Collective masses for fusion/fission dynamics

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Macroscopic model of sub-barrier fusion and spontaneous fission

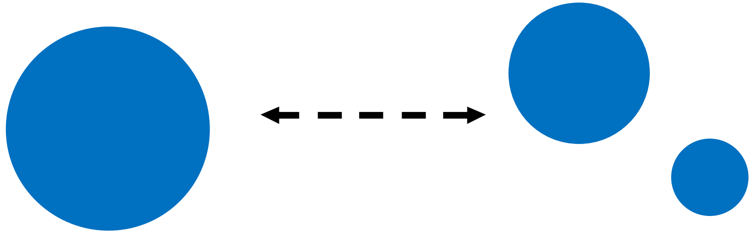
Macroscopic (collective) variables: $\{\beta^i, p_i\}_{i=1,\cdots M}$

$$H = \frac{1}{2}\sum_i B^{ij}(\beta)p_i p_j + V(\beta)$$

Quantization: $H \rightarrow \hat{H}$

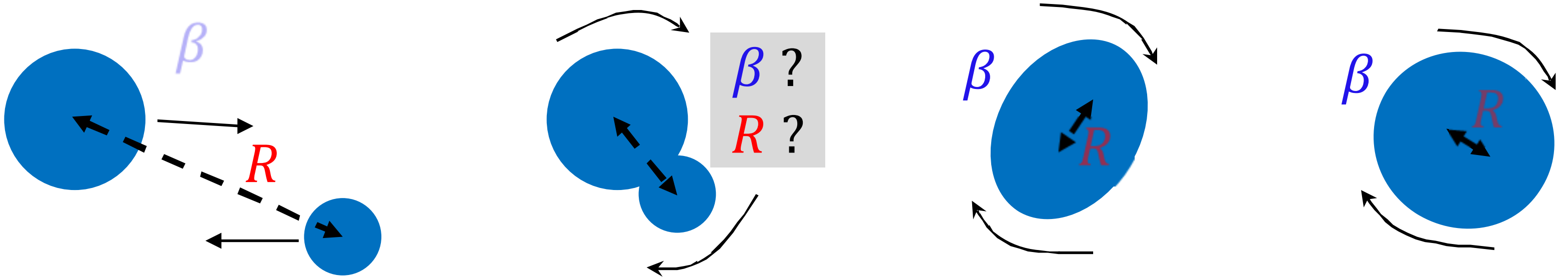
Asymptotically, the variables β^i should describe

- Shape d.o.f.
- Relative motion between two fragments



Relative coordinates vs Shape variables

- Low-energy nuclear reaction (fusion)
  - From relative motion between two nuclei
  - Into motion in shape and rotational d.o.f.



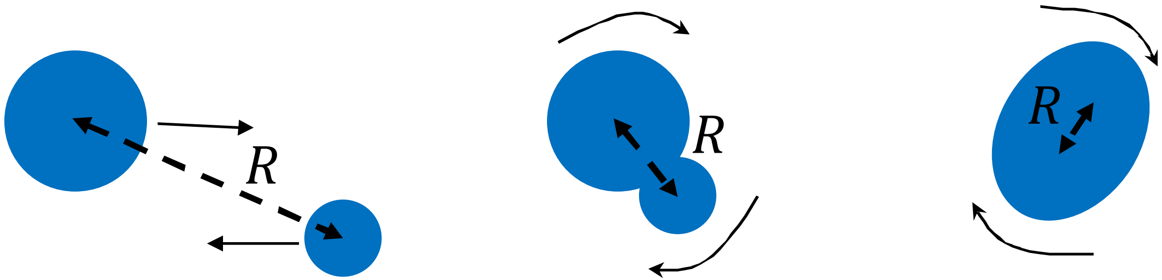
The relative coordinate R or the shape parameter β ?

Relative coordinates vs Shape variables

- <u>One-to-one correspondence</u>: Point transf: $R(\beta)$, $\beta(R)$

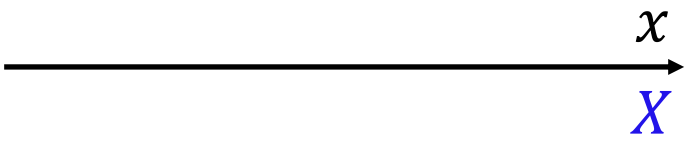
  The relative coordinate R can be a possible choice

$$V(\beta) = V(\beta(R)) = \bar{V}(R)$$



The choice, R or β, influences the inertial masses

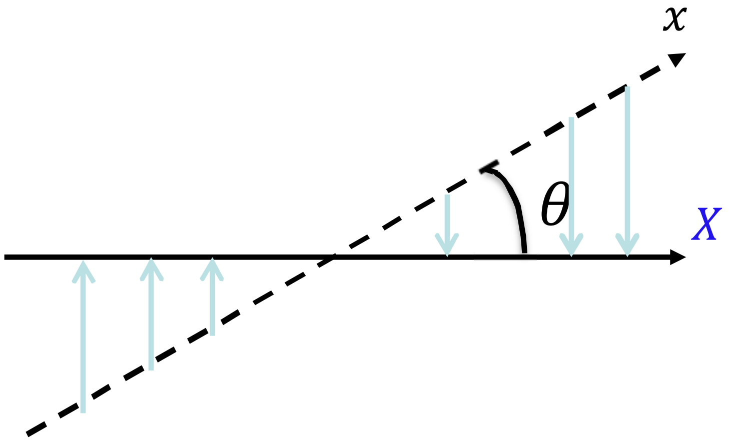
Mass & scale of coordinate



- A particle moving along x axis
  - $L = \frac{1}{2} m_x \dot{x}^2 - V(x)$
- Looking at the motion in X axis
  - Scale transf.: $x = \alpha X$
  - $L = \frac{1}{2} m_X \dot{X}^2 - \bar{V}(X)$
  - $\bar{V}(X) \equiv V(\alpha X)$
  - $\boxed{m_X = \alpha^2 m_x}$

Mass & choice of coordinate

- A particle moving along x axis
  - $L = \frac{1}{2} m_x \dot{x}^2 - V(x)$
- Looking at the motion in X axis
  - Point transf.: $x = f(X) = X/\cos\theta$
  - $L = \frac{1}{2} m_X \dot{X}^2 - \bar{V}(X)$
  - $\bar{V}(X) \equiv V(f(X))$
  - $m_X = \frac{m_x}{(\cos\theta)^2}$



Equivalent description with X and x

m_X depends on the choice of X.

$\theta \to \pi/2$, the coordinate X is no longer valid.

