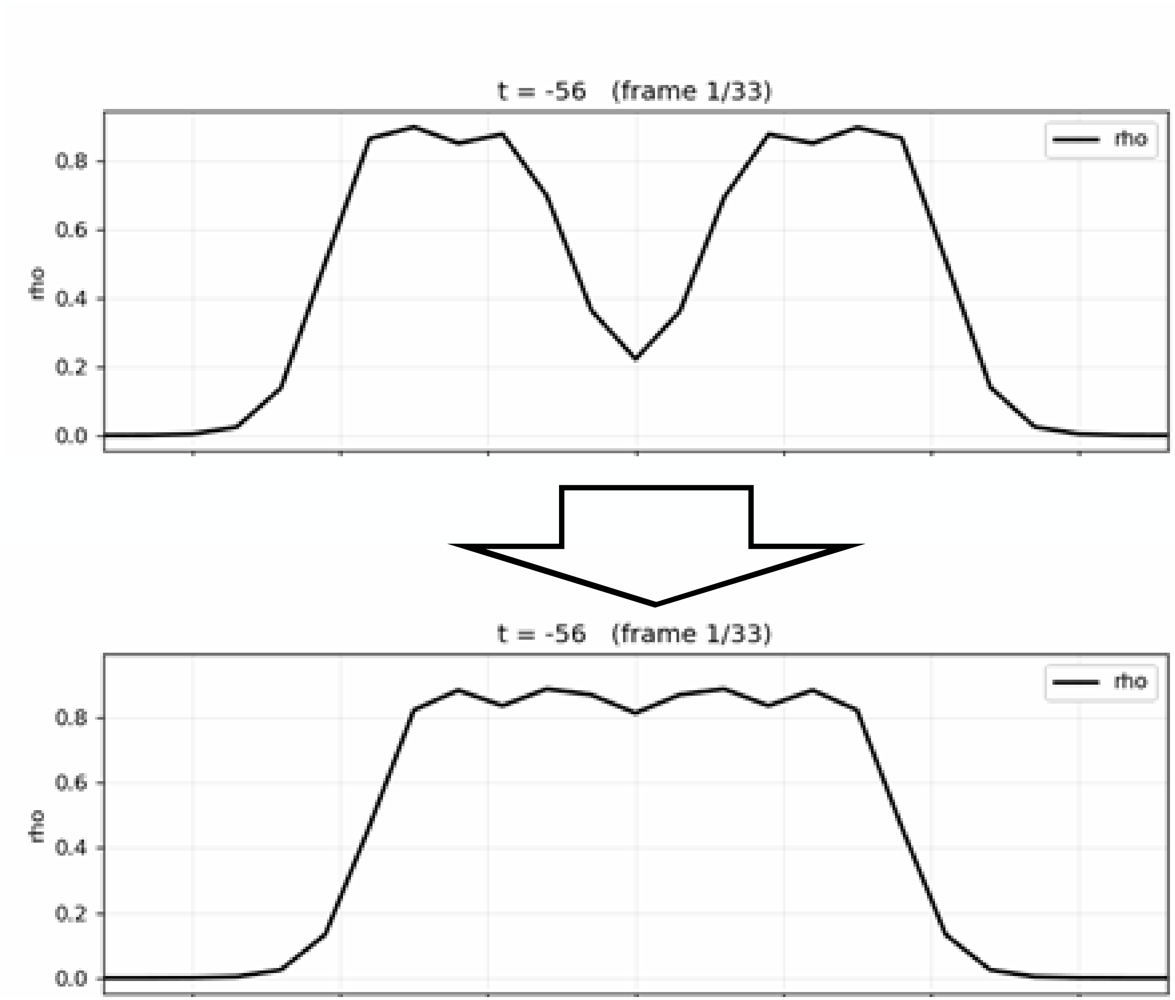
Results

After ~1000 iterations, the density converged to the following:



