第四届全国核物理及核数据中的机器学习应用研讨会 2025年10月30日-11月3日 湖南衡阳

（2022，江西瑞昌（主持讨论）；2023，广西桂林（挂职）；2024，浙江湖州（主办））

基于机器学习与贝叶斯分析的核物质状态方程研究

李庆峰

湖州师范学院

合作者：魏国俊、李志龙、高泽鹏、李甫鹏、王永佳

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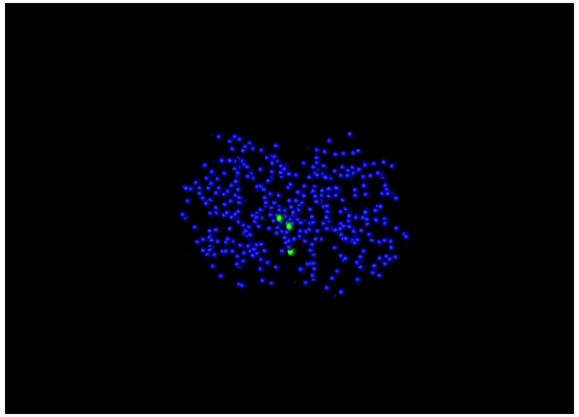
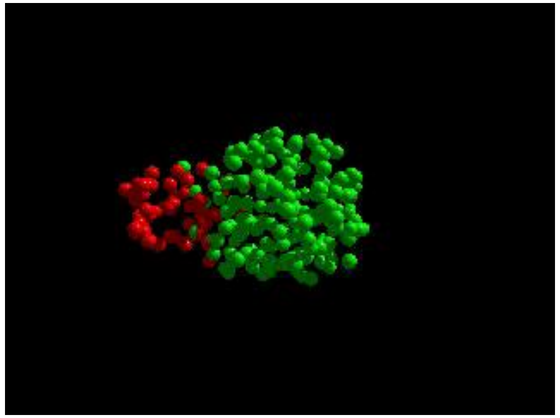
I、 Background & Motivation

ρ: Overview of HIC

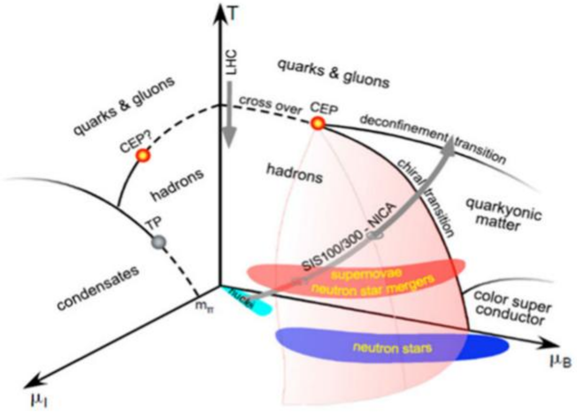
◆ intermediate energies HIC



◆ Low energy fusion reaction



~100 MeV- a few GeV/nucleon



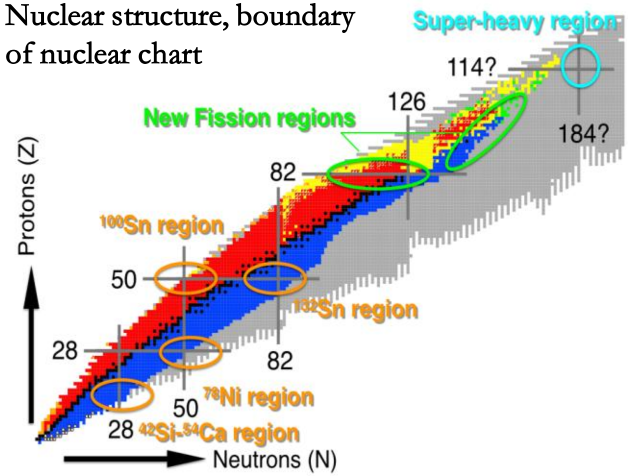
QCD phase diagram,
QGP properties

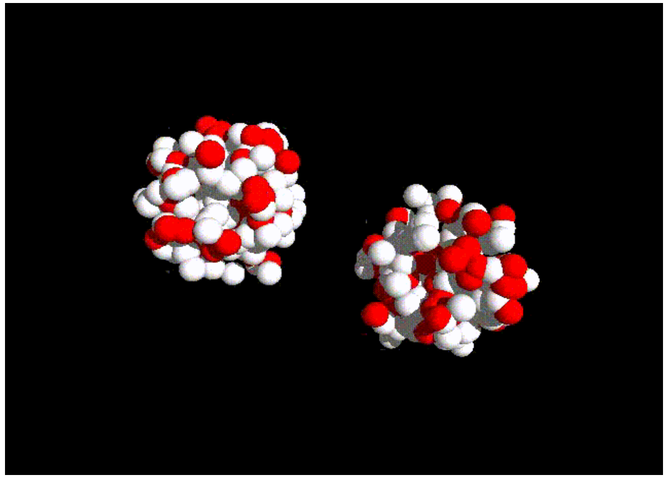
~100 MeV/nucleon

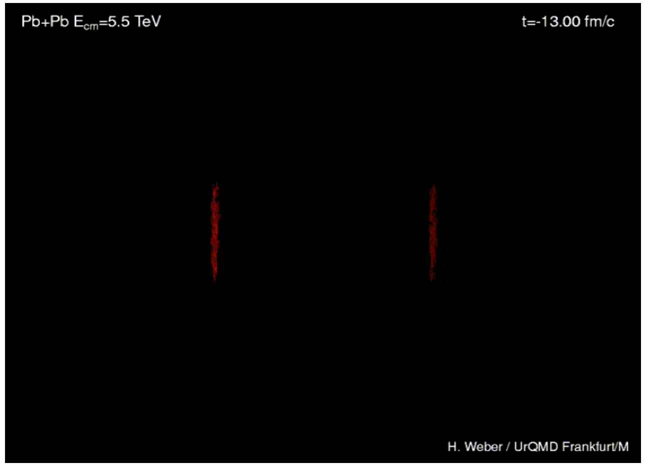
> hundreds of GeV/nucleon

beam energy

Nuclear structure, boundary of nuclear chart







◆ Relativistic HIC

I、Background & Motivation

K: Isospin-dependent EOS---parameters

- "equation of state"(EOS): energy/nucleon vs. density

$$\varepsilon(\rho,T) = \varepsilon_T(\rho,T) + \varepsilon_C(\rho,T=0) + \varepsilon_0 \qquad (\varepsilon = E/A)$$

thermal compressional ground state energy

- nuclear equation-of-state at T=0: the "compressional" energy

$$E/A(\rho,T=0) = \frac{1}{\rho}\int U(\rho)d\rho$$

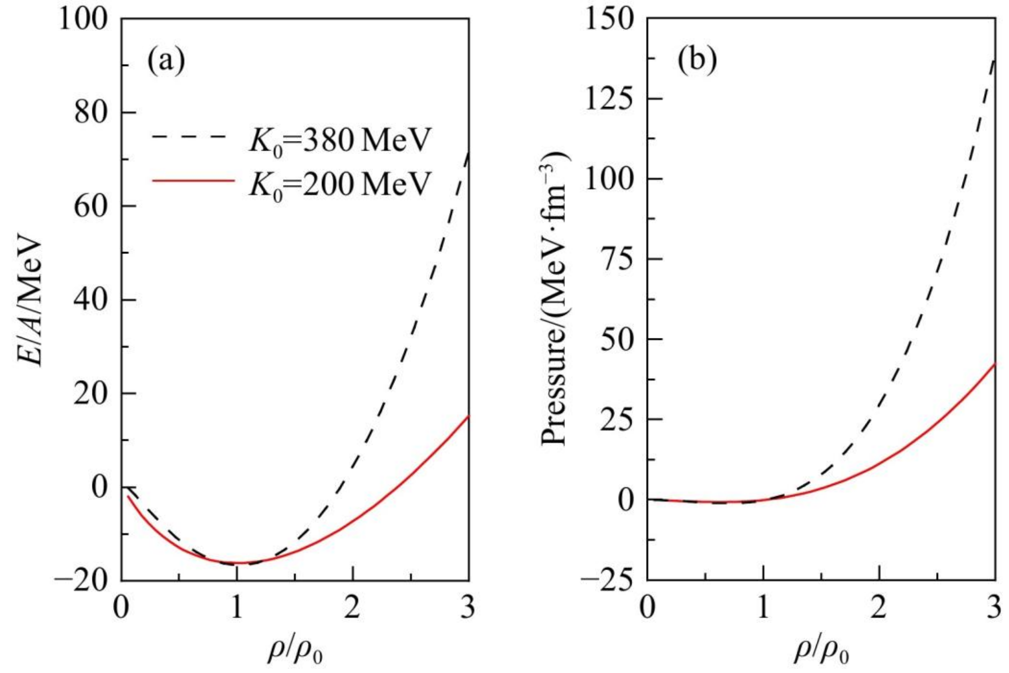
- $U(\rho)$ density dependent local potential

- compression modulus κ

$$\kappa = \left(9\rho^2 \frac{\partial^2 E/A(\rho,T=0)}{\partial \rho^2}\right)_{\rho=\rho_0}$$

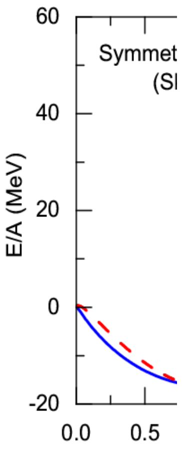
- constraints $\varepsilon(\rho=\rho_0, T=0) = -16\ MeV$

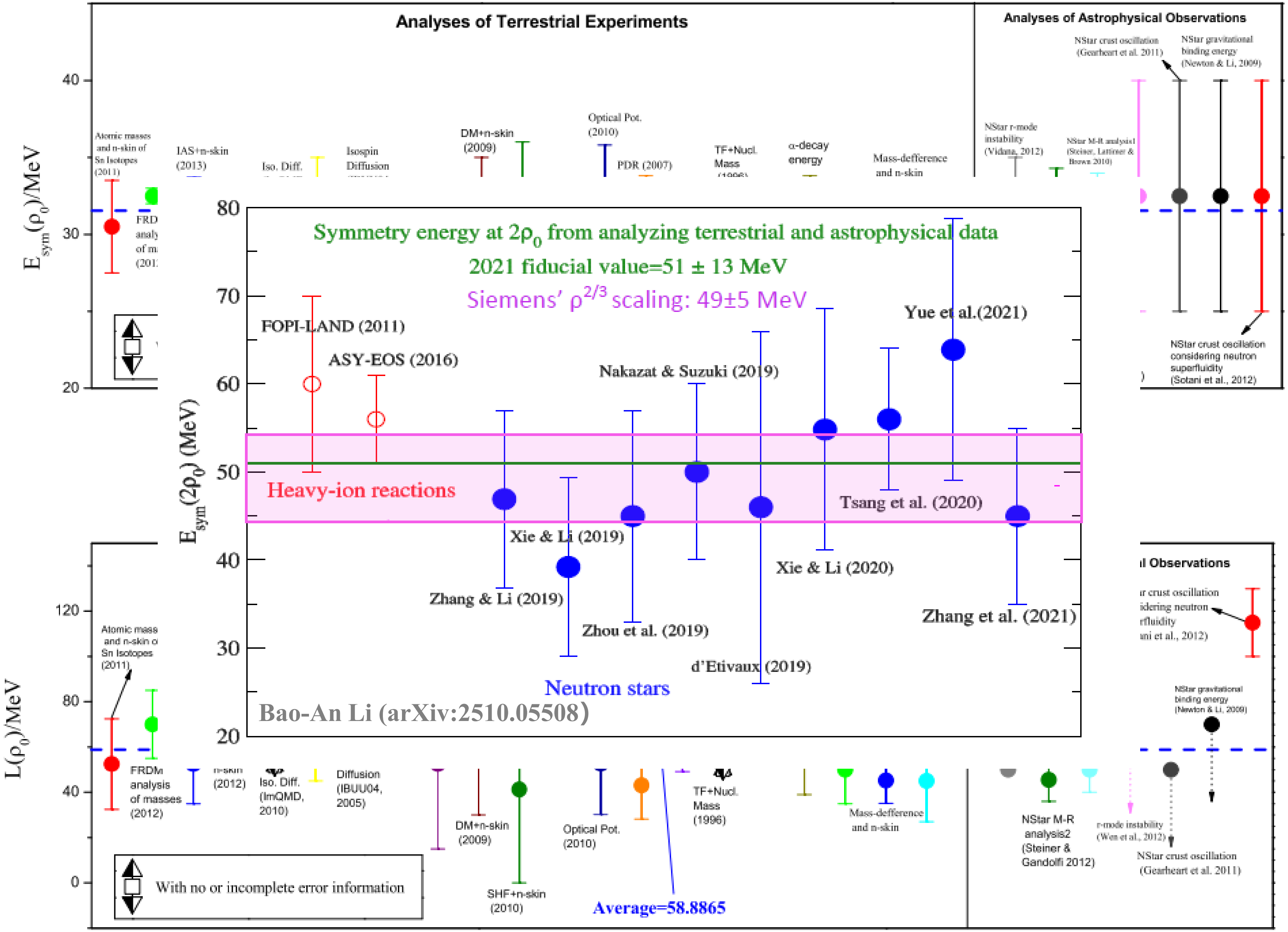
$$\left(\frac{\partial \varepsilon(\rho,T=0)}{\partial \rho}\right)_{\rho=\rho_0} = 0$$

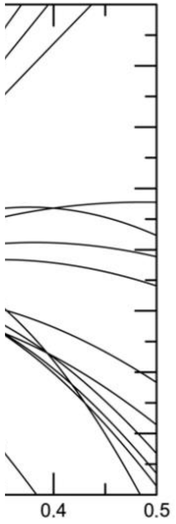


I、Bac

Sym: Isos
parameter







E_{sym} (

E_{sym} (

$L \equiv$ 3

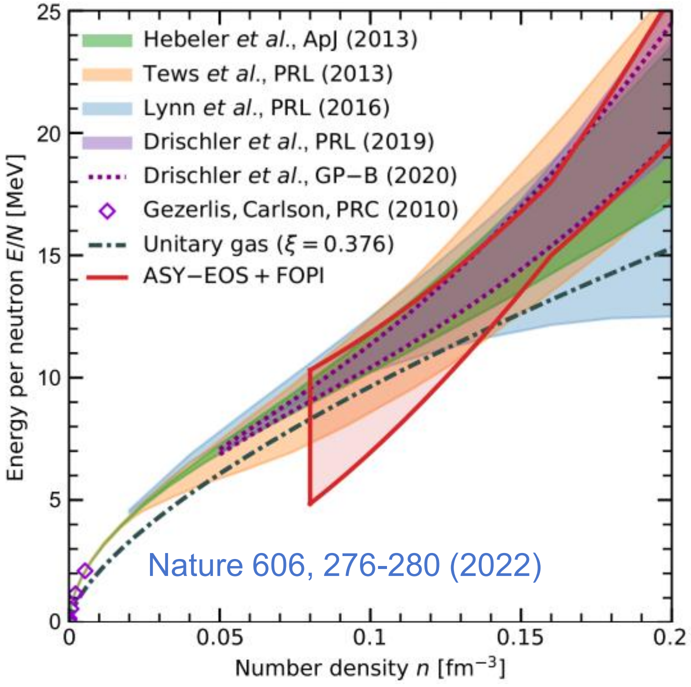
K_{sym}

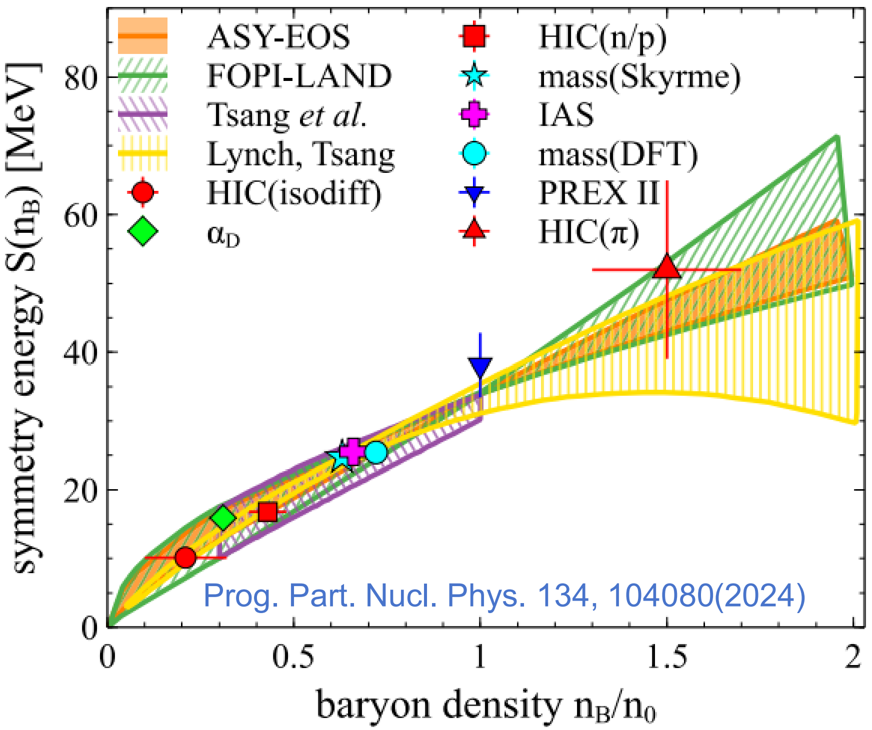
I、Background & Motivation

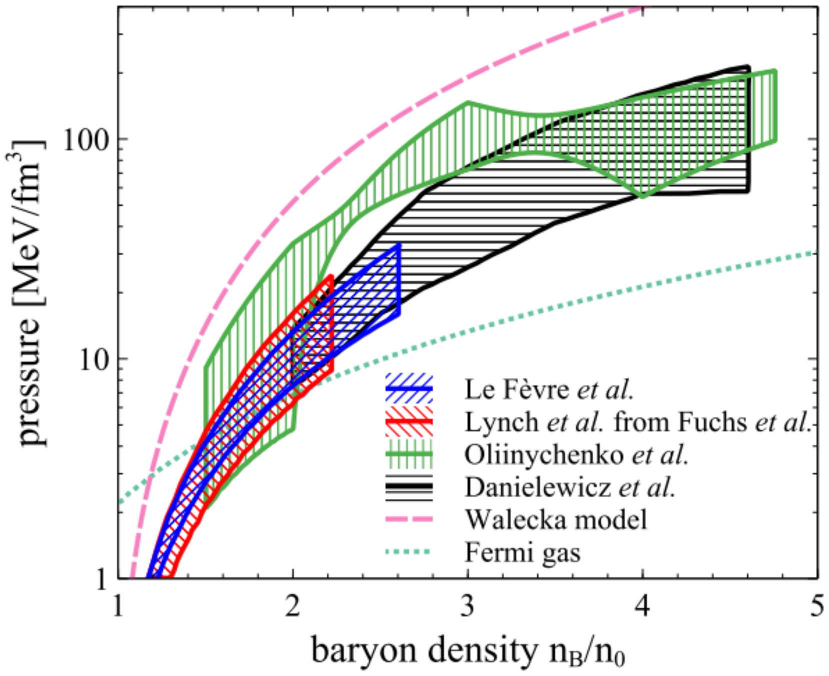
IEOS: phase diagram and the paths

- EoS→**I**EoS

$$E(\rho,\delta) = E(\rho,0) + E_{sym}(\rho)\delta^2 + O(\delta^4), \qquad \delta = (\rho_n - \rho_p/\rho)$$





