

香港中文大學(深圳)  
The Chinese University of Hong Kong, Shenzhen

High Energy Nuclear Physics  
meets  
Discriminative and Generative AI

**Kai Zhou (CUHK Shenzhen)**

第4届核物理与核数据中的机器学习应用  
研讨会， 南华大学， 衡阳， 湖南 2025

# Overview : Nuclear Physics meets Machine Learning

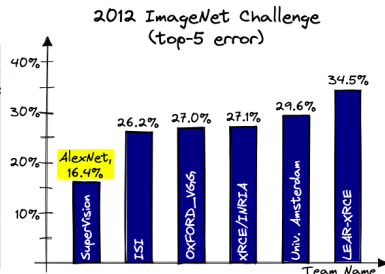
- **2012** : Discovery of Higgs boson



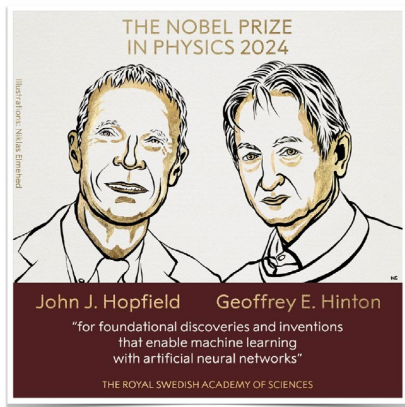
The New York Times

*Physicists Find Elusive Particle Seen as Key to Universe*

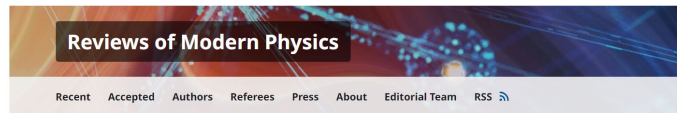
- AlexNet - Birth of Deep Learning



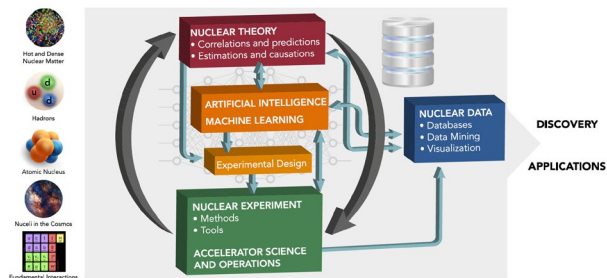
- **2024** : Nobel Prize in Physics



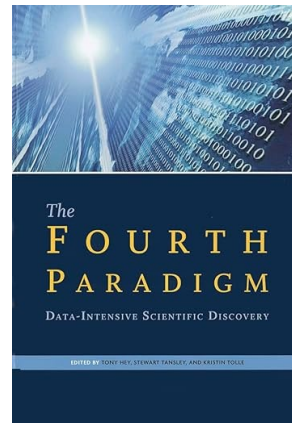
- ML4Physics, AI4Science



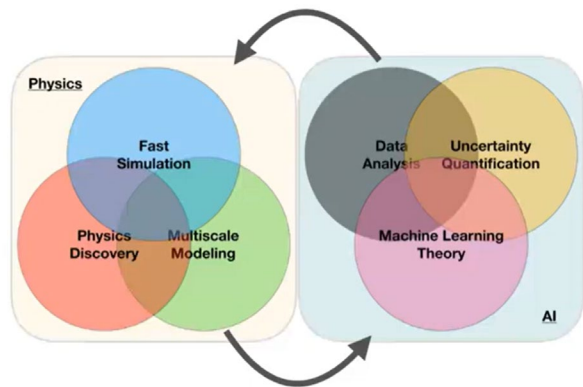
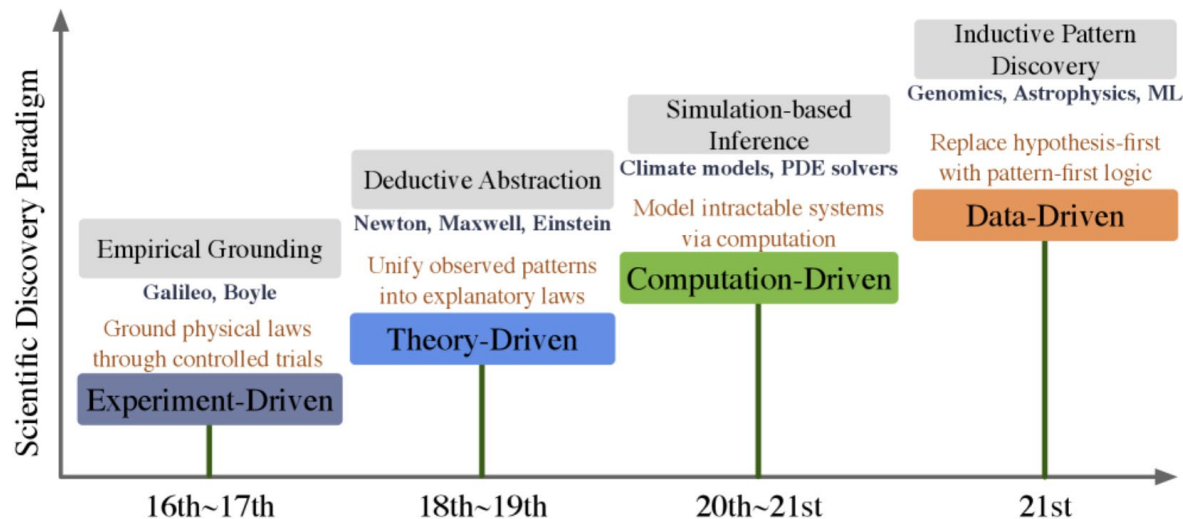
## Colloquium: Machine learning in nuclear physics



# Overview : Fifth paradigm for scientific discovery



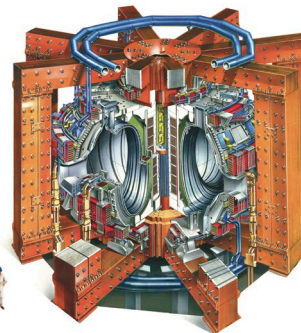
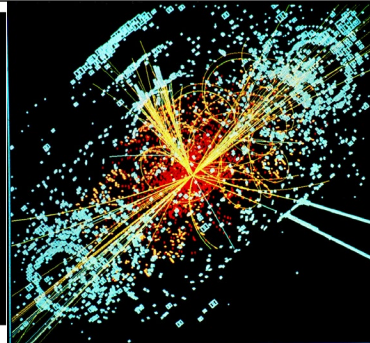
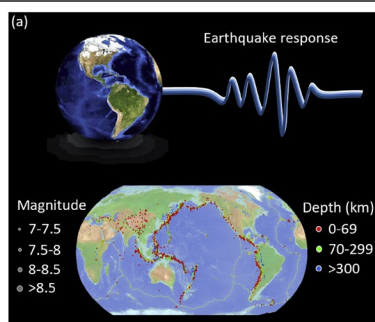
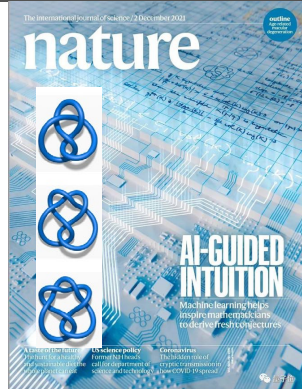
- Scientific discovery were driven by series of **methodological paradigm** evolution
- **Machine Learning & Synthetic data** forms the fifth methodological paradigm



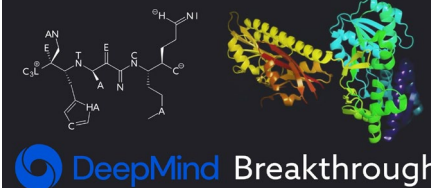
# Overview : Machine- and Deep-Learning



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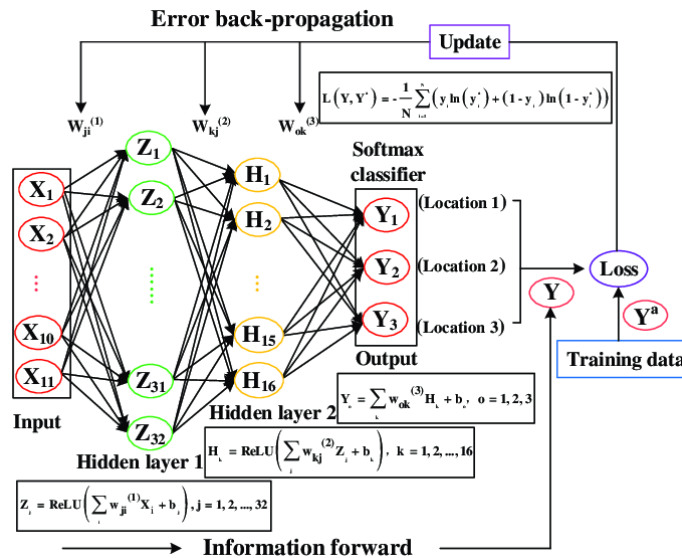
## Protein Folding AI



Find and Decode the  
mapping/representations  
into  
Deep Neural Network

→ **Function approximator**

Universal approximator  
(Hastad et al 86 & 91)



Differentiable programming

Backward Propagation

Gradient Descent Algorithm

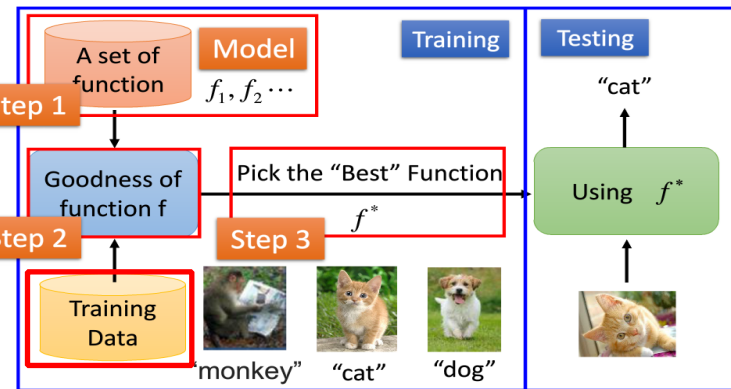


# Overview: Machine learning, Deep Neural Networks, Representation learning

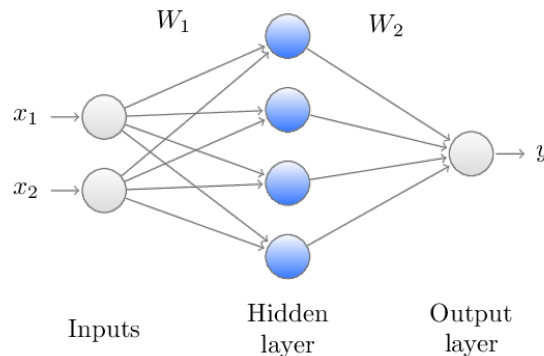
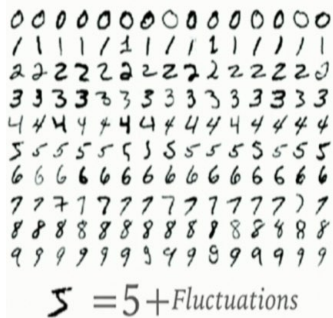
## Framework