Discussion & Conclusion

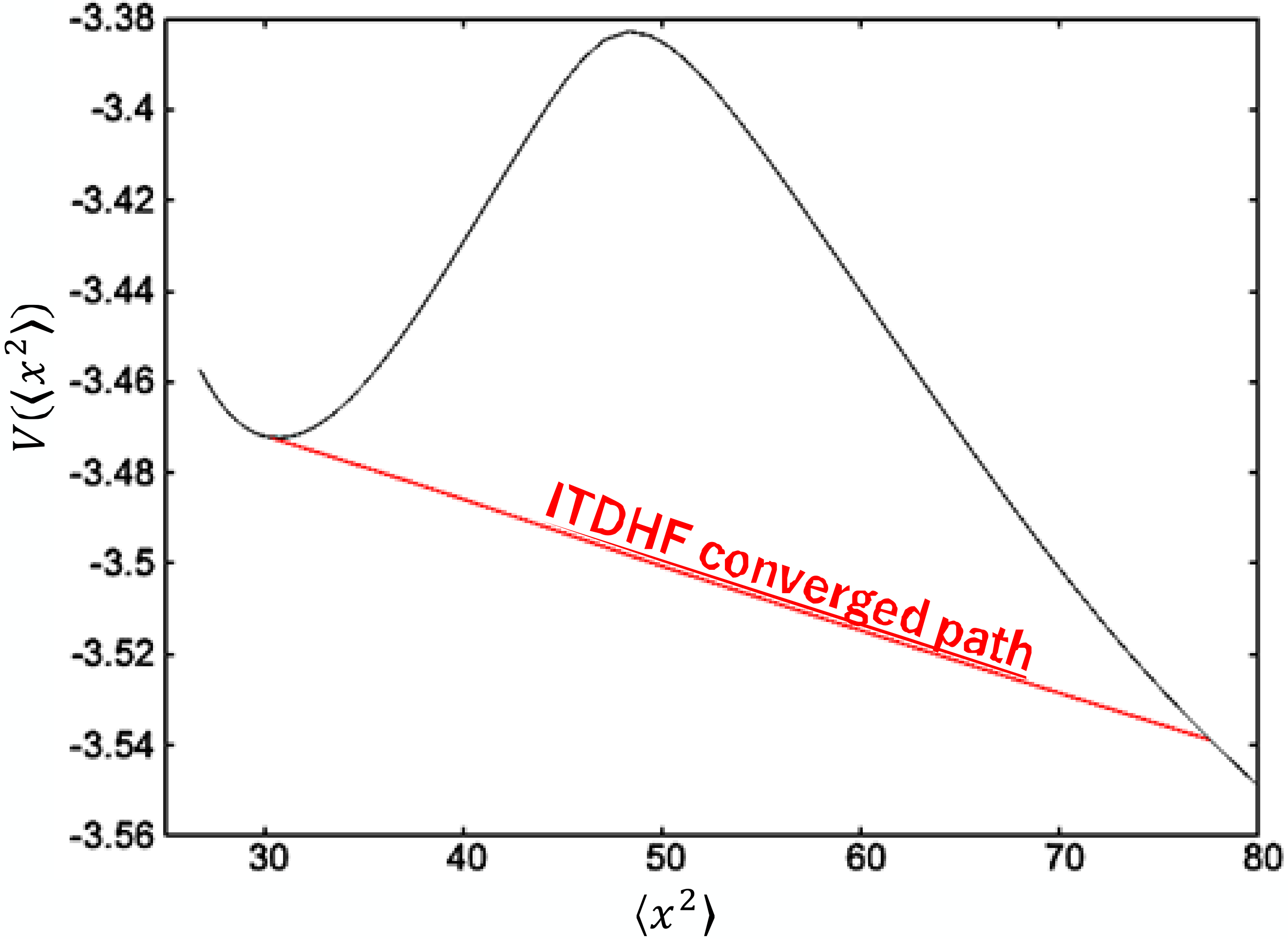
Discussion①



The amplitude of $\langle x^2 \rangle$ is very small in initial path,
but it **become large in converged path**.

Discussion②

Potential energy surface



In a converged path, it appears that energy is not conserved.

Conclusion & Future Work

Conclusion

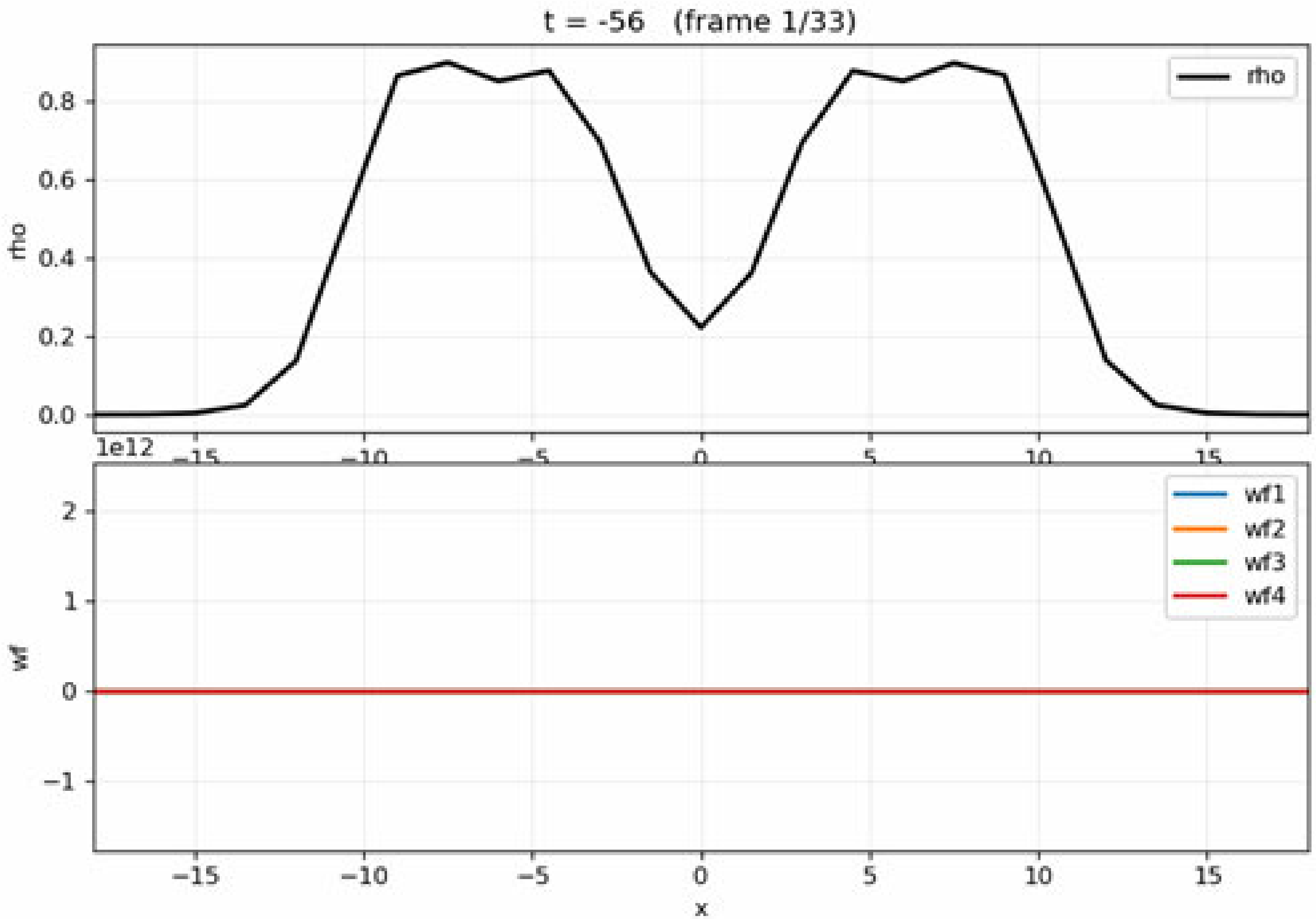
- Our research purpose is to describe spontaneous fission from nucleon degrees of freedom.
- Time-Dependent Hartree-Fock（TDHF) method is known as a method for microscopically describing the dynamics of nuclei, however, since mean-field motion is classical, quantization of TDHF is necessary to describe spontaneous fission.
- We quantized TDHF by periodic Imaginary TDHF and calculated simple 1D systems.