Macroscopic reaction model at low energy

One-to-one correspondence in collective variable: $\beta \leftrightarrow R$

$$\left\{-\frac{1}{2}\frac{d}{dR}\frac{1}{M(R)}\frac{d}{dR}+\frac{L(L+1)}{2\,I(R)}+V(R)\right\}\psi_L(R)=E_L\psi_L(R)$$

- Microscopic construction requires
  - Determination of reaction path
  - Calculation of the potential $V(R)$
  - Calculation of the mass $M(R)$ & M.o.I $I(R)$

Adiabatic Self-consistent Collective Coordinate (ASCC) method

TN, PTEP 2012, 01A207 (2012)
TN, et al., RMP 88, 045004 (2016)

ASCC method

- microscopically determines a collective subspace (*reaction path*)
- produces collective potential $V(R)$ and collective inertias $M(R), I(R)$

Requantization on the collective subspace leads to

$$\left\{-\frac{d}{dR}\frac{1}{2M(R)}\frac{d}{dR}+\frac{L(L+1)}{2I(R)}+V(R)\right\}\psi_L(R)=E_L\psi_L(R)$$

Finite amplitude method for collective mass

Wen, TN, PRC 94, 054618 (2016)
Washiyama, Hinohara, TN, PRC 103, 014306 (2021)
TN, Hinohara, EPJA, in press

$$\begin{pmatrix} A & B \\ B^* & A^* \end{pmatrix} \begin{pmatrix} X^i \\ Y^i \end{pmatrix} = \Omega_i \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} X^i \\ Y^i \end{pmatrix}$$

- Normal eigenmodes: $\frac{\partial x}{\partial \rho_{ph}}, m_x$
- For a given (*intuitive*) one-body operator: X

$$m_X = m_x \left(\frac{dx}{dX}\right)^2 = m_x \sum_{p,h} \left(\frac{\partial x}{\partial \rho_{ph}} \frac{\partial \rho_{ph}}{\partial X}\right)^2$$

- Respect symmetries by inclusion of residual effects

Energy density functional

$$E[\rho] = \int \frac{1}{2m}\tau(\mathbf{r})d\mathbf{r} + \int d\mathbf{r}\left\{\frac{3}{8}t_0\rho^2(\mathbf{r}) + \frac{1}{16}t_3\rho^3(\mathbf{r})\right\}$$

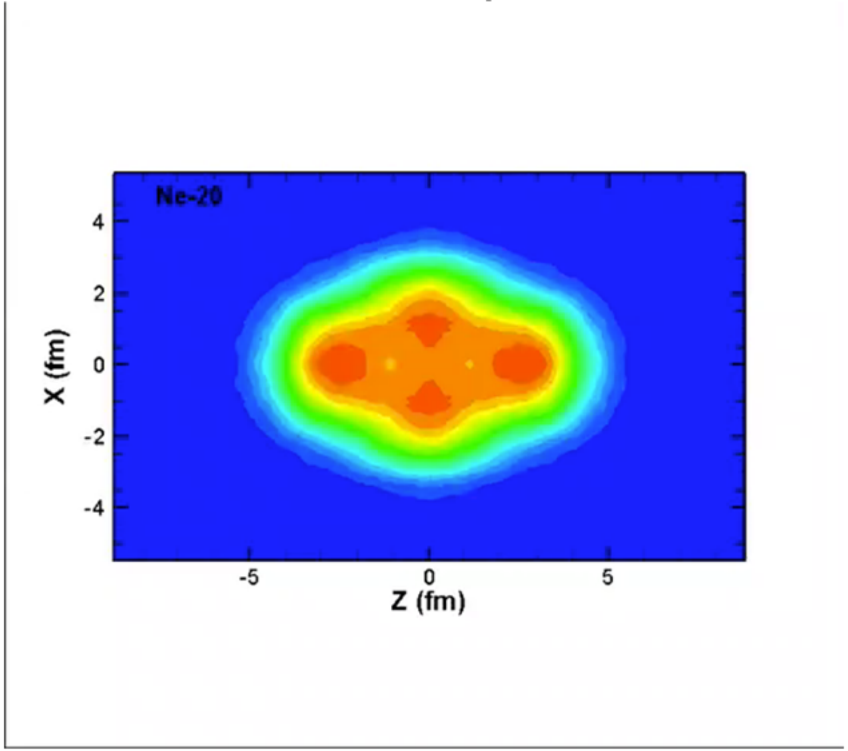
$$+ \iint d\mathbf{r}d\mathbf{r}'\rho(\mathbf{r})v(\mathbf{r}-\mathbf{r}')\rho(\mathbf{r}')$$

$$\hat{h}_{\mathrm{HF}}(\mathbf{r}) = -\nabla\frac{1}{2m^*(\mathbf{r})}\nabla + \frac{3}{4}t_0\rho(\mathbf{r}) + \frac{3}{16}t_3\rho^2(\mathbf{r})$$

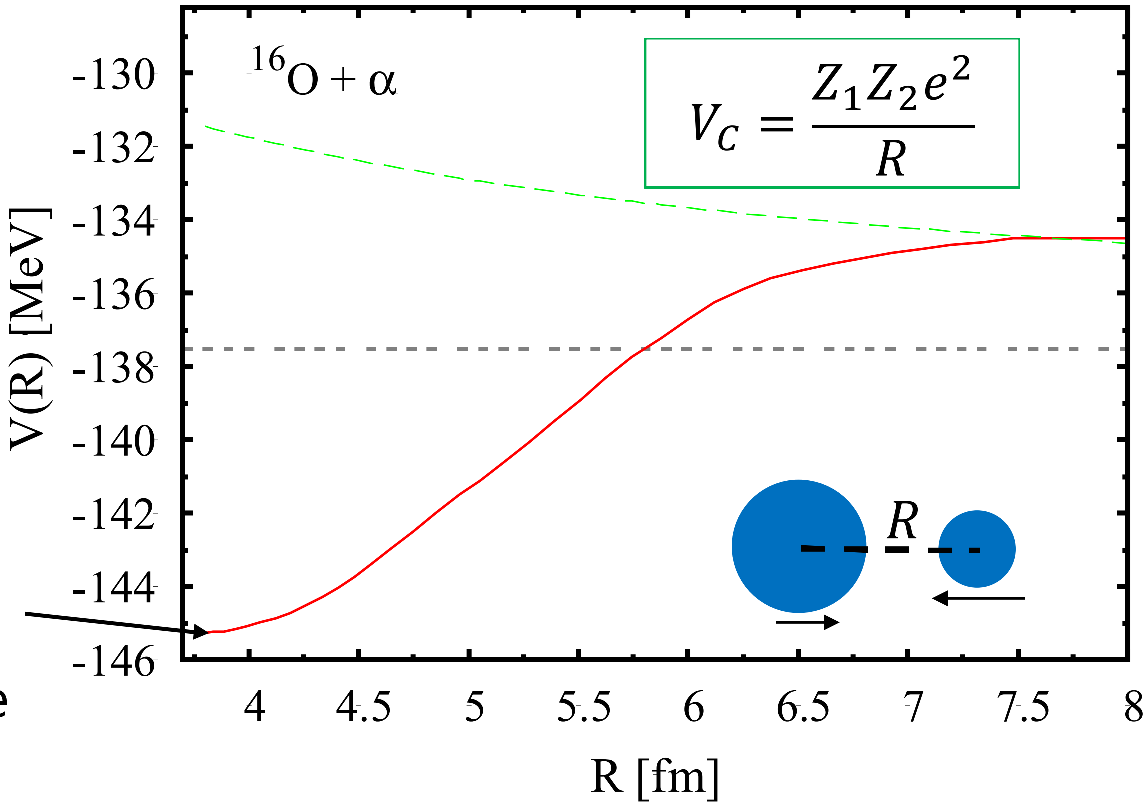
$$+ \int d\mathbf{r}'v(\mathbf{r}-\mathbf{r}')\rho(\mathbf{r}')$$

