A Quasi-Parton Model by Physics Informed Neural Network

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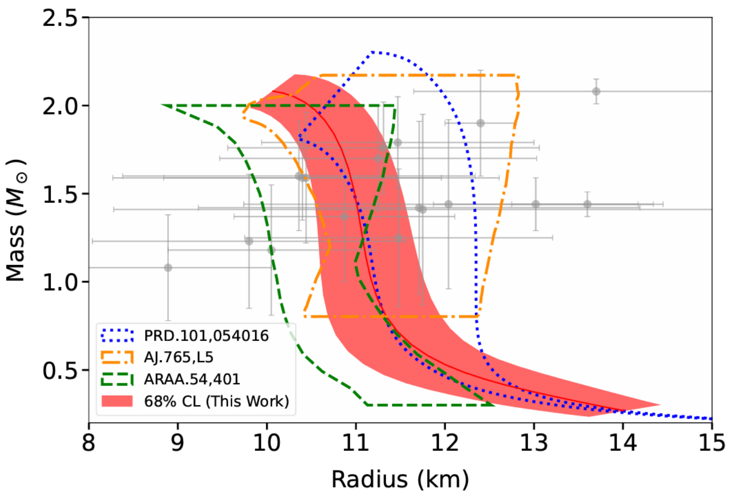
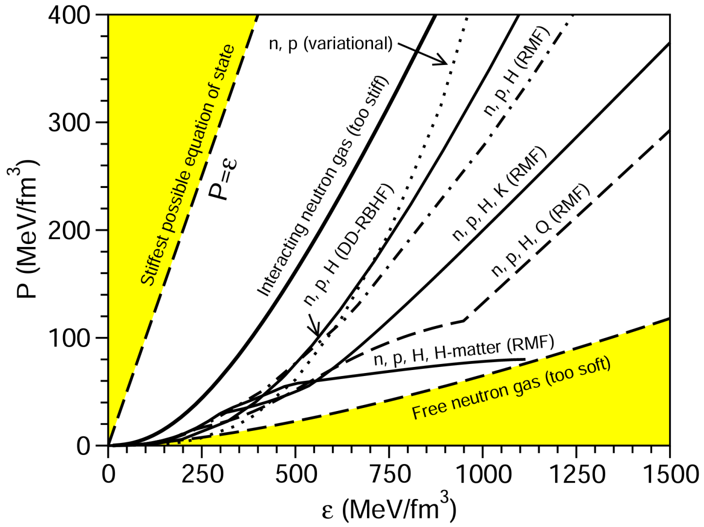
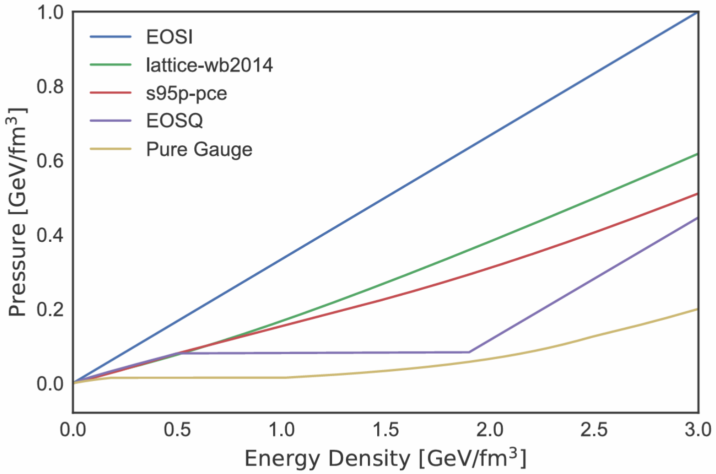
https://github.com/leefp29

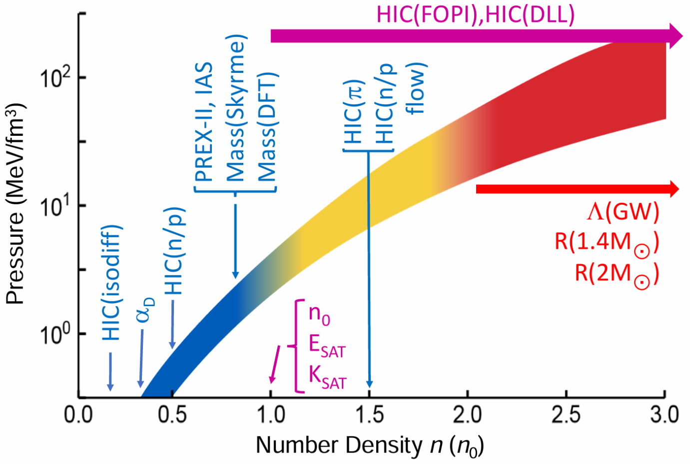
Nuclear EoS

- Crucial for understanding the **evolution of early universe**, **neutron star** and **properties of QGP**
- Also constrains **many-body nuclear interactions** and **non-perturbative QCD**
- One of the **key physical objectives** of heavy-ion collision experiments









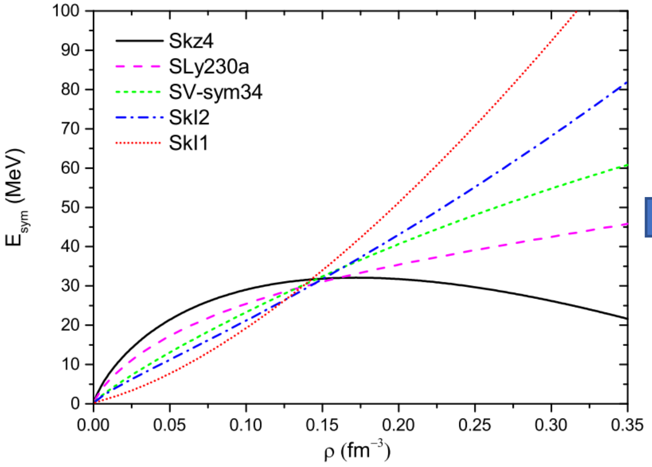
Shriya Soma, Lingxiao Wang, Shuzhe Shi. et al. Phys.Rev.D 107 (2023) 8, 083028
Tsang, C.Y., Tsang, M.B., Lynch, W.G. et al. Nat Astron 8, 328–336 (2024)
Long-Gang Pang, Hannah Petersen, Xin-Nian Wang. Phys. Rev. C 97, 064918(2018)
F. Weber, R. Negreiros, P. Rosenfield Prog.Part.Nucl. Phys.59:94-113,2007

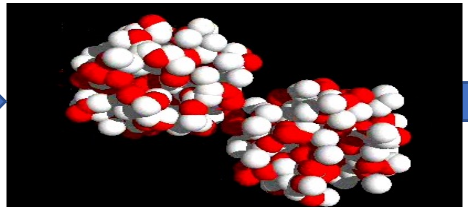
How: Traditionally *vs* DL

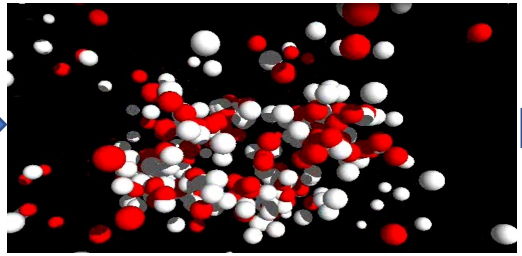
For a student

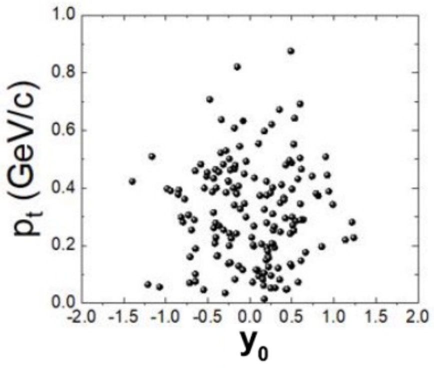
Learning data(>>one hour)

Tell me the answer



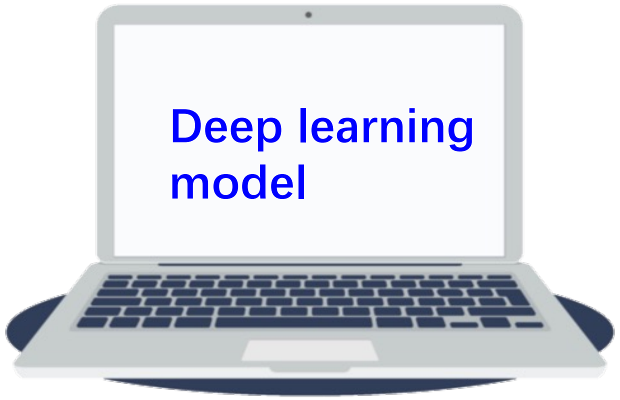






Learning data(≈one hour)

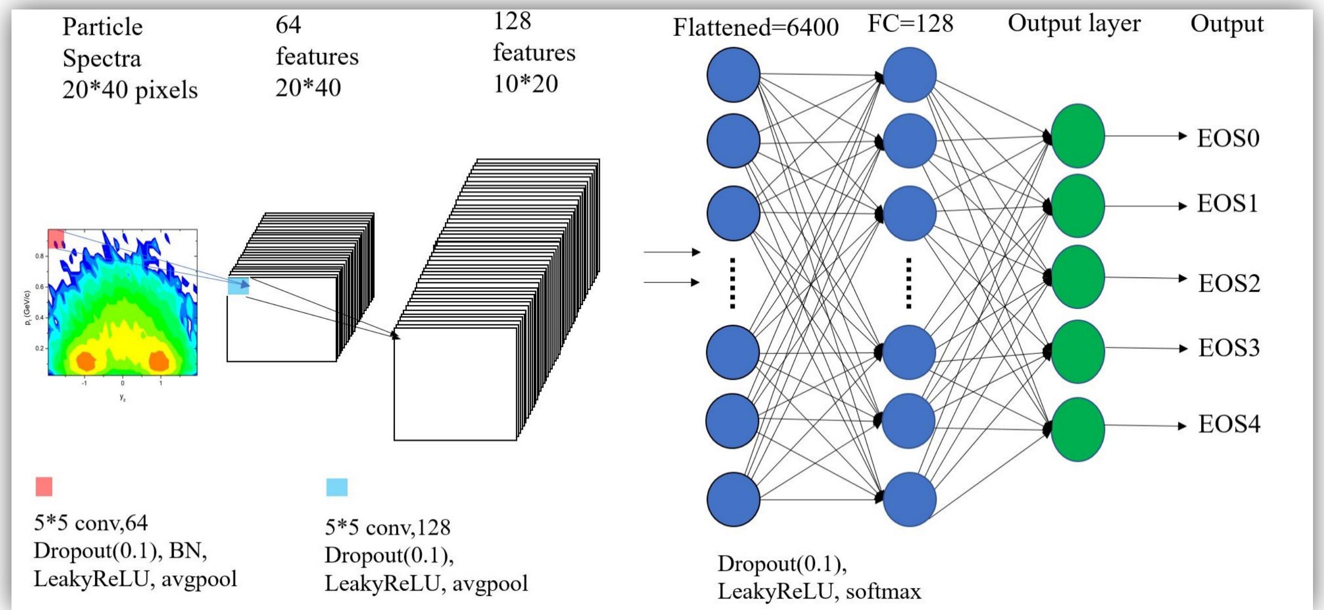
For a DL model



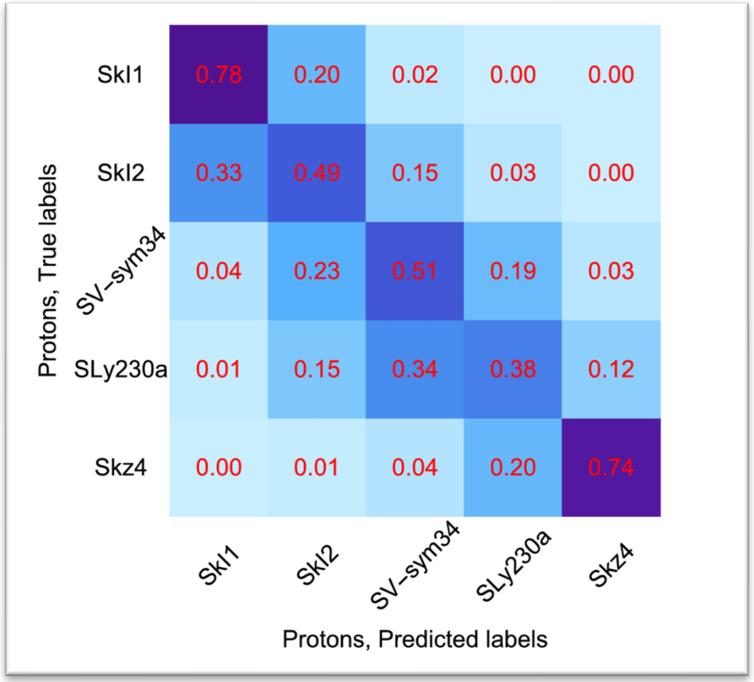
Tell me the answer

FPL, Yong-Jia Wang & Ze-Peng Gao, et al. Phys.Rev.C 104 (2021) 3, 034608
Yong-Jia Wang, **FPL** & Qing-Feng Li, et al. Phys.Lett.B 822,136669(2021)