Future Work

- Investigate why energy is not conserved.
- Calculate the half-life in the 1D system and compare it with the half-lives calculated using other theories.

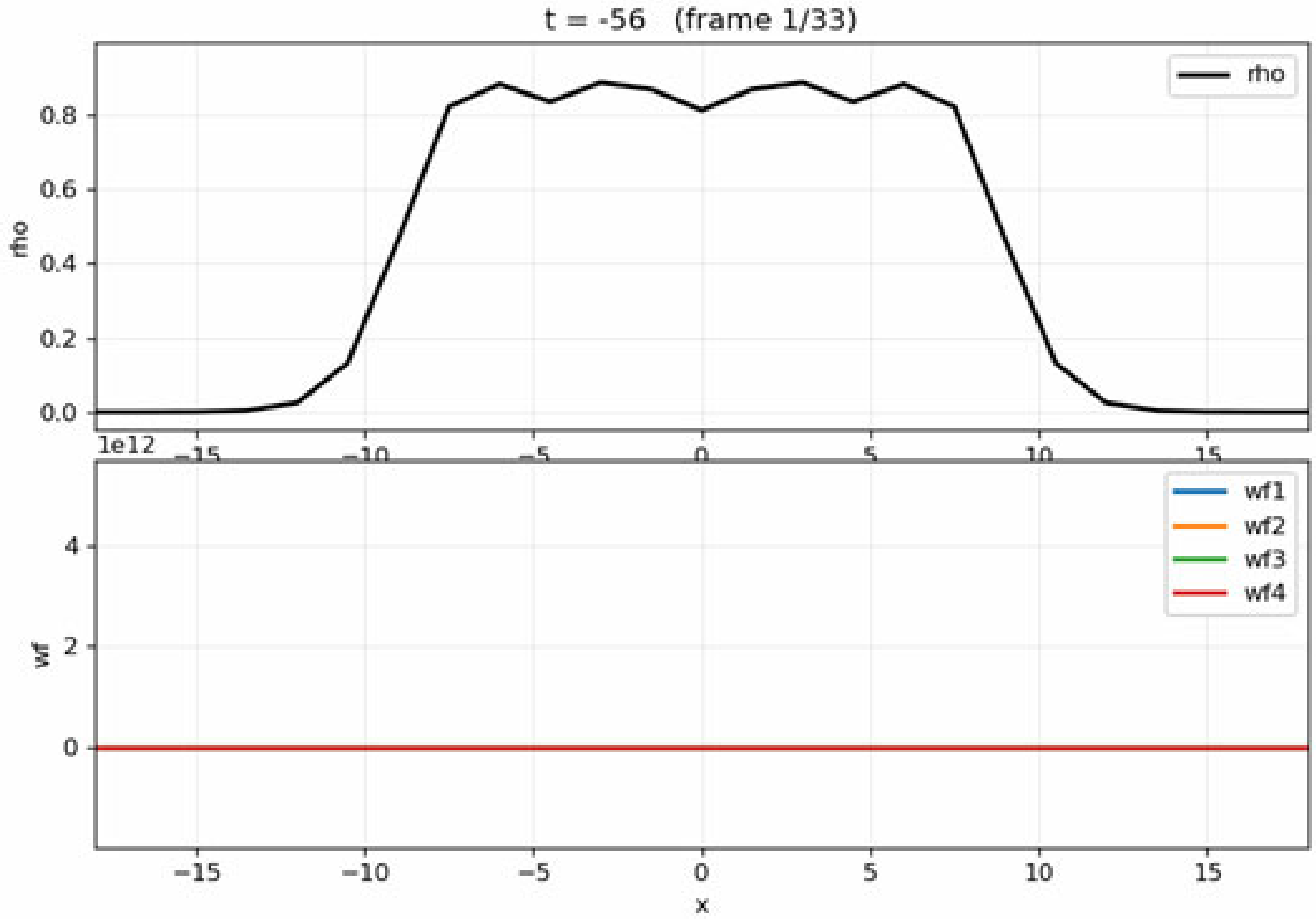
Back up

Back up



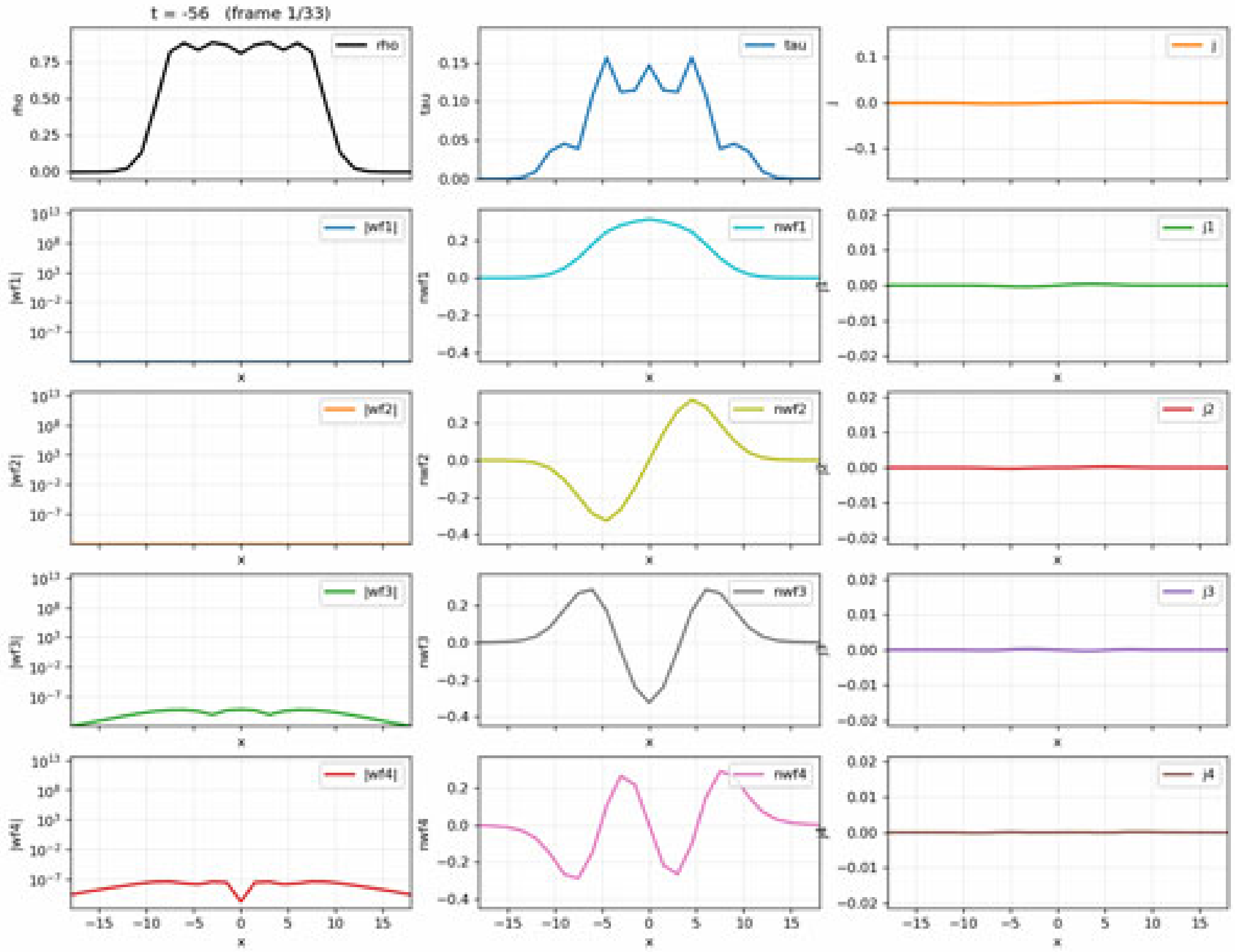
t = -56 (frame 1/33)
rho
0.8
0.6
0.4
0.2
0.0
1e12
2
1
0
−1
wf
wf1
wf2
wf3
wf4
−15
−10
−5
0
5
10
15
x