Image Recognition:

$$f(\text{cat image}) = \text{"cat"}$$



modified from Hung-Yi.Lee

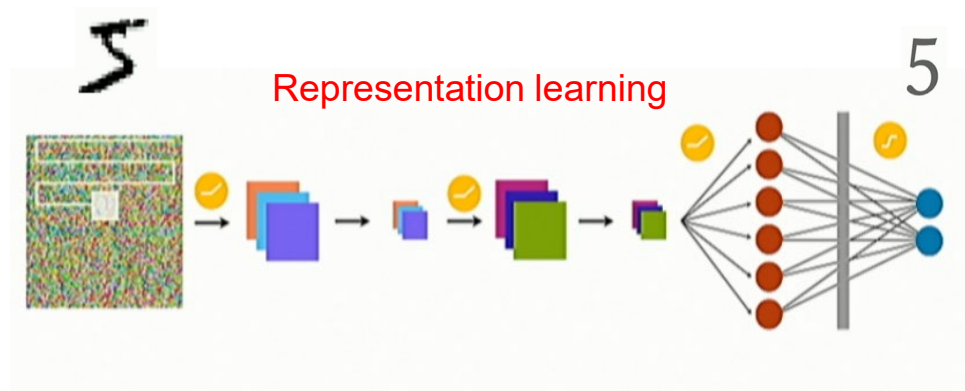


**Composing :**

Linear affine transformation  
+  
Non-linear activation

Layer by layer

$$f_{NN}(x; \theta) = h_2(w_2 h_1(w_1 x + b_1) + b_2)$$



- Discriminative Learning : **prediction**

function fitting  $y = f(x)$

conditional probability  $p_{\theta}(y|x) \rightarrow p(y|x)$

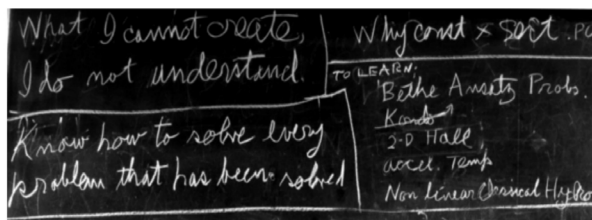


- Generative Modelling : **understand**

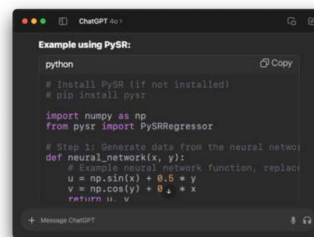
Joint probability distribution  $p_{\theta}(x, y) \rightarrow p(x, y)$



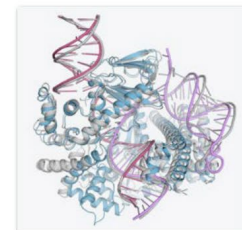
$p(X)$   
pixels, words, atoms, ...



DaLL-E



ChatGPT



AlphaFold3

“What I can not create, I do not understand”

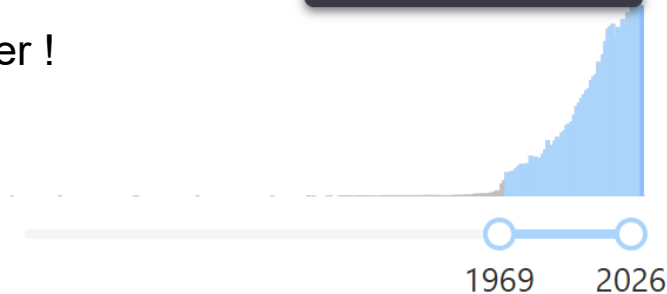
# Overview : super **biased** introduction in this talk in following

literature ▾ ("machine learning" or "deep learning" or "AI" or ""neural network") in nucl-th 🔍

Date of paper

Selected Papers: 52,331  
Total Papers: 52,331  
Year: 2025

- Above 50k paper !



- This talk will selectively (biasedly) focus on high energy nuclear physics studies with:
- Discriminative ML to Physics Unfolding and Generative ML for Physics obs Generation

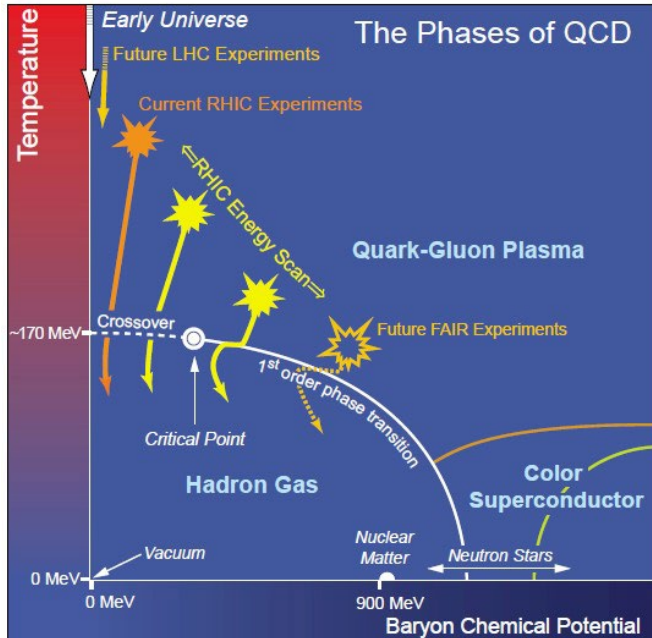
# Overview : **Golden Age** of QCD matter in extreme

- **Phases** of matter : solid, liquid, gas, plasma
- Matter in extreme conditions reveals its **constituents** :  
nuclear matter → quark matter



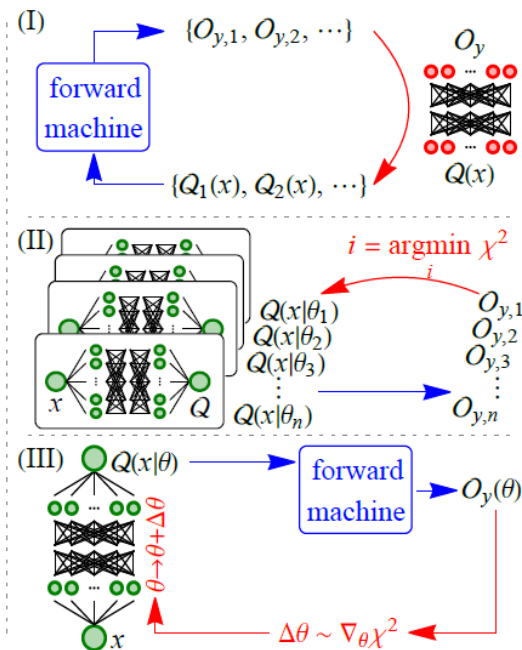
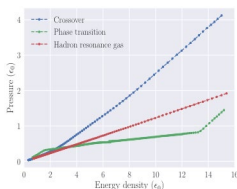
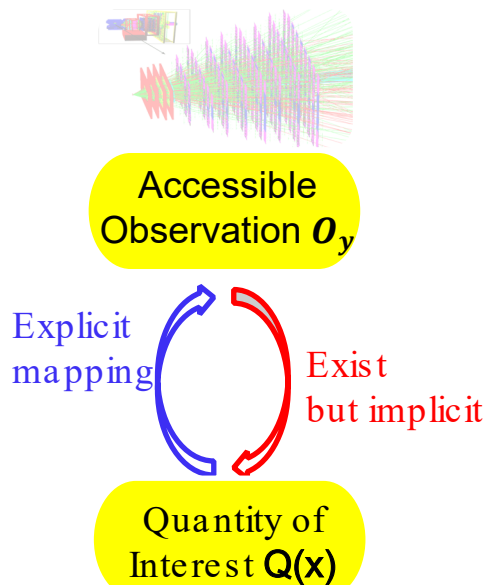
To study the most elementary particle matter :

- **Nuclear Collisions** : heat & compress matter
- **Neutron Star** : dense matter, astronomy constraints
- **Lattice Field Theory / fQCD / Effective models**





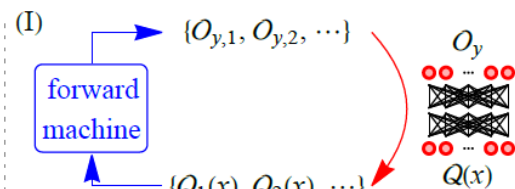
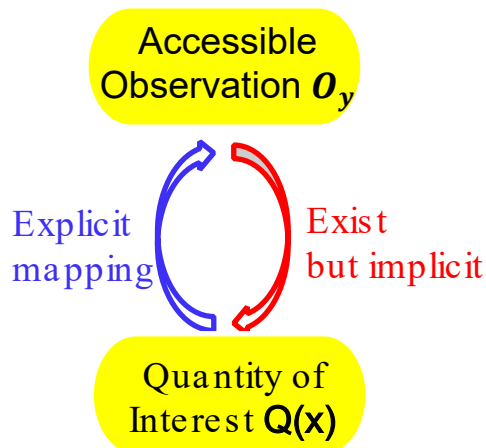
- **Discriminative AI** for unfolding physics in high-energy nuclear physics  
physics priors need to be embedded in solving inverse problems
- **Generative AI** for speeding up physics simulations (IQFT, HIC modelling)