Result



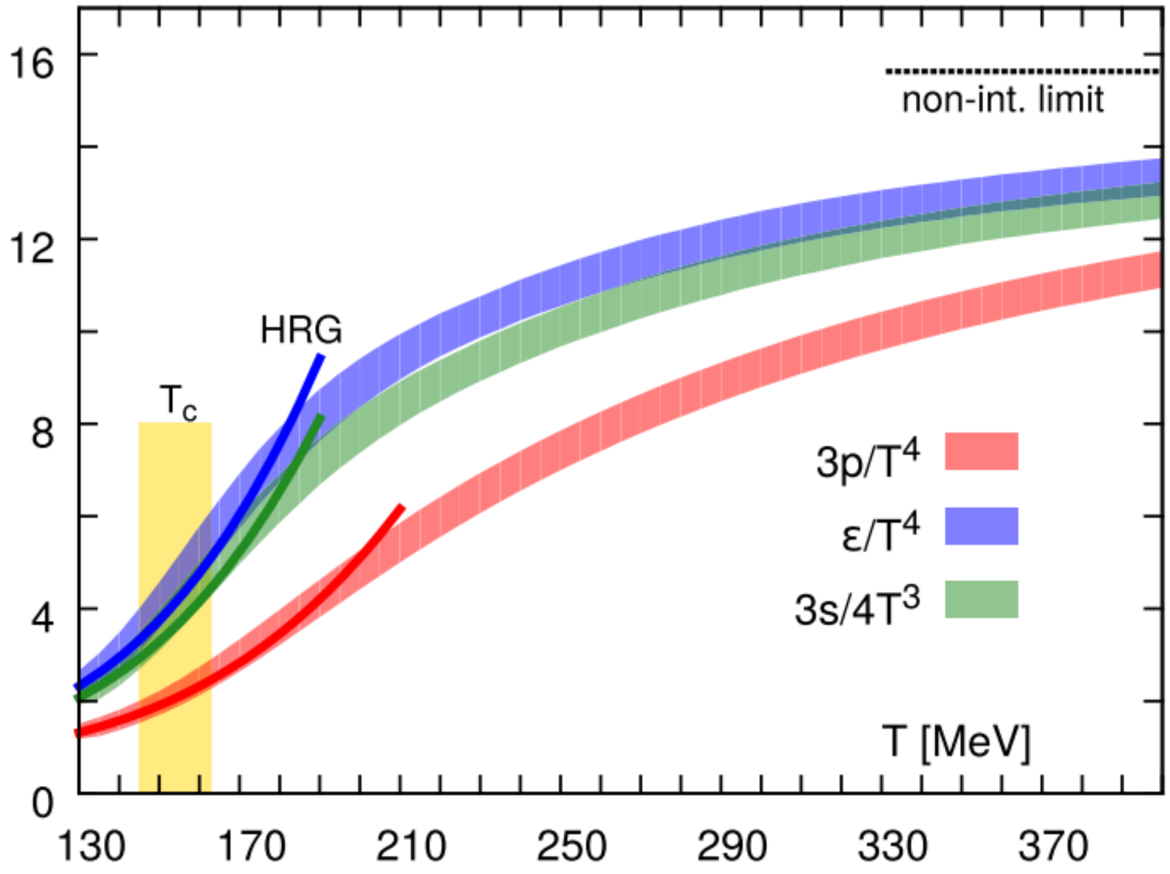
The confusion matrix for five-class classification task.



- The diagonal entries show the fraction of correctly classified testing data.
- The larger the value on the diagonal, the better the network will perform.

FPL, Yong-Jia Wang & Ze-Peng Gao, et al. Phys.Rev.C 104 (2021) 3, 034608
Yong-Jia Wang, **FPL** & Qing-Feng Li, et al. Phys.Lett.B 822,136669(2021)

QCD equation of state

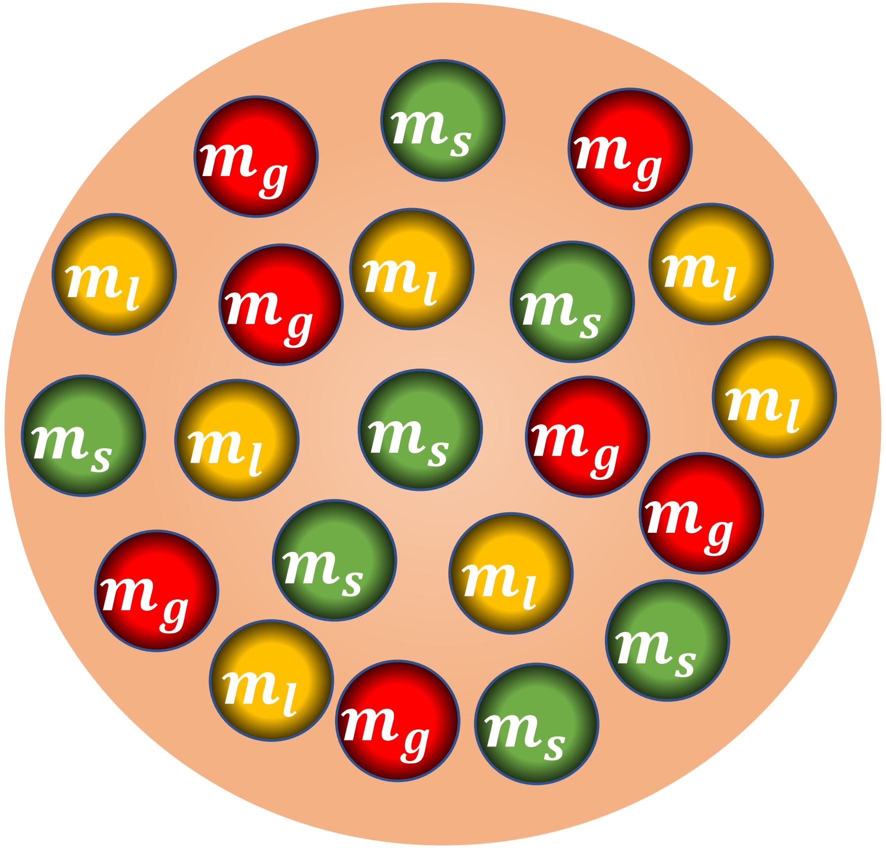


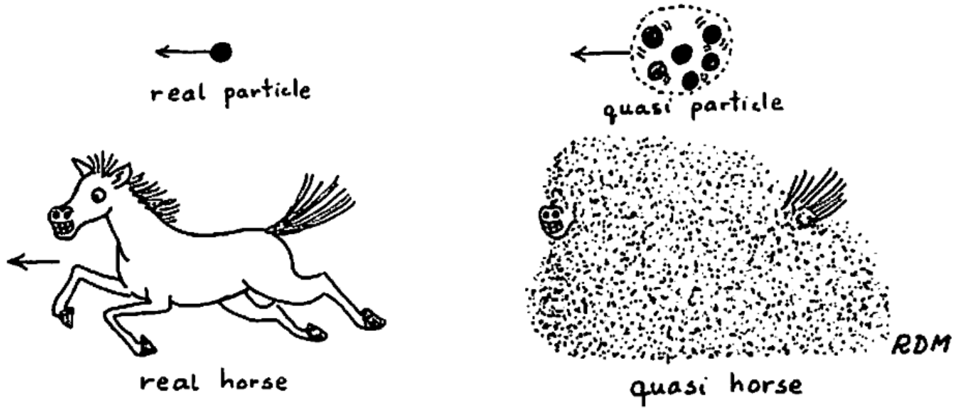
HRG model: the hot and dense QCD matter is considered as non-interaction hadrons.

- T < 200 MeV: QCD equation of state is well described by HRG.
- T > 200 MeV: nuclear matter transitions into the QGP phase.

HotQCD, Phys Rev D 90, 094503 (2014)

Quasi-particle method





quasi particles of this particular system. Many different types of systems of interacting particles may be described in this manner, and in general we have

real particle + 'coat' or 'cloud' of other particles = quasi particle. (0.1)

Sometimes this same equation is stated in a more powerful terminology coming from quantum field theory:

'bare' particle + 'clothing' or 'cloud' = 'dressed' or 'clothed' or 'physical' or 'renormalized' particle. (0.2)

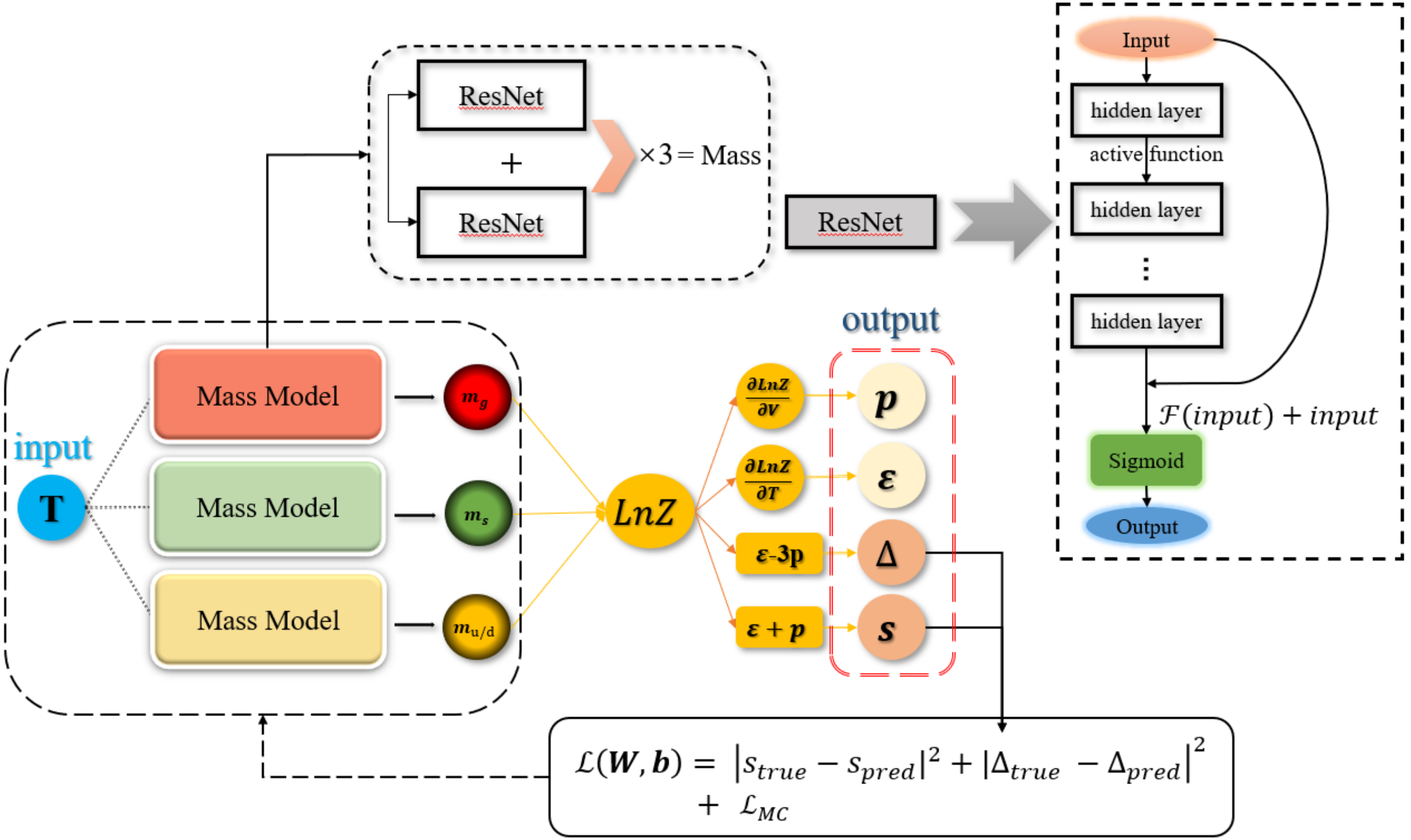
- Absorb the interacting potential into the mass.
- We construct a weakly interacting quasi-parton-gas model, which is an effective theory for strongly coupled QGP.