Back up

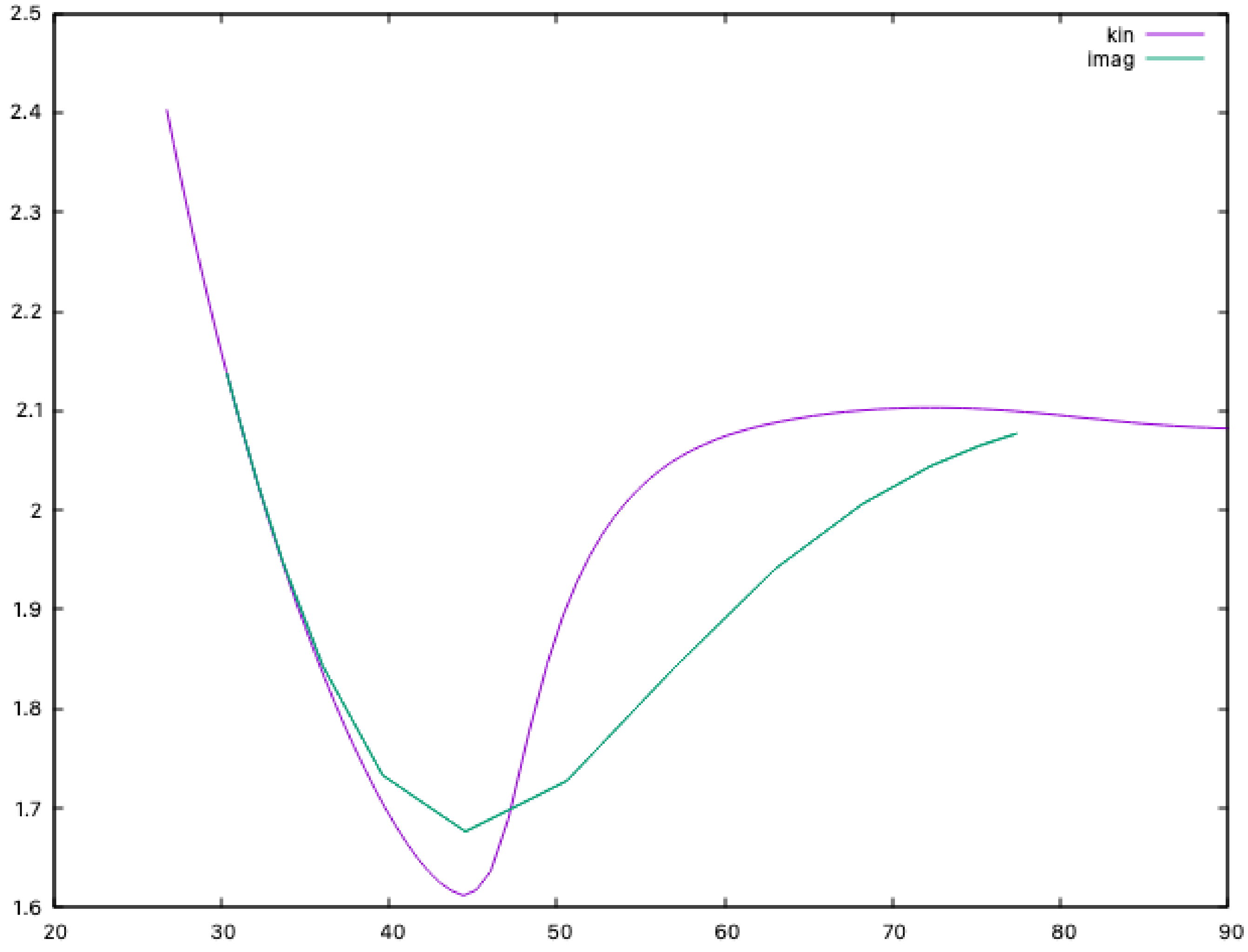


t = -56 (frame 1/33)
rho
0.8
0.6
0.4
0.2
0.0
1e12
wf1
wf2
wf3
wf4
wf
4
2
0
−15
−10
−5
0
5
10
15
x

Back up


t = -56 (frame 1/33)
rho
tau
j
|wf1|
nwf1
j1
|wf2|
nwf2
j2
|wf3|
nwf3
j3
|wf4|
nwf4
j4
x

Back up



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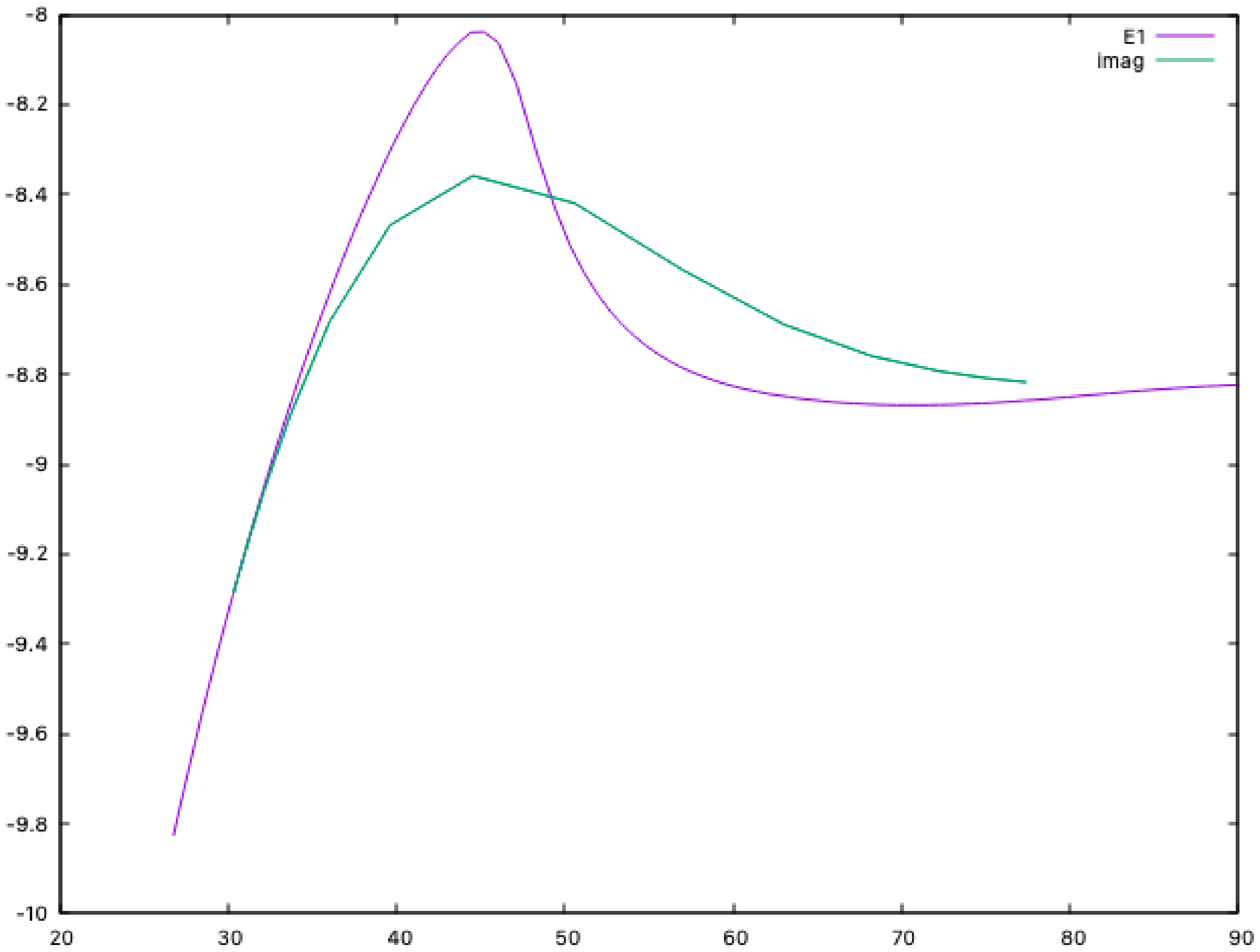