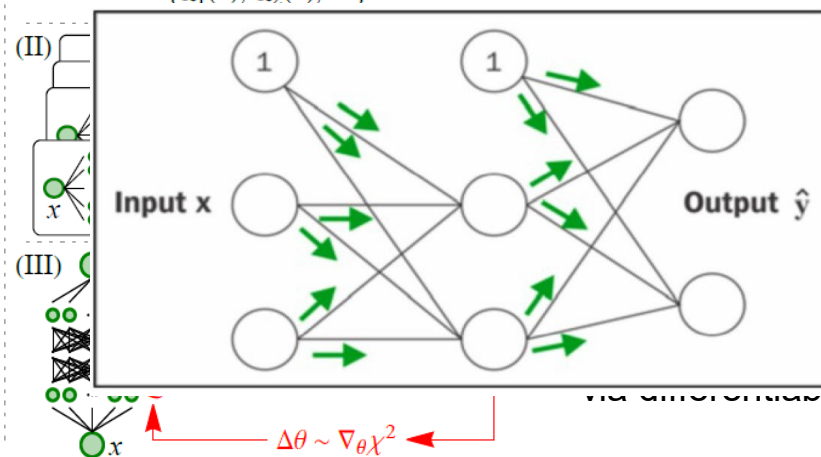
- **Direct inverse mapping capturing :**  
with Supervised Learning
- **Statistical approach to  $\chi^2$  fitting :**  
Bayesian Reconstruction for posterior or Heuristic (Generic) Algorithm to min.  

$$\chi^2 = \sum_y \left( \frac{\mathcal{F}_y[\mathcal{Q}_{NN}(x|\theta)] - O_y}{\Delta O_y} \right)^2$$
- **Automatic Differentiation :**  
fuse physical prior into reconstruction via differentiable programming strategy

$$\frac{1}{2} \nabla_{\theta} \chi^2 = \sum_y \frac{\mathcal{F}_y[\mathcal{Q}_{NN}(x|\theta)] - O_y}{(\Delta O_y)^2} \int dx \frac{\delta \mathcal{F}_y[\mathcal{Q}(x)]}{\delta \mathcal{Q}(x)} \bigg|_{\mathcal{Q}(x)=\mathcal{Q}_{NN}(x|\theta)} \nabla_{\theta} \mathcal{Q}_{NN}(x|\theta)$$



- Direct inverse mapping capturing :**  
with Supervised Learning



**approach to  $\chi^2$  fitting :**  
reconstruction for posterior  
(generic) Algorithm to min.

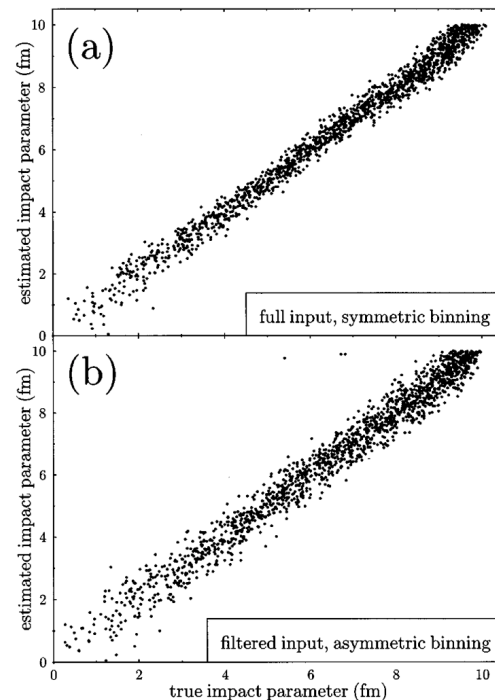
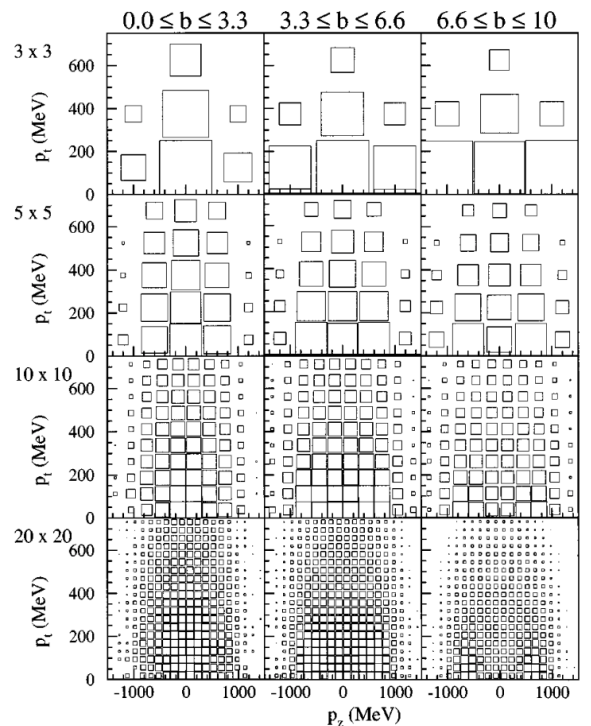
$$\frac{[\mathcal{Q}_{NN}(x|\theta) - \mathcal{O}_y]^2}{\Delta \mathcal{O}_y}$$

**Differentiation :**  
al prior into reconstruction  
via differentiable programming strategy

$$\frac{1}{2} \nabla_{\theta} \chi^2 = \sum_y \frac{\mathcal{F}_y[\mathcal{Q}_{NN}(x|\theta)] - \mathcal{O}_y}{(\Delta \mathcal{O}_y)^2} \int dx \frac{\delta \mathcal{F}_y[\mathcal{Q}(x)]}{\delta \mathcal{Q}(x)} \bigg|_{\mathcal{Q}(x) = \mathcal{Q}_{NN}(x|\theta)} \nabla_{\theta} \mathcal{Q}_{NN}(x|\theta)$$

# Early attempts : impact parameter determination

**Simple DNN**  
**Trained on**  
**QMC data**  
**Input 5X5**



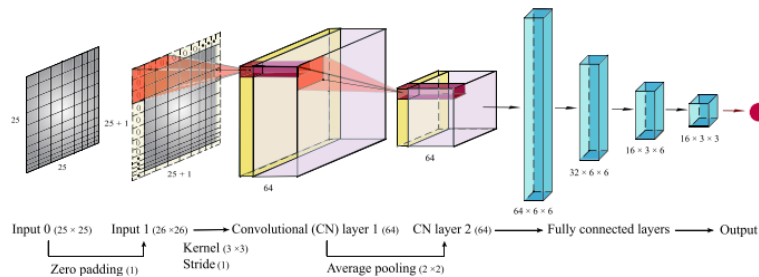
S. A. Bass, A. Bischoff, J. A. Maruhn, H. Stöcker, and W. Greiner, Phys. Rev. C 53, 2358 (1996)



# Further dev for impact parameter determination

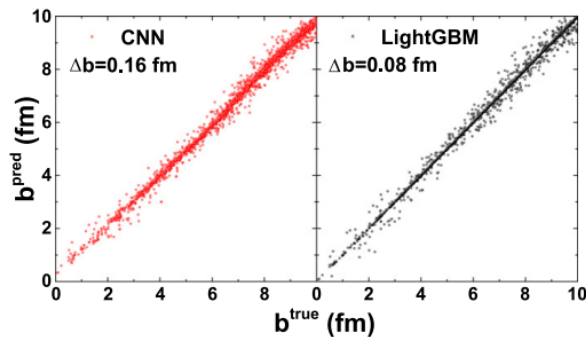
P. Xiang, Y. Zhao, X. Huang,  
**Chi. Phys. C** 53, 2358 (2022)

**MLP and CNN**  
(on AMPT event)



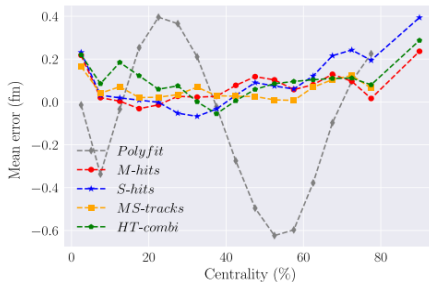
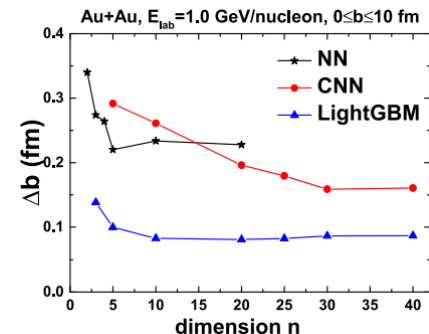
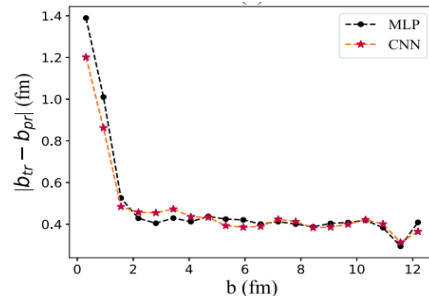
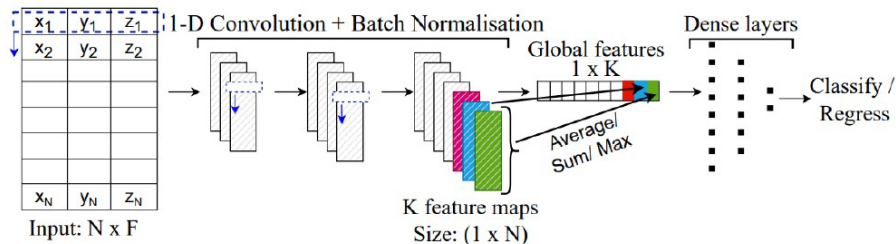
F. Li, Y. Wang, H. Lue, P. Li,  
Q. Li, F. Liu, **JPG** 47, 115104 (2020)

**CNN and LightGBM**  
(on UrQMD event)



M. OK, J. S, K. Z, H. S,  
**PLB** 811, 135872 (2020)

**PointCloud Network**  
(on UrQMD + CBMR00t event)  
End-to-end b estimation

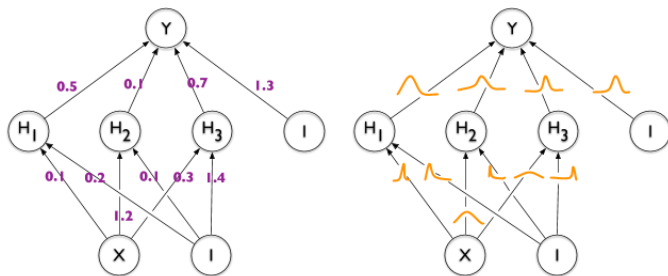


# Initial clustering structure identification in HICs

PHYSICAL REVIEW C **104**, 044902 (2021)