$$H = T + V_{eff} = \frac{P^2}{2M_{real}} + V_{eff}$$

$$H = \frac{P^2}{2M_{quasi}}$$

A Guide to Feynman Diagrams in the Many-Body Problem(1992), R.D.Mattuck

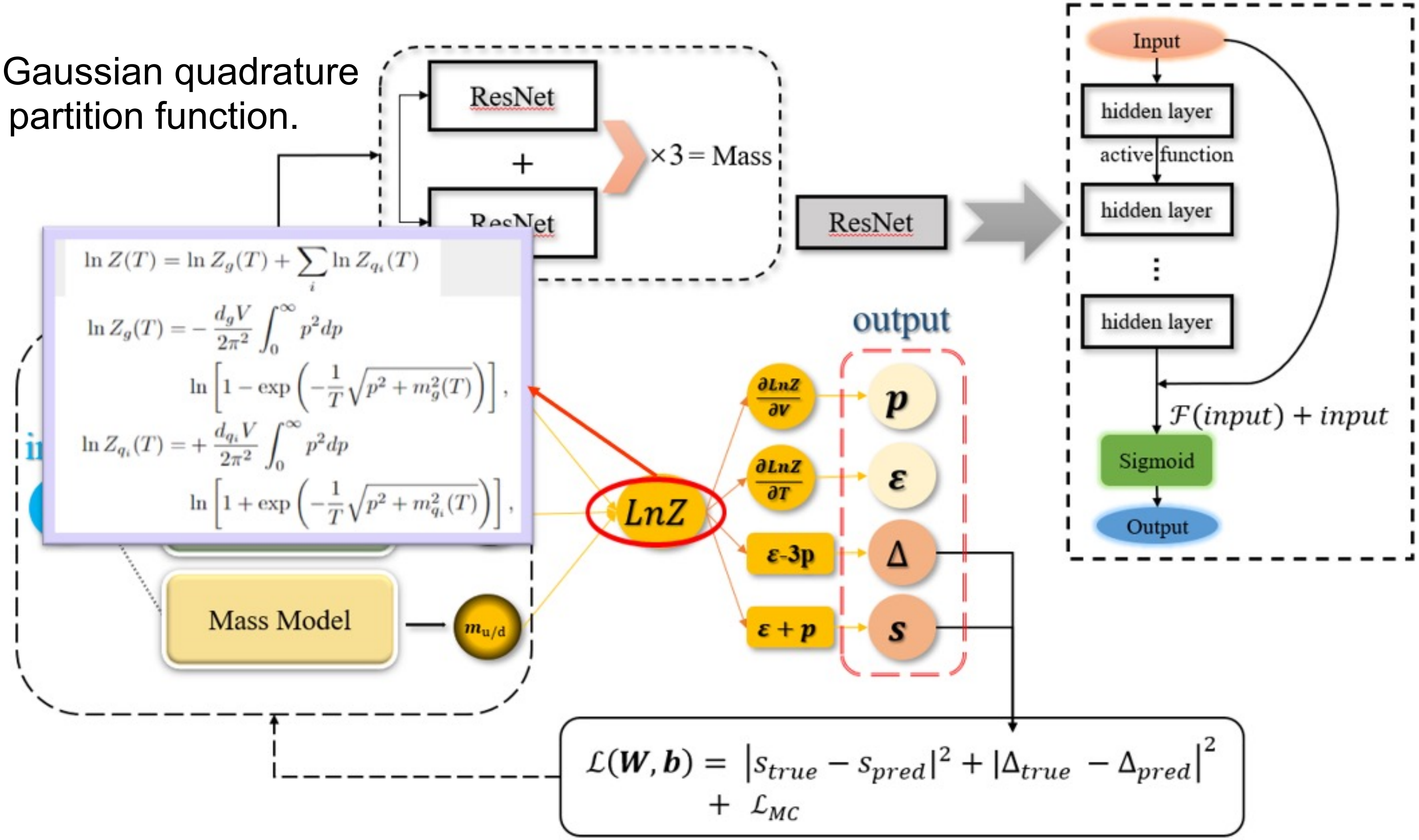
The framework of DNN: $\mu_B = 0$



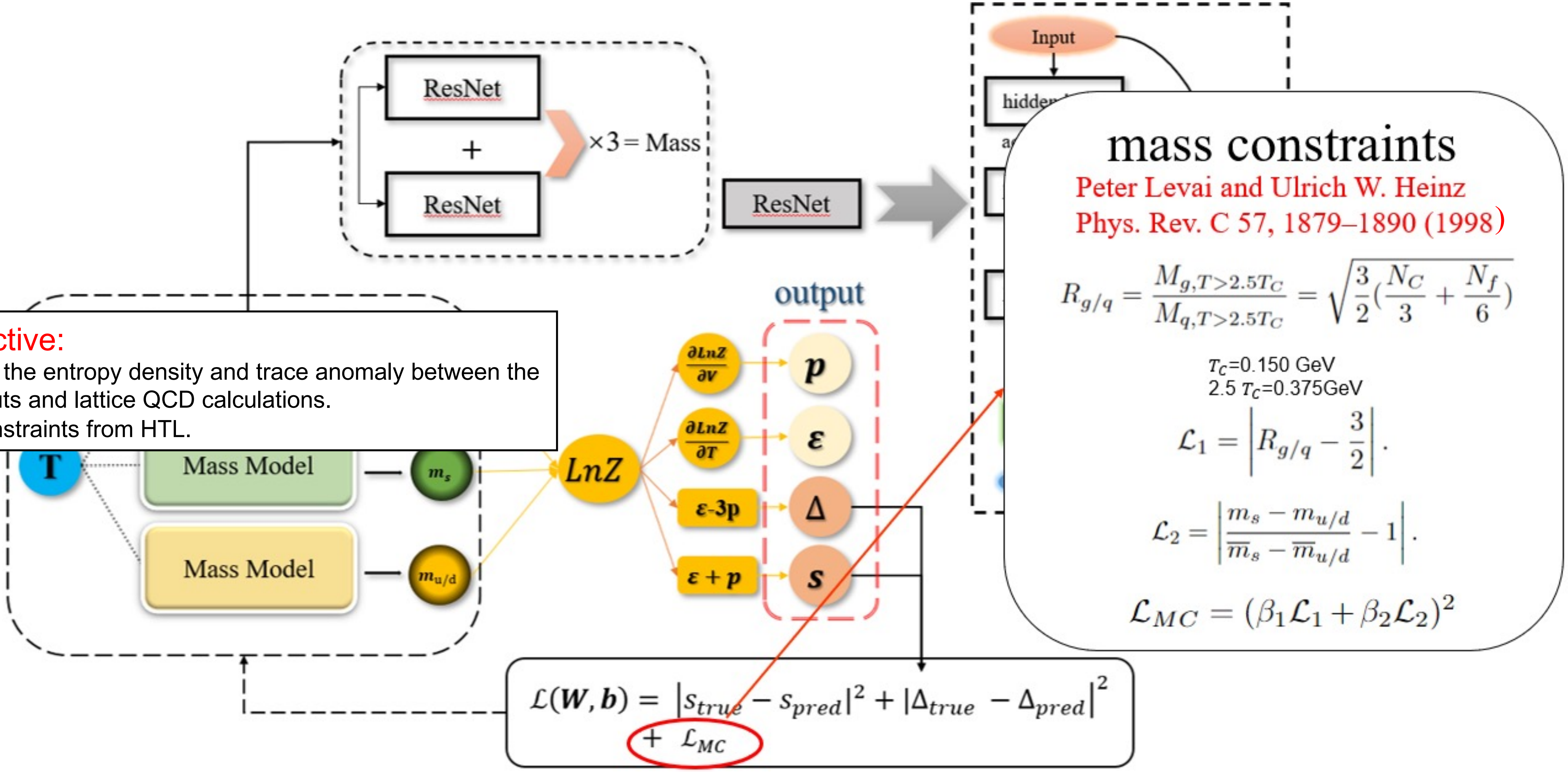
FPL, Hong-Liang Lü, Long-Gang Pang & Guang-You Qin, Phys.Lett.B 844 (2023), 138088

The framework of DNN

✓ Using 25-point Gaussian quadrature to compute the partition function.