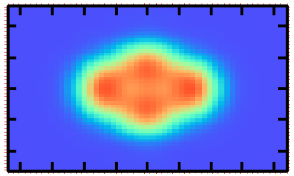
BKN functional
PRC 13, 1226 (1976)

"Fictious" fission of ^{20}Ne
α-^{16}O fusion (alpha reaction)



$V(R) \quad (m^*/m = 1)$

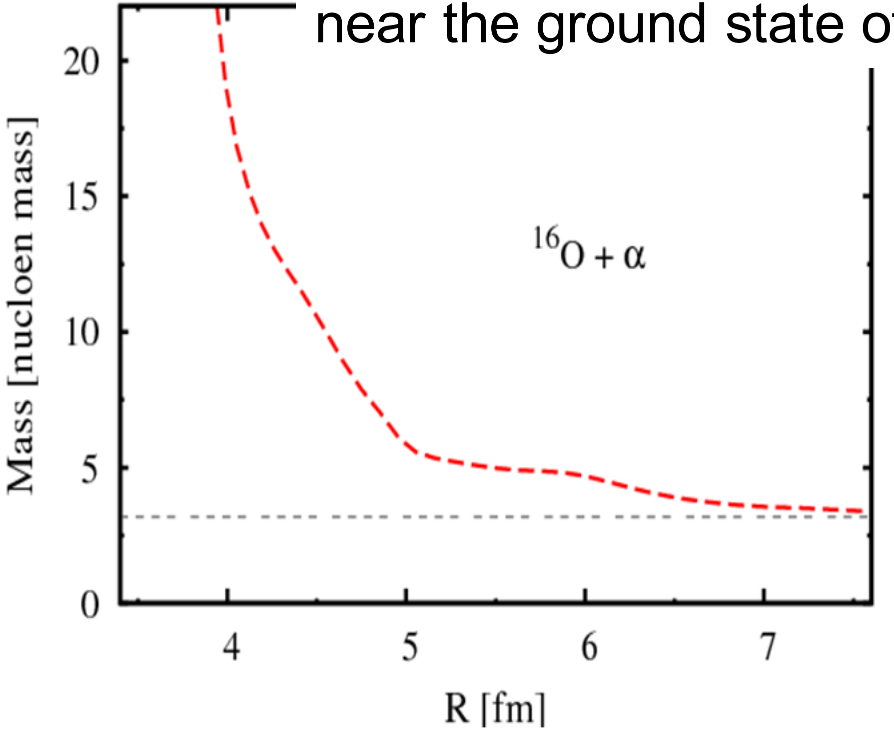


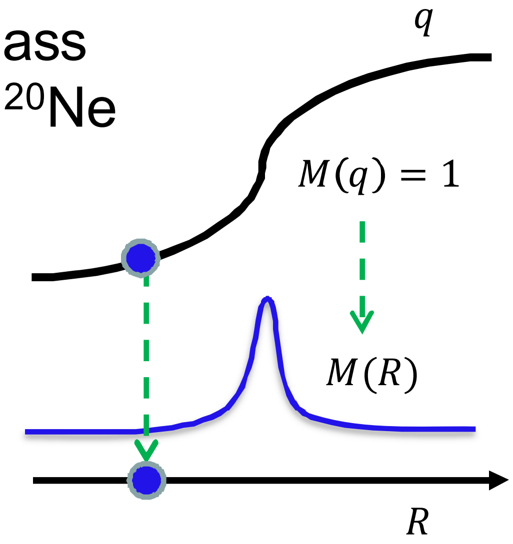


Ground state of ^{20}Ne

^{20}Ne: Inertial mass for relative motion

Strong increase in the mass near the ground state of ^{20}Ne





Reduced mass

$$M(R) \to 3.2m = \mu_{\rm red}$$

Effective mass

- Velocity-dependent potential
- Nucleonic effective mass
  - $\frac{m^*}{m} \sim 0.7 - 0.8$
- Does this affect the inertial mass?
  - For CoM translation, $M = Am \quad (\neq Am^*)$

Recovery of Galilean invariance

Energy density functional

$$E[\rho] = \int \frac{1}{2m}\tau(\mathbf{r})d\mathbf{r} + \int d\mathbf{r}\left\{\frac{3}{8}t_0\rho^2(\mathbf{r}) + \frac{1}{16}t_3\rho^3(\mathbf{r})\right\}$$
$$+ \iint d\mathbf{r}d\mathbf{r}'\rho(\mathbf{r})v(\mathbf{r}-\mathbf{r}')\rho(\mathbf{r}')$$
$$+ B_3\int d\mathbf{r}\{\rho(\mathbf{r})\tau(\mathbf{r}) - \mathbf{j}^2(\mathbf{r})\},$$

BKN functional
PRC 13, 1226 (1976)

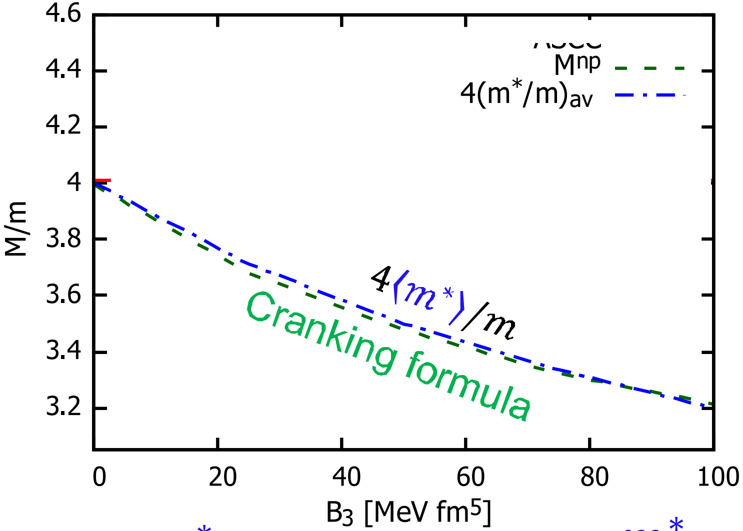
$$\hat{h}_{\mathrm{HF}}(\mathbf{r}) = -\nabla\frac{1}{2m^*(\mathbf{r})}\nabla + \frac{3}{4}t_0\rho(\mathbf{r}) + \frac{3}{16}t_3\rho^2(\mathbf{r})$$
$$+ \int d\mathbf{r}'v(\mathbf{r}-\mathbf{r}')\rho(\mathbf{r}') + B_3[\tau(\mathbf{r}) + i\nabla\cdot\mathbf{j}(\mathbf{r})]$$
$$+ 2iB_3\mathbf{j}(\mathbf{r})\cdot\nabla,$$