## Machine-learning-based identification for initial clustering structure in relativistic heavy-ion collisions

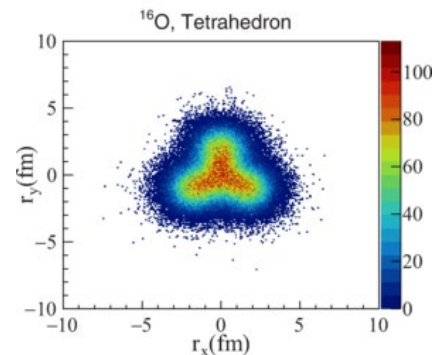
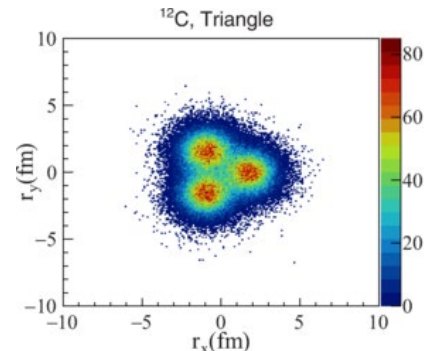
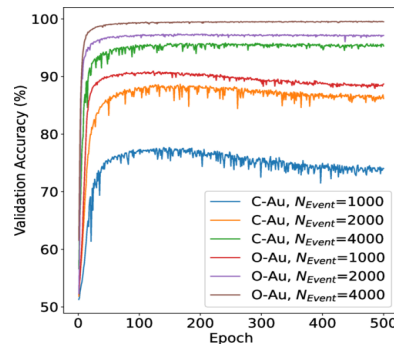
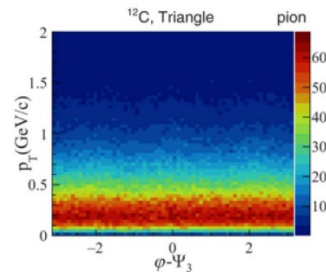
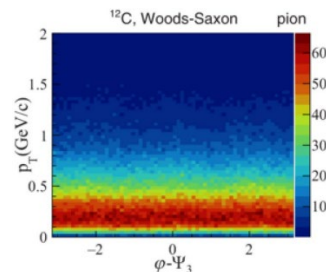
Junjie He (何俊杰)<sup>①,1,2</sup> Wan-Bing He (何万兵)<sup>②,3,\*</sup> Yu-Gang Ma (马余刚)<sup>②,3,†</sup> and Song Zhang (张松)<sup>②,3</sup>



### Bayesian CNN

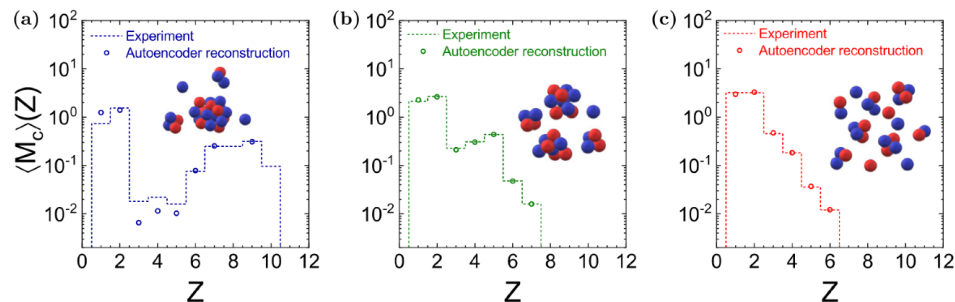
on AMPT events (multiple-event basis)  
Charged pions ( $\phi$ ,  $p_T$ ) from  $^{12}\text{C}/^{16}\text{O}$   
+  $^{197}\text{Au}$  collisions at 200 GeV

Multiple event basis



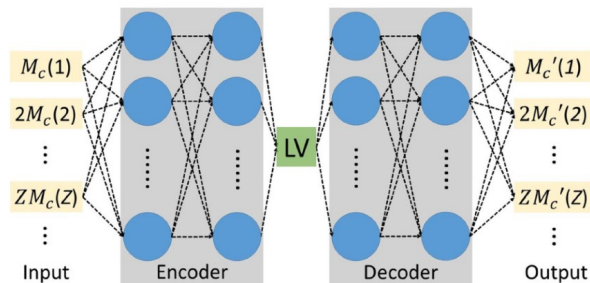
## Nuclear liquid-gas phase transition with machine learning

Rui Wang<sup>①,1,2,\*</sup> Yu-Gang Ma<sup>1,2,†</sup> R. Wada,<sup>3</sup> Lie-Wen Chen<sup>②,4</sup> Wan-Bing He,<sup>1</sup> Huan-Ling Liu,<sup>2</sup> and Kai-Jia Sun<sup>3,5</sup>



EbE charge-weighted charge multiplicity distribution of quasi-projectile as input  $\rightarrow$

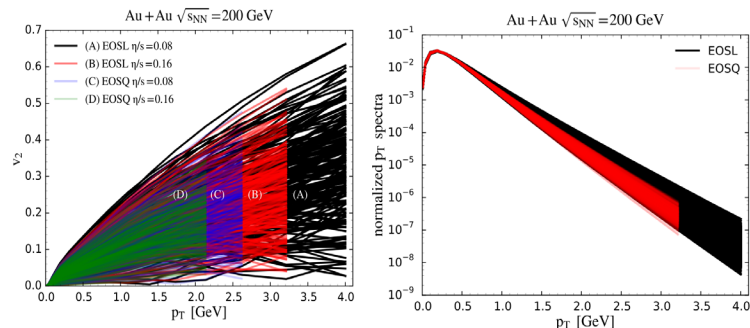
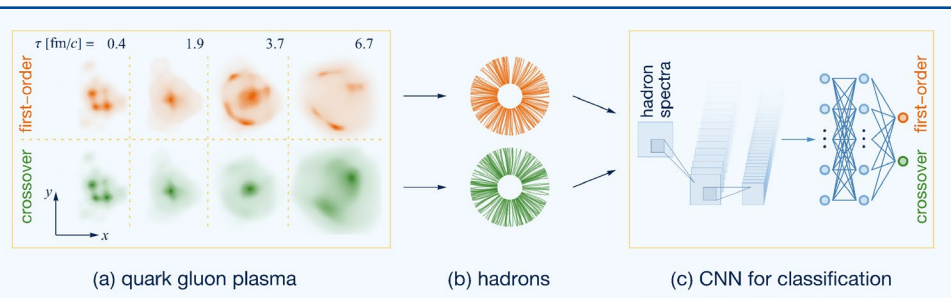
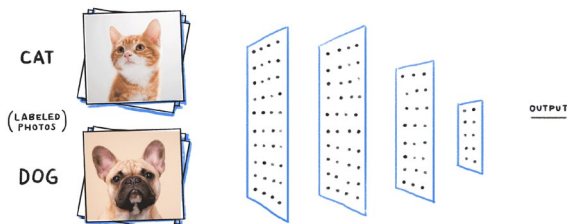
**Autoencoder + confusion scheme**  
(on NIMROD experiment)



# Direct inverse mapping with CNN for identifying QCD transition

Data-driven  
Inverse Mapping

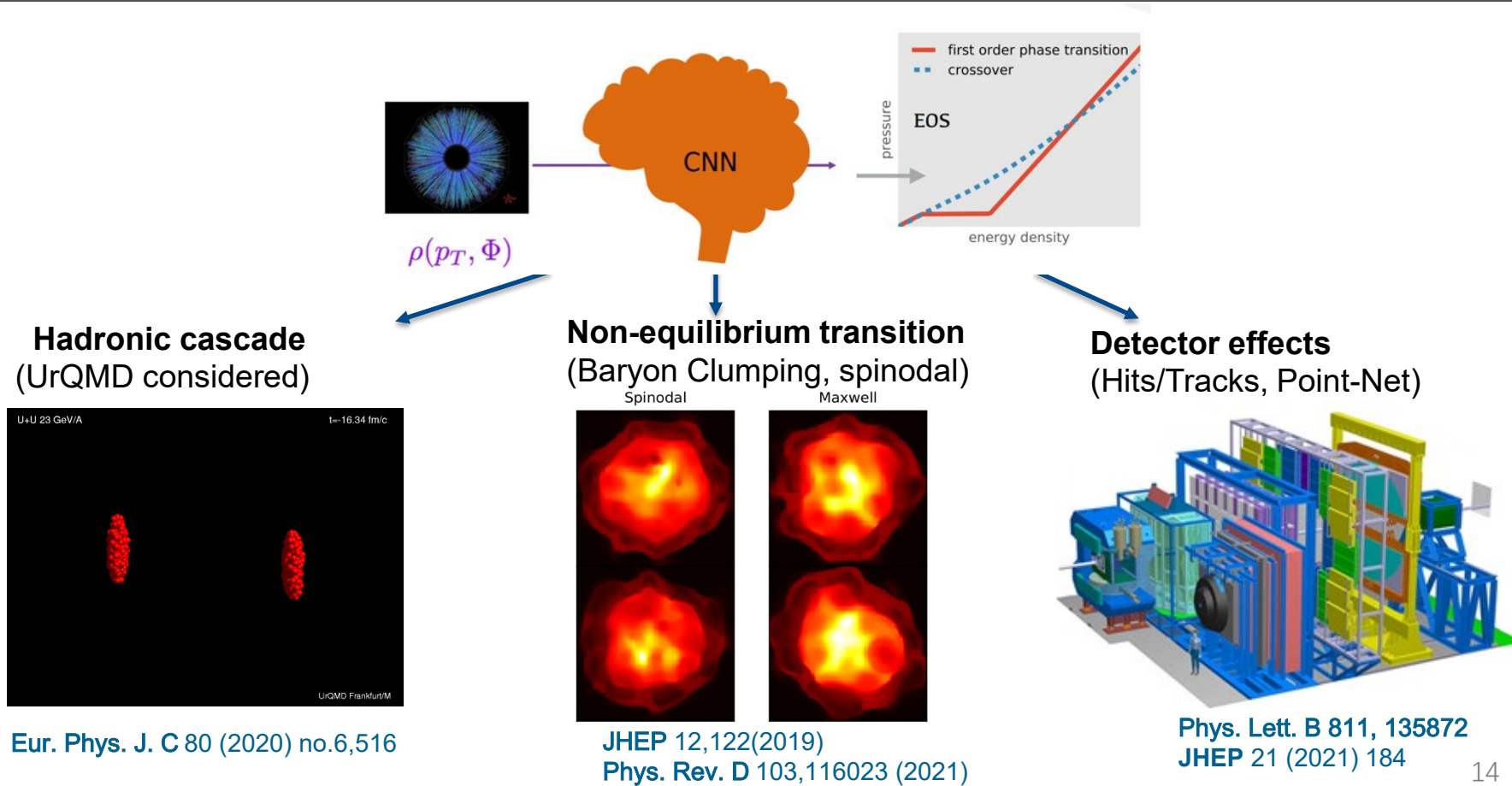
Physics Simulation  
provide the Prior



- Conventional obs. hard to distinguish
- Strongly influence from initial fluctuations and other uncertainties
- CNN : 95% event-by-event accuracy!
- Robust to initial conditions, eta/s