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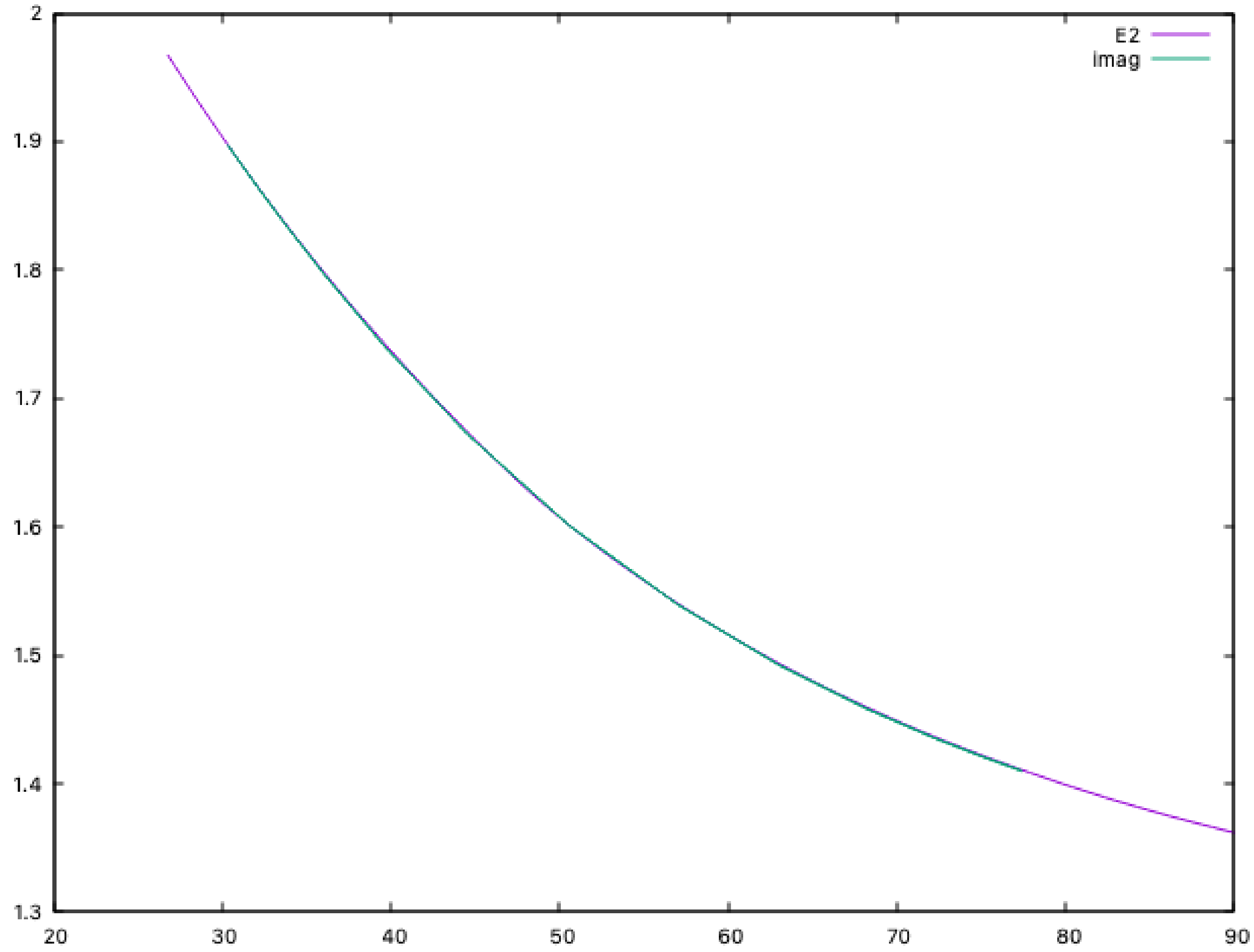