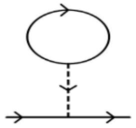


I、Background & Motivation

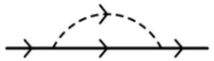
How to describe the transport process self-consistently ---theories (e.g., RBUU)

Transport Equation

$$\{[\partial_x^\mu - \Sigma_{HF}^{\mu\nu}(x,p,\tau)\partial_\nu^p - \partial_p^\nu \Sigma_{HF}^\mu(x,p,\tau)\partial_\nu^x]p_\mu + m^*_{N,\Delta}[\partial_\nu^x \Sigma_{HF}^S(x,p,\tau)\partial_p^\nu - \partial_p^\nu \sum\nolimits_{HF}^S(x,p,\tau)\partial_\nu^x]\}\frac{f_{N,\Delta}(x,p,\tau)}{E^*} = C(x,p,\tau).$$



Hartree



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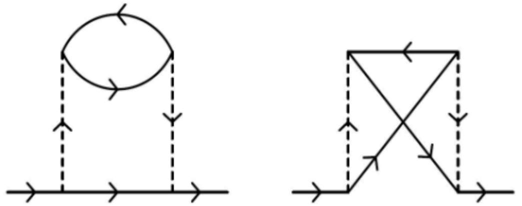


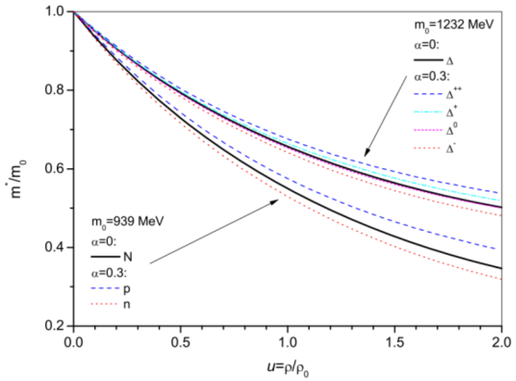
Bonn

G. J. Mao, Relativistic microscopic quantum transport equation, Nova Science Publishers (2005)

Mean Field

$$\begin{aligned} L_F = {} & \bar{\Psi}[i\gamma_\mu\partial^\mu - m_N]\Psi + \bar{\Psi}_{\Delta\nu}[i\gamma_\mu\partial^\mu - m_\Delta]\Psi_\Delta^\nu \\ & + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{4}F_{\mu\nu}\cdot F^{\mu\nu} + \frac{1}{2}\partial_\mu\boldsymbol{\delta}\partial^\mu\boldsymbol{\delta} \\ & - \frac{1}{4}L_{\mu\nu}\cdot L^{\mu\nu} + \frac{1}{2}\partial_\mu\boldsymbol{\pi}\partial^\mu\boldsymbol{\pi} \\ & - U(\sigma) + U(\omega) - U(\boldsymbol{\delta}) + U(\boldsymbol{\rho}) - U(\boldsymbol{\pi}), \end{aligned}$$

$$\boxed{E_{\mathrm{Sym}}(\rho_B) = \frac{1}{6}\frac{k_f^2}{E_f} + \frac{1}{2}\left(\frac{\Gamma_\rho^2}{4} - \Gamma_\delta^2 \frac{M^{*2}}{{E_f}^2[1+\Gamma_\delta^2 A(k_f,M^*)]}\right)\rho_B}$$



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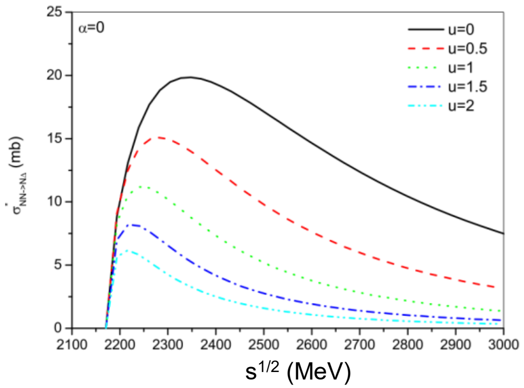
Q. F. Li, Z. X. Li, Phys. Lett. B 773, 557-562 (2017).
Q. F. Li, Z. X. Li, Sci. China-Phys. Mech. Astron. 62, 972011 (2019).

Collision Term

$$\begin{aligned} L_I = {} & g_{NN}^\sigma\bar{\Psi}\Psi\sigma - g_{NN}^\omega\bar{\Psi}\gamma_\mu\Psi\omega^\mu + g_{NN}^\delta\bar{\Psi}\boldsymbol{\tau}\cdot\Psi\boldsymbol{\delta} \\ & - \frac{1}{2}g_{NN}^\rho\bar{\Psi}\gamma_\mu\boldsymbol{\tau}\cdot\Psi\boldsymbol{\rho}^\mu \\ & + g_{\Delta\Delta}^\sigma\bar{\Psi}_{\Delta\nu}\Psi_\Delta^\nu\sigma - g_{\Delta\Delta}^\omega\bar{\Psi}_{\Delta\nu}\gamma_\mu\Psi_\Delta^\nu\omega^\mu + g_{\Delta\Delta}^\delta\bar{\Psi}_{\Delta\nu}\boldsymbol{\tau}\cdot\Psi_\Delta^\nu\boldsymbol{\delta} \\ & - \frac{1}{2}g_{\Delta\Delta}^\rho\bar{\Psi}_{\Delta\nu}\gamma_\mu\boldsymbol{\tau}\cdot\Psi_\Delta^\nu\boldsymbol{\rho}^\mu \\ & + g_{NN}^\pi\bar{\Psi}\gamma_\mu\gamma_5\boldsymbol{\tau}\cdot\Psi\partial^\mu\boldsymbol{\pi} - g_{N\Delta}^\pi\bar{\Psi}_{\Delta\mu}\partial^\mu\boldsymbol{\pi}\cdot S^+\Psi \\ & - g_{N\Delta}^\pi\bar{\Psi}S\Psi_{\Delta\mu}\cdot\partial^\mu\boldsymbol{\pi} \qquad (5 \end{aligned}$$

$$\begin{aligned} & C_{el,in}(x,p,\tau) \\ & = \frac{1}{2}\int\frac{d^3p_2}{(2\pi)^3}\int\frac{d^3p_3}{(2\pi)^3}\int\frac{d^3p_4}{(2\pi)^3}(2\pi)^4\delta^{(4)}(p_1+p_2-p_3-p_4) \\ & \quad \times W_{el,in}(p_1,p_2,p_3,p_4)[F_2 - F_1] \\ & = \frac{1}{2}\int\frac{d^3p_2}{(2\pi)^3}d\Omega\sigma^*_{el,in}(s,t,\alpha)v[F_2 - F_1]. \end{aligned}$$



I、 Background & Motivation

How to describe the transport process numerically ---models (e.g., UrQMD)

➢ UrQMD model

➢ **Cascade mode**; Prog. Part. Nucl. Phys. 41 (1998) 225; J. Phys. G25 (1999) 1859–1896.
Considering particle productions, scatterings, decays, but no interactions

➢ **Hybrid mode**; Phys. Rev. C 78 (2008) 044901; Phys. Rev. C 84, 045208 (2011); Phys. Lett. B 674 (2009) 111–116
with the help of macroscopic hydrodynamics with different EOS

➢ **Potential-field mode.** Phys. Rev. C 83, 044617 (2011); Front. Phys. 15, 44302 (2020); Eur. Phys. J. C (2022) 82:427 ; Sci. China-Phys. Mech. Astron. 66(2), 222011 (2023); ibid 66(3), 232011 (2023). Considering the mean-field, density functional, or chiral…

More model-comparisons, please see:

Progress in Particle and Nuclear Physics 125 (2022) 103962

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Progress in Particle and Nuclear Physics

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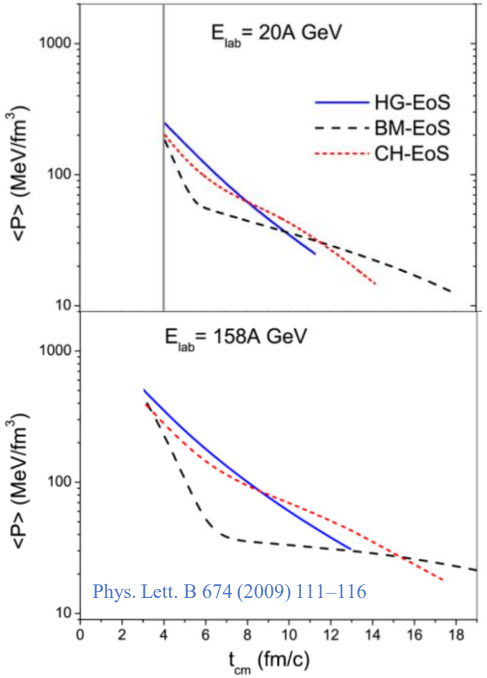
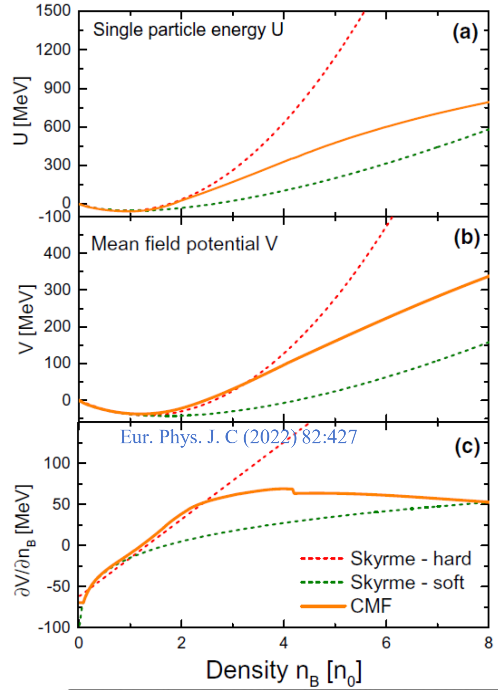
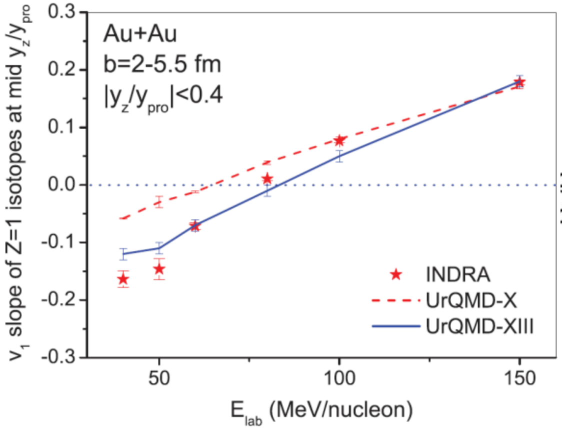
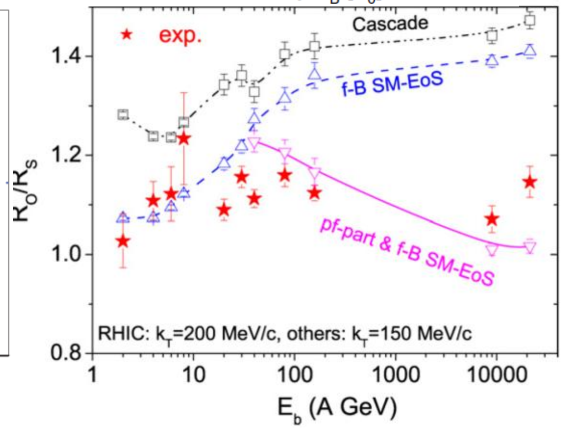
Review

Transport model comparison studies of intermediate-energy heavy-ion collisions









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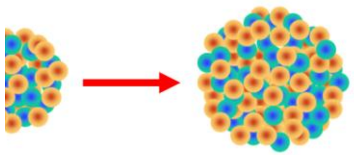
Experimentally done by He



国家重大科技基础设施强流重离子加速器装置调试成功

中国科学院近代物理研究所 2025年10月28日 19:16 甘肃

10月28日，位于广东省惠州市的国家重大科技基础设施——强流重离子加速器装置（HIAF）调试成功，实现束流全线贯通。

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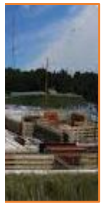

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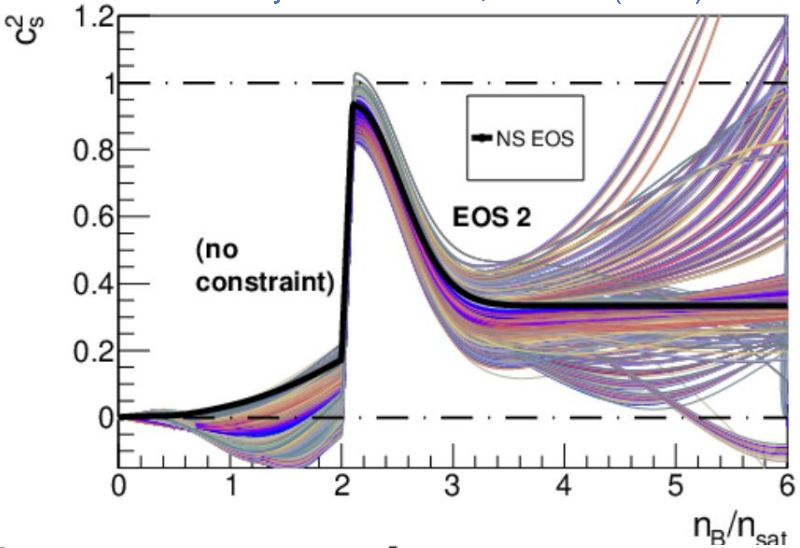
Ⅰ、Background & Motivation

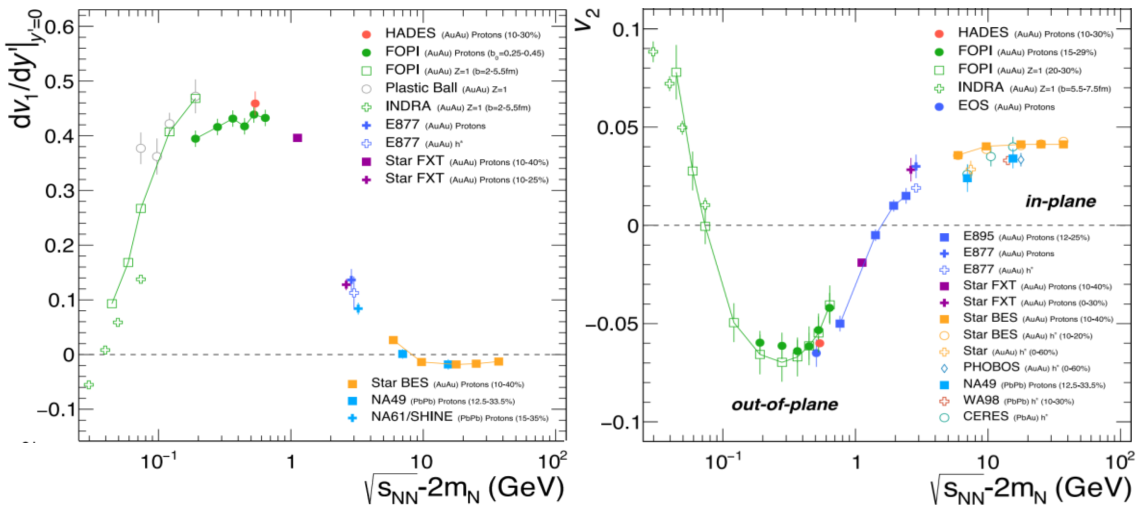
① 更高的密度

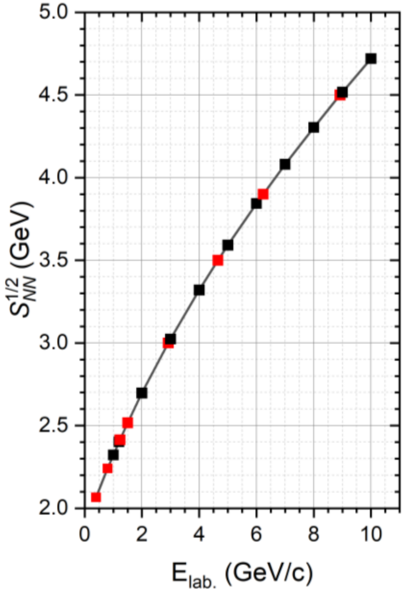
Phys. Rev. C 109, 065803 (2024)

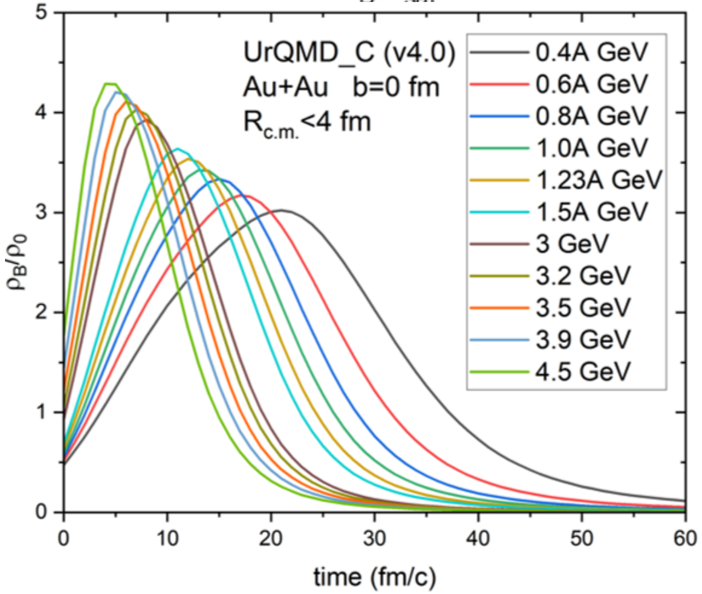


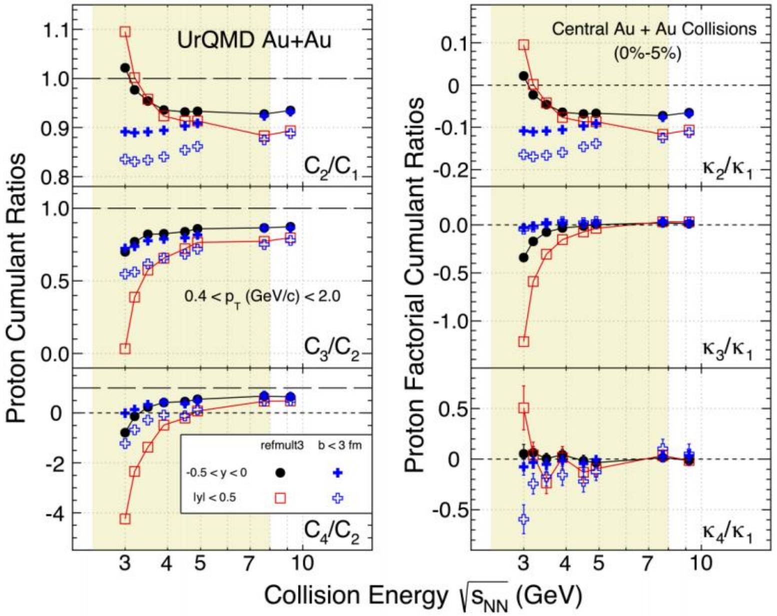
② 更精确的对比

Prog. Part. Nucl. Phys. 134, 104080(2024)









arXiv:2506.18832v3

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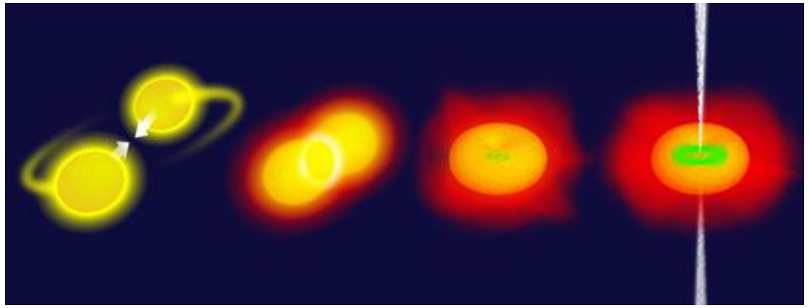
③ 学科交叉：HICs + astrophysical