**Conclusion** : Information of early dynamics can **survive** to the end of hydrodynamics and encoded within the final state raw spectra, immune to evolution's uncertainties, **with deep CNN we can decode it back.**



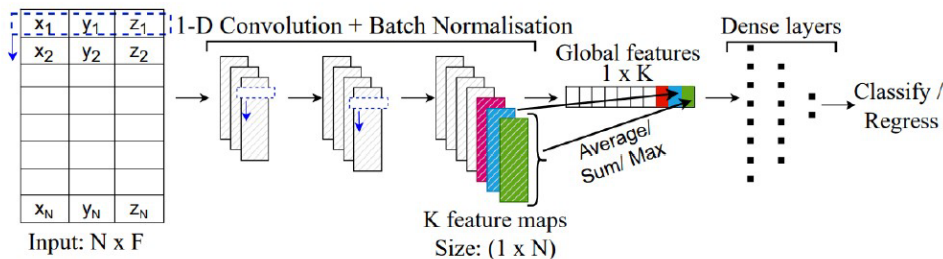
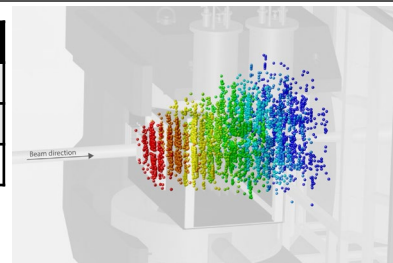


# Point Cloud Network for Physics online analysis for HICs

- Experimental data has inherent **point cloud structure**
  - collection of particles** as 2D array :
- PointNet** based models learn directly from point clouds.
  - respects the **order invariance** of point clouds
  - direct processing of experimental data from detector  $\Rightarrow$  **ideal online analysis algorithm**
  - optimal for higher dimensional data

X1	y1	Z1
X2	y2	Z2
.	.	.
.	.	.
.	.	.
.	.	.
Xn	yn	Zn

E	Px	Py	Pz	pid
6.84	1.07	4.5	6.83	211
40.4	0.06	0.54	40	321
...	...	...	...	...



## • Collision Centrality Regression

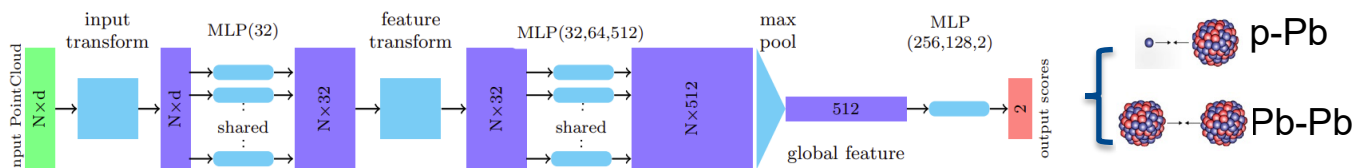
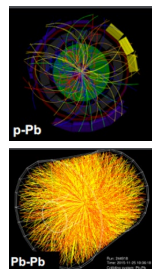
M. OK, J. S, K. Zhou, H. S, *Phys.Lett.B* 811 (2020) 135872

## • EoS Classification

M. OK, K. Zhou, J. S, H. S, *JHEP* 10(2021) 184

## • Small/ Large-system Identification

Manjunath O.K. and Kai Zhou, etc. *Phys.Lett.B* 811 (2020) 135872; *JHEP*10(2021)184.

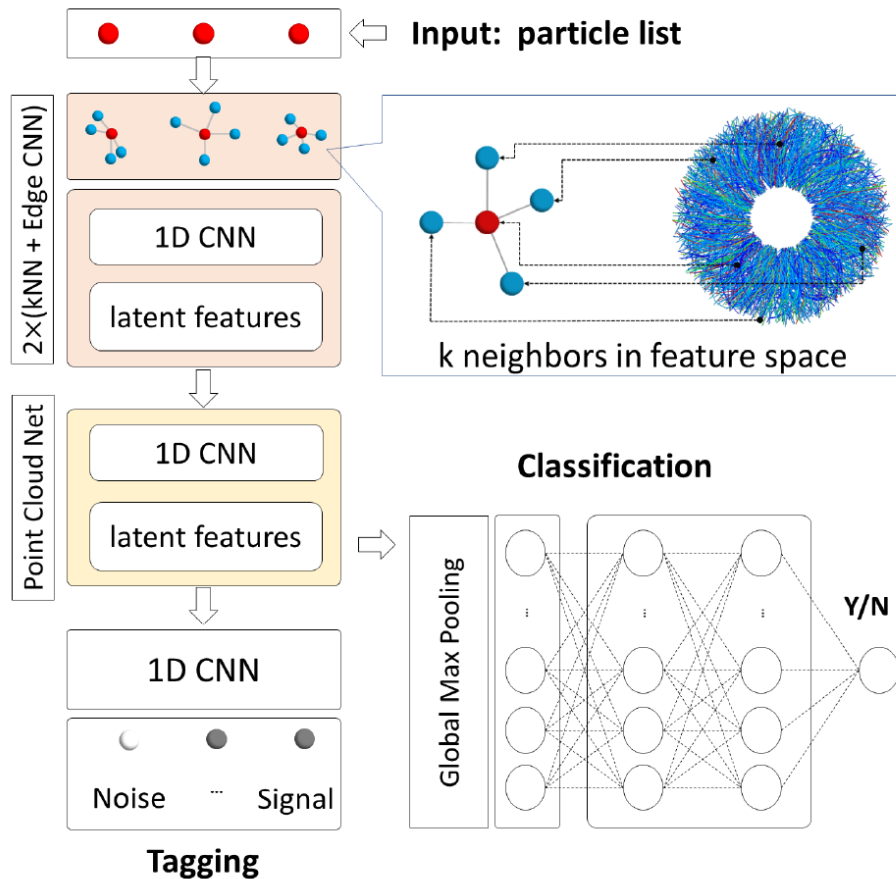


S.Guo, H. Wang, K. Zhou, G. Ma, *Phys.Rev.C* 2024

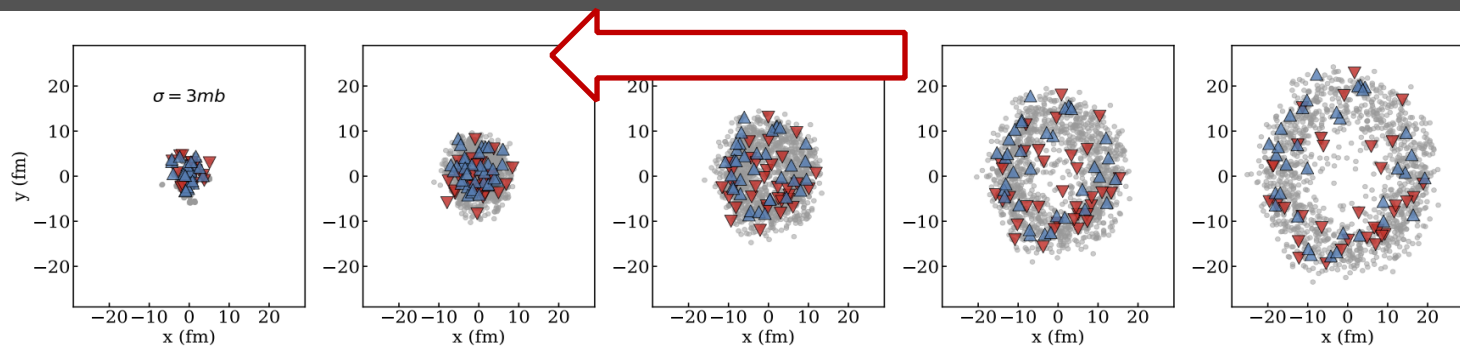
# Modern **dynamical edge CNN** + PCN for self similarity searching

dynamical edge convolution network followed by a point cloud net is used to identify self-similarity and critical fluctuations in HIC

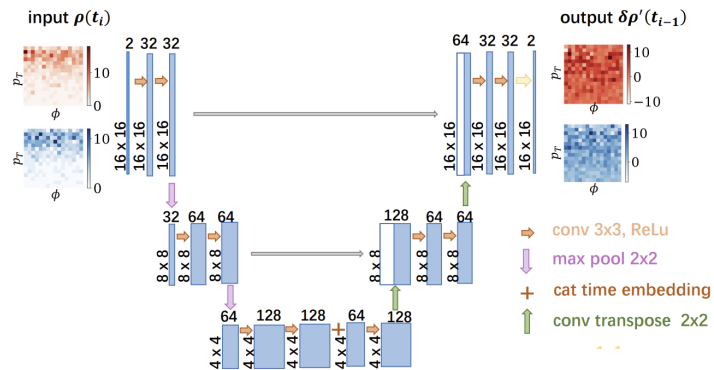
Repeating the **KNN** and **edge convolution** blocks twice helps to find long-range multi-particle correlations that are the key to searching for critical fluctuations.