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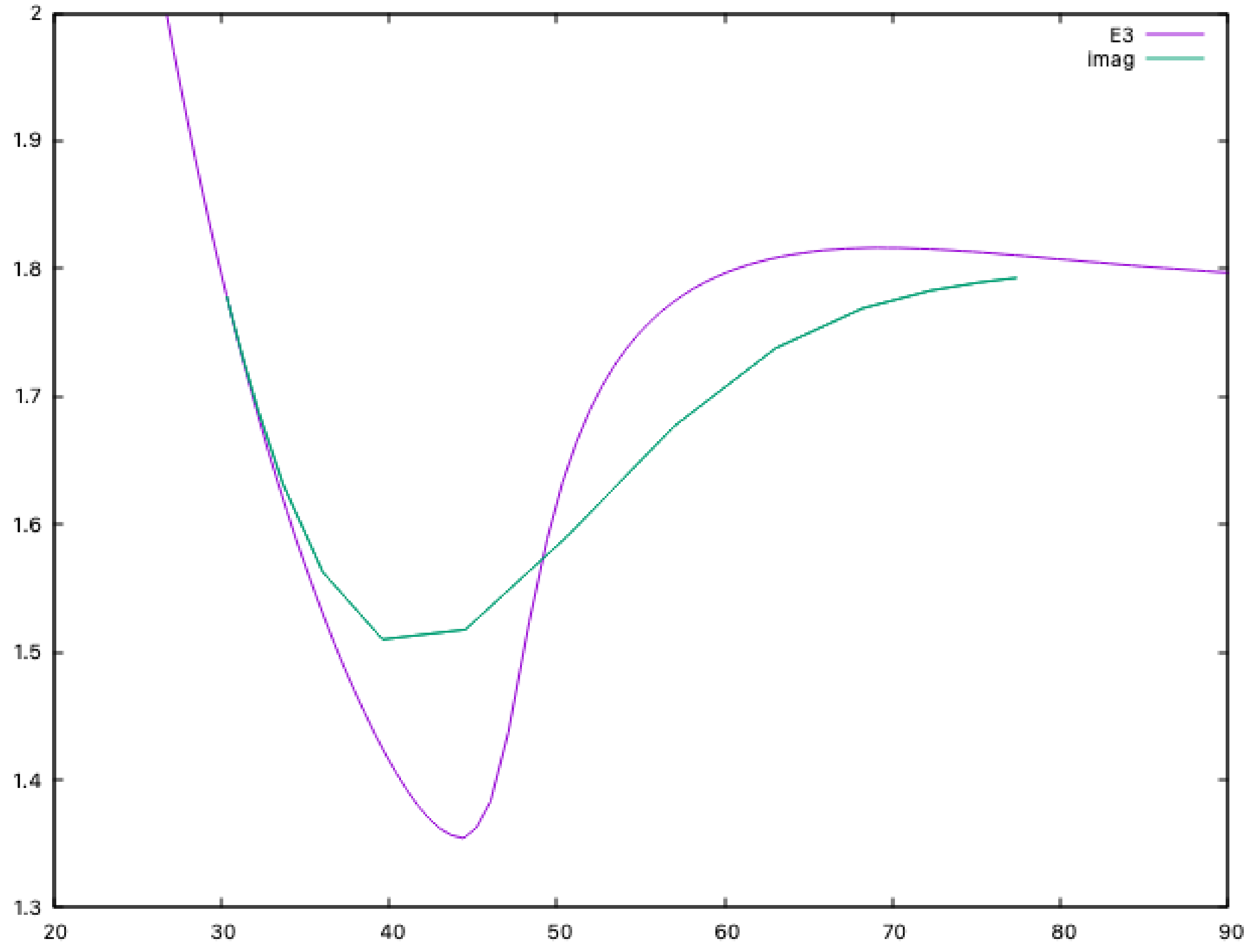

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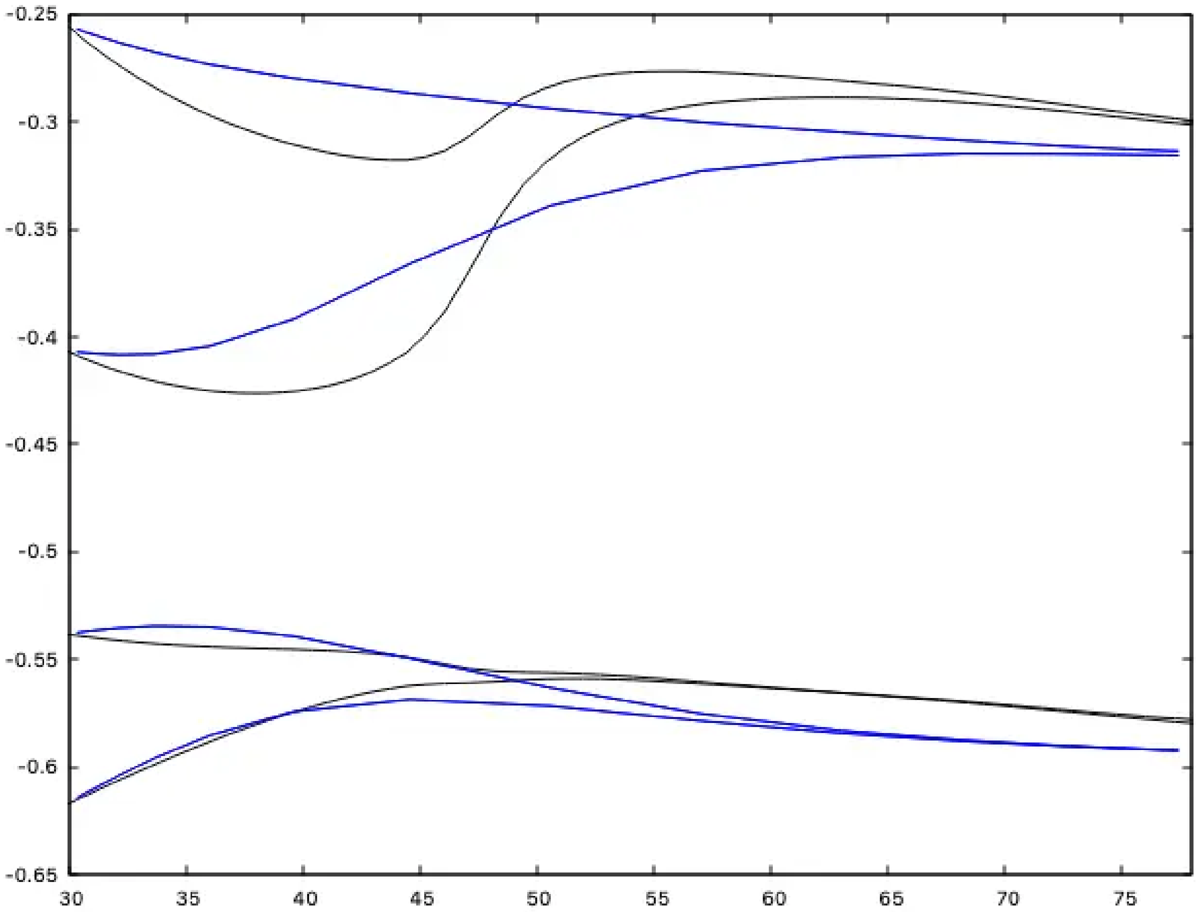