The framework of DNN



$$\mathcal{L}(\boldsymbol{W},\boldsymbol{b}) = \left|s_{true} - s_{pred}\right|^2 + \left|\Delta_{true} - \Delta_{pred}\right|^2 + \mathcal{L}_{MC}$$

Training objective:

1. The RMSE of the entropy density and trace anomaly between the network outputs and lattice QCD calculations.
2. The mass constraints from HTL.

mass constraints

Peter Levai and Ulrich W. Heinz
Phys. Rev. C 57, 1879–1890 (1998)

$$R_{g/q} = \frac{M_{g,T>2.5T_C}}{M_{q,T>2.5T_C}} = \sqrt{\frac{3}{2}\left(\frac{N_C}{3} + \frac{N_f}{6}\right)}$$

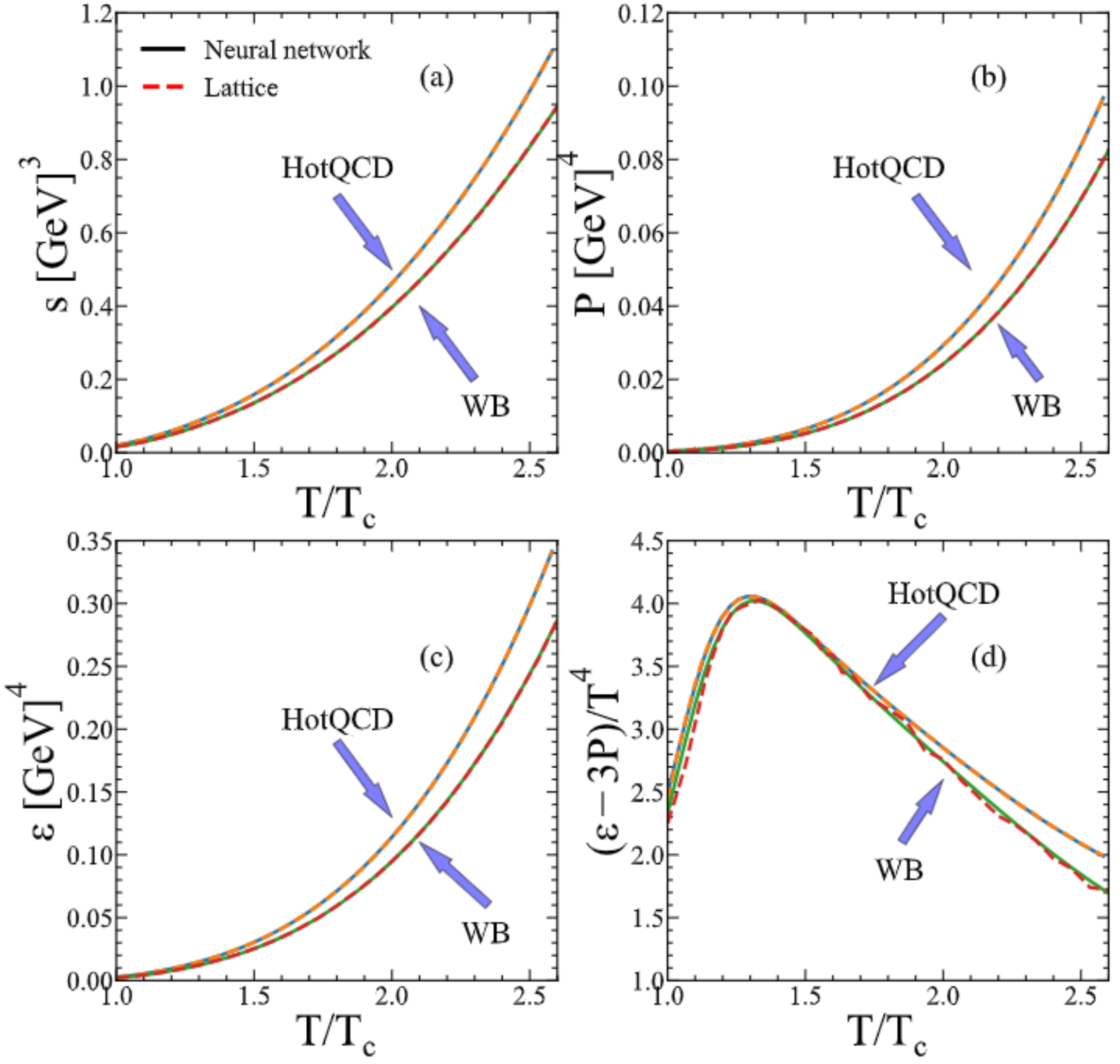
T_C=0.150 GeV
2.5 T_C=0.375GeV

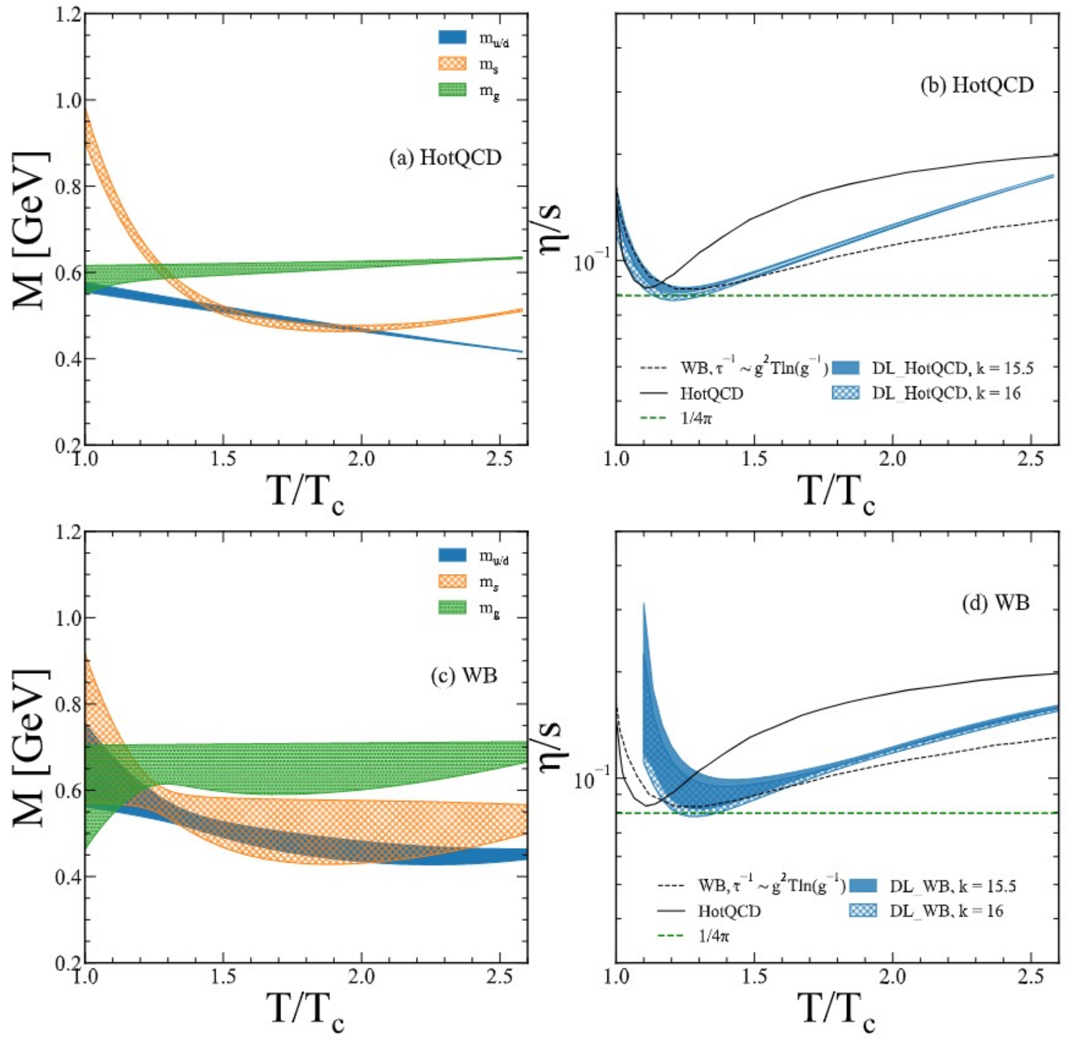
$$\mathcal{L}_1 = \left|R_{g/q} - \frac{3}{2}\right|.$$

$$\mathcal{L}_2 = \left|\frac{m_s - m_{u/d}}{\overline{m}_s - \overline{m}_{u/d}} - 1\right|.$$

$$\mathcal{L}_{MC} = (\beta_1\mathcal{L}_1 + \beta_2\mathcal{L}_2)^2$$

Result





FPL, Hong-Liang Lü, Long-Gang Pang & Guang-You Qin, Phys.Lett.B 844 (2023), 138088

Result: Extended work

DLQPM (data-driven):

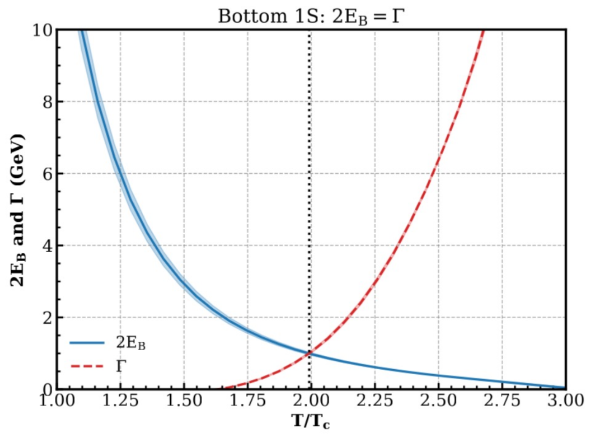
- Train ResNet on lattice-QCD EoS (s, $\varepsilon - 3p$) in $T/T_c \in [1, 3]$.
- Outputs quasi-parton masses $m_g(T), m_q(T)$.
- Infer $\alpha_s(T)$ via thermal-mass relations; then $m_D(T) = T\sqrt{4\pi\alpha_s(T)\left(\frac{N_c}{3} + \frac{N_f}{6}\right)}$.

Radial Schrödinger (S-wave):

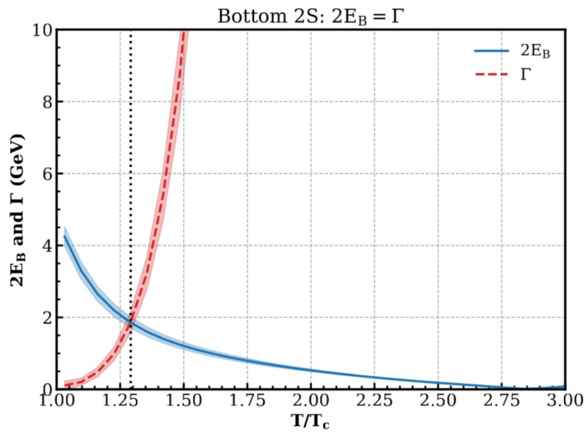
$$\left[-\frac{1}{2\mu}\frac{d^2}{dr^2} + \frac{\ell(\ell+1)}{2\mu r^2} + \mathrm{Re}\, V(r,T)\right]\psi_n = E_n(T)\psi_n, \quad \mu = \frac{m_Q}{2}.$$

Thermal width (first-order perturbation):

$$\Gamma_n(T) = -\int_0^\infty 4\pi r^2\,|\psi_n(r)|^2\,\mathrm{Im}\,V(r,T)\,dr.$$



$\Upsilon(1S)$: width criterion.



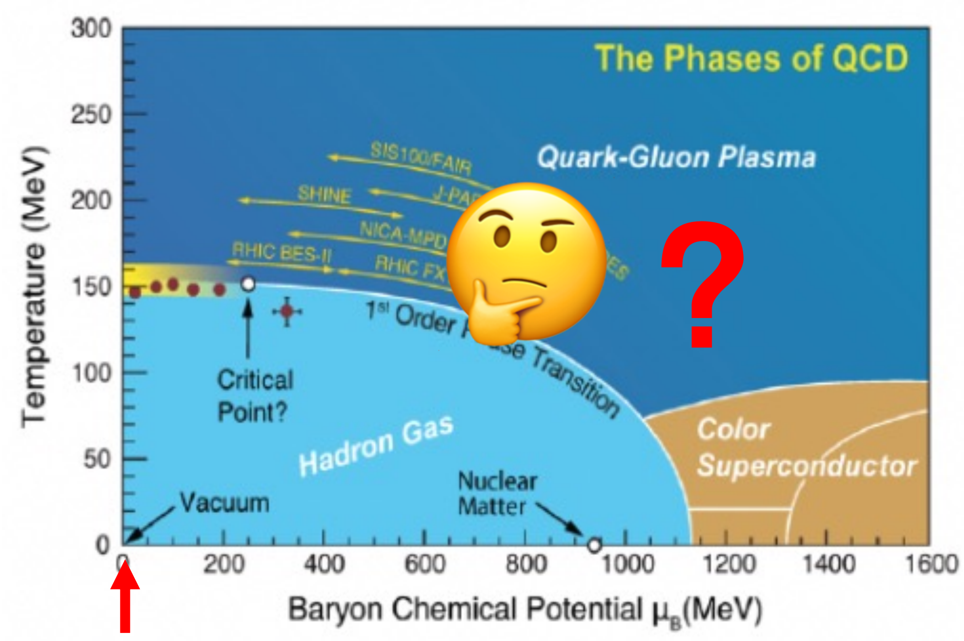
$\Upsilon(2S)$: width criterion.