# Unfold CME in HICs with time-embedded UNet

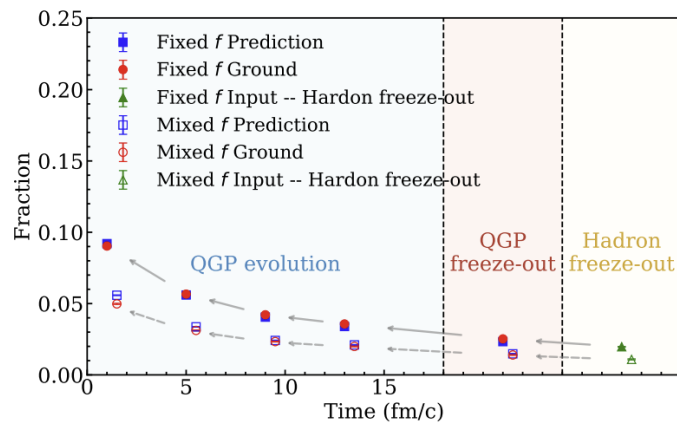


$$f = \frac{N_{\uparrow(\downarrow)}^{\pm} - N_{\downarrow(\uparrow)}^{\pm}}{N_{\uparrow(\downarrow)}^{\pm} + N_{\downarrow(\uparrow)}^{\pm}}$$



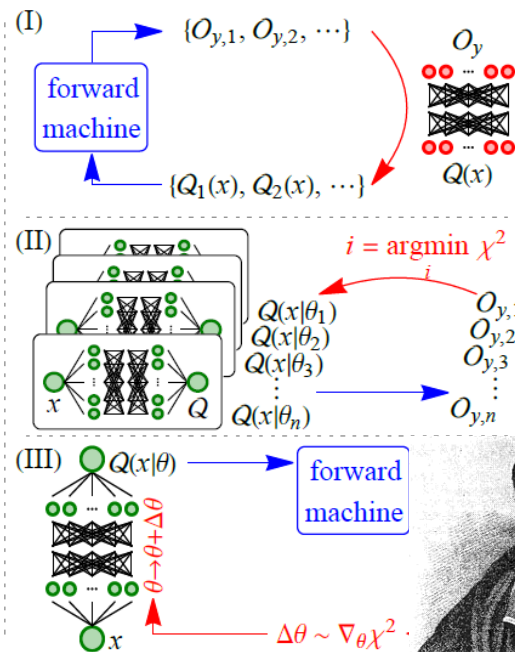
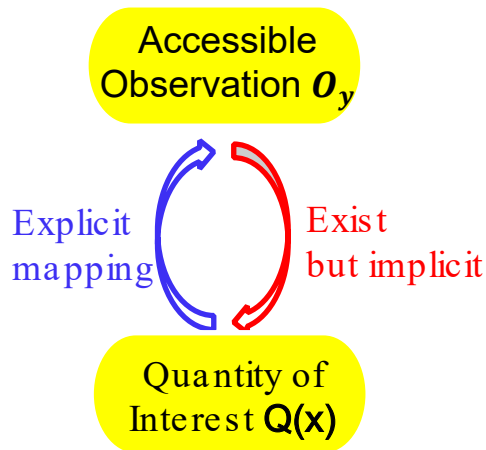
$$\delta\rho(t_{i-1}) = \rho(t_{i-1}) - \rho(t_i)$$

$$\rho'(t_{i-1}) = \delta\rho'(t_{i-1}) + \rho(t_i)$$



See talk of **Shuang Guo** this afternoon, 14:50pm

- **Low-level features** (raw data) might work better in DL era !
- **Point Cloud** representation for nuclear collision analysis ! Theory-Data
- **Physics prior** info is important !
- Need to have well-defined problem : classification, regression
- Benchmark datasets release from our community?
- Interpretable ? How to connect to fundamental physics?



- Direct inverse mapping capturing :**  
with Supervised Learning

- Statistical approach to  $\chi^2$  fitting :**  
Bayesian Reconstruction for posterior or Heuristic (Generic) Algorithm to min.

$$\chi^2 = \sum_y \left( \frac{\mathcal{F}_y[Q_{NN}(x|\theta)] - O_y}{\Delta O_y} \right)^2$$

## Bayes' Theorem

$$\overbrace{P(\theta | y)}^{\text{Posterior}} \propto \prod_i^N \underbrace{P(y_i | \theta)}_{\text{Data Likelihood}} \overbrace{P(\theta)}^{\text{Prior}}$$





## Constraining the Equation of State of Superhadronic Matter from Heavy-Ion Collisions

Scott Pratt,<sup>1</sup> Evan Sangaline,<sup>1</sup> Paul Sorensen,<sup>2</sup> and Hui Wang<sup>2</sup>

<sup>1</sup>*Department of Physics and Astronomy and National Superconducting Cyclotron Laboratory Michigan State University, East Lansing, Michigan 48824, USA*

<sup>2</sup>*Brookhaven National Laboratory, Upton, New York 11973, USA*

(Received 19 January 2015; published 19 May 2015)

$$P(D|\theta) = \prod_i \exp(-(z_i(\theta) - z_{i,\text{exp}})^2/2),$$

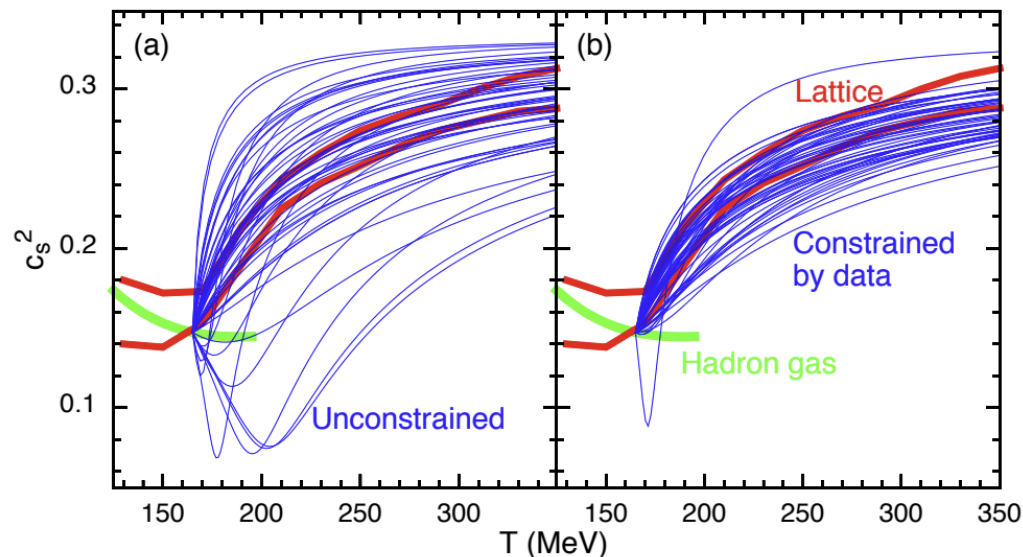
14 model parameters

speed of sound squared slightly  
softer than lattice EoS

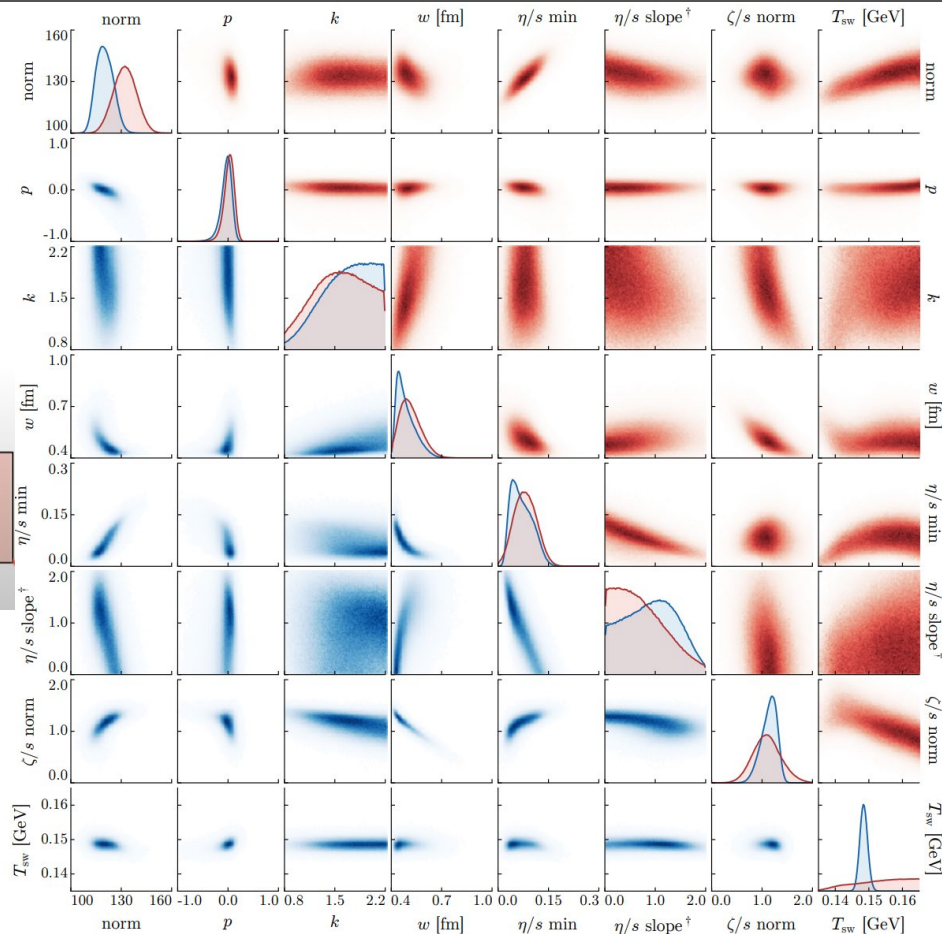
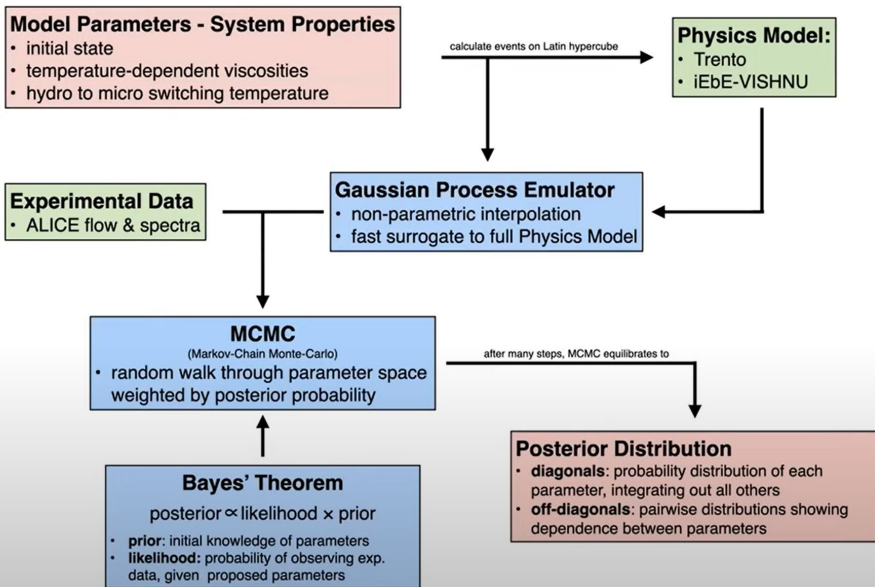
**But significantly overlap**

$$c_s^2(\epsilon) = c_s^2(\epsilon_h) + \left(\frac{1}{3} - c_s^2(\epsilon_h)\right) \frac{X_0 x + x^2}{X_0 x + x^2 + X'^2},$$

$$X_0 = X' R c_s(\epsilon) \sqrt{12}, \quad x \equiv \ln \epsilon / \epsilon_h,$$