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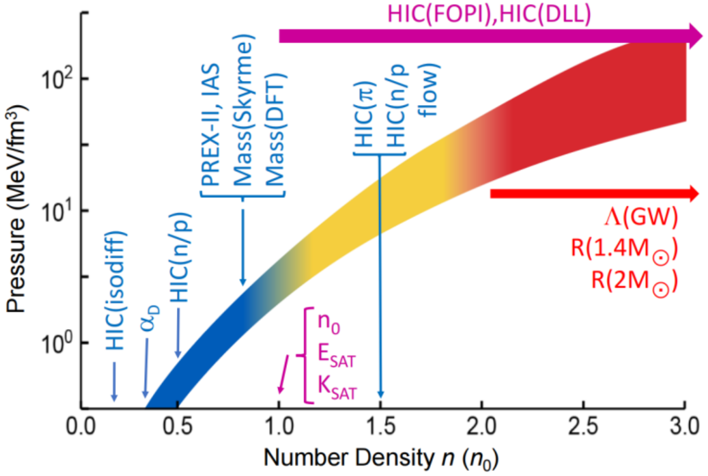
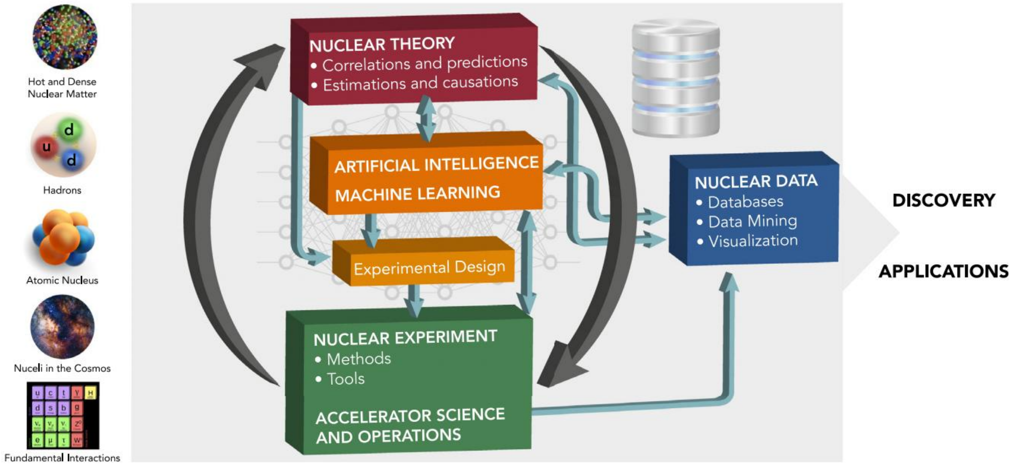
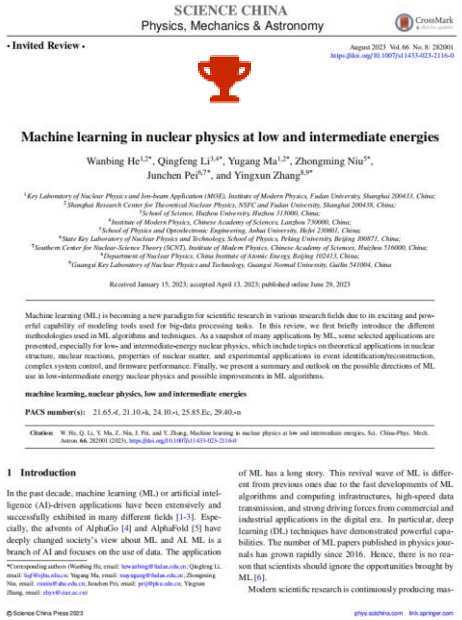
→C. Y. Tsang, et al. Nature Astron. 8, 328-336 (2024).



④ 方法创新：AI4science

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• Invited Review •

Machine learning in nuclear physics at low and intermediate energies

Wanbing He, Qingfeng Li, Yugang Ma, Zhongming Niu, Junchen Pei, and Yingxun Zhang

1 Introduction

→ A. Boehnlein, M. Diefenthaler, N. Sato, et al. Rev. Mod. Phys. 94, 031003 (2022).
→ W. B. He, Q. F. Li, Y. G. Ma, Z. M. Niu, J. C. Pei and Y. X. Zhang, Sci. China Phys. Mech. Astron. 66, 282001 (2023).

I、Background & Motivation

Physical Application

Springer Link

Review | Published: 22 March 2021

Eur. Phys. J. A 57, 100 (2021)

A.I. for nuclear physics

Paulo Bedaque, Amber Boehnlein, Mario Cromaz, Markus Diefenthaler, Latifa Elouadrhiri, Tanja Horn, Michelle Kuchera, David Lawrence, Dean Lee, Steven Lidia, Robert McKeown, Wally Melnitchouk, Witold Nazarewicz, Kostas Orginos, Yves Roblin, Michael Scott Smith, Malachi Schram & Xin-Nian Wang

The European Physical Journal A 57, Article number: 100 (2021) | Cite this article

Open Access Review

Symmetry, 16(11): 1426 (2024)

Review of Deep Learning in High-Energy Heavy-Ion Collisions

by Shiqi Zheng [1,*] and Jiamin Liu [2,*]

1 School of Engineering, Brown University, Providence, RI 02912, USA
2 Department of Physics, Tianjin University, Tianjin 300354, China
* Authors to whom correspondence should be addressed.

Symmetry **2024**, *16*(11), 1426; https://doi.org/10.3390/sym16111426

Submission received: 3 September 2024 / Revised: 17 October 2024 / Accepted: 22 October 2024 / Published: 26 October 2024

PROCEEDINGS OF SCIENCE

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Volume 419 - FAIR next generation scientists - 7th Edition Workshop (FAIRness2022) - Main session

Deep Learning for inverse problems in nuclear physics

K. Zhou*, L. Pang, S. Shi and H. Stoecker

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PoS FAIRness2022 64 (2023)

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Rev. Mod. Phys. 94(3), 031003 (2022)

Rev. Mod. Phys. **94**, 031003 – Published 8 September 2022

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arXiv > hep-ph > arXiv:2509.26374

High Energy Physics - Phenomenology

[Submitted on 30 Sep 2025]

Machine learning approach to QCD kinetic theory

Sergio Barrera Cabodevila, Aleksi Kurkela, Florian Lindenbauer

arxiv:2509.26374 (2025)

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Jia-Jie Li [1,*], Yu Tian [1], and Armen Sedrakian [2,3,†]

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Physical Review C, 111(5): 055804 (2025).

Phys. Rev. C **111**, 055804 – **Published 21 May, 2025**

DOI: https://doi.org/10.1103/PhysRevC.111.055804

International Journal of Modern Physics E | Vol. 33, No. 06, 2430009 (2024) | Review Article

Studying high-energy nuclear physics with machine learning

Long-Gang Pang

https://doi.org/10.1142/S0218301324300091 | **Cited by:** 5 **(Source: Crossref)**

International Journal of Modern Physics E, 33(06): 2430009 (2024)

Journal of Physics G: Nuclear and Particle Physics

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Journal of Physics G: Nuclear and Particle Physics 51(10): 103001 (2024)

Volume 51

Number 10, October 2024

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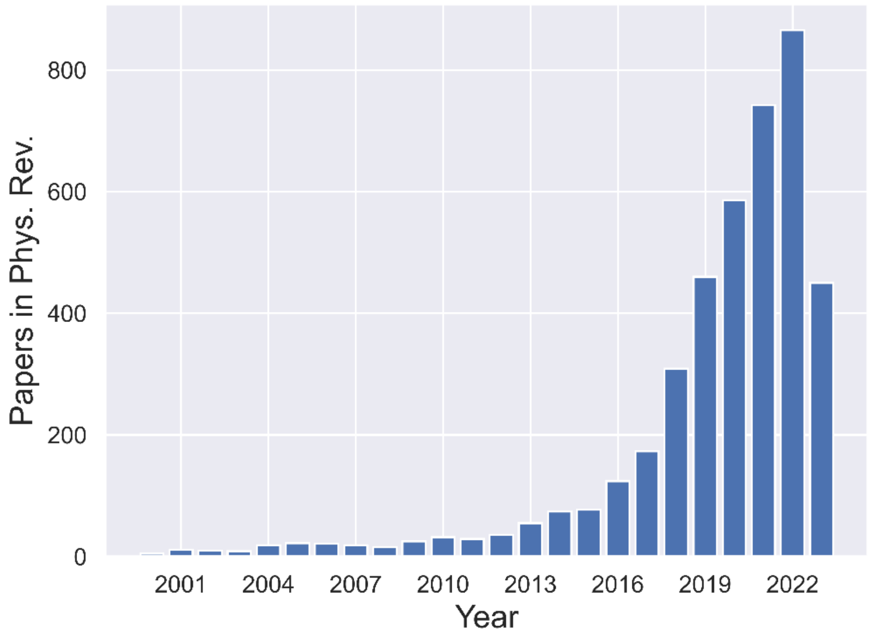
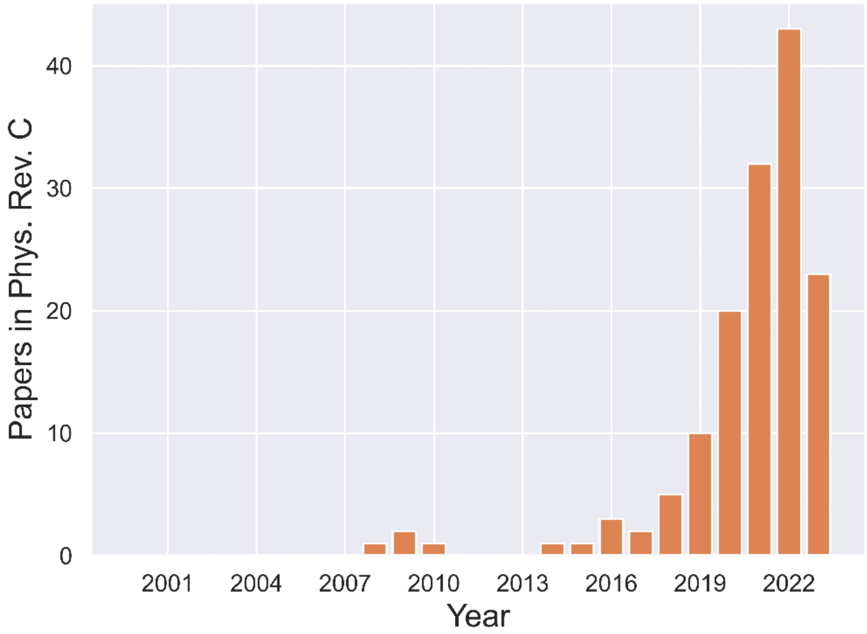
I、Background & Motivation

机器学习:最新进展（物理应用：以PR为例，截止至 2023.6 ）

搜索关键词“machine learning”统计结果





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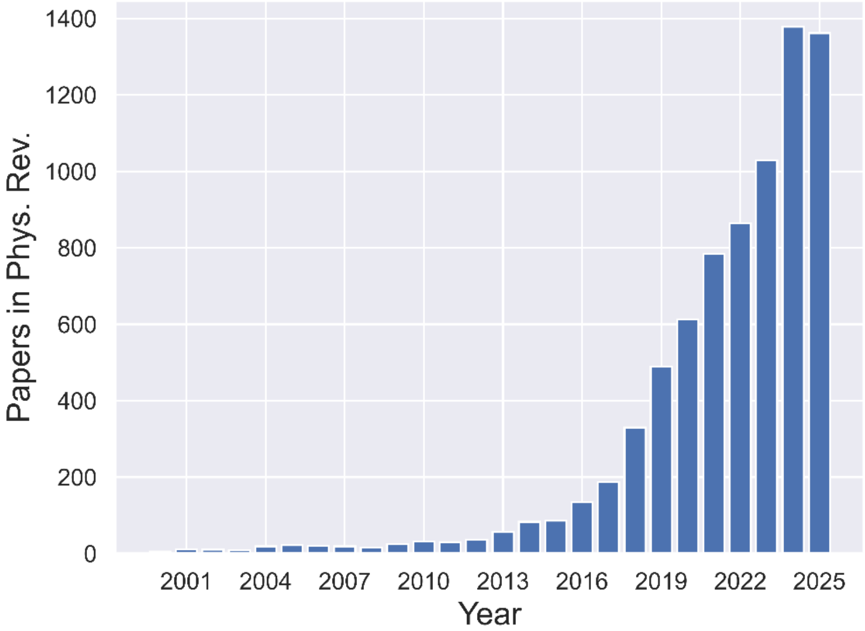

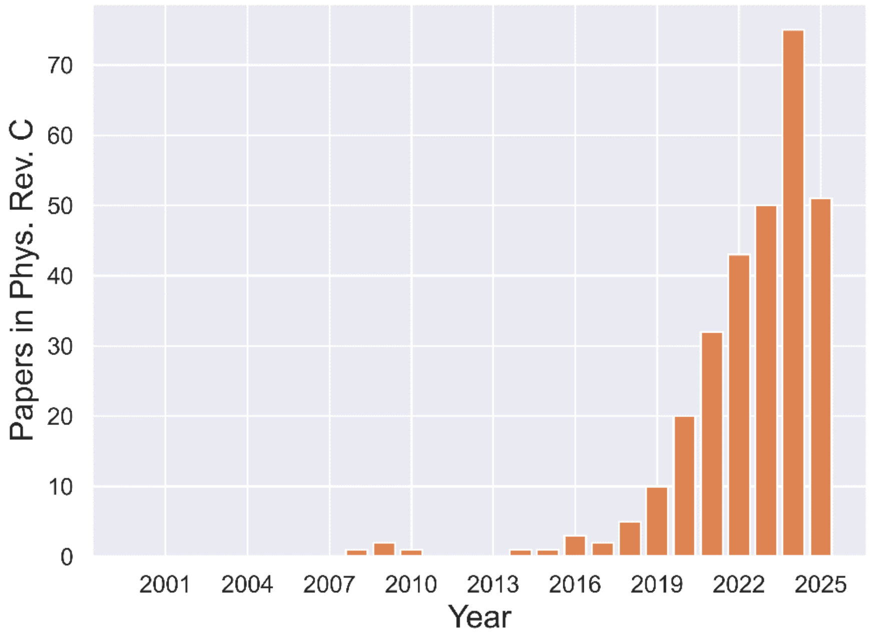

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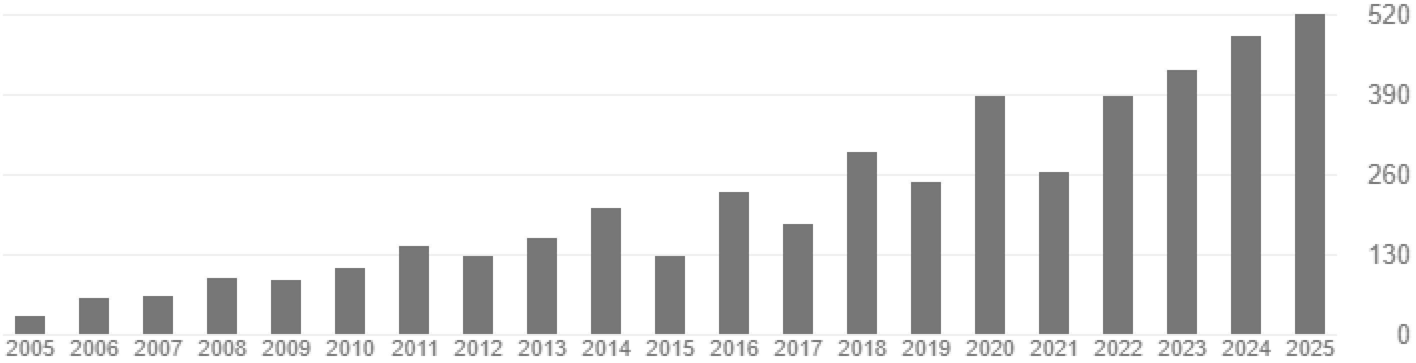
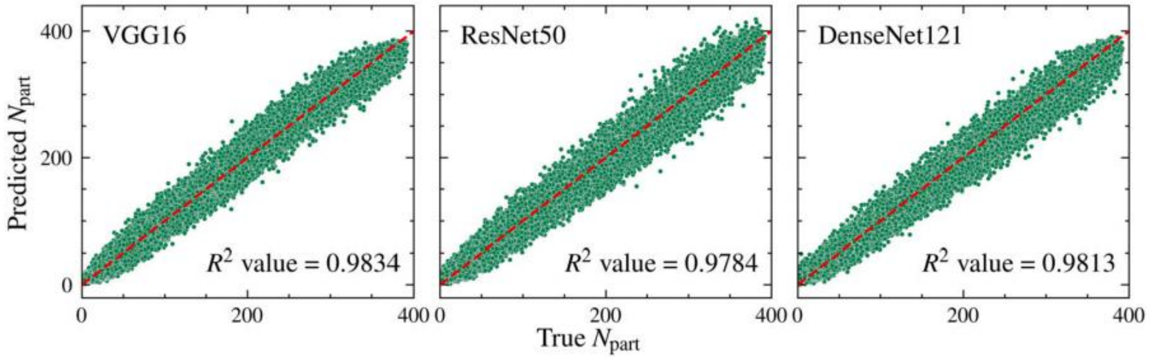
标题	引用次数	年份
Symmetry energy from elliptic flow in 197Au+ 197Au P Russotto, PZ Wu, M Zoric, M Chartier, Y Leifels, RC Lemmon, Q Li, ... Physics Letters B 697 (5), 471-476	304	2011
Results of the ASY-EOS experiment at GSI: The symmetry energy at suprasaturation density P Russotto, S Gannon, S Kupny, P Lasko, L Acosta, M Adamczyk, ... Physical Review C 94 (3), 034608	293	2016
Understanding transport simulations of heavy-ion collisions at $100A$ and $400A$ MeV: Comparison of heavy-ion transport codes under controlled conditions J Xu, LW Chen, MYB Tsang, H Wolter, YX Zhang, J Aichelin, M Colonna, ... Physical review C 93 (4), 044609	180	2016
Comparison of heavy-ion transport simulations: Collision integral in a box YX Zhang, YJ Wang, M Colonna, P Danielewicz, A Ono, MB Tsang, ... Physical Review C 97 (3), 034625	154	2018
Transport model comparison studies of intermediate-energy heavy-ion collisions H Wolter, M Colonna, D Cozma, P Danielewicz, CM Ko, R Kumar, A Ono, ... Progress in Particle and Nuclear Physics 125, 103962	150	2022
Machine learning the nuclear mass ZP Gao, YJ Wang, HL Lü, QF Li, CW Shen, L Liu Nuclear Science and Techniques 32 (10), 109	131	2021