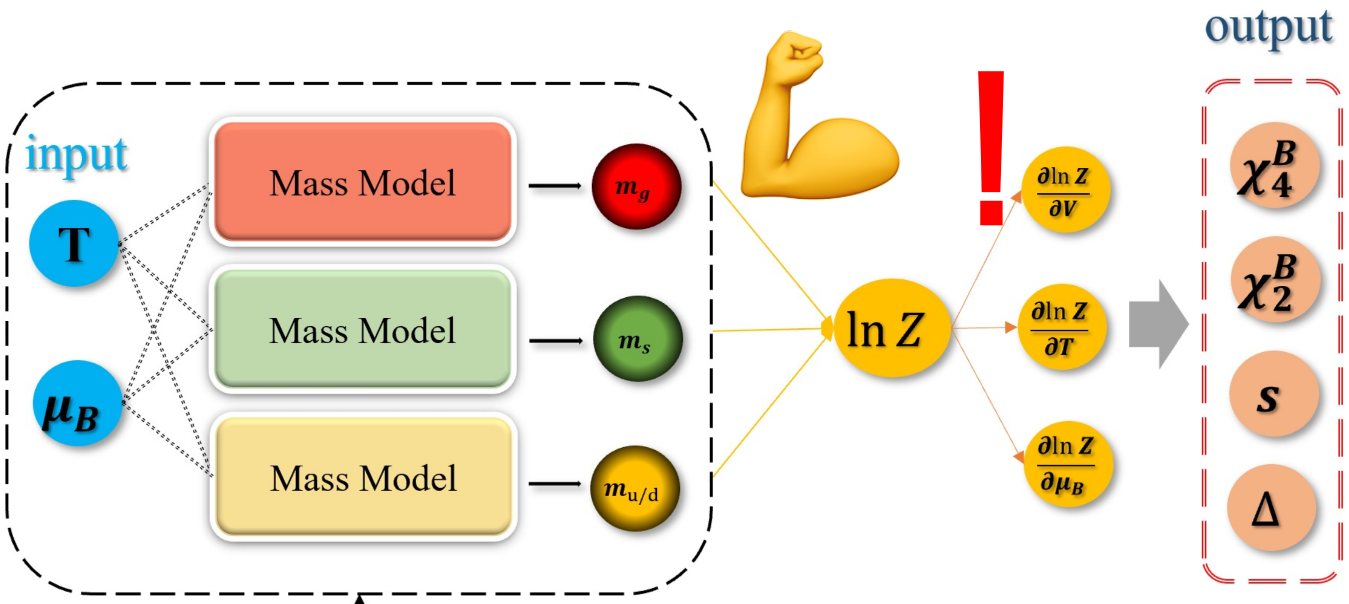
Method / Reference	Dissociation Temperatures T_d (in units of T_c)			
	$\Upsilon(1S)$	$\Upsilon(2S)$	J/ψ	$\psi(2S)$
This work: ($2E_B = \Gamma$)	1.99	1.29	1.30	≤ 1.00
This work: ($E_B = 3T$)	1.38	1.10	1.13	≤ 1.00
Mocsy and Petreczky [31]	2.00	1.20	1.20	≤ 1.00
Digal et al. [41]	2.31	1.10	1.10	< 1.00
Satz [42]	> 4.00	1.60	2.10	1.12
Blaschke et al. [43]	2.25	1.05	1.20	< 1.00
Rethika et al. [44]	0.77	0.82	1.47	1.62
Meng et al. [45]	5.81	1.56	2.06	1.13
Jamal et al. [33]	2.96	1.47	1.52	< 1.00
Agotiya et al. [46]	2.60	2.10	1.90	1.70

TABLE I: Comparison of the dissociation temperatures T_d for selected quarkonium states. The first two rows show the results of this work using the thermal width criterion ($2E_B = \Gamma$) and the lower bound criterion ($E_B = 3T$). The lower rows present representative theoretical predictions from the literature [31, 33, 41–46]. All temperatures are in units of the QCD critical temperature T_c.

Mohammad Yousuf Jamal, **FPL**, Long-Gang Pang & Guang-You Qin, arXiv:2509.14970

The framework of DNN: $\mu_B > 0$





$$\mathcal{L}(\boldsymbol{W}, \boldsymbol{b}) = |s_{NN} - s_{input}| + \left|\frac{\Delta_{NN} - \Delta_{input}}{T}\right| + |\chi^B_{2,NN} - \chi^B_{2,input}| + |\chi^B_{4,NN} - \chi^B_{4,input}| + \mathcal{L}_{MC}$$

$$\ln Z_g(T,\mu_B) = -\frac{d_g V}{2\pi^2}\int_0^\infty p^2 dp \ln\left[1 - \exp\left(-\frac{1}{T}\sqrt{p^2 + m_g^2(T,\mu_B)}\right)\right],$$

$$\ln Z_{q(\bar{q})_i}(T,\mu_B) = +\frac{d_{q(\bar{q})_i} V}{2\pi^2}\int_0^\infty p^2 dp \ln\left[1 + \exp\left(-\frac{1}{T}\left(\sqrt{p^2 + m^2_{q(\bar{q})_i}(T,\mu_B)} - \mu_{q(\bar{q})_i}\right)\right)\right],$$

$$\mu_u = \frac{1}{3}\mu_B + \frac{2}{3}\mu_Q,$$

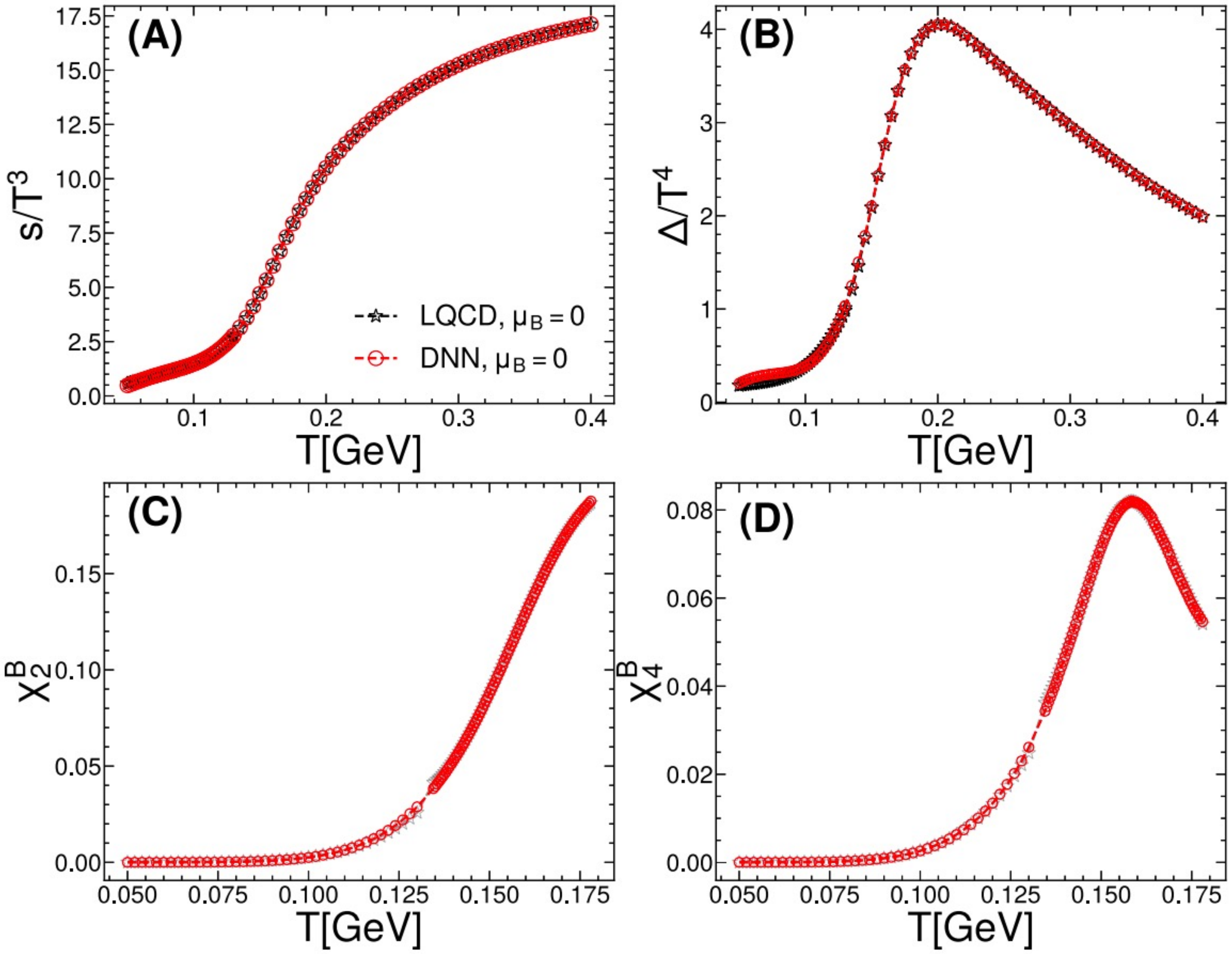
$$\mu_d = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q,$$

$$\mu_s = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q - \mu_S,$$

$$\chi_i^B = \left.\frac{\partial P(T,\hat{\mu}_B)/T^4}{\partial \hat{\mu}_B^i}\right|_{\hat{\mu}_B=0}, \hat{\mu}_B = \mu_B/T.$$

FPL, Long-Gang Pang & Guang-You Qin, Phys. Lett. B 868 (2025) 139692

The training data&result

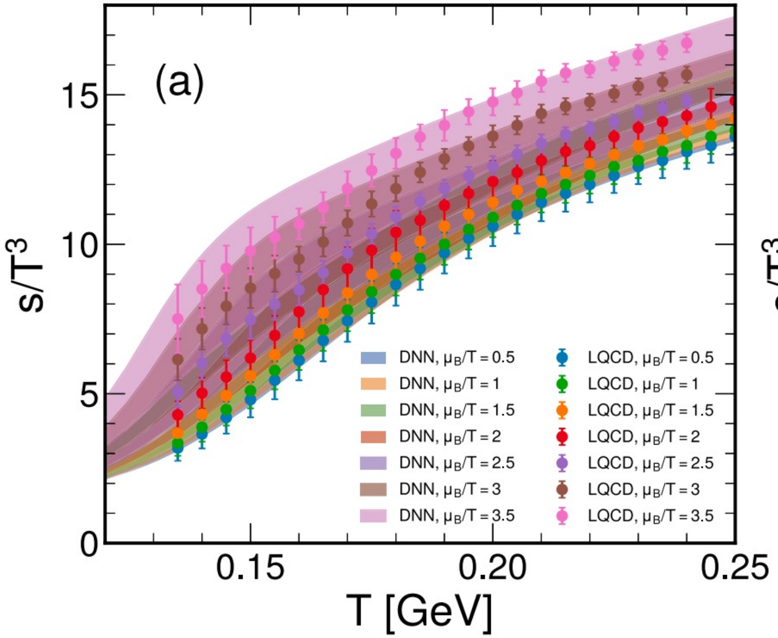


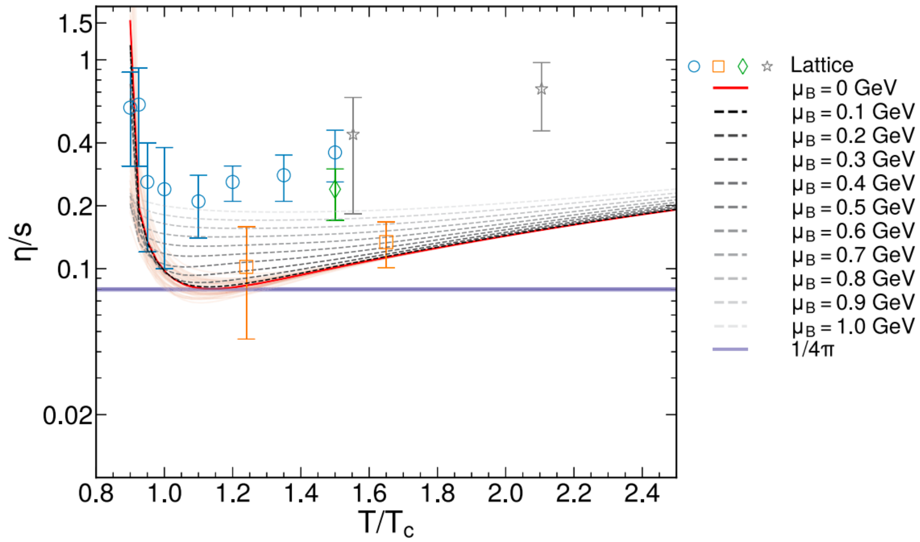
Training data:

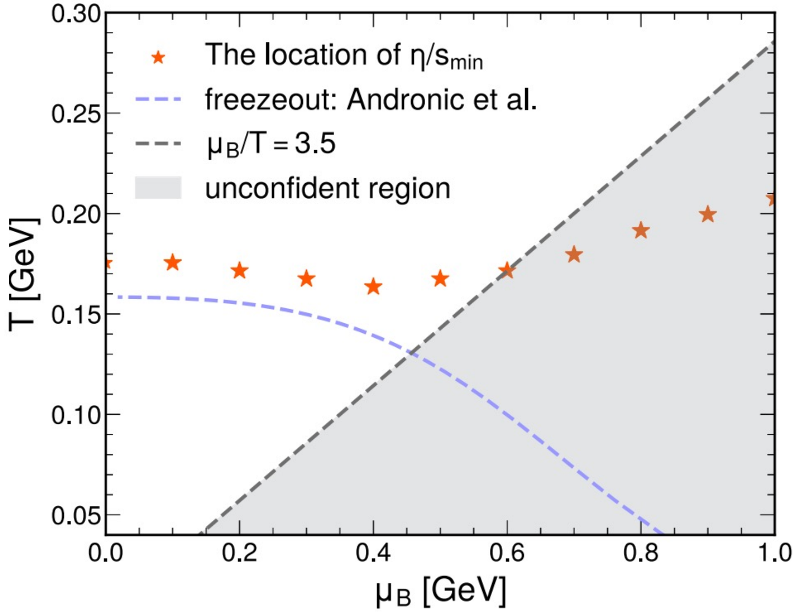
Phys Rev D 95, 054504 (2017)
Phys Rev Lett 118, 182301 (2017)
Phys Rev D 90, 094503 (2014)

FPL, Long-Gang Pang & Guang-You Qin, Phys. Lett. B 868 (2025) 139692

Result

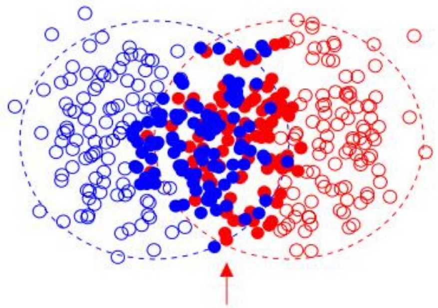






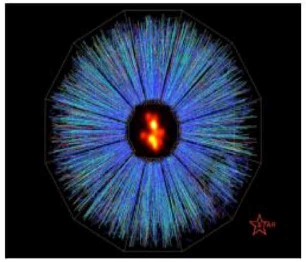
FPL, Long-Gang Pang & Guang-You Qin, Phys. Lett. B 868 (2025) 139692

The framework of DNN: 4D DLQPM



$$\nabla_\mu T^{\mu\nu} = 0$$

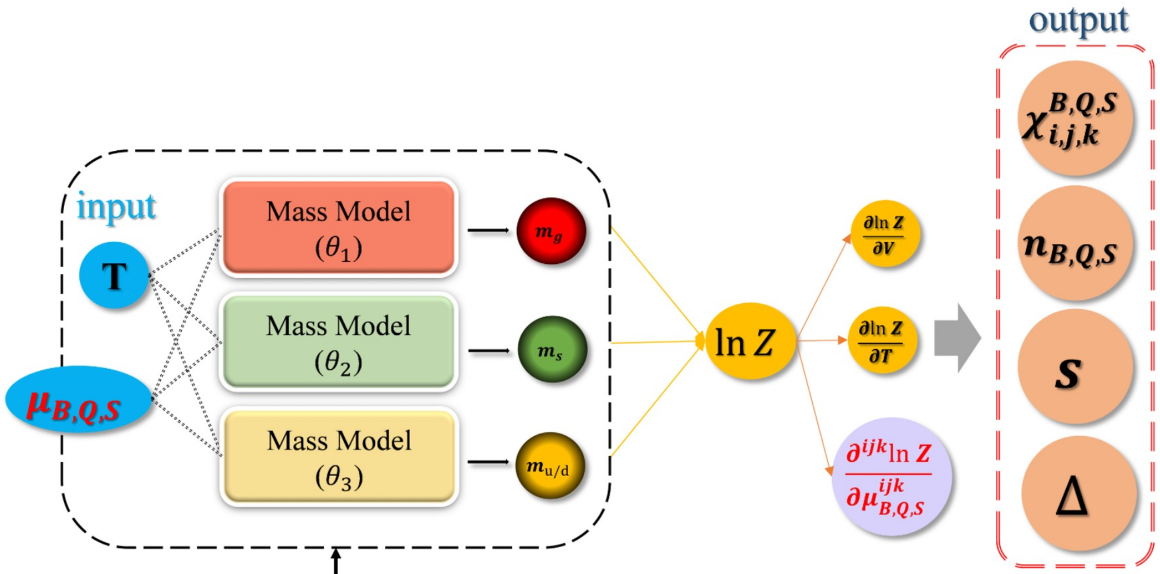
$$T^{\mu\nu} = (\varepsilon + P + \Pi)u^\mu u^\nu - (P + \Pi)g^{\mu\nu} + \pi^{\mu\nu}$$



Initial condition

EoS Bulk Viscosity

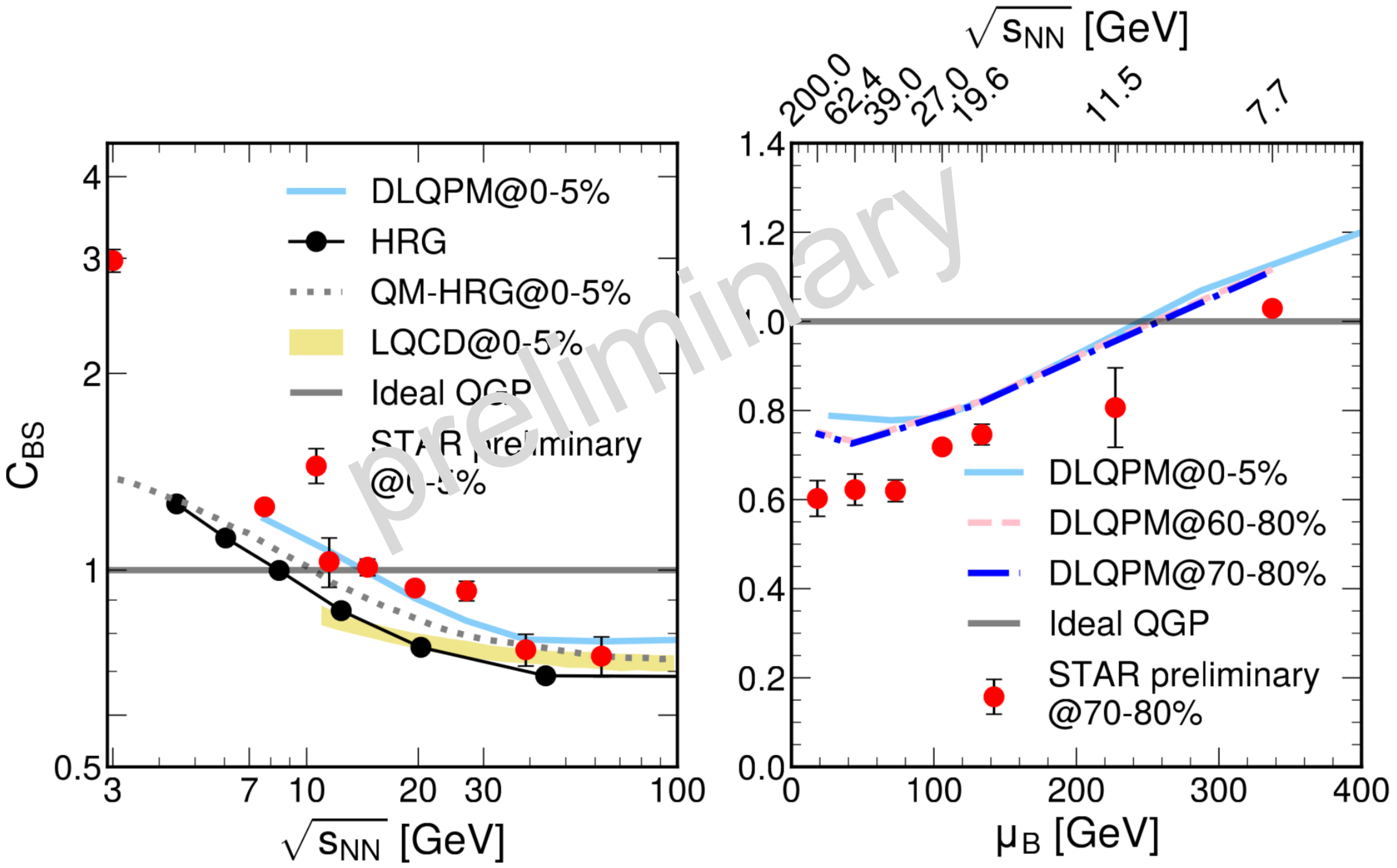
Shear Viscosity



$$\mathcal{L}(\theta_1, \theta_2, \theta_3) = |s_{DNN} - s_{LQCD}| + \left|\frac{\Delta_{DNN} - \Delta_{LQCD}}{T}\right| + |\chi^{B,Q,S}_{i,j,k,DNN} - \chi^{B,Q,S}_{i,j,k,LQCD}| + |n_{B,Q,S}| + |\ | + \mathcal{L}_{MC}$$