# Bayesian (Statistical) global fit on HICs



Trento + IEBE-VishNew + UrQMD

J. Bernhard, J. Morel, S. Bass, **Nat. Phys.** 15, 1113 (2019)

G. Nijs, W. Schee, U. Guersoy, R. Snellings, **PRC**103,054909;

JETSCAPE, **PRL**126,242301; U. Heinz+, 2302.14184 (VAH)

M. R. Heffernan, C. Gale, S. Jeon, J. Paudyal, **PRC**109,065207;

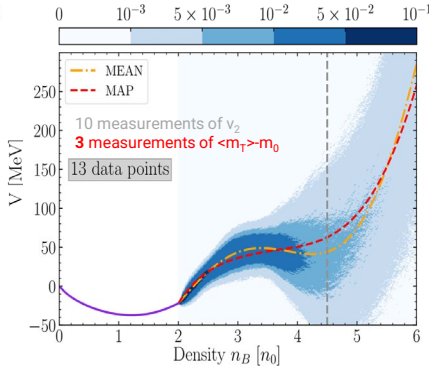
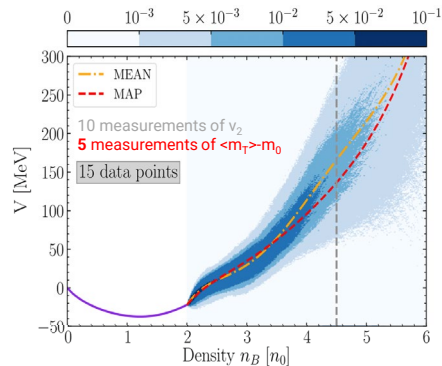
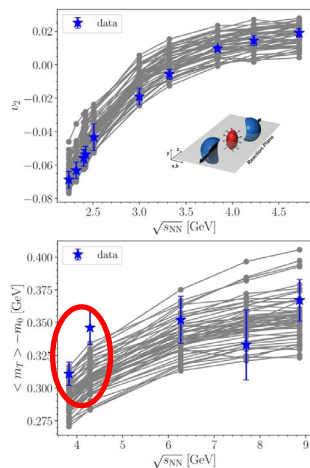
... ..

**Jet quenching/diffusion:**

Y. He, L. Pang, X. Wang, **PRL** 122 (25) 252302

M. Xie, W. Ke, H. Zhang, X. Wang, **PRC**108 (2023) L011901; ... ..

# Bayesian Inference Dense Matter EoS from HIC and Holography

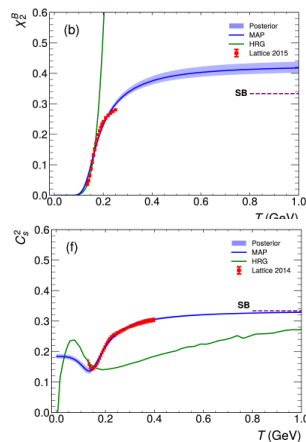
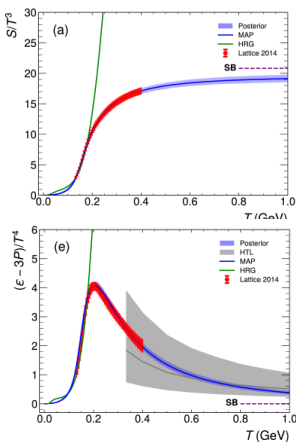


• Comprehensive Bayesian inference necessary for unambiguous solution

• Tension between data-data or model (UrQMD)-data

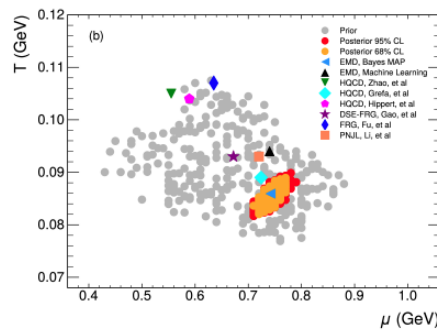
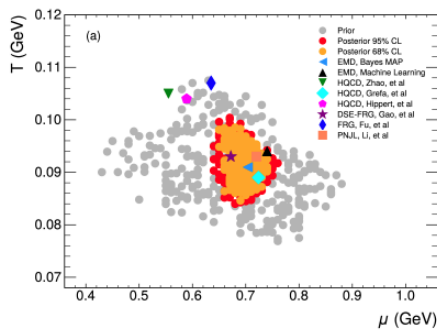
• Next-gen experiments will provide immense amount of high precision data

M.OK, J. Steinheimer, K. Zhou, H. Stoecker, **PRL131,202303(2023)**



$$S_E = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \left[ R - \frac{f(\phi)}{4} F^2 - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right]$$

$$A(z) = d \ln(a z^2 + 1) + d \ln(b z^4 + 1), f(z) = e^{c z^2 - A(z) + k}$$



See talk of **Li-qiang Zhu** tomorrow 16:40pm

• Critical endpoint from **holography (EMD)** via Bayesian Inference

L. Zhu, X. Chen, K. Zhou, H. Zhang, M. Huang, **Phys. Rev. D112(2025) 026019**

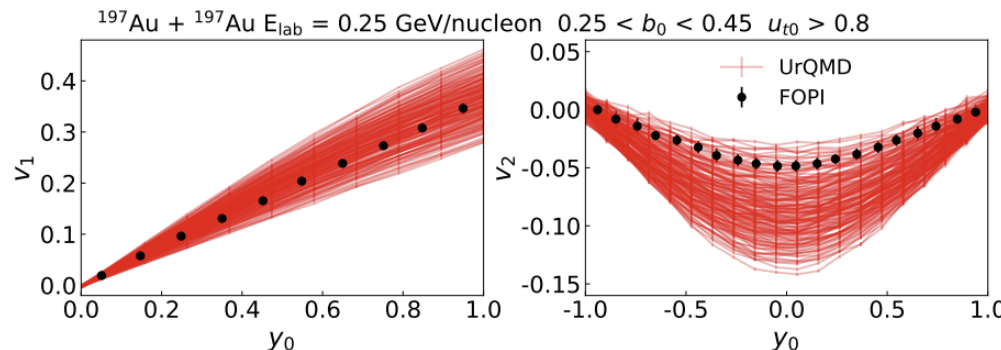
# Incompressibility K from Bayesian Inference with low-energy HIC

Bayesian analysis of properties of nuclear matter with the FOPI experimental data

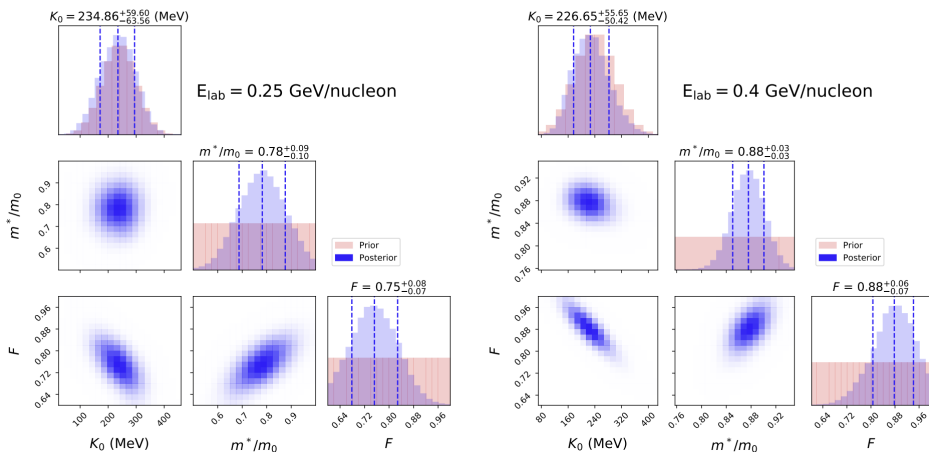
Guojun Wei,<sup>1,2</sup> Manzi Nan,<sup>1,3</sup> Pengcheng Li,<sup>1</sup> Yongjia Wang,<sup>1,4,\*</sup> Qingfeng Li,<sup>1,†</sup> Gaochan Yong,<sup>3</sup> and Fuhu Liu<sup>2</sup>

**arXiv:2509.03406**




**UrQMD simulation of proton  $v_1$  and  $v_2$   
@Au+Au at E=0.25, 0.4 GeV/Nucleon**



See talk of **Qing-feng Li**  
this morning 11:00am



## Bayesian inference of nuclear incompressibility from proton elliptic flow in central Au+Au collisions at 400 MeV/nucleon

J. M. Wang (汪金梅),<sup>1,2</sup> X. G. Deng (邓先概) ,<sup>1,2,\*</sup> W. J. Xie (谢文杰),<sup>3</sup> B. A. Li (李宝安) ,<sup>4,†</sup> and Y. G. Ma (马余刚) ,<sup>1,2,‡</sup>

***Chinese Physics C* 49, 124105 (2025)**

### IQMD simulation of proton $v_2$ Au+Au at E=400 MeV/Nucleon

### MDI: momentum dependent Interaction

$$E/A = \frac{\alpha}{2} \frac{\rho}{\rho_0} + \frac{\beta}{\gamma+1} \left( \frac{\rho}{\rho_0} \right)^\gamma + \frac{3}{10m} \left( \frac{3\pi^2 \hbar^3 \rho}{2} \right)^{2/3} + \frac{1}{2} t_4 \frac{\rho}{\rho_0} \int f(\vec{p}) \ln^2 \left[ 1 + t_5 (\vec{p} - \langle \vec{p}' \rangle)^2 \right] d^3 p, \quad (1)$$

$$P = \rho^2 \frac{\partial E/A}{\partial \rho} = \frac{\alpha}{2} \frac{\rho^2}{\rho_0} + \frac{\beta \gamma \rho}{\gamma+1} \left( \frac{\rho}{\rho_0} \right)^\gamma + \frac{1}{5m} \left( \frac{3}{2} \pi^2 \hbar^3 \right)^{2/3} \rho^{5/3} + \frac{t_4}{2} \frac{\rho^2}{\rho_0} \ln^2 (1 + t_5 P_F^2), \quad (2)$$

$$K = 9\rho^2 \frac{\partial^2 E/A}{\partial \rho^2} \Big|_{\rho_0} = -\frac{3}{5m} \left( \frac{3\pi^2 \hbar^3 \rho_0}{2} \right)^{2/3} + \frac{9\beta\gamma(\gamma-1)}{\gamma+1} + \ln(1 + t_5 P_F^2) \frac{6t_4 t_5 P_F^2}{1 + t_5 P_F^2}, \quad (3)$$