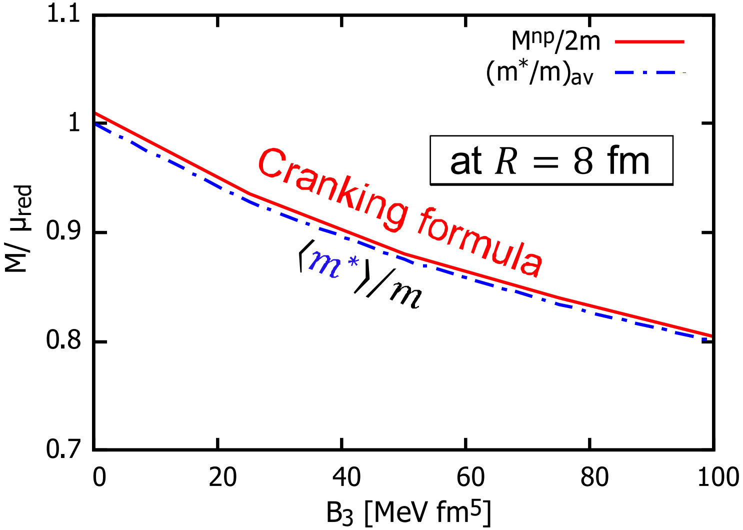
$$B_3 = 0 \quad \rightarrow \quad m^* = m$$
$$B_3 > 0 \quad \rightarrow \quad m^* < m$$

Failure of cranking formula

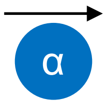
$$M_{\mathrm{cr}}^{\mathrm{np}} = 2 \sum_{n \in p, j \in h} \frac{|\langle n | \hat{p}_x | j \rangle|^2}{e_n - e_j}$$

$$M_{\mathrm{cr}}^{\mathrm{np}} = 2 \sum_{n \in p, j \in h} \frac{|\langle n | \partial / \partial R \, | j \rangle|^2}{e_n - e_j}$$

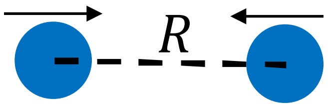




$m = m^*$



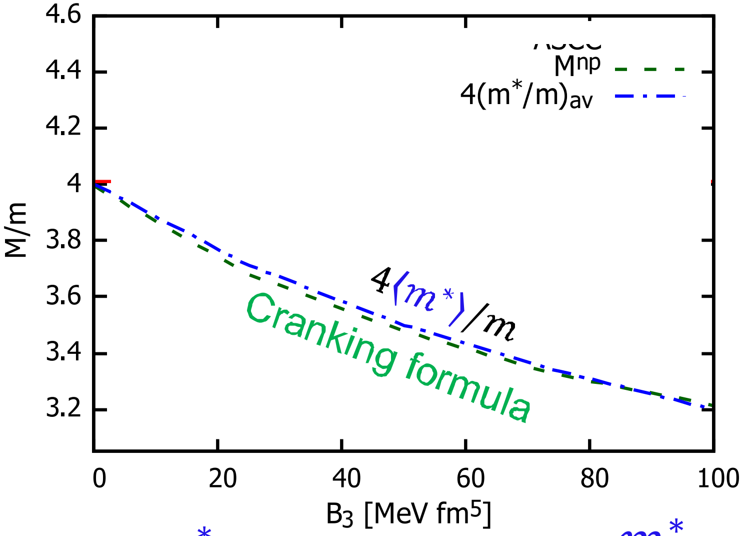
$$\frac{m^*}{m} = 0.8$$

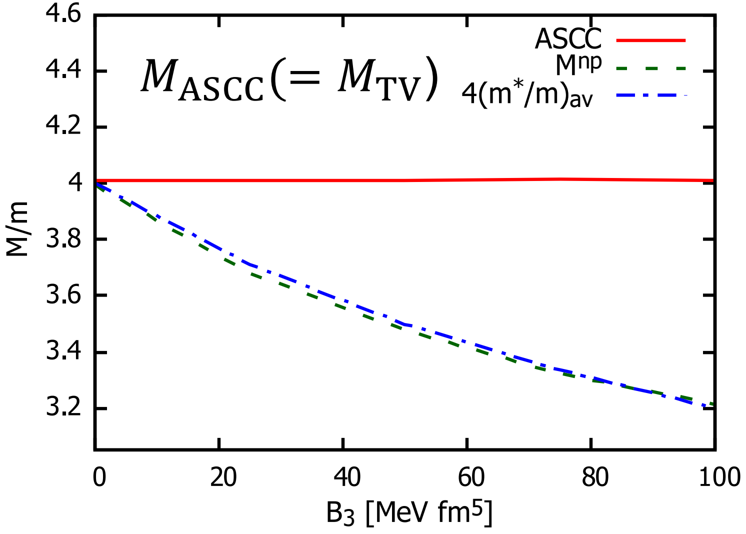


Recovery of Galilean invariance

$$M_{\mathrm{cr}}^{\mathrm{np}} = 2 \sum_{n\in p, j\in h} \frac{|\langle n|\hat{p}_x|j\rangle|^2}{e_n - e_j}$$

Inclusion of residual interaction (time-odd mean fields)