I、Background & Motivation

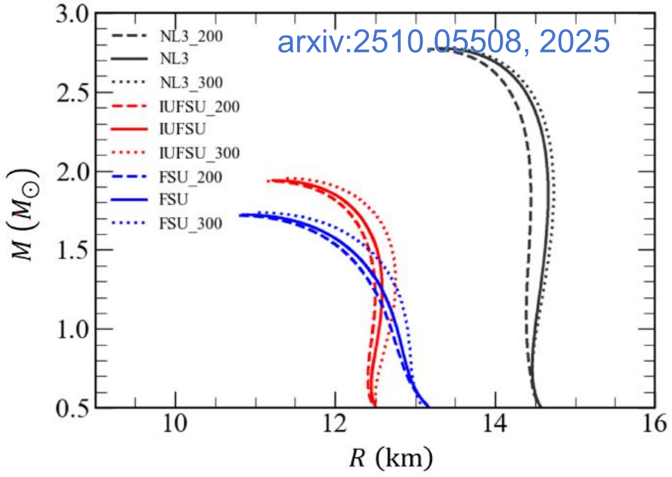
Physical Application：most recent

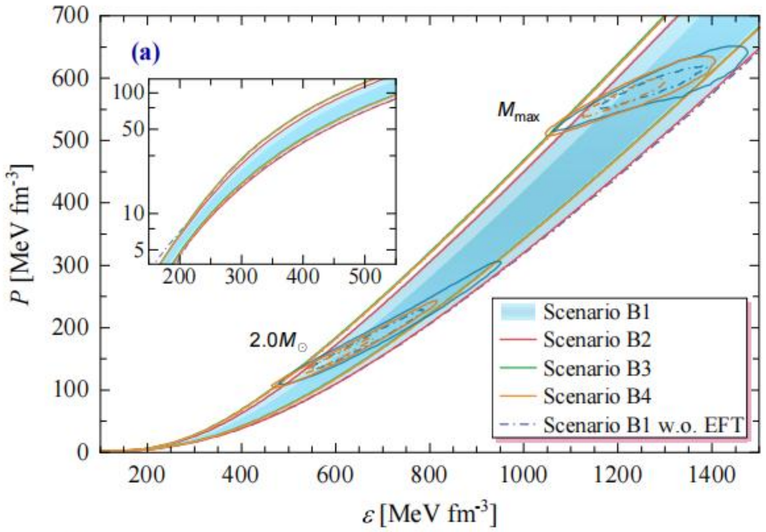
Nucl.Phys.A 1057 123043 (2025)



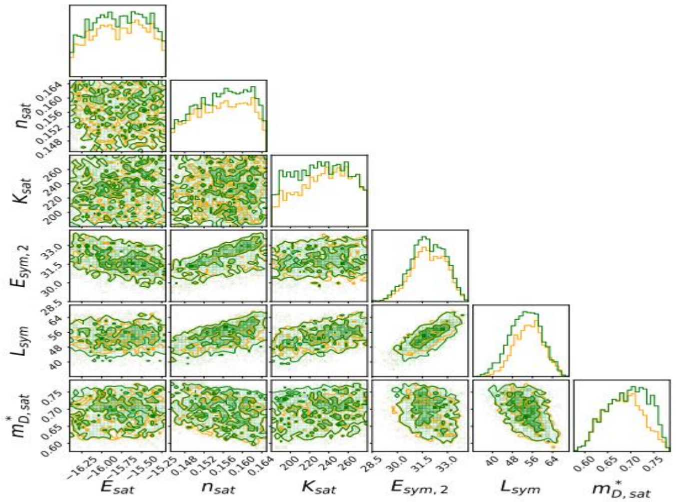
Predicted number of participants (N_{part}) : centrality

arxiv:2510.05508, 2025





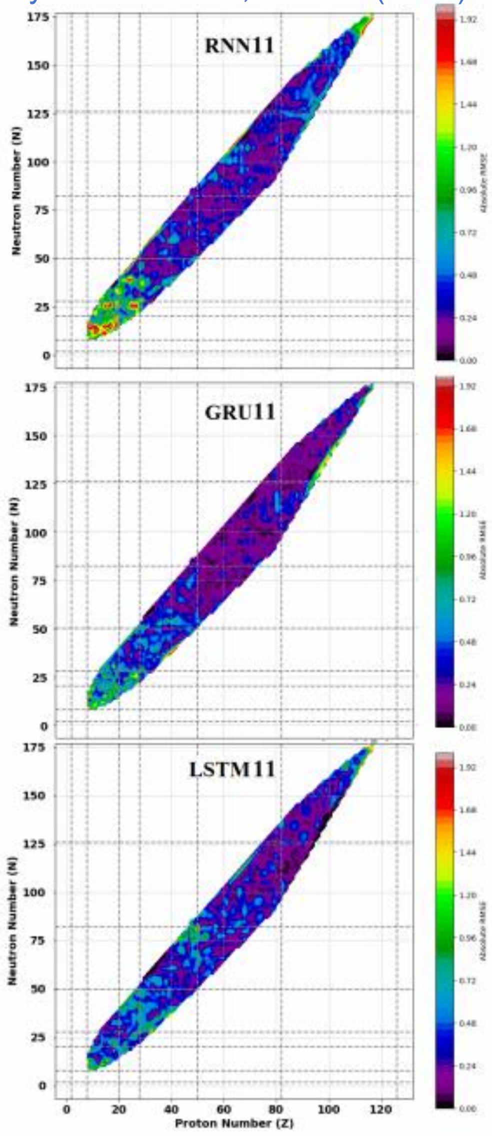
Phys. Rev. C 111(5), 055804 (2025)



Phys. Rev. C 112(3), 035805 (2025)

Posteriors for the nucleonic compact-star EoS distribution

Phys.Rev.C 112 2, 024305 (2025)



RMSE of nuclear binding energy

II、ML & Previous Work Review

Successful AI Applications in nuclear physics so far from my group

Publications	Task	Data	AI	Features
		Exp./Sim.		BF/PI
Y2020 by FPLi; JPG47, 115104	Impact Parameter	UrQMD	CNN LightGBM	BF(2)
Y2021 by FPLi; PRC104,034608	Impact Parameter	UrQMD	ANN、CNN LightGBM	BF(7)
Y2021 by ZPGao; NST32, 109	Nuclear Mass	Exp.	LightGBM	BF(10)+PI(4)
Y2021 by YJWang; PLB822, 136669	Symmetry Energy	UrQMD	CNN	BF(4)
Y2022 by YJWang; PLB835, 137508	Symmetry Energy	UrQMD	LightGBM	BF(30)
Y2022 by ZPGao; SSPMA52, 252010	quadrupole deformation	UrQMD	LightGBM	BF(6)
Y2024 by ZLLi; PRC109,024604	Fusion CS	Exp.	LightGBM	BF(17)+PI(3)
Y2025 by ZLLi; PRC112, 014312	nuclear charge radius	Exp.	LightGBM SVGP	BF(10)+PI(1)
Y2025 by GJWei JPG52, 015107	Medium Modified NN elastic CS：F	UrQMD	LightGBM	BF(8)
Y2025 by GJWei arxiv:2509.03406	K_0, m*, F	UrQMD，Exp.	Bayesian analysis	v_{11}, v_{20}, vartl

每人各2篇

钱以斌、田俊龙、李剑、刘健

曹政（高密）

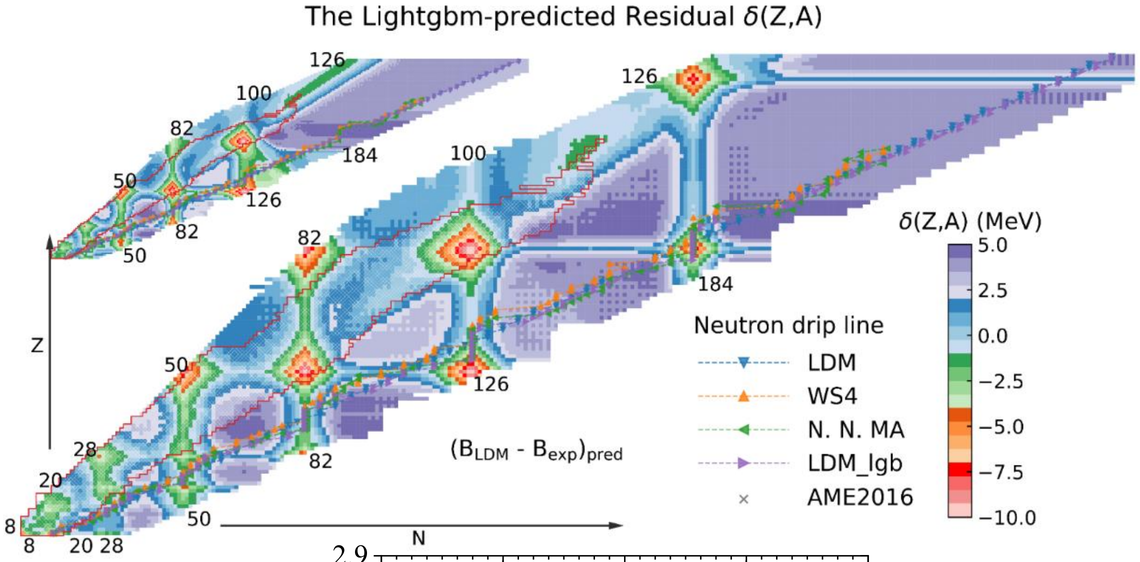
吕梦蛟（结团）

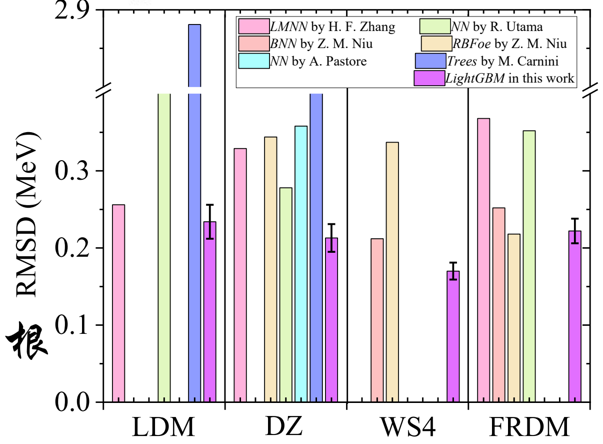
罗文（中子俘获）

李甫鹏（高能）、夏鋮君（中子星）

II、ML & Previous Work Review

Machine Learning Research on Nuclear Mass





均方根

Nucl. Sci. Tech. 32, 109, 2021

- All four mass models can provide satisfactory corrections.
- The correction for the droplet model can identify the magic numbers near the driplines.

Accuracy is crucial !!!

σ_{pre}		LDM 2.462 ± 0.023	DZ 0.613 ± 0.007	WS4 0.302 ± 0.003	FRDM 0.599 ± 0.009
	RBF by Wang [11]	–	–	0.170	–
	KRR by Wu [40]	–	–	0.199	–
	RBFs by Ma [59]	–	–	0.130	0.209
Training set	LMNN by Zhang [39]	0.235	0.325	–	0.348
	BNN by Niu [27]	–	–	0.176	0.187
	RBFoe by Niu [41]	–	0.171	0.140	0.182
	NN by Utama [25]	0.466	0.274	–	0.342
	NN by Pastore [58]	–	0.324	–	–
	Trees by Carnini [44]	2.070	0.471	–	–
	LightGBM in this work	0.058 ± 0.011	0.066 ± 0.010	0.055 ± 0.011	0.077 ± 0.013
Test set	LMNN by Zhang	0.256	0.329	–	0.368
	BNN by Niu	–	–	0.212	0.252
	RBFoe by Niu	–	0.344	0.337	0.218
	NN by Utama	0.486	0.278	–	0.352
	NN by Pastore	–	0.358	–	–
	Trees by Carnini	2.881	0.569	–	–
	LightGBM in this work	0.234 ± 0.022	0.213 ± 0.018	0.170 ± 0.011	0.222 ± 0.016

II、 ML & Previous Work Review

Machine Learning Research on Nuclear Mass

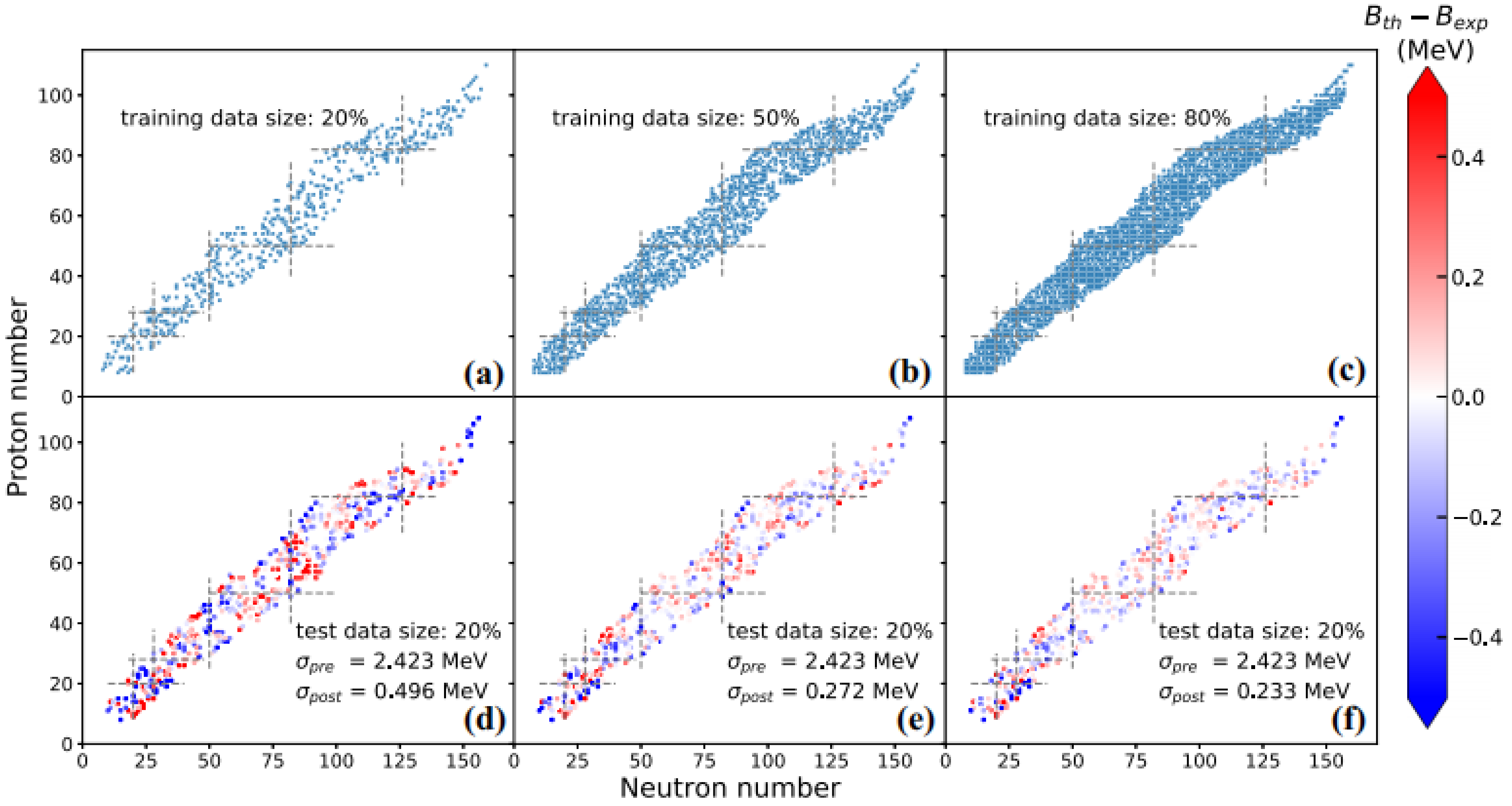
Nucl. Sci. Tech. 32, 109, 2021

Table 2 Selection of characteristic quantities

Feature	Description
A	Mass number
Z	Proton number
N	Neutron number
N/Z	Ratio of neutrons to protons
B_{LDM}	Theoretical value from the LDM
$N_{\mathrm{pair}} = 0,1,2$	Dependence on pair effect;
	for odd–odd, odd–even, even–even
$Z_m = 1, 2, \ldots$	Shell of the last proton;
	for $8 \leqslant Z < 20$, $20 \leqslant Z < 28$, $\cdots$
$N_m = 1, 2, \ldots$	Shell of the last neutron;
	for $8 \leqslant N < 20$, $20 \leqslant N < 28$, $\cdots$
$\lvert Z - m \rvert$	Distance between proton number
	and the nearest magic number;
	$m \in \{8, 20, 28, 50, 82, 126\}$
$\lvert N - m \rvert$	Distance between neutron number
	and the nearest magic number;
	$m \in \{8, 20, 28, 50, 82, 126, 184\}$