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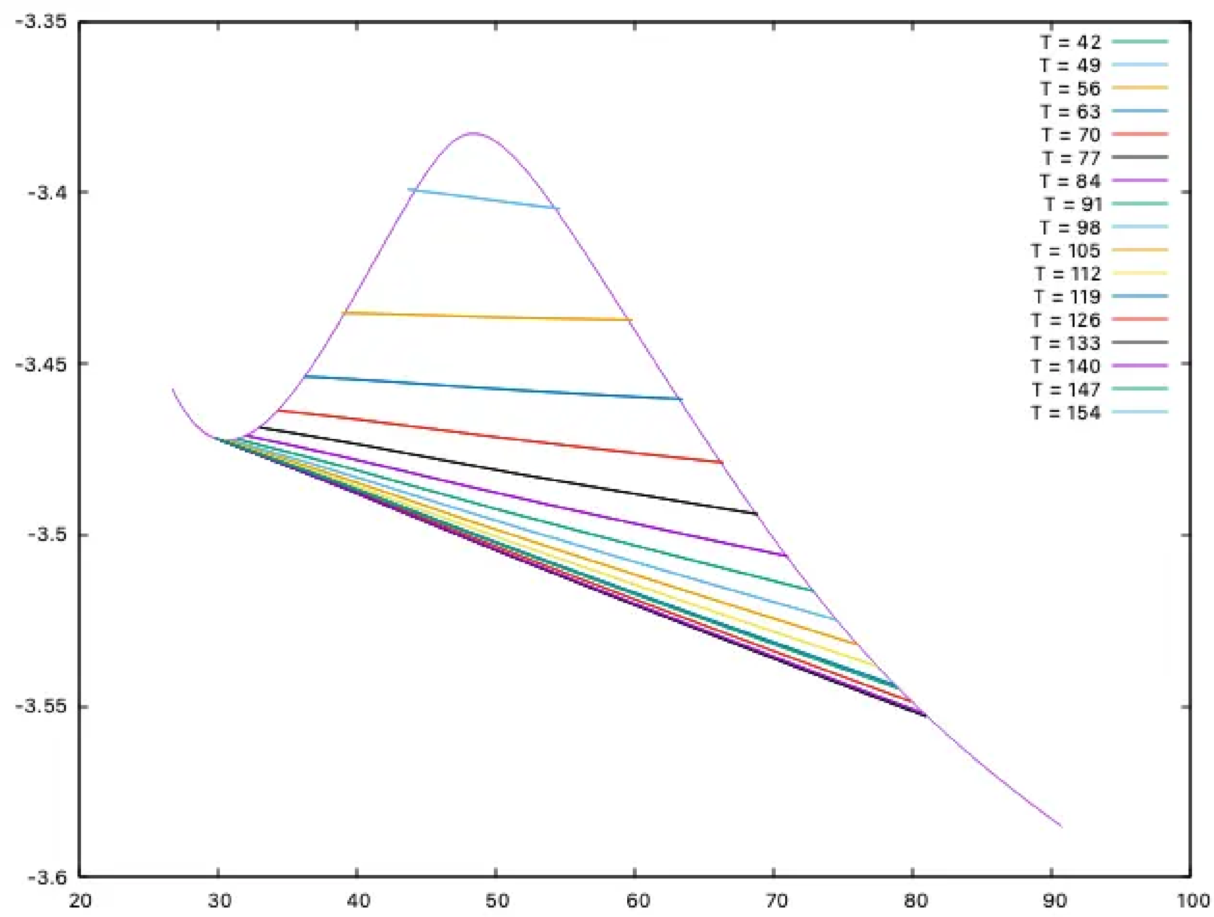

-3.35
-3.4
-3.45
-3.5
-3.55
-3.6
20
30
40
50
60
70
80
90
100
T = 42
T = 49
T = 56
T = 63
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T = 77
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T = 126
T = 133
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T = 154