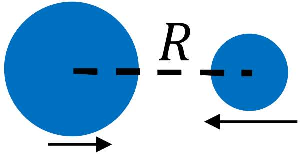
$m = m^*$

$\frac{m^*}{m} = 0.8$

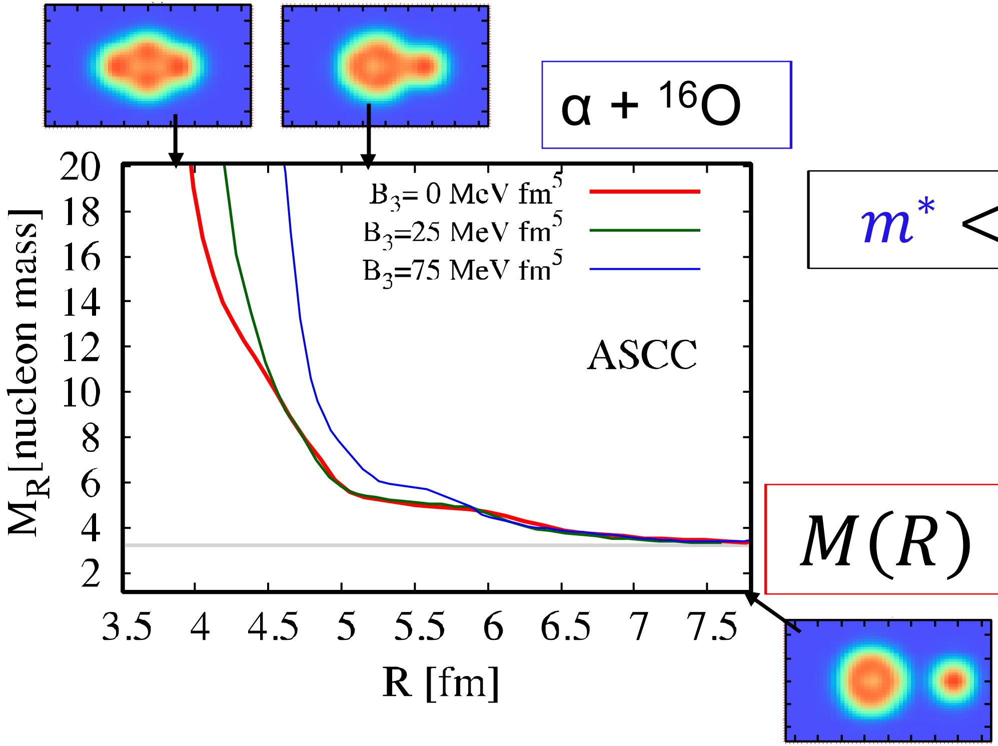
α

α $4m$

$M(R) \quad (m^*/m \leq 1)$



$\alpha + {}^{16}O$

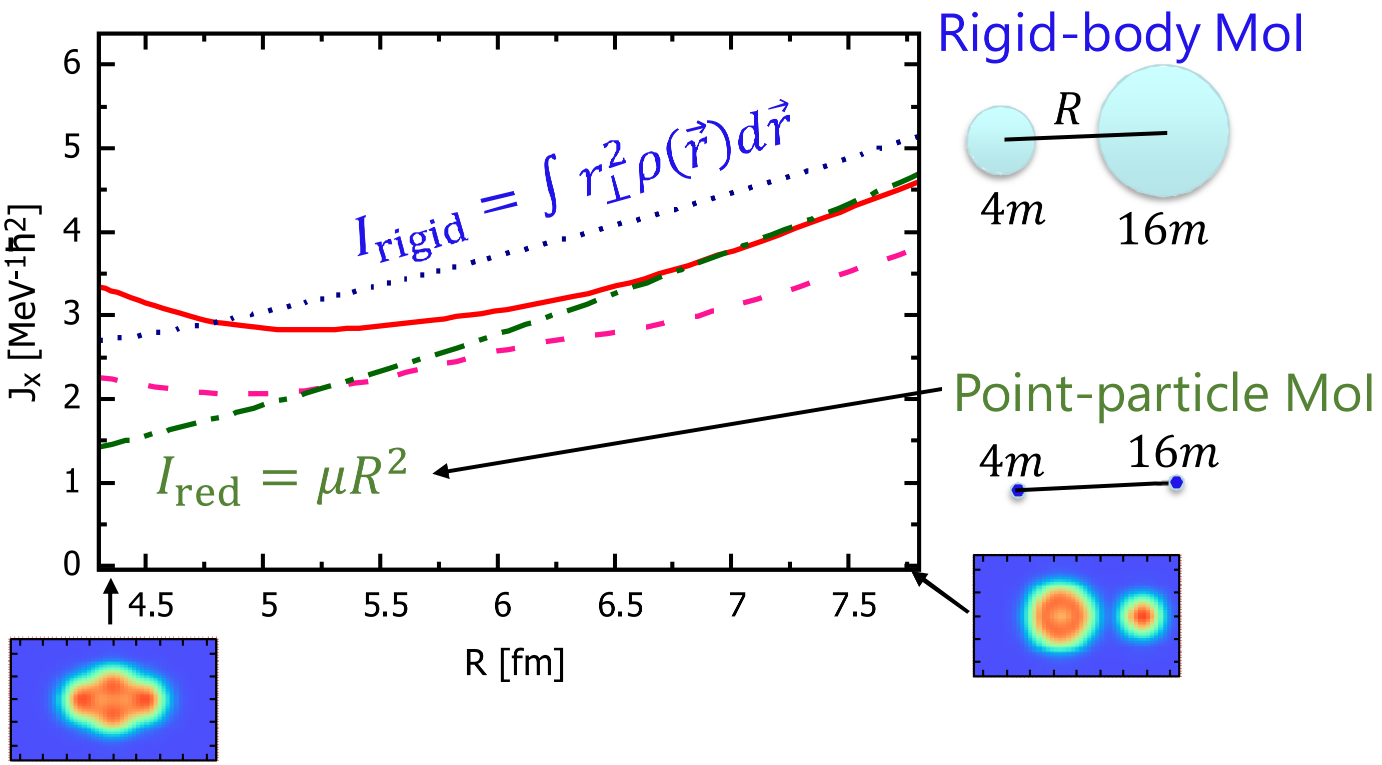


$m^* < m^* < m^* = m$

Reduced mass

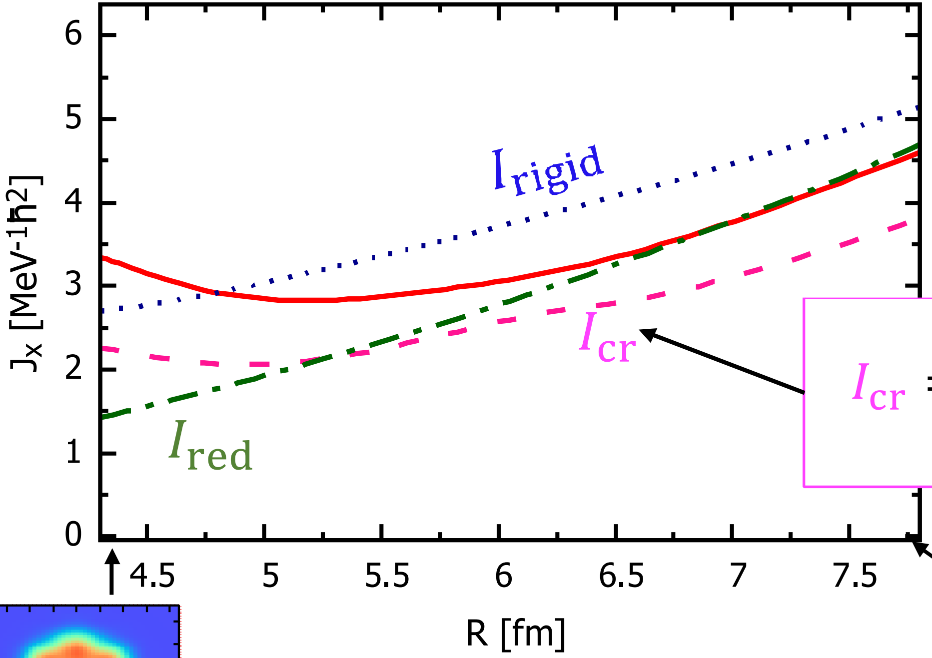
$$M(R) = \mu_R \quad (R \to \infty)$$

$I(R)$ $(m^*/m < 1)$



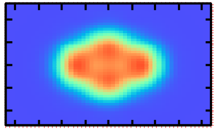
Ground state of ^{20}Ne

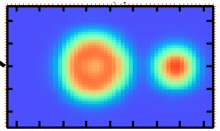
$I(R)$ $(m^*/m < 1)$



Cranking formula

$$I_{cr} = 2 \sum_{n\in p, j\in h} \frac{|\langle n|\hat{j}_x|j\rangle|^2}{e_n - e_j}$$