LightGBM (Light Gradient Boosting Machine)

developed by Microsoft in 2016

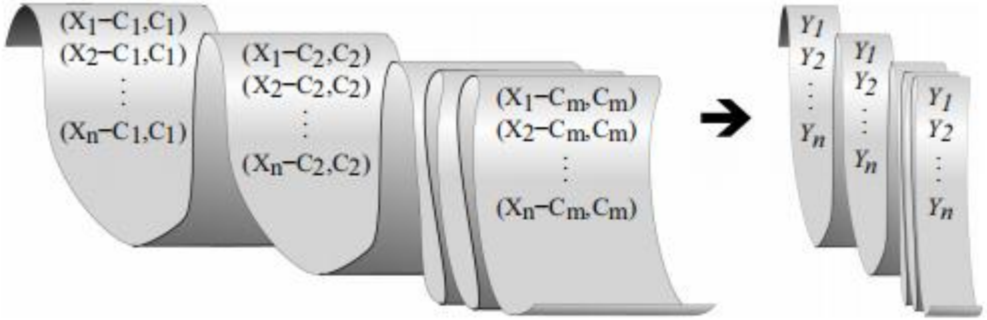


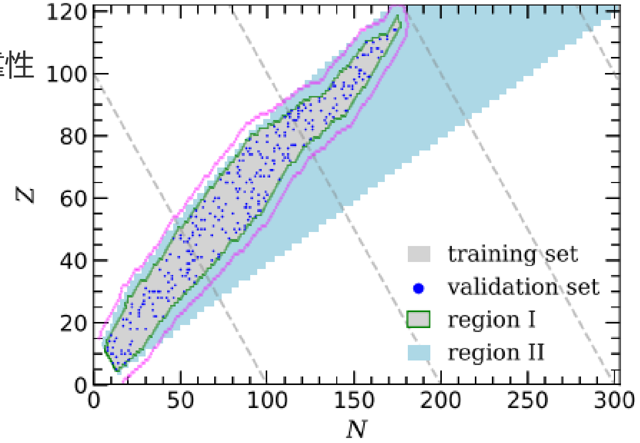
II、ML & Previous Work Review

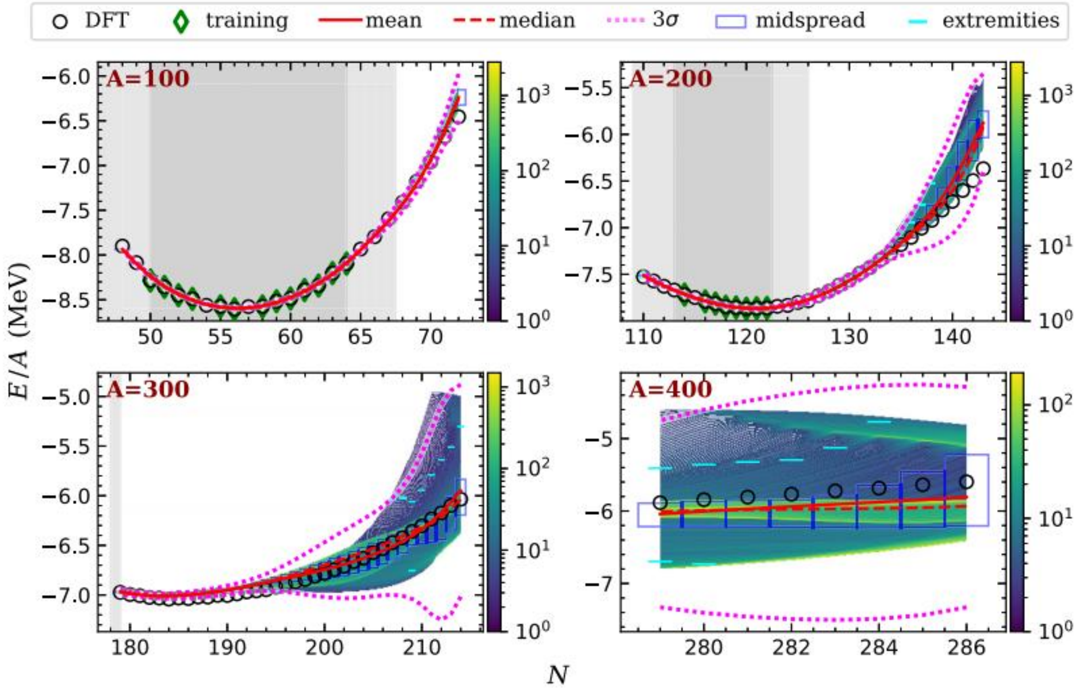
Machine Learning Research on Nuclear Mass

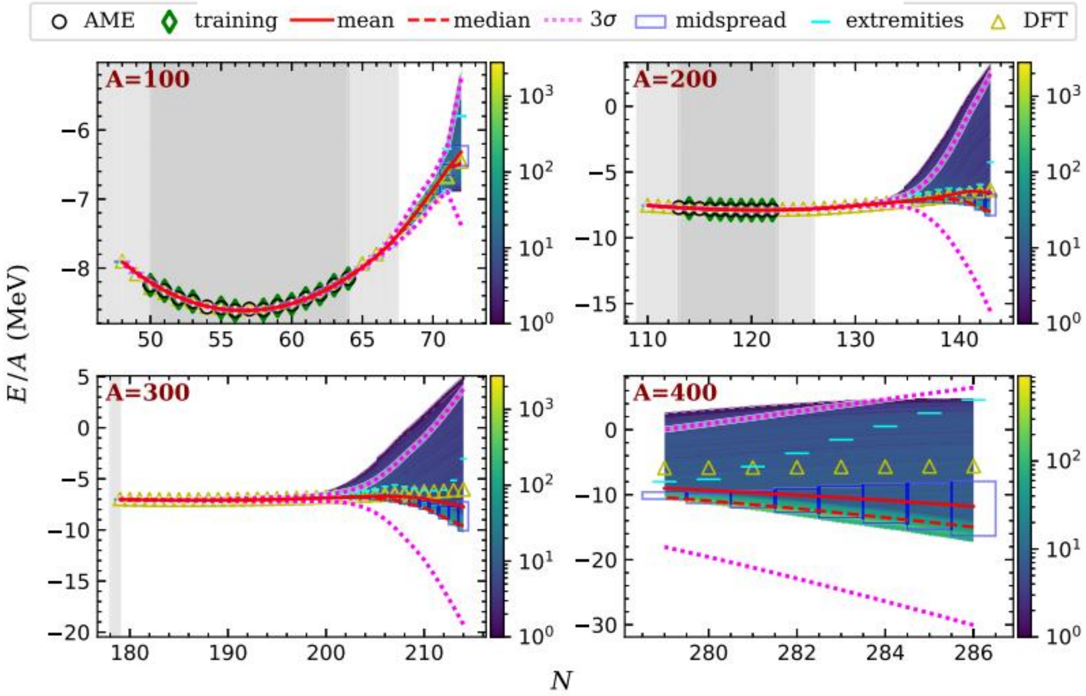
MY Huang, etal, Phys. Rev. C, 112(3): 034317 (2025).

不确定度量化Δ-UQ mapping方法：系统化评估测量不确定度的来源及影响，提高决策或预测的可靠性









II、ML & Previous Work Review

Machine Learning Research on Fusion Cross Section

Phys. Rev. C 109(2): 024604 (2024)

training set + validation set

Table 1 – continued from previous page

System	Data points	Energy range (MeV)	System	Data pointts	Energy range (MeV)	System	Data pointts	Energy range (MeV)
^{16}O+^{176}Yb	15	59-89	^{32}S+^{98}Mo	11	73-91	^{36}S+^{106}Pd	5	80-90
^{16}O+^{186}W	35	62-83	^{32}S+^{100}Mo	8	72-91	^{36}S+^{108}Pd	6	79-95
^{16}O+^{204}Pb	12	66-98	^{32}S+^{100}Ru	8	77-91	^{36}S+^{110}Pd	6	79-95
^{16}O+^{208}Pb	38	65-110	^{32}S+^{101}Ru	6	80-91	^{36}S+^{204}Pb	26	138-179
^{16}O+^{209}Bi	14	71-95	^{32}S+^{102}Ru	9	76-92	^{36}S+^{238}U	5	147-173
^{16}O+^{232}Th	12	70-150	^{32}S+^{103}Rh	8	78-92	^{37}Cl+^{24}Mg	12	25-35
^{16}O+^{238}U	14	80-157	^{32}S+^{104}Pd	6	81-92	^{37}Cl+^{25}Mg	12	26-36
^{17}O+^{144}Sm	25	54-90	^{32}S+^{104}Ru	9	77-92	^{37}Cl+^{26}Mg	12	26-38
^{18}O+^{44}Ca	17	19-43	^{32}S+^{105}Pd	5	81-92	^{37}Cl+^{59}Co	13	55-71
^{18}O+^{74}Ge	26	30-50	^{32}S+^{106}Pd	7	80-92	^{37}Cl+^{70}Ge	12	62-76
^{18}O+^{112}Sn	17	42-78	^{32}S+^{108}Pd	8	79-92	^{37}Cl+^{72}Ge	14	63-77
^{18}O+^{118}Sn	17	44-79	^{32}S+^{110}Pd	9	78-93	^{37}Cl+^{73}Ge	12	63-77
^{18}O+^{124}Sn	17	44-79	^{32}S+^{138}Ba	13	99-135	^{37}Cl+^{74}Ge	9	63-75
^{18}O+^{192}Os	7	73-113	^{32}S+^{154}Sm	9	101-128	^{37}Cl+^{76}Ge	14	62-76
^{18}O+^{208}Pb	18	68-95	^{32}S+^{182}W	15	122-170	^{37}Cl+^{93}Nb	7	81-93
^{37}Cl+^{98}Mo	7	82-95	^{40}Ca+^{194}Pt	31	159-200	^{58}Ni+^{54}Fe	25	85-110
^{37}Cl+^{100}Mo	8	83-95	^{40}Ca+^{197}Au	10	162-223	^{58}Ni+^{58}Ni	17	95-109
^{40}Ar+^{112}Sn	15	96-126	^{40}Ca+^{208}Pb	5	169-208	^{58}Ni+^{64}Ni	27	89-110
^{40}Ar+^{116}Sn	14	95-128	^{48}Ca+^{48}Ca	29	46-63	^{58}Ni+^{124}Sn	10	143-162
^{40}Ar+^{122}Sn	17	94-130	^{48}Ca+^{90}Zr	33	90-115	^{64}Ni+^{64}Ni	23	89-107
^{40}Ar+^{144}Sm	11	116-140	^{48}Ca+^{96}Zr	38	88-113	^{64}Ni+^{124}Sn	11	144-170
^{40}Ar+^{148}Sm	12	112-150	^{48}Ca+^{154}Sm	51	125-192	^{124}Sn+^{40}Ca	22	107-150
^{40}Ar+^{154}Sm	15	108-145	^{48}Ca+^{197}Au	8	164-245	^{132}Sn+^{40}Ca	15	109-136
^{40}Ca+^{40}Ca	15	49-70	^{48}Ca+^{208}Pb	5	171-206	^{134}Te+^{40}Ca	10	115-140
^{40}Ca+^{44}Ca	15	47-75	^{48}Ti+^{58}Fe	27	64-85	^{124}Sn+^{48}Ca	13	109-170
^{40}Ca+^{46}Ti	25	54-80	^{48}Ti+^{58}Ni	14	72-91	^{132}Sn+^{48}Ca	12	109-170
^{40}Ca+^{48}Ca	14	48-67	^{48}Ti+^{60}Ni	15	72-93	^{132}Sn+^{58}Ni	10	152-200
^{40}Ca+^{48}Ti	28	53-82	^{46}Ti+^{64}Ni	18	71-96	^{132}Sn+^{64}Ni	9	147-200
^{40}Ca+^{50}Ti	30	53-83	^{48}Ti+^{64}Ni	18	71-96	^{208}Pb+^{26}Mg	7	109-190
^{40}Ca+^{58}Ni	21	67-95	^{48}Ti+^{122}Sn	19	121-160	^{50}Ti+^{90}Zr	8	100-118
^{40}Ca+^{64}Ni	23	63-94	^{50}Ti+^{60}Ni	14	72-91	^{40}Ca+^{192}Os	37	151-200
^{40}Ca+^{90}Zr	40	90-120	^{46}Ti+^{93}Nb	10	100-120			
^{40}Ca+^{94}Zr	59	88-115	^{50}Ti+^{93}Nb	9	100-120	Total systems	220	
						Total points	3635	

Data from Atom.Data Nucl.Data Tabl. 114 (2017) 281-370 • e-Print: 1504.00756 [nucl-th]

220 systems

Name	Data points
training set + validation set	3635

test set

Table 2. Test set.

System	Data points	energy range (MeV)	Ref.
^{35}Cl + ^{130}Te	16	90-125	[70]
^{37}Cl + ^{130}Te	17	90-125	[71]
^{37}Cl + ^{68}Zn	11	60-90	[72]
^{16}O + ^{61}Ni	8	25-41	[73]
^{18}O + ^{61}Ni	16	25-41	[73]
^{18}O + ^{62}Ni	15	25-42	[73]
^{18}O + ^{116}Sn	21	40-75	[74]
^{30}Si + ^{156}Gd	14	90-116	[75]
^{28}Si + ^{100}Mo	13	65-98	[76]
^{36}S + ^{50}Ti	23	40-60	[77]
^{36}S + ^{51}V	22	40-60	[77]
^{12}C + ^{182}W	14	40-80	[78]
^{12}C + ^{184}W	14	40-80	[78]
^{12}C + ^{186}W	14	40-80	[78]
^{40}Ca + ^{92}Zr	16	89-108	[79]
^{48}Ca + ^{116}Cd	20	104-130	[80]
^{48}Ca + ^{118}Sn	16	104-130	[80]
^{48}Ca + ^{120}Te	10	104-130	[80]
Total systems	18		
Total points	280		

The test set system is not included in the training set.

Name	Data points
test set	280

18 systems

Ⅱ、ML & Previous Work Review

Machine Learning Research on Fusion Cross Section

Table 3. Selection of basic features (BF).

features	Description
$E_{c.m}$	collision center-of-mass energy (MeV)
Z_1	charge of the first reaction partner
N_1	number of neutrons of the first reaction partner
A_1	mass number of the first reaction partner
Z_2	charge of the second reaction partner
N_2	number of neutrons of the second reaction partner
A_2	mass number of the second reaction partner
Z_3	charge of the compound nucleus
N_3	number of neutrons of the compound nucleus
A_3	mass number of the compound nucleus
B_1	binding energy of the first reaction partner
B_2	binding energy of the second reaction partner
B_3	binding energy of the compound nucleus
Q	fusion Q-value (MeV)
S_p	one-proton separation energy of the compound nucleus (MeV)
S_{2p}	two-proton separation energy of the compound nucleus (MeV)
S_n	one-neutron separation energy of the compound nucleus (MeV)
S_{2n}	two-neuturn separation energy of the compound nucleus (MeV)

$$\text{MAE} = \frac{1}{N}\sum_{i=1}^{N} |\log_{10}(\sigma_{\text{pred}}) - \log_{10}(\sigma_{\text{exp}})|$$

Table 4. Physical-informed quantities

features	Description
sig_ECC	$\log_{10}$ of CS obtained by ECC model
sig_W	$\log_{10}$ of CS obtained by Wong formula
sig_E	$Z_1Z_2/E_{c.m}$

Table 5. Different modes with different input features.

Mode name	Input features
Mode_BF	BF
Mode_ECC	BF+sig_ECC
Mode_W	BF+sig_W
Mode_E	BF+sig_E

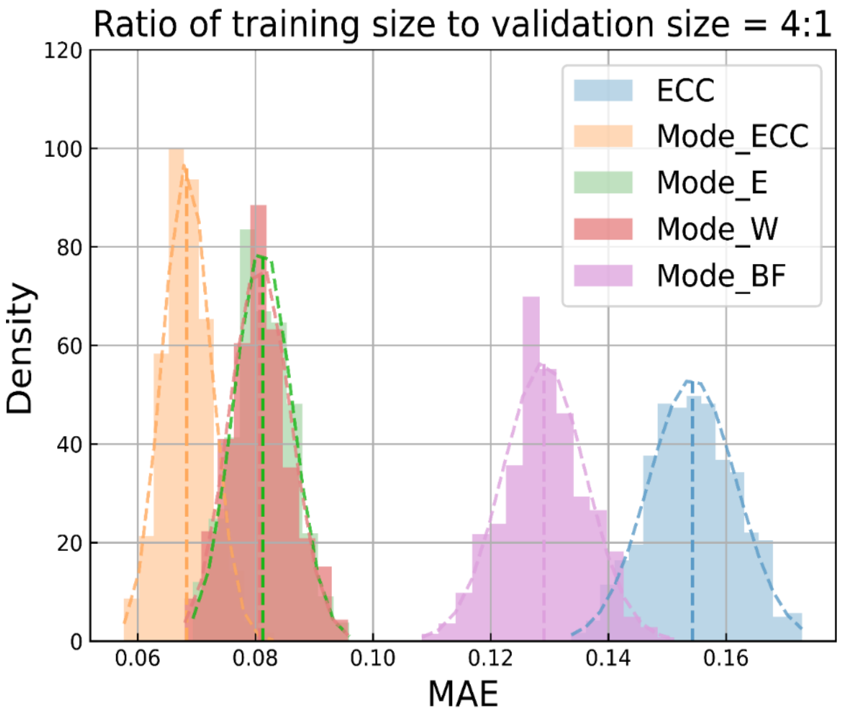
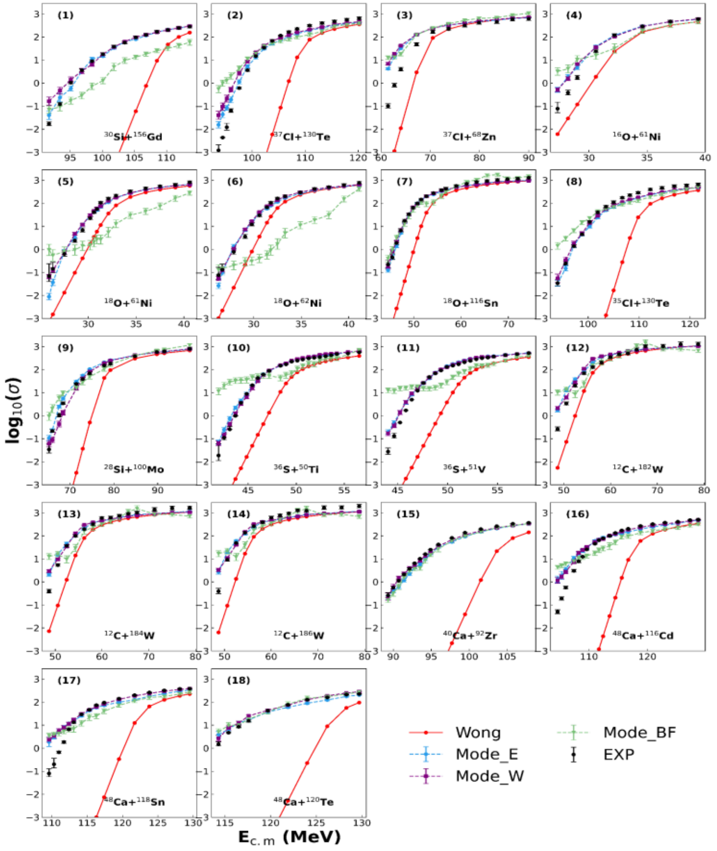


Table 6. The average MAE on the validation set obtained from different modes and from Wong formula and ECC model. The ratio of training set to test set is 4:1.