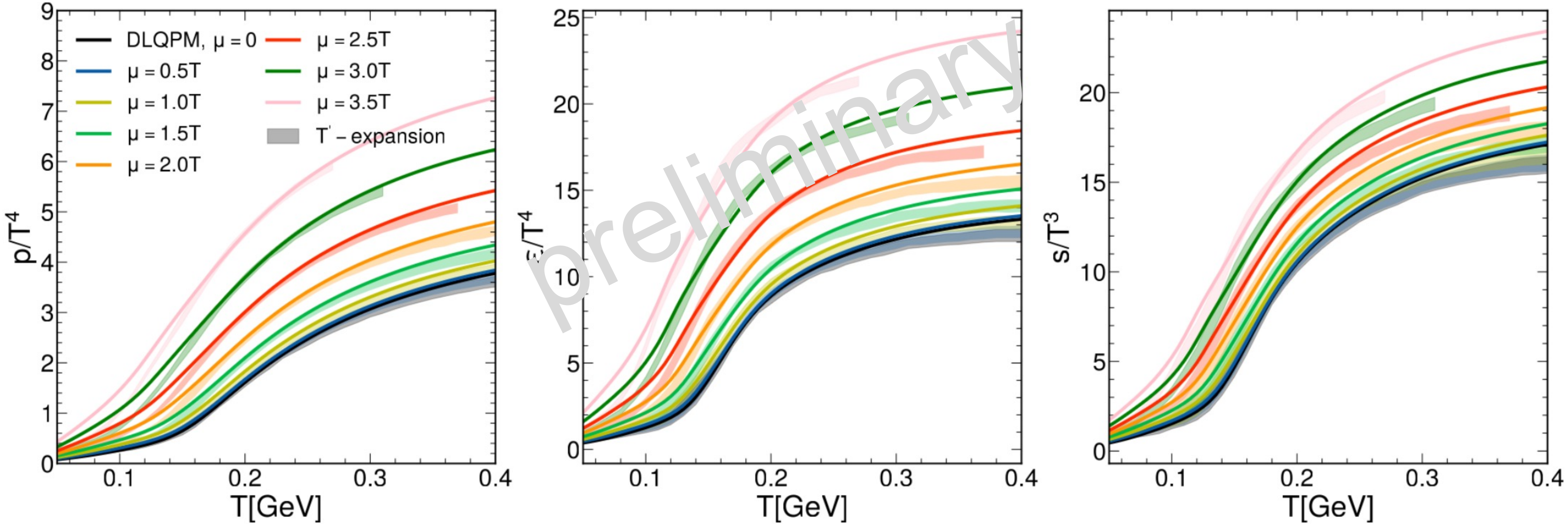
FPL, Long-Gang Pang & Guang-You Qin, in preparation

Result



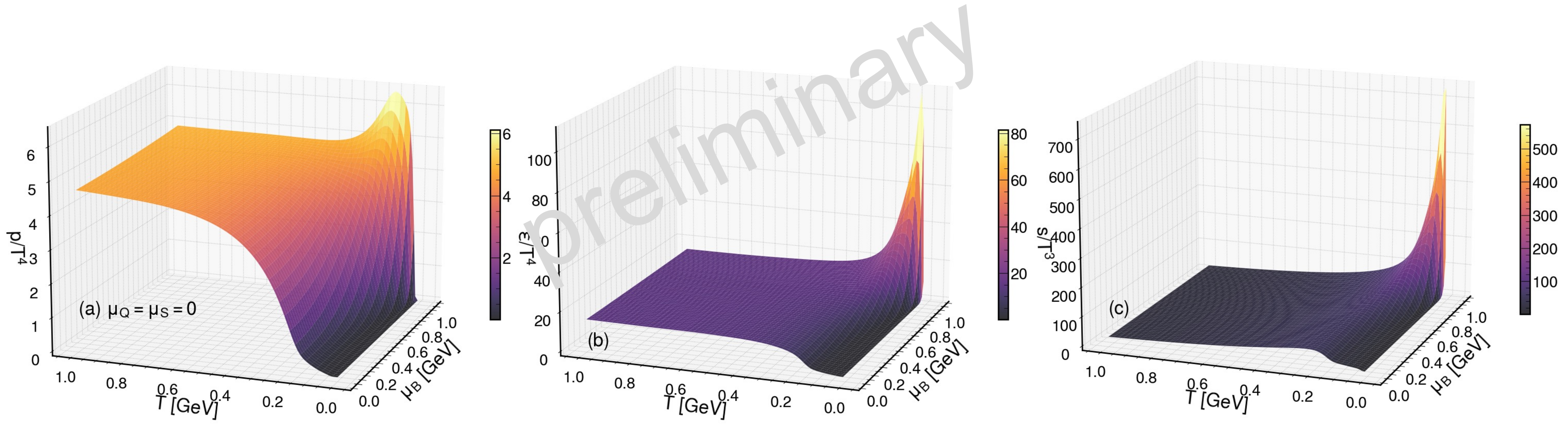
FPL, Long-Gang Pang & Guang-You Qin, in preparation

Result



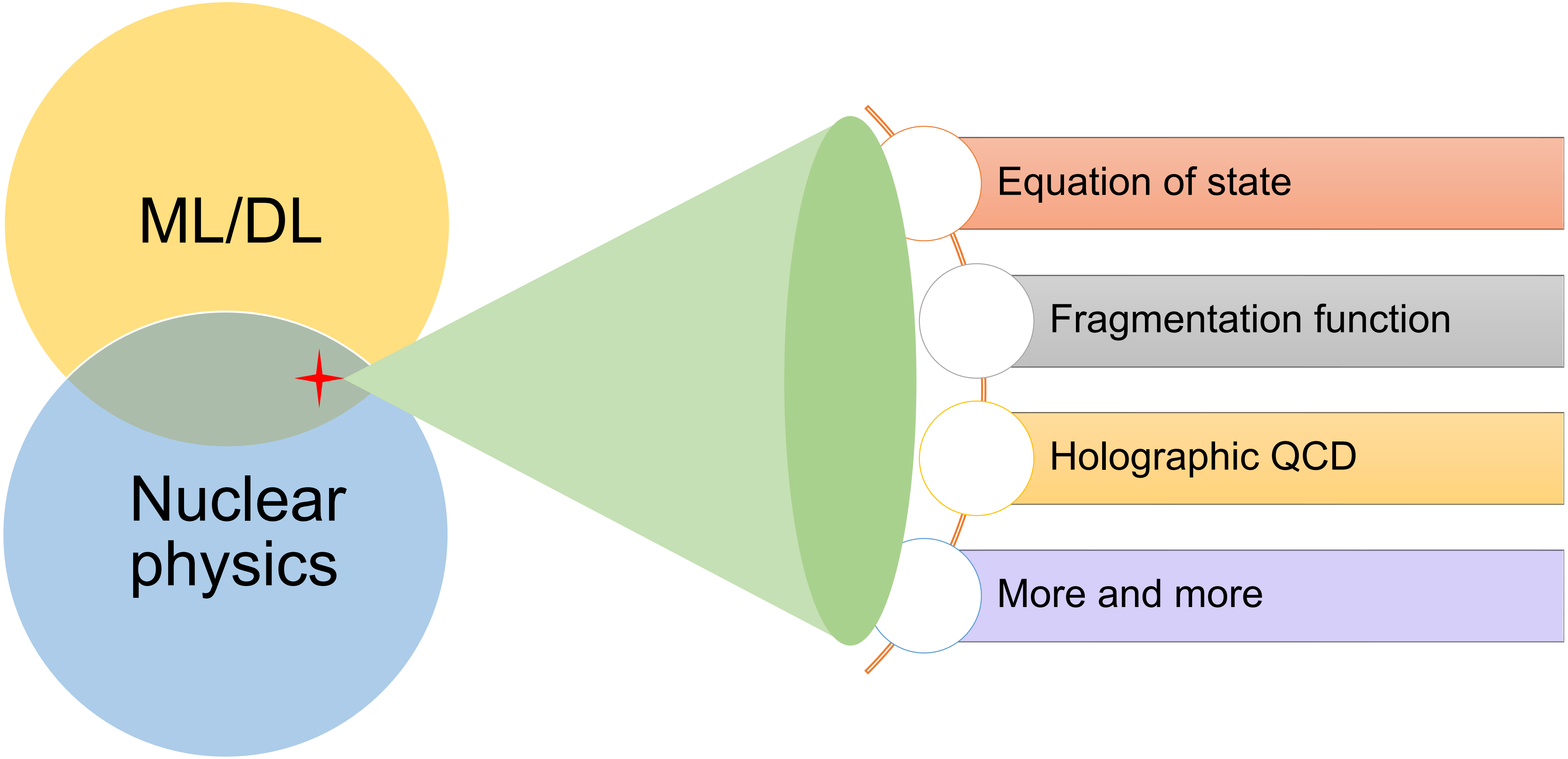
FPL, Long-Gang Pang & Guang-You Qin, in preparation

Result



FPL, Long-Gang Pang & Guang-You Qin, in preparation

Inverse problems in nuclear physics



Reviews:
Exploring QCD matter in extreme conditions with Machine Learning(2024)
Colloquium: Machine learning in nuclear physics(2022)
Modern Machine Learning and Particle Physics(2021)
Machine learning and the physical sciences (2019)

DL for Parton Fragmentation Functions

Framework of the neural networks

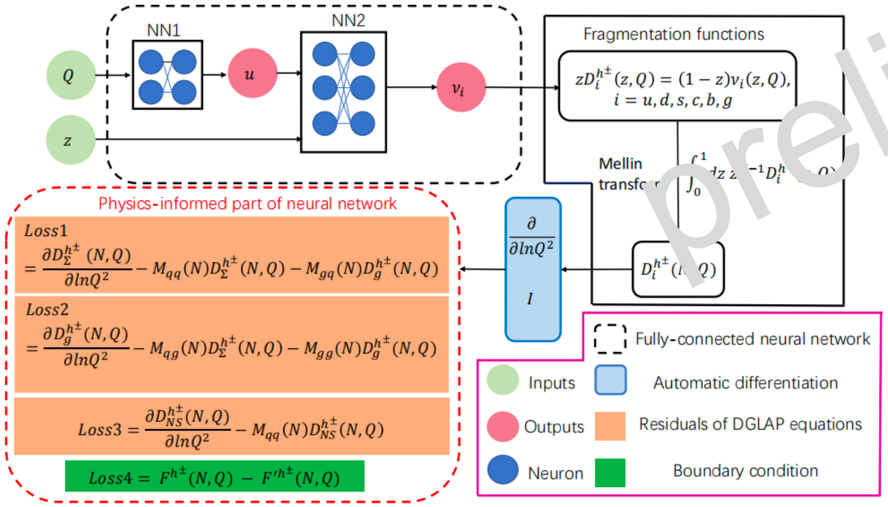
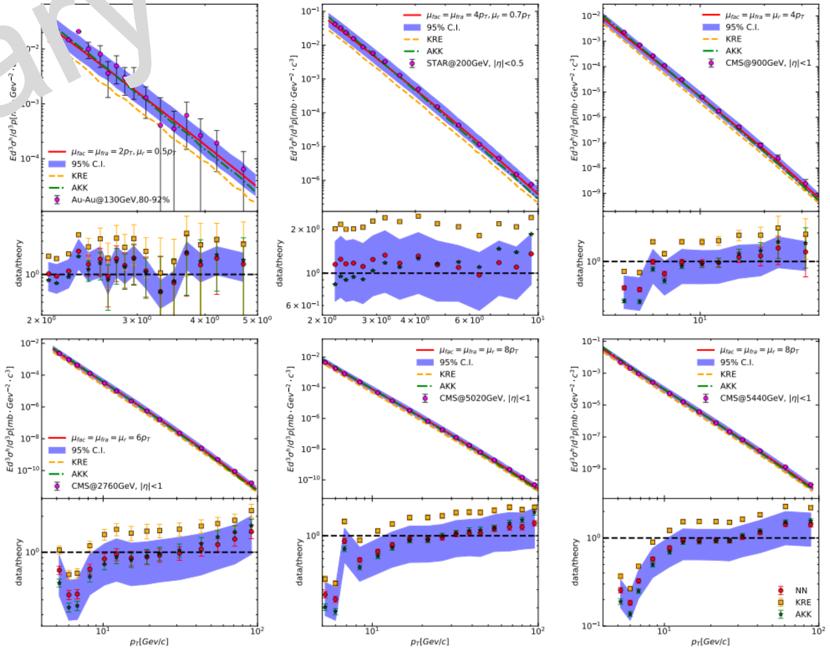


Figure 1. The neural network framework for FFs extractions.