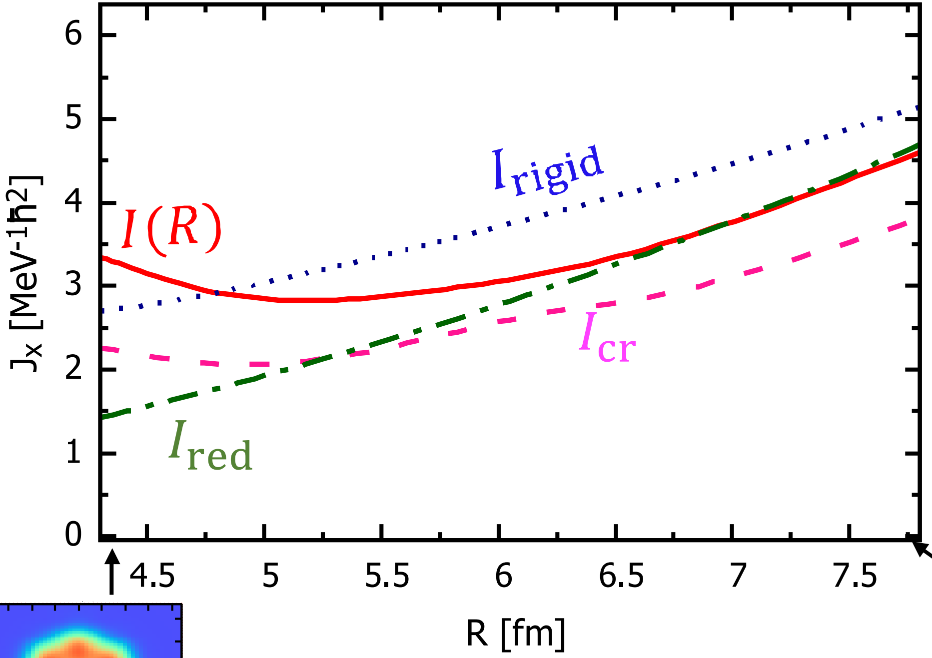




$I_{cr} < I_{rigid}$

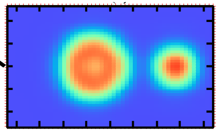
$I_{cr} \neq I_{red}$ at large R

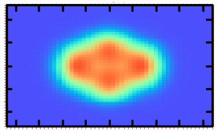
$I(R)$ $(m^*/m < 1)$



$I(R) = I_{red}$

at large R





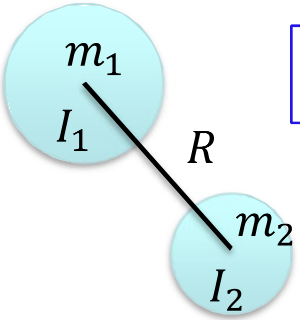
$I(R) \approx I_{rigid}$

$I(R)$ decreases with R ($R < 5.5$ fm)

Transition from I_{rigid} to I_{red}

Quantum effect

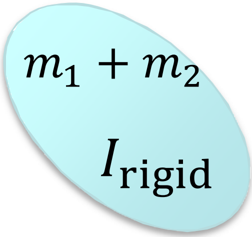
Quantum spherical systems: $I = 0$ (Vanishing MoI)



$$I(R) = I_{\text{red}}(R) + I_1 + I_2$$

$$I_{\text{red}}(R) = \mu_R R^2 = \frac{m_1 m_2}{m_1 + m_2} R^2$$

I_i: MoI of nucleus i w.r.t. its CoM



$$I(R) = I_{\text{red}}(R) + I_{\text{rigid}}^{(1)} + I_{\text{rigid}}^{(2)}$$

Moments of inertia (HO potential model)

Bohr, Mottelson, *Nuclear Structure* Vol.2

$$\mathscr{I}_1 = \frac{\hbar}{2\omega_2\omega_3}\left(\frac{(\omega_2+\omega_3)^2}{\omega_2-\omega_3}(\Sigma_3-\Sigma_2)+\frac{(\omega_2-\omega_3)^2}{\omega_2+\omega_3}(\Sigma_2+\Sigma_3)\right)$$

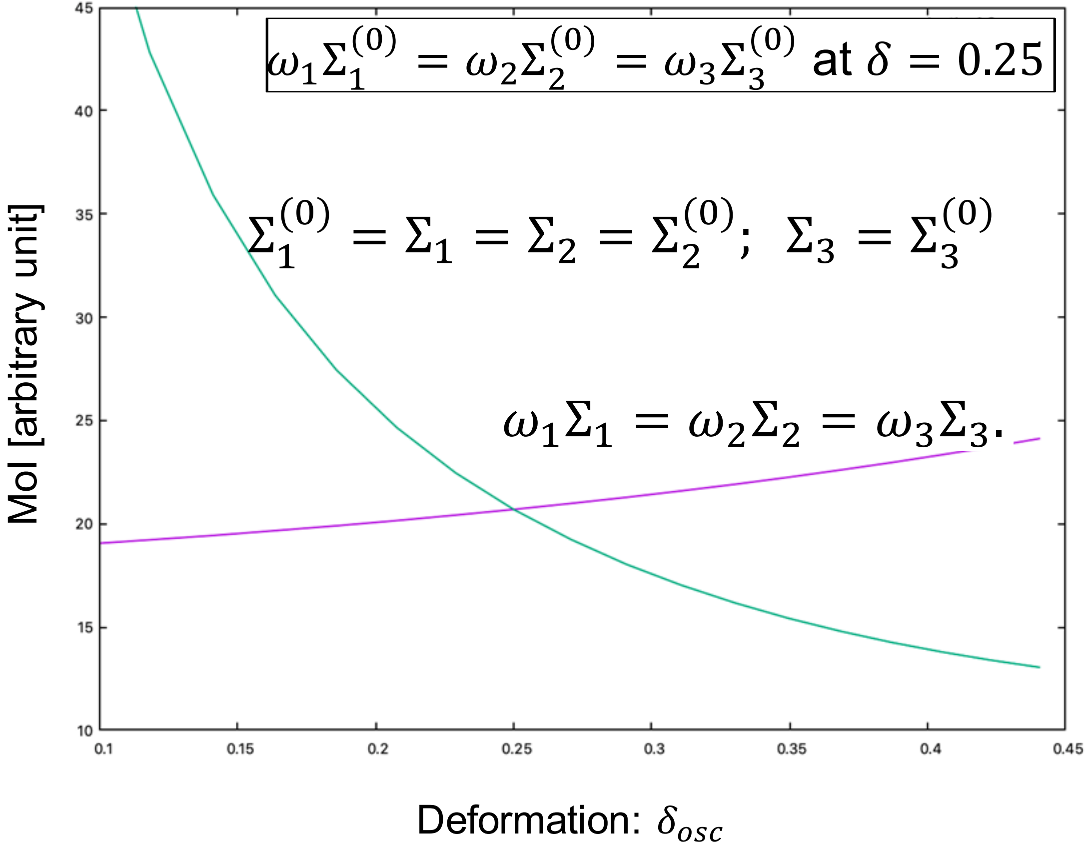
$$\omega_1\Sigma_1=\omega_2\Sigma_2=\omega_3\Sigma_3 \qquad \mathscr{I}_1=\left\langle\sum_{k=1}^{A} M(x_2^2+x_3^2)_k\right\rangle=\mathscr{I}_{\text{rig}}$$