Model	MAE
Wong Formula	2.38 ± 0.09
ECC model	0.154 ± 0.008
Mode_BF	0.129 ± 0.007
Mode_ECC	0.068 ± 0.004
Mode_W	0.081 ± 0.005
Mode_E	0.081 ± 0.005

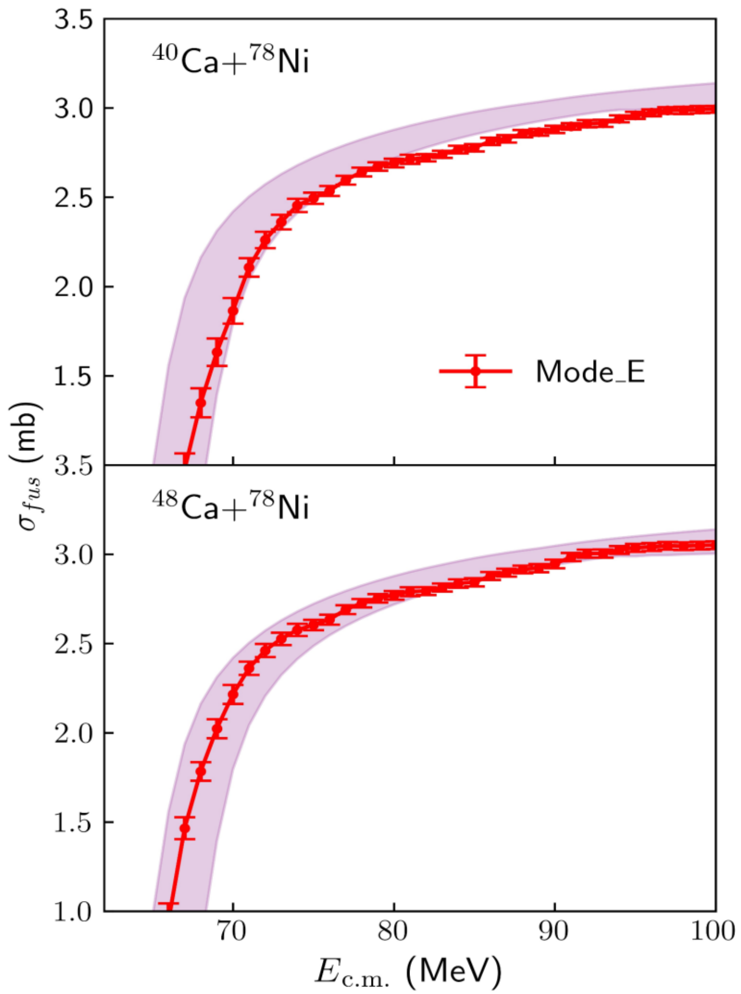
II、ML & Previous Work Review

Machine Learning Research on Fusion Cross Section



Name	18 systems	Data points
test set		280

Comparison of Mode_E and DC-TDHF [1]



[1] Physical Review C, 105, 034601 (2022)

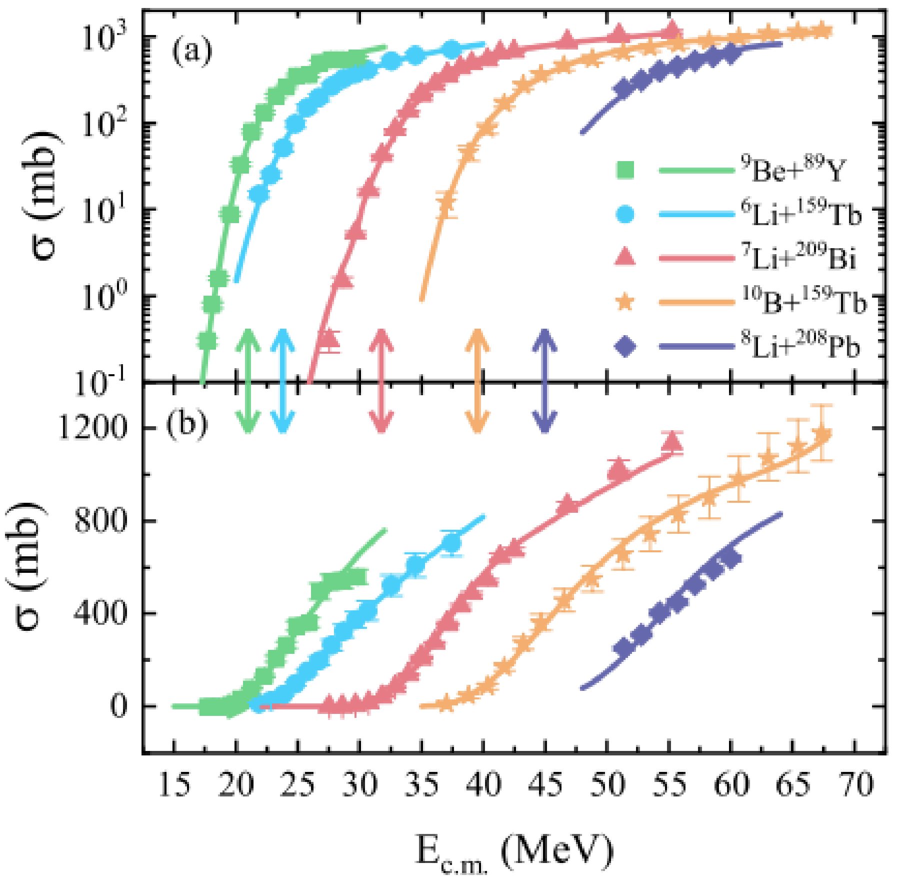
II、ML & Previous Work Review

马春旺组 Nucl. Sci. Tech. 36(10): 194 (2025):根据新证据不断调整先验认知

Reaction	E_{cm}/V_B	N_{exp}	F_{BNN}	F_{exp}	Ref	Reaction	E_{cm}/V_B	N_{exp}	F_{BNN}	F_{exp}	Ref
^{6}Li+^{64}Ni	0.85–2.06	15	0.87	0.88	[41]	^{7}Li+^{159}Tb	1.07–1.66	5	0.71	0.73	[57]
^{6}Li+^{90}Zr	0.82–1.65	8	0.67	0.7	[17]	^{7}Li+^{165}Ho	0.86–1.69	10	0.79	0.74	[15]
^{6}Li+^{94}Zr	0.89–1.68	5	0.52	0.49	[7]	^{7}Li+^{197}Au	0.81–1.50	8	0.84	0.86	[50]
^{6}Li+^{96}Zr	0.90–1.58	7	0.77	0.77	[42]	^{7}Li+^{198}Pt	0.79–1.52	6	0.72	0.77	[58]
^{6}Li+^{120}Sn	0.74–1.32	13	0.78	0.81	[43, 44]	^{7}Li+^{205}Tl	0.82–1.31	10	0.77	0.74	[59]
^{6}Li+^{124}Sn	0.83–1.70	15	0.72	0.66	[45]	^{7}Li+^{209}Bi	0.83–1.67	21	0.75	0.77	[53]
^{6}Li+^{144}Sm	0.79–1.58	11	0.55	0.54	[46]	^{9}Be+^{89}Y	0.83–1.39	15	0.78	0.75	[60]
^{6}Li+^{152}Sm	0.80–1.60	20	0.63	0.62	[47]	^{9}Be+^{93}Nb	0.85–1.45	7	0.85	0.90	[61]
^{6}Li+^{154}Sm	1.04–1.45	6	0.64	0.71	[48]	^{9}Be+^{124}Sn	0.90–1.33	13	0.73	0.75	[62]
^{6}Li+^{159}Tb	0.87–1.50	13	0.65	0.66	[49]	^{9}Be+^{144}Sm	0.89–1.31	10	0.92	0.94	[63]
^{6}Li+^{197}Au	0.84–1.35	16	0.61	0.60	[50]	^{9}Be+^{169}Tm	0.93–1.33	12	0.78	0.80	[64]
^{6}Li+^{198}Pt	0.67–1.14	10	0.75	0.75	[51]	^{9}Be+^{181}Ta	0.94–1.34	13	0.66	0.68	[65]
^{6}Li+^{208}Pb	0.92–1.28	20	0.67	0.69	[52]	^{9}Be+^{186}W	1.08–1.40	4	0.59	0.57	[66]
^{6}Li+^{209}Bi	0.83–1.53	14	0.65	0.68	[53]	^{9}Be+^{187}Re	0.93–1.28	12	0.75	0.76	[64]
^{7}Li+^{64}Ni	0.87–2.06	16	0.90	0.90	[54]	^{9}Be+^{197}Au	0.83–1.17	12	0.78	0.70	[67]
^{7}Li+^{93}Nb	1.29–1.63	4	0.75	0.75	[55]	^{9}Be+^{208}Pb	0.88–1.24	16	0.78	0.79	[53]
^{7}Li+^{119}Sn	0.72–1.30	15	0.93	0.94	[43, 44]	^{9}Be+^{209}Bi	0.88–1.21	19	0.98	0.98	[52]
^{7}Li+^{124}Sn	0.79–1.86	23	0.71	0.73	[56]	^{10}B+^{159}Tb	0.91–1.66	16	0.87	0.87	[57]
^{7}Li+^{144}Sm	0.88–1.59	14	0.63	0.63	[18]	^{10}B+^{209}Bi	1.06–1.44	5	0.88	0.89	[68]
^{7}Li+^{152}Sm	0.81–1.61	16	0.66	0.69	[18]						

$$F_{BNN} = \frac{\sigma_{BNN}}{\sigma_{BPM}} \quad \text{or} \quad F_{exp} = \frac{\sigma_{exp}}{\sigma_{BPM}},$$

Bayesian neural network

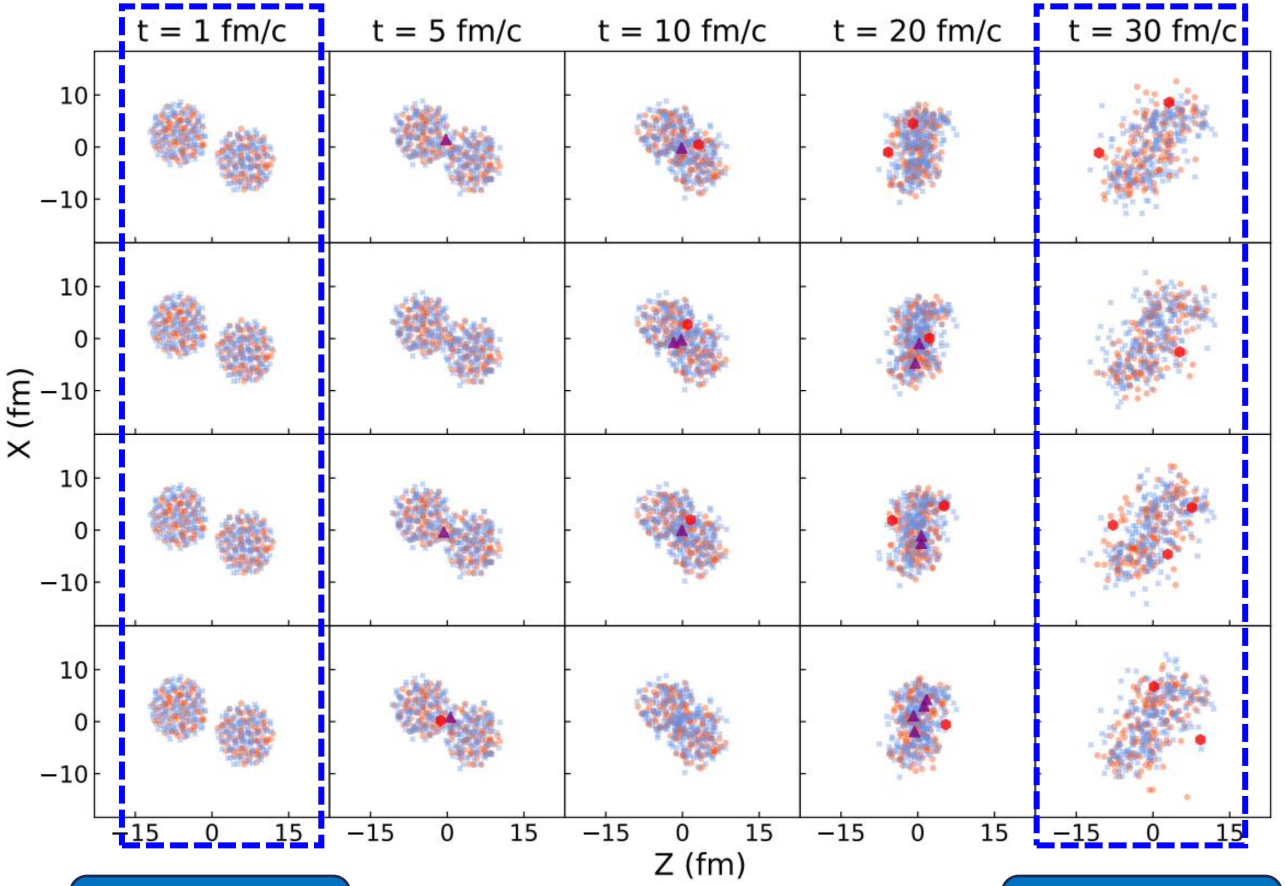


III、 Results on EoS

Y2025 by GJWei JPG52, 015107	Medium Modified NN elastic CS	UrQMD	LightGBM	BF(8)
Y2025 by GJWei arxiv:2509.03406	K_0, m*, F	UrQMD, Exp.	Bayesian analysis	v_{11}, v_{20}, vartl

Event-by-event analysis

重离子碰撞：逐事件



The same initial state

Different final states

Each collision between the projectile nucleus and the target nucleus is called an event. Due to the initial state fluctuations of the atomic nuclei, random nucleon-nucleon scattering, and other factors, even under exactly the same initial conditions, the values of the final state observables will be different. In current research, the average value of multiple event observables is generally the focus of attention.

In fact, both theory and experiment can obtain information about the observables event by event.

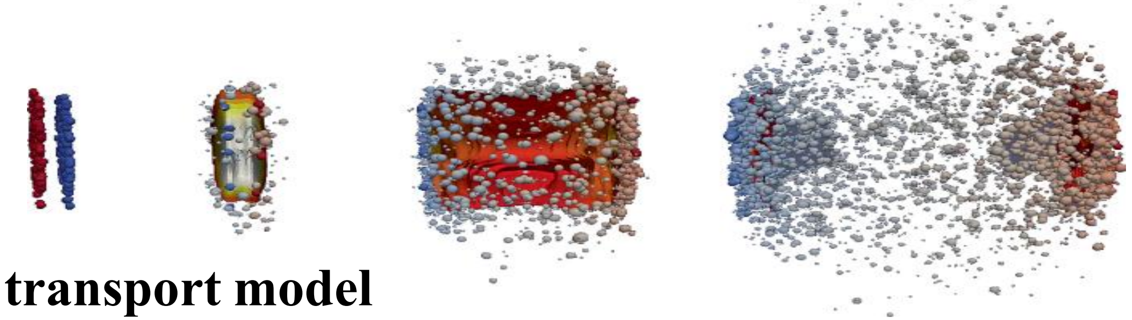
Event-by-event analysis has been widely employed in the field of relativistic heavy-ion collisions to obtain information about critical points.

Ⅲ、 Results on EoS

Machine learning combined with heavy ion collisions

Transport model emulates heavy-ion collisions

Initial state： Collision system, beam energy, EoS, impact parameters, etc.



UrQMD transport model

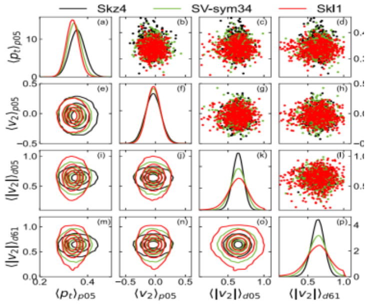
Analyze the prediction results

EoS、symmetry energy, etc.

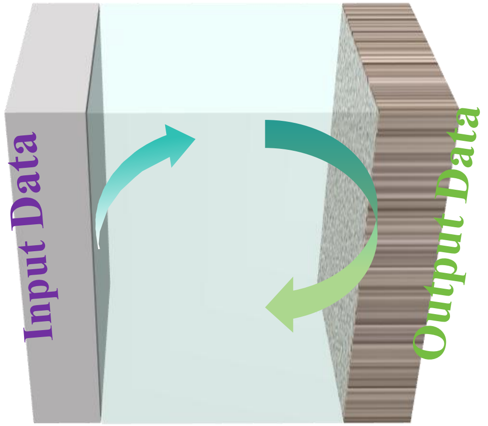
Shapley additive explanations

Feature Importance

Event-by-event data



Observables



Machine learning algorithm

Light gradient boosting machine (LightGBM)

Artificial Neural Network (ANN)

Training and testing of machine learning models

III、 Results on EoS

Machine learning the in-medium correction factor on nucleon-nucleon elastic cross section

the in-medium correction factor

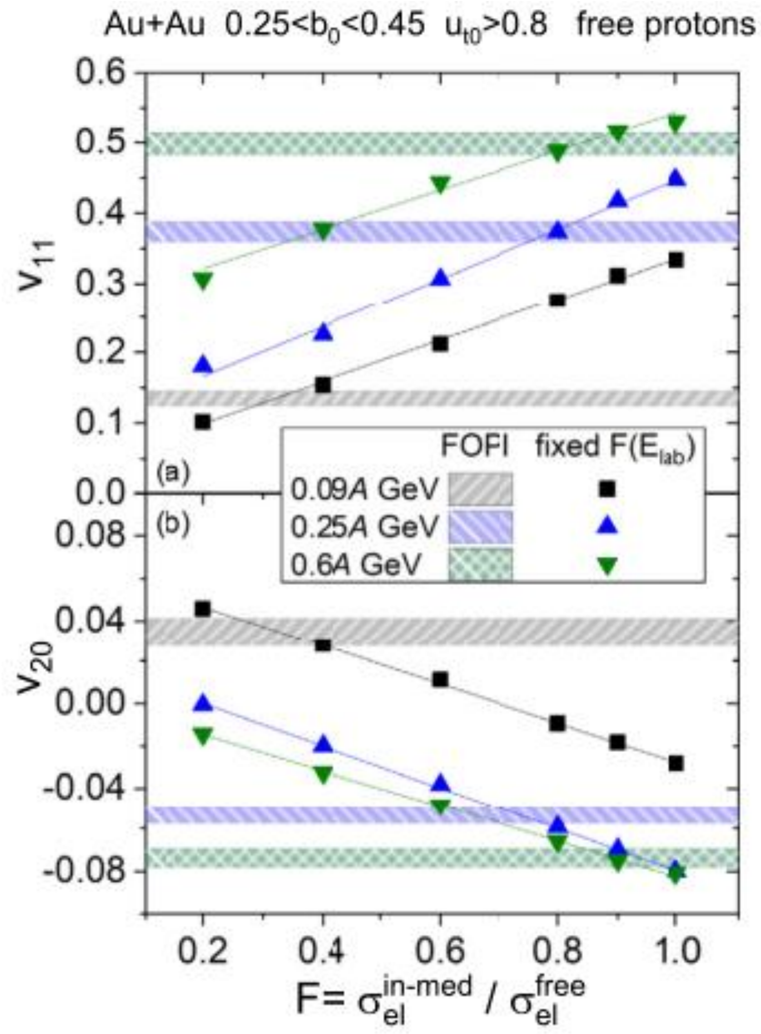
$$\sigma_{\rm el}^{\text{in-med}} = \mathcal{F}(\rho, p) \cdot \sigma_{\rm el}^{\rm free}$$

$$\mathcal{F}(\rho, p) = \begin{cases} f_0, & p_{NN} > 1\ \mathrm{GeV}/c, \\ \frac{\lambda + (1-\lambda)e^{-\frac{\rho}{\zeta\rho_0}} - f_0}{1+(p_{NN}/p_0)^{\kappa}} + f_0, & p_{NN} \leq 1\ \mathrm{GeV}/c. \end{cases}$$

2-σ confidence limit

E_{lab} (A GeV)	0.09	0.15	0.25	0.4	0.6	0.8
fixed F	0.32	0.57	0.76	0.86	0.87	0.87
error	±0.07	±0.06	±0.06	±0.07	±0.10	±0.09

Machine learning further reduce the error



Phys. Lett. B, 828: 137019 (2022).