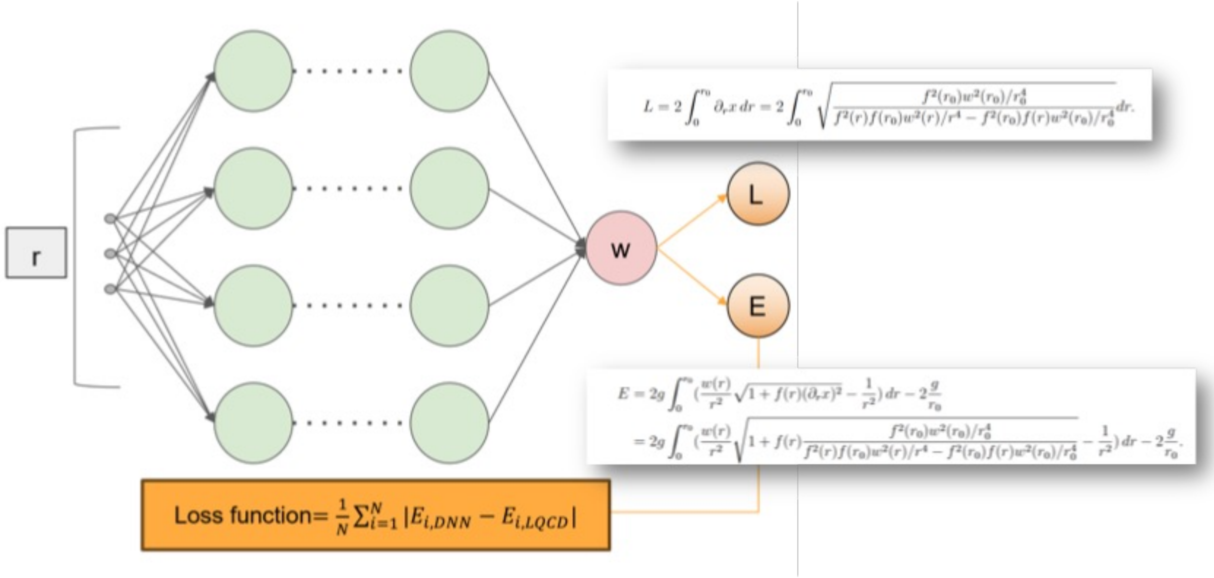
Fitting hadron spectra in pp collisions



per panels: comparison of our NLO results for single-inclusive charged hadron production $pp \to h + X$, where $h = h^+ + h^-$, with data from the STAR, PHENIX and CMS data for various c.m.s energies $\sqrt{s}$. Also shown are the results obtained with the KRE and AKK parameterizations. Lower panels: data/theory comparison for our results and the KRE and AKK parameterizations.

Si-Wei Dai, **FPL**, Long-Gang Pang, Xin-Nian Wang, Ben-Wei Zhang & Han-Zhong Zhang, in preparation

DL for hQCD



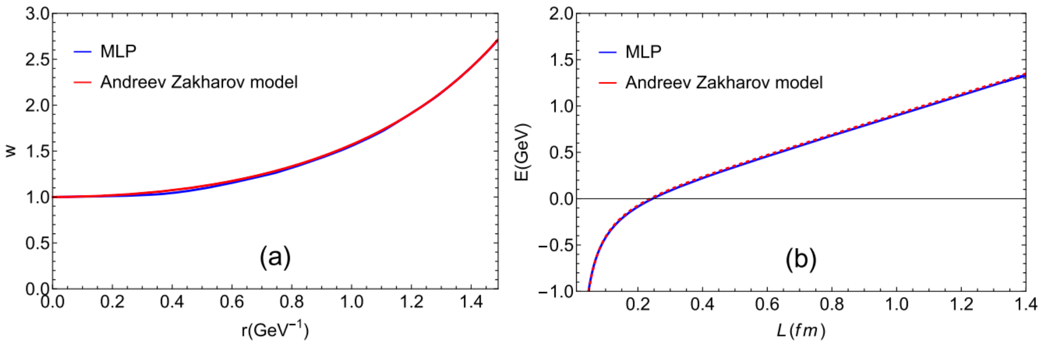


Fig. 3 **a** The comparison of $w(r)$ from the MLP and the Andreev–Zakharov model. **b** The comparison of $E(L)$ from Andreev–Zakharov model with $E(L)$ from MLP at vanishing temperature and chemical potential

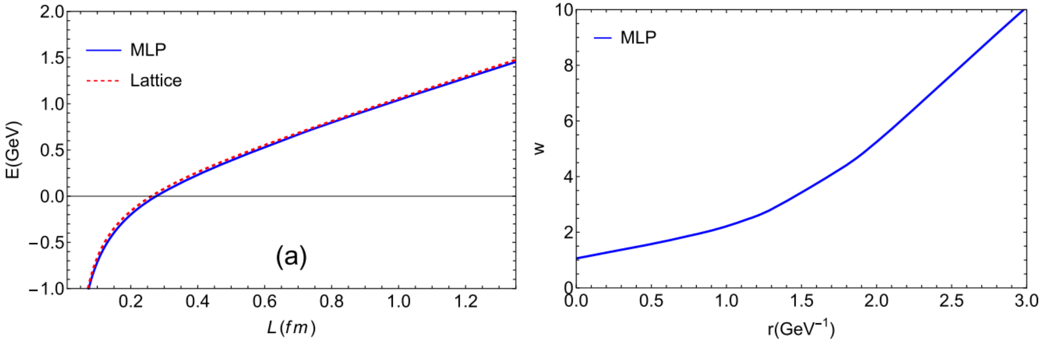
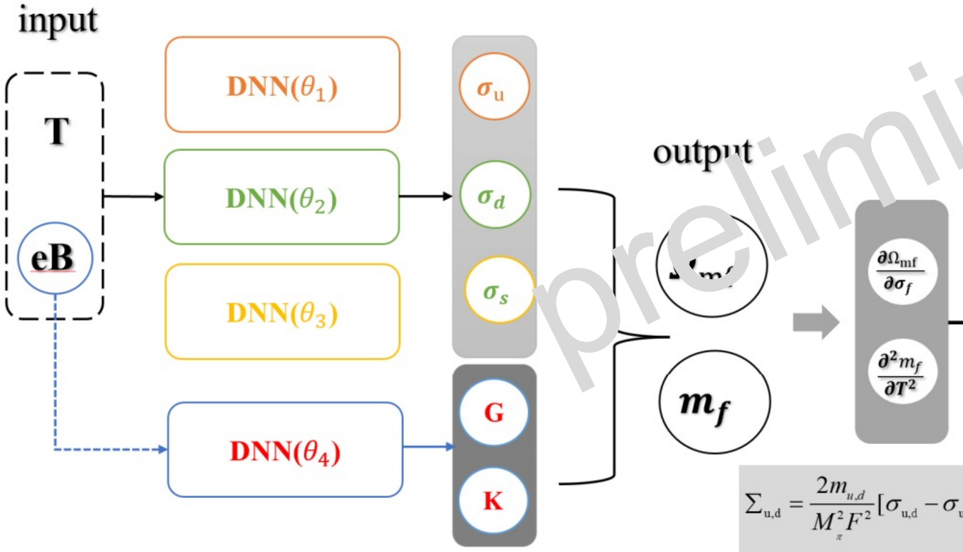


Fig. 4 **a** Comparison of lattice QCD results [108] with MLP performance on the training dataset, illustrating the relationship between L and E at vanishing temperature and chemical potential. **b** Reconstruction of the function $w(r)$ from MLP, with g set to 0.176

Ou-Yang Luo, Xun Chen, **FPL**, Xiao-Hua Li & Kai Zhou, Eur.Phys.J.C 85 (2025) 6, 637

DL for QCD in external magnetic fields



$$
\begin{aligned}
m_u &= m_0^u - 4G\sigma_u + 2K\sigma_d\sigma_s,\\
m_d &= m_0^d - 4G\sigma_d + 2K\sigma_u\sigma_s,\\
m_s &= m_0^s - 4G\sigma_s + 2K\sigma_u\sigma_d,
\end{aligned}
$$

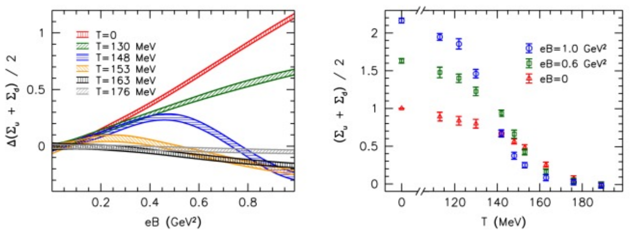
$$E_f = \sqrt{p_z^2 + 2l|Q_f B| + m_f^2}$$

$$
\begin{aligned}
\Omega_{\rm mf} &= 2G(\sigma_u^2+\sigma_d^2+\sigma_s^2) - 4K\sigma_u\sigma_d\sigma_s + \Omega_q,\\
\Omega_q &= -3\sum_{f=u,d,s}\frac{|Q_f B|}{2\pi}\sum_l \alpha_l \int\frac{dp_z}{2\pi}\\
&\quad \left[E_f + 2T\ln\left(1+e^{-\frac{E_f}{T}}\right)\right],
\end{aligned}
$$

$$\Sigma_{u,d} = \frac{2m_{u,d}}{M_\pi^2 F^2}[\sigma_{u,d} - \sigma_{u,d}(0,0)] + 1,$$

$$\Delta\Sigma_{u,d} = \frac{2m_{u,d}}{M_\pi^2 F^2}[\sigma_{u,d} - \sigma_{u,d}(0,T)]$$

$$\mathcal{L}(\theta_1,\theta_2,\theta_3,\theta_4) = \frac{\partial\Omega_{\rm mf}}{\partial\sigma_f} + \frac{\partial^2 m_f}{\partial T^2} + |\Sigma_{i,DNN} - \Sigma_{i,LQCD}|$$



$$\Sigma_{u,d}(B,T) = \frac{2m_{ud}}{M_\pi^2 F^2}[\bar\psi\psi_{u,d}(B,T) - \bar\psi\psi_{u,d}(0,0)] + 1,$$

$$\Delta\Sigma_{u,d}(B,T) = \Sigma_{u,d}(B,T) - \Sigma_{u,d}(0,T) = \frac{2m_{ud}}{M_\pi^2 F^2}[\bar\psi\psi_{u,d}(B,T) - \bar\psi\psi_{u,d}(0,T)].$$

FPL, Shi-Jun Mao & Lingxiao Wang, in preparation

Summary

➢ Deep learning methods can be used for inverse problems in NP:

I. **Identifying** the nuclear equation of state
   - The DL model can assist us in mapping the relationship between final state observables and initial nuclear equation of state.

II. **Reconstructing** the QCD equation of state
   - We use three neural networks to represent the quasi-particles masses can well reproduce the lattice QCD EoS at zero chemical potential.
   - We can calculate the entropy density at finite baryon chemical potentials, which is consisted with Lattice QCD result using Taylor expansions.
   - The QCD equation of state at finite chemical potential can be used in relativistic hydrodynamics simulations to study the QCD matter produced in the BESII region.

III. **Reconstructing** the parton FFs, the metric of hQCD and ME.

Last but not least

- ML/DL provides us with promising methods and ideas for studying physics.

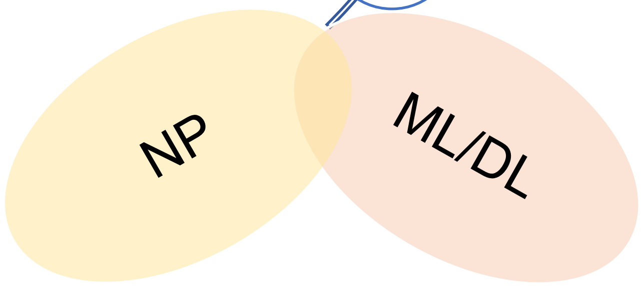
for others

for small x physics

for Quantum Computing

for nuclear ab-initio calculation

for QCD equation of state



Thank you for your attention!