Rigid body

$$\Sigma_1=\Sigma_2=\Sigma_3; \qquad \mathscr{I}_1 = M\frac{\left\langle\sum_k (x_2^2-x_3^2)_k\right\rangle^2}{\left\langle\sum_k (x_2^2+x_3^2)_k\right\rangle}$$

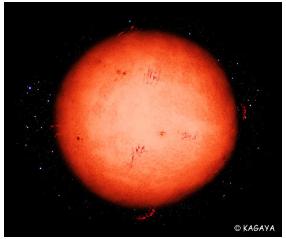
Irrotational flow

Decreasing function w.r.t. deformation



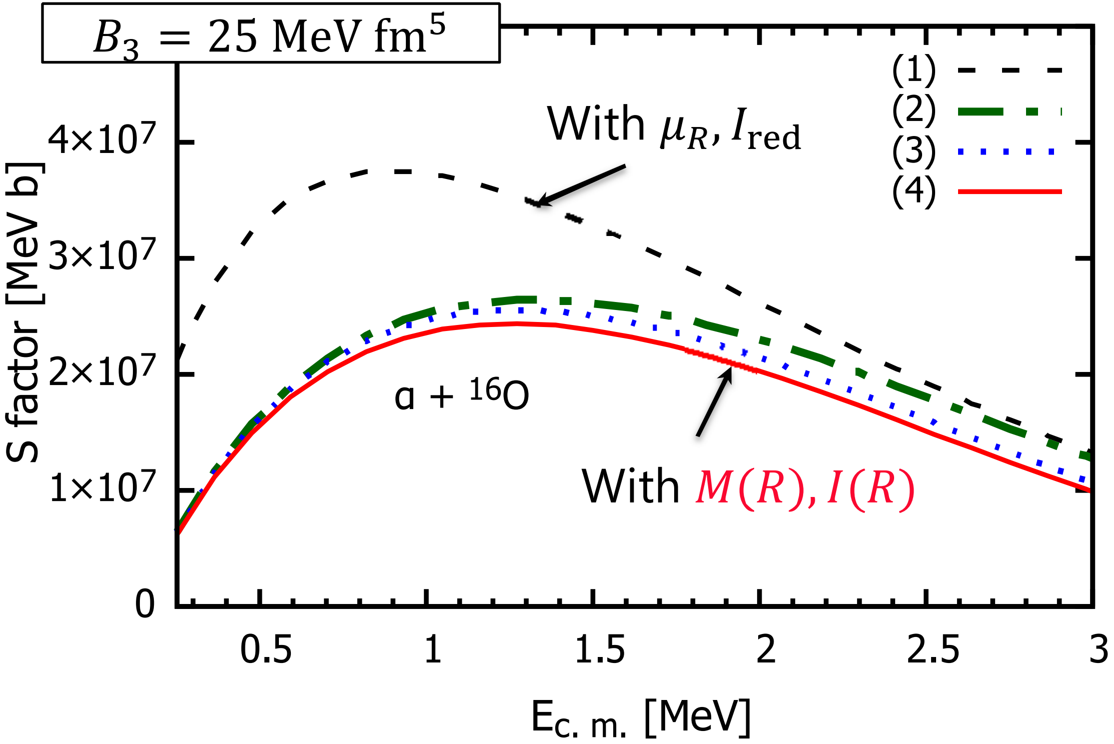
Alpha reaction : $^{16}O + \alpha$

Synthesis of ^{20}Ne



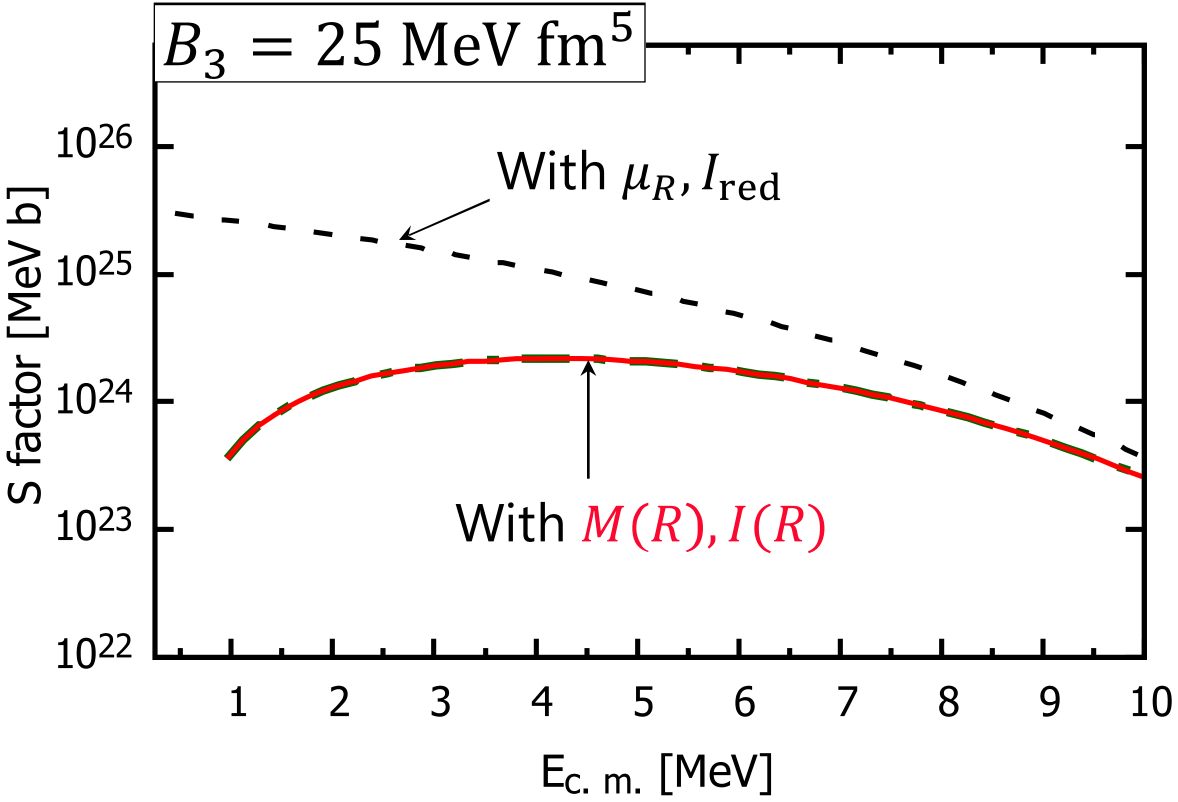
Fusion reaction:
Astrophysical S-factor

$$\sigma(E) = \frac{1}{E} P(E) \times S(E)$$



$^{16}O + ^{16}O$ fusion reaction

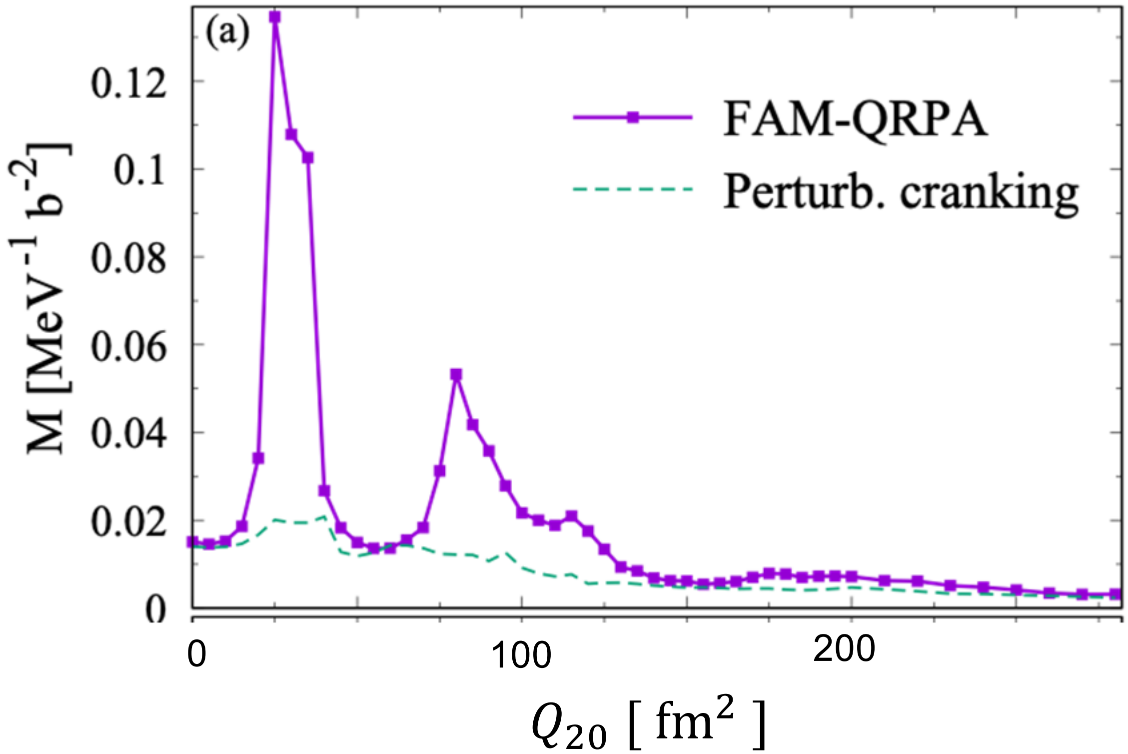
$$\sigma(E) = \frac{1}{E} P(E) \times S(E)$$



Strong fusion hindrance due to dynamical change of the inertial mass

Collective mass for fission of ^{240}Pu

Washiyama, Hinohara, TN, PRC 103, 014306 (2021)



Collective coordinate
= Mass quadrupole moment

Strong enhancement over the cranking mass

Increase of fission lifetime by several order of magnitude

Reduced-basis method

N. Hinohara, X. Zhang, and J. Engel, in preparation