III、 Results on EoS

Machine learning the in-medium correction factor on nucleon-nucleon elastic cross section

Initial state → UrQMD → Observables

^{132}Sn + ^{124}Sn

E_{lab} = 0.27 GeV/nucleon

$b_0 = 0$

F = 0.4 - 0.8 (随机选取)

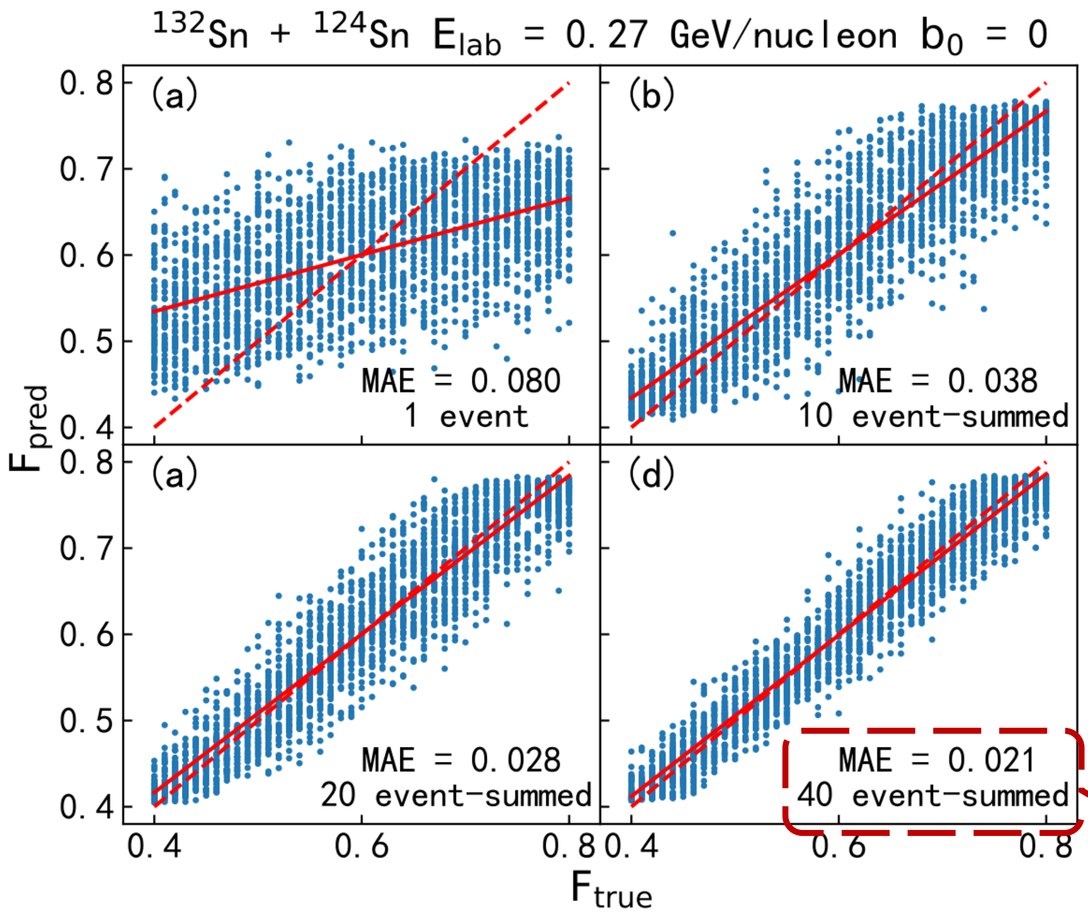
$$E_\perp = \sum \frac{p_t^2}{2m}$$

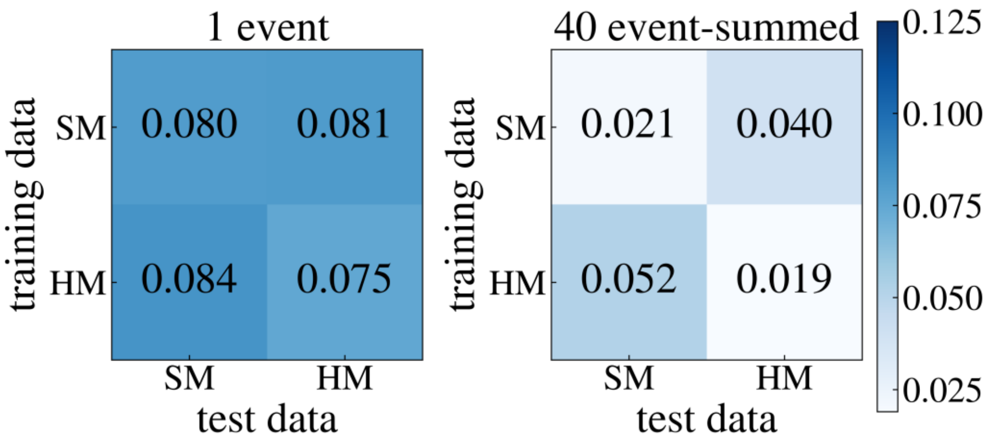
$$R_p = \frac{2}{\pi}\frac{\sum p_t}{\sum |p_z|}$$

Features	Description
N_p	Yield of proton
$(E_\perp)_p$	Transverse energy of proton
$<p_t>_p$	Mean transverse momentum per nucleon of proton
$(R_P)_p$	Ratio of transverse momentum to longitudinal momentum of proton
N_d	Yield of deuterium
$(E_\perp)_d$	Transverse energy of deuterium
$<p_t>_d$	Mean transverse momentum per nucleon of deuterium
$(R_P)_d$	Ratio of transverse momentum to longitudinal momentum of proton

III、 Results on EoS

Machine learning the in-medium correction factor on nucleon-nucleon elastic cross section





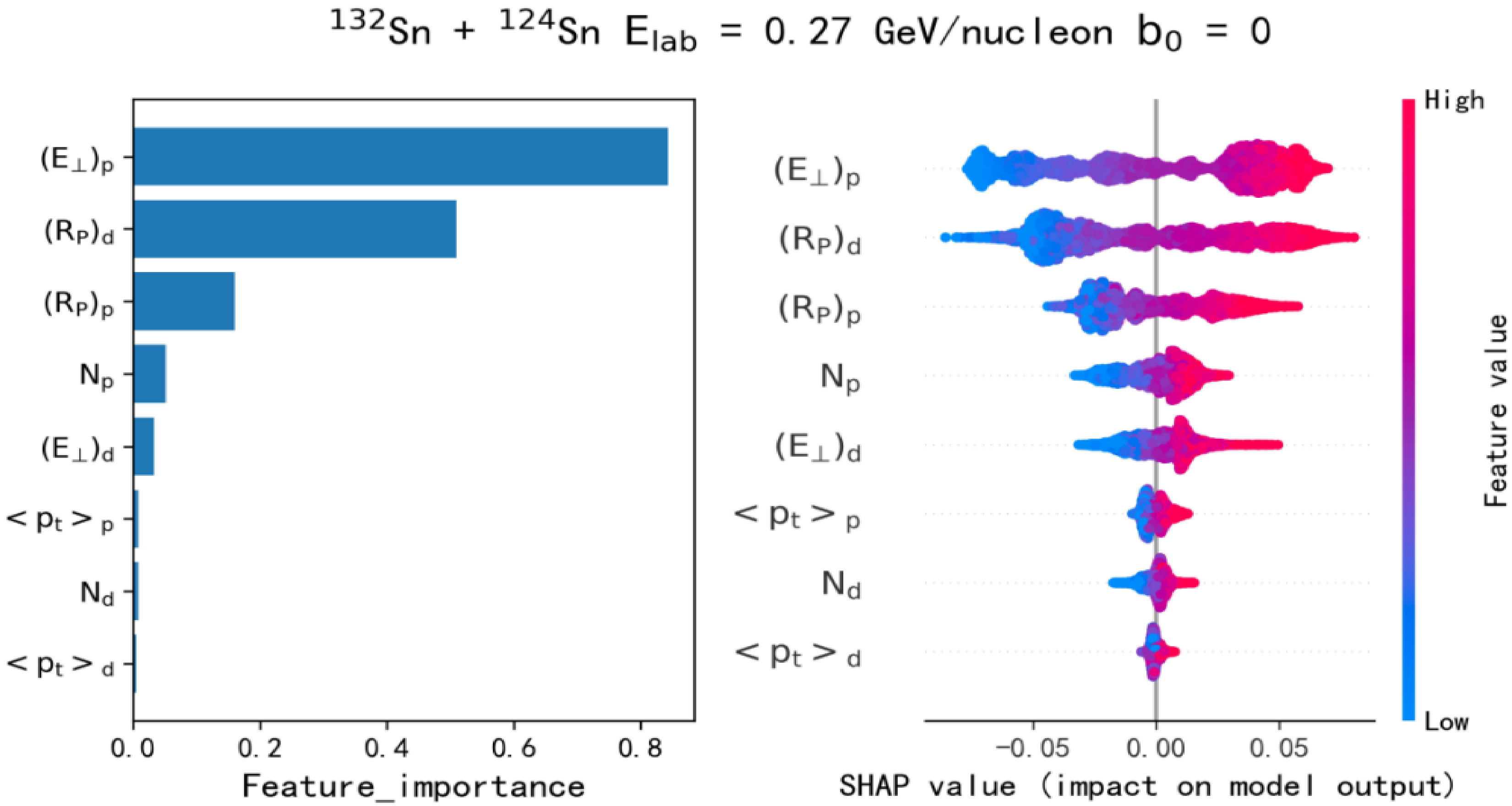
SM： K_0 = 200 MeV

HM： K_0 = 380 MeV

When the average of a small number of events is used as the observables of the sample, the influence of fluctuations is significantly reduced and the error is greatly decreased.

III、Results on EoS

Machine learning the in-medium correction factor on nucleon-nucleon elastic cross section



- ✓ The contribution of each input feature
- ✓ Sorting of sensitive observables
- ✓ Presenting the positive and negative correlations between observables and F.

Ⅲ、 Results on EoS

Bayesian analysis constraints EoS

Thomas Bayes

Constrain multiple parameter distributions from multiple experimental data simultaneously.

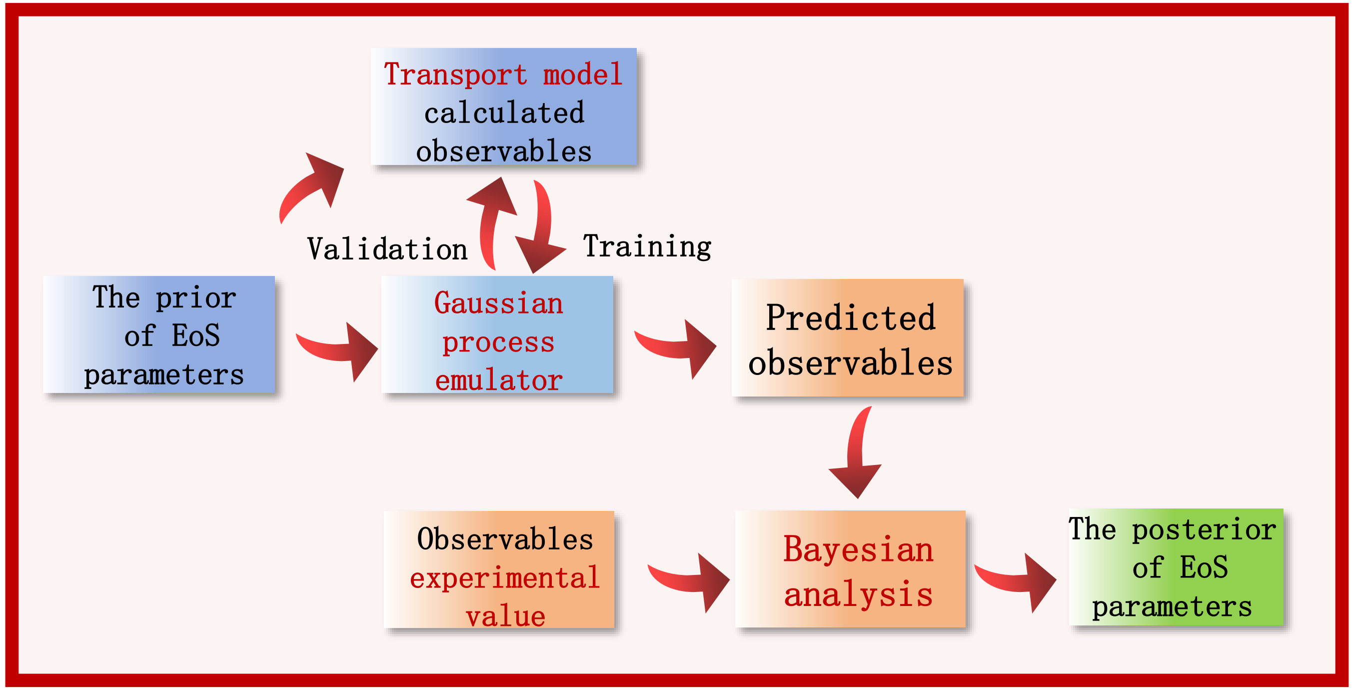
$$\mathcal{P}(\boldsymbol{\theta} \mid \mathbf{y}_{\text{exp}}) \propto \mathcal{P}(\mathbf{y}_{\text{exp}} \mid \boldsymbol{\theta})\mathcal{P}(\boldsymbol{\theta}).$$

Posterior

Likelihood

Prior

$$p(y_{exp}|\theta) \propto \prod_i exp(-\frac{(y_i-\mu_i)^2}{2\sigma_i^2})$$



III、 Results on EoS

Bayesian analysis constraints EoS

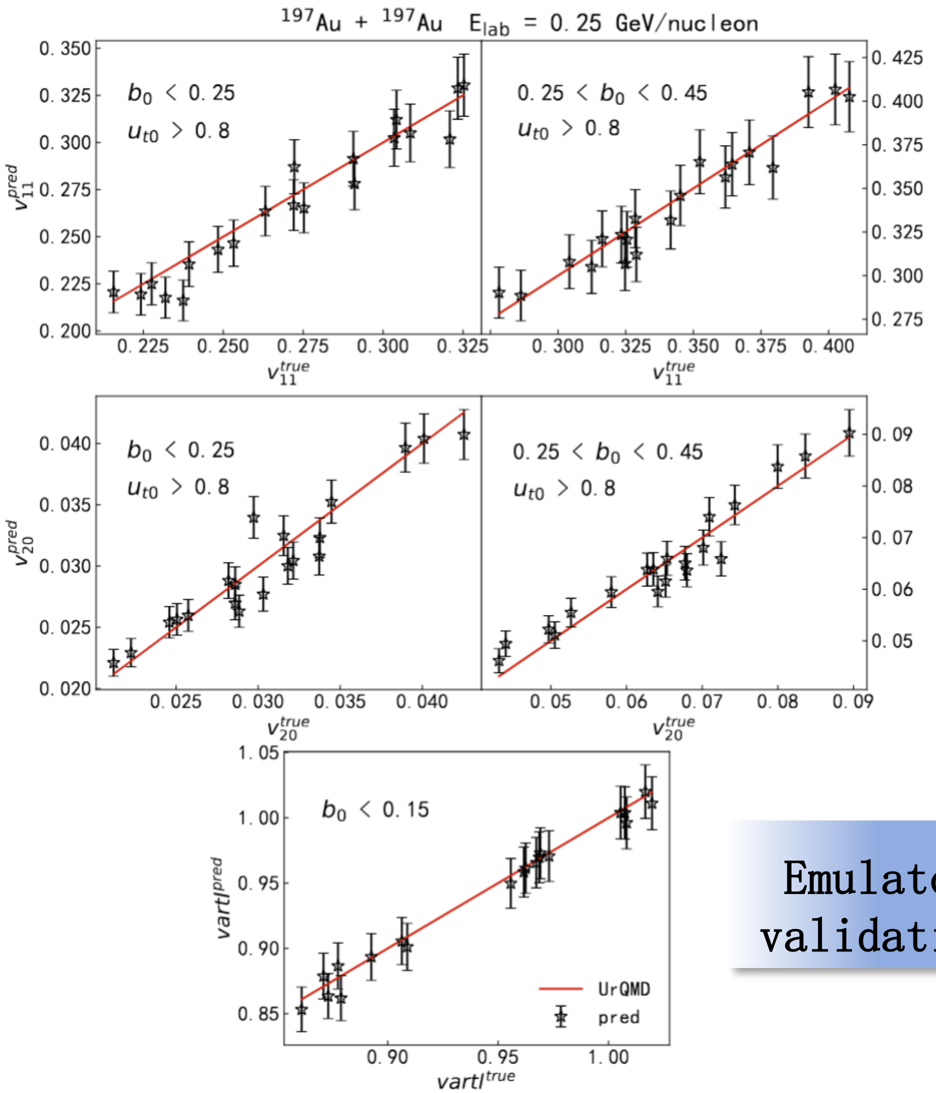
The prior of EoS parameters

参数	先验分布
incompressibility K_0 (MeV)	N(240， 60^2)
effective mass m^*/m_0	U(0.5, 0.95)
the in-medium correction factor F	U(0.6, 1.0)