Application to calculation of collective mass

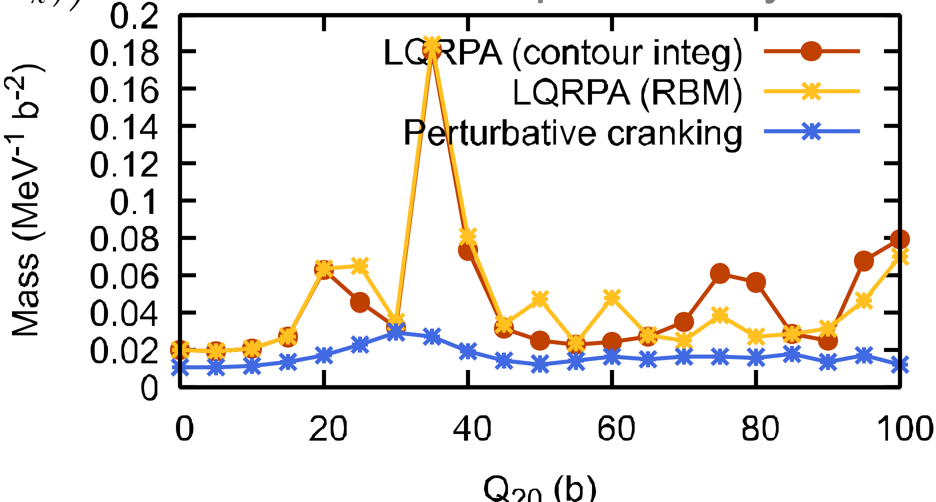
Using solutions of the finite amplitude method for n different frequencies ω_k

$$\begin{pmatrix} X_{\mu\nu}(\omega) \\ Y_{\mu\nu}(\omega) \end{pmatrix} = \sum_{k=1}^{n} a_k(\omega) \begin{pmatrix} X_{\mu\nu}(\omega_k) \\ Y_{\mu\nu}(\omega_k) \end{pmatrix} + b_k(\omega) \begin{pmatrix} Y^*_{\mu\nu}(\omega_k) \\ X^*_{\mu\nu}(\omega_k) \end{pmatrix}$$

$2n$-dimensional RPA matrix

Result with $n = 10$

Hinohara, preliminary



Summary

- Self-consistent description of fusion/fission
  - Reaction path affects the inertial mass
  - Effective mass: Clear failure of the cranking formula
  - Vanishing MoI in spherical nuclei (quantum effect)
  - Strong reduction of astrophysical S-factor and fission probability (fusion/fission hindrance due to mass effect)
  - Developments in RBM

K. Wen, TN., Phys. Rev. C 105 (2022) 034603; Front. Phys. 8 (2020) 16

Washiyama, Hinohara, TN, PRC 103, 014306 (2021)

Hinohara, et al., in preparation