Observables experimental value

E_{lab}	Observable	impact parameter	u_{t0} cut	value
0.25	v_{11}	$b_0 < 0.25$	$u_{t0} > 0.8$	0.23±0.01
		$0.25 < b_0 < 0.45$	$u_{t0} > 0.8$	0.37±0.01
0.25	$-v_{20}$	$b_0 < 0.25$	$u_{t0} > 0.8$	0.026±0.001
		$0.25 < b_0 < 0.45$	$u_{t0} > 0.8$	0.046±0.005
		$0.45 < b_0 < 0.55$	$u_{t0} > 0.8$	0.064±0.002
0.25	$vartl$	$b_0 < 0.15$	—	0.891±0.041
0.4	v_{11}	$b_0 < 0.25$	$u_{t0} > 0.8$	0.31±0.01
		$0.25 < b_0 < 0.45$	$u_{t0} > 0.8$	0.45±0.02
		$b_0 < 0.15$	$u_{t0} > 0.4$	0.23±0.01
		$b_0 < 0.25$	$u_{t0} > 0.4$	0.28±0.01
		$0.25 < b_0 < 0.45$	$u_{t0} > 0.4$	0.39±0.02
		$0.45 < b_0 < 0.55$	$u_{t0} > 0.4$	0.43±0.01
0.4	$-v_{20}$	$b_0 < 0.25$	$u_{t0} > 0.8$	0.041±0.002
		$0.25 < b_0 < 0.45$	$u_{t0} > 0.8$	0.075±0.005
		$0.45 < b_0 < 0.55$	$u_{t0} > 0.8$	0.111±0.002
		$0.25 < b_0 < 0.45$	$u_{t0} > 0.4$	0.054±0.003
0.4	$vartl$	$b_0 < 0.15$	—	0.955±0.045

arXiv:2509.03406

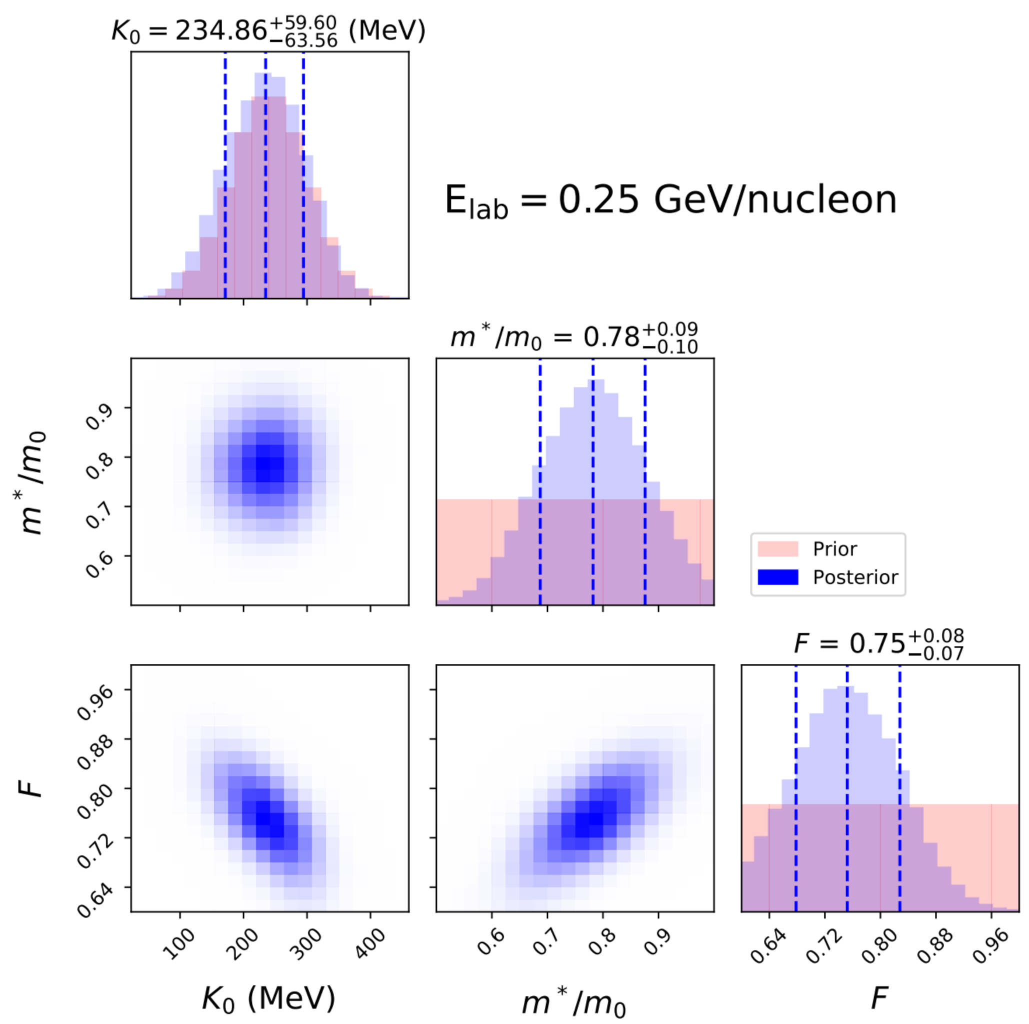


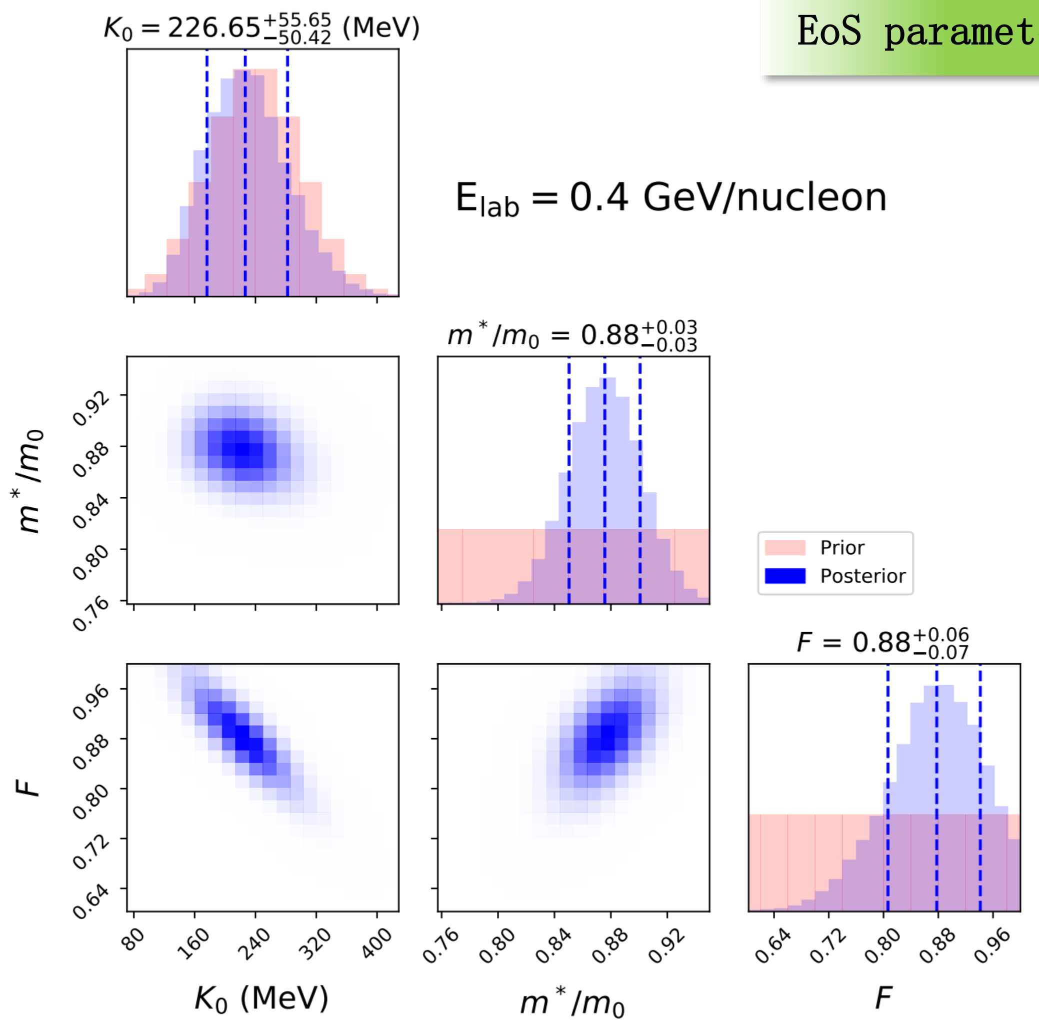
Emulator validation

III、 Results on EoS

Bayesian analysis constraints EoS

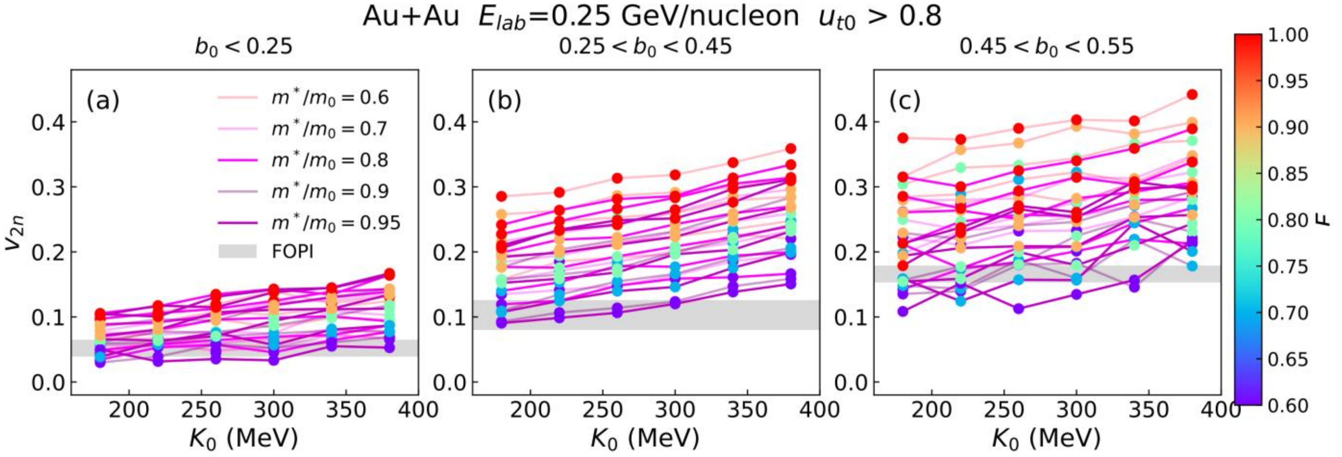
The posterior of EoS parameters

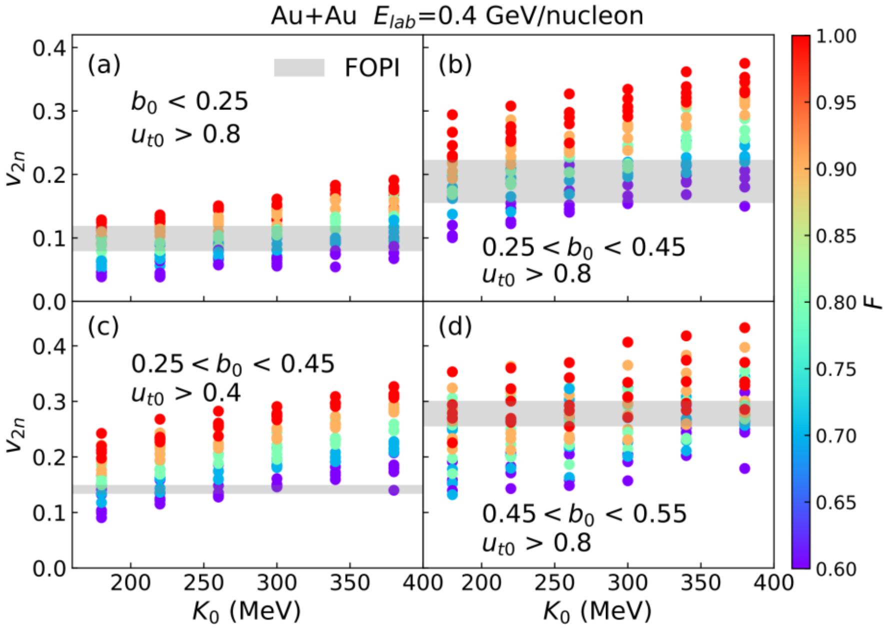




Ⅲ、Results on EoS

Bayesian analysis constraints EoS





$$v_{2n} = |v_{20}| + |v_{22}|$$

Posterior

E_{lab} = 0.25 GeV/nucleon

$K_0 = 234.86^{+59.60}_{-63.56}$ MeV

$m^*/m_0 = 0.78^{+0.09}_{-0.10}$

$F = 0.75^{+0.08}_{-0.07}$

E_{lab} = 0.4 GeV/nucleon

$K_0 = 226.65^{+55.65}_{-50.42}$ MeV

$m^*/m_0 = 0.88^{+0.33}_{-0.33}$

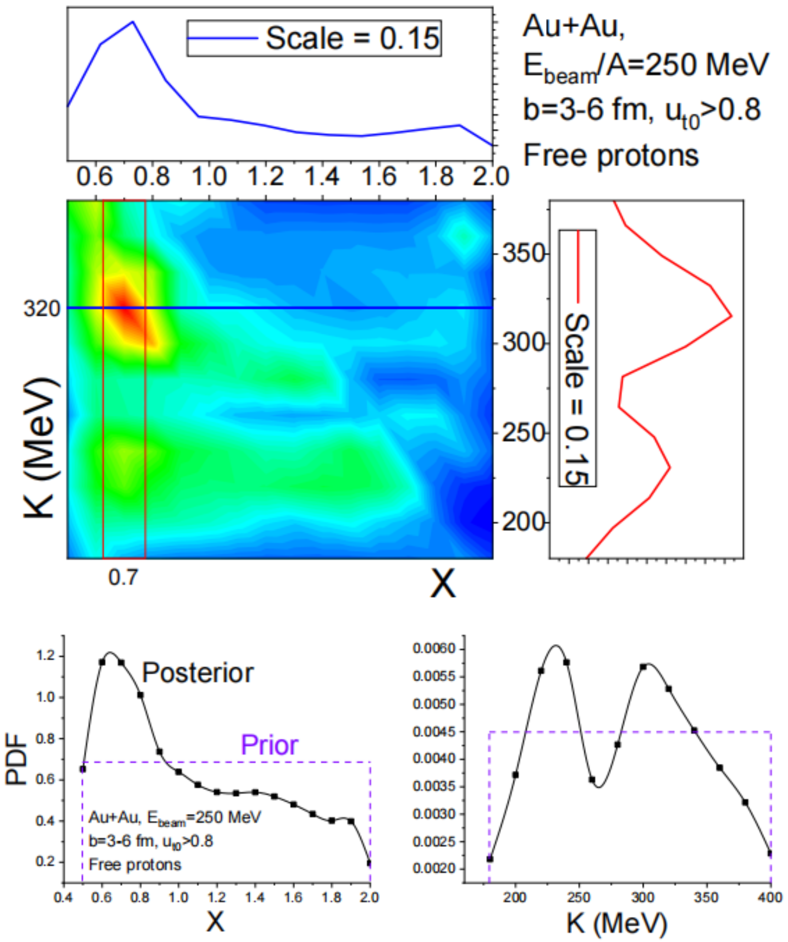
$F = 0.88^{+0.06}_{-0.07}$

III、Results on EoS

Bayesian analysis constraints EoS

BA LI & WJ Xie, Phys. Rev. C, 111(5): 054602 (2025)

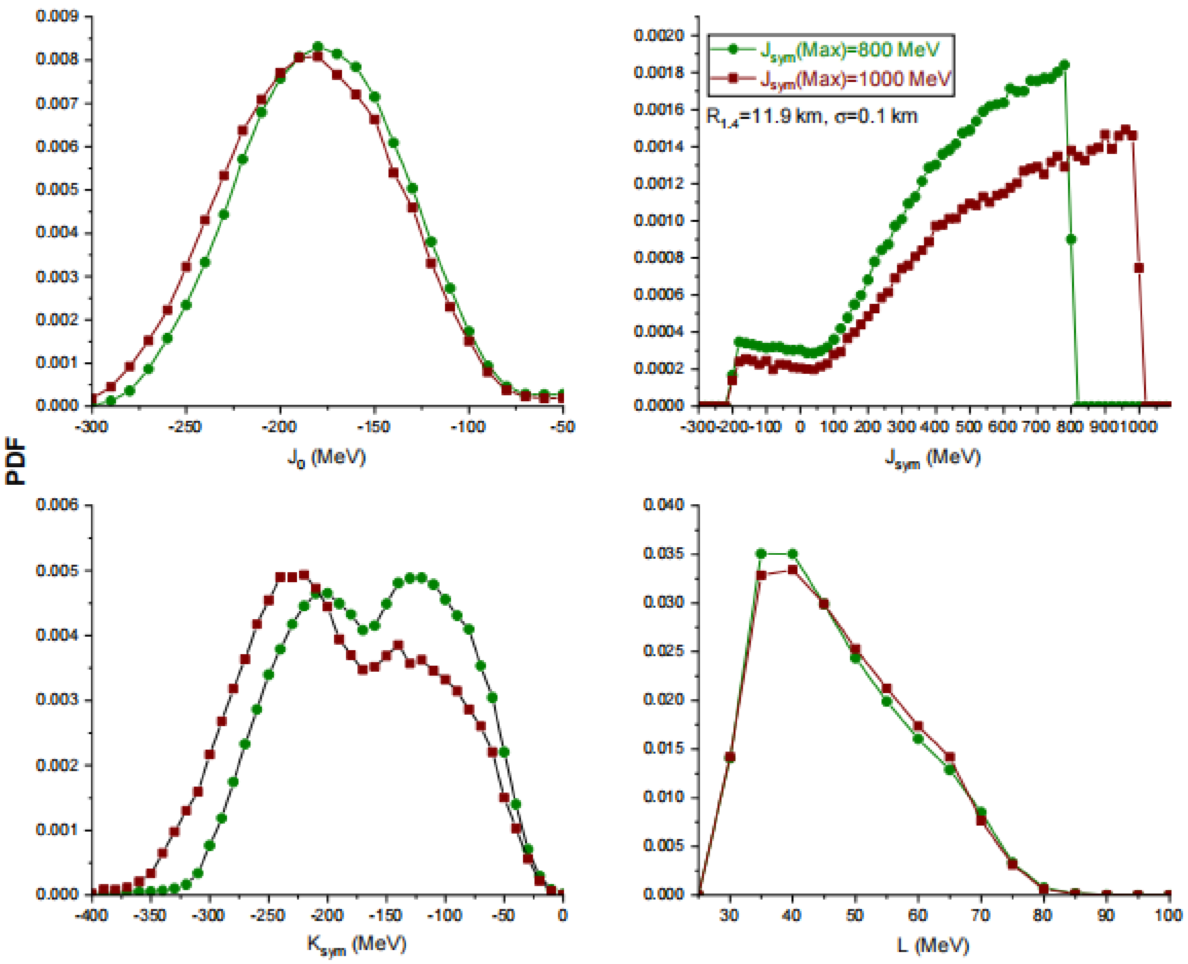
模型依赖性



$$X \equiv \sigma_{NN}^{med} / \sigma_{NN}^{free}$$

Bo-An LI, arXiv:2510.05508, 2025

& Phys. Rev. D 110, 103040 (2024)： 中子星更高阶的关联



IV、Summary & Outlook

- Incorporating physical information features in the input can significantly reduce the MAE. So, Physical information plays a crucial role in predicting using machine learning, which requires deeper nuclear theoretical investigations.

- Machine learning + Bayesian analysis have become important tools for constraining and understanding the nuclear EoS, bringing new perspectives and breakthroughs to the research. However, there is still some uncertainty regarding the constraints for K_0.

- Machine learning and Bayesian analysis should be combined with more sensitive observables and research fields, such as the properties of dense nuclear matter under extreme conditions and the comprehensive analysis of multi-messenger astronomical data, in order to promote the continuous development of the research on the EoS.

Thanks for listening!
And welcome to Huzhou!

Huzhou University Qingfeng Li liqf@zjhu.edu.cn