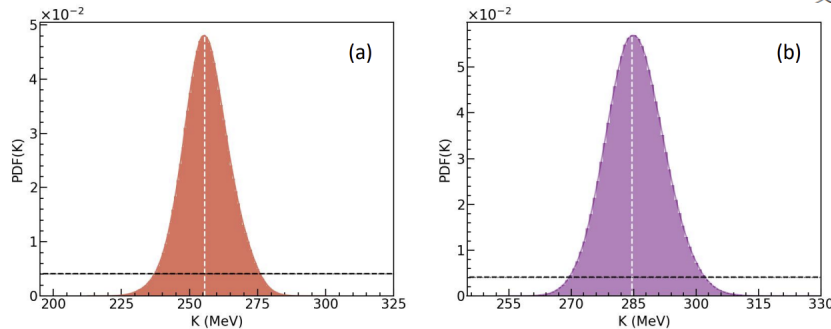


FIG. 4. Without considering the MDI: the posterior PDFs of  $K$ . Left: using the observables  $-v_2(y_0)$  and  $-v_2(u_{10})$ , right: observables  $-v_2(y_0)$ ,  $-v_2(u_{10})$  and  $v_1(p_t^{(0)})$ .

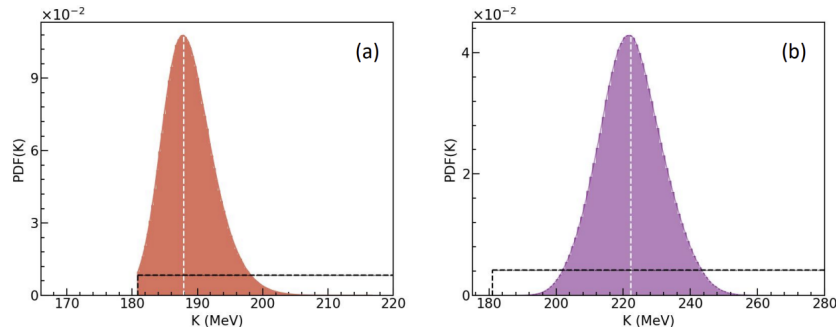
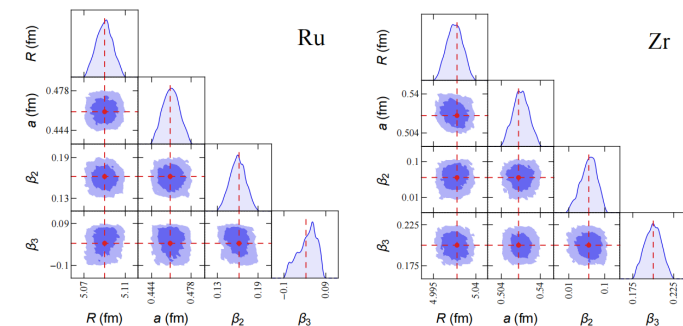
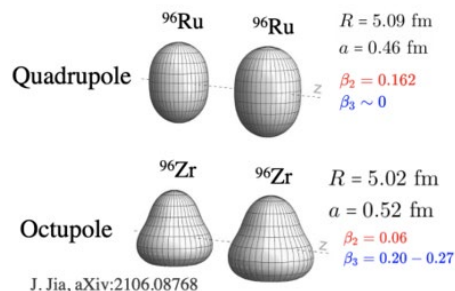


FIG. 6. Considering the MDI: the posterior PDFs of  $K$ . Left: observables  $-v_2(y_0)$  and  $-v_2(u_{10})$ , right: observables  $-v_2(y_0)$ ,  $-v_2(u_{10})$  and  $v_1(p_t^{(0)})$ .

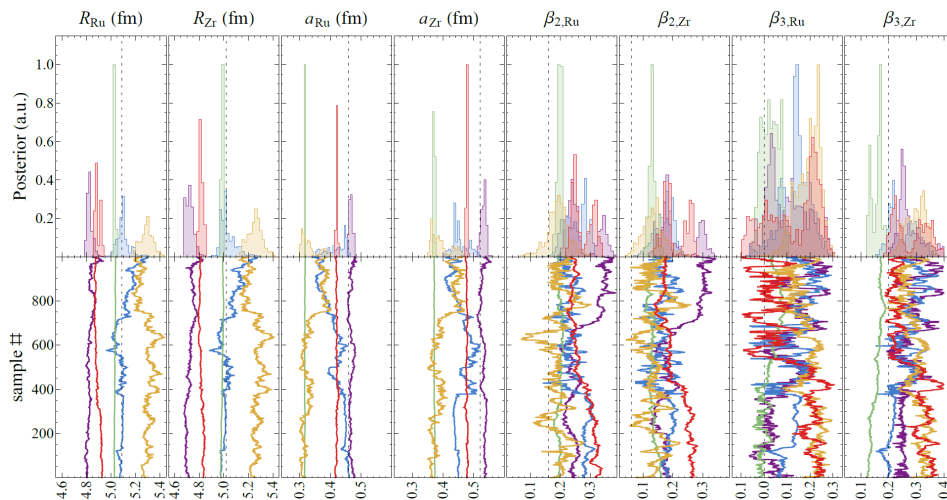
# Bayesian Imaging for Nuclear Structure in Isobar Collisions

- Nuclear Structure imaging for single system ? (caveat: model dependent)
- Simultaneous inference for isobar systems with ratio?
- Bayesian Inference:** Gaussian Process emulator + PCA dim reduction + MCMC  
Data: MC-Glauber + Matching (linear response approximation)

$$\mathbf{y}_{\text{Ru}} \equiv \{P_a^{\text{Ru}}, \varepsilon_{2,a}^{\text{Ru}}, \varepsilon_{3,a}^{\text{Ru}}, d_{\perp,a}^{\text{Ru}}\}_{a=1,\dots,40}$$



Single system works good



With purely the Isobar-Ratios:

MCMC can not converge to a stationary inference of the nuclear structure

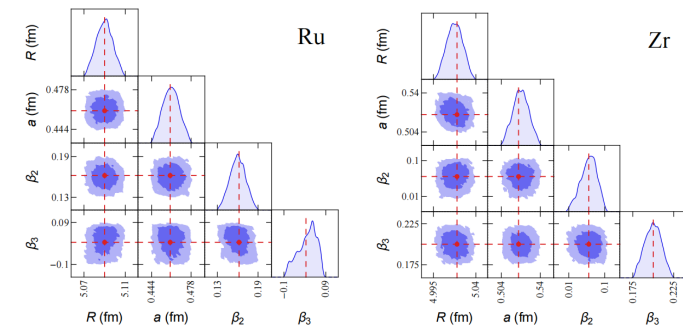
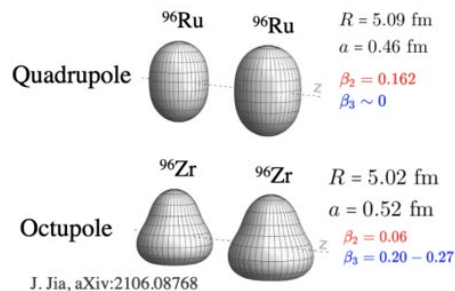
$$\mathbf{y}_{\text{r},1} \equiv \{R_{P,a}, R_{\varepsilon_{2,a}}, R_{\varepsilon_{3,a}}, R_{d_{\perp,a}}\}_{a=1,\dots,40}$$



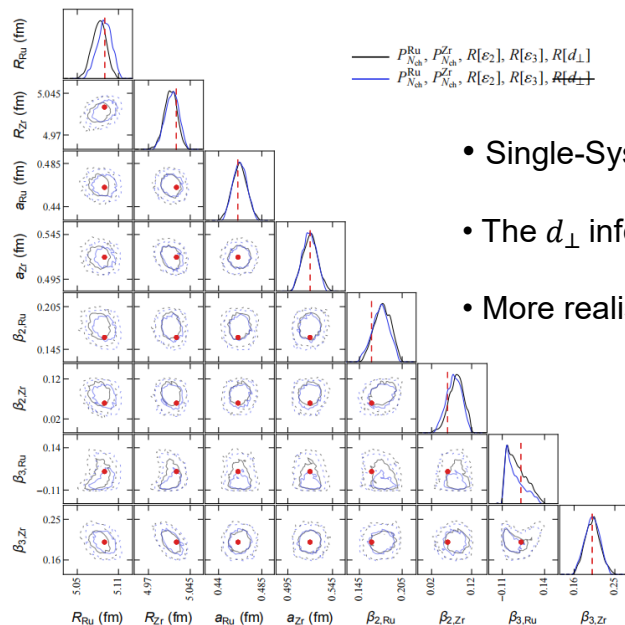
# Bayesian Imaging for Nuclear Structure in Isobar Collisions

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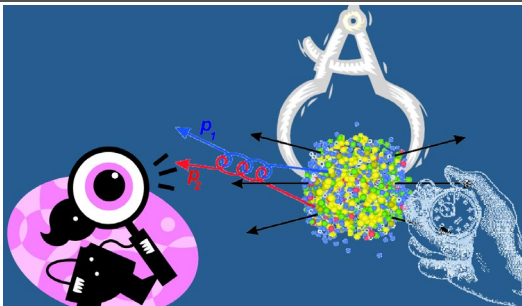
Single system works good



- Single-System Multiplicity makes it possible
- The  $d_{\perp}$  information is redundant
- More realistic analysis with AMPT in progress

$$\mathbf{y}_{\text{r},2} \equiv \{P_a^{\text{Ru}}, P_a^{\text{Zr}}, R_{\varepsilon_2,a}, R_{\varepsilon_3,a}, R_{d_{\perp},a}\}_{a=1,\dots,40}$$

# Bayesian reconstruction for h-h interaction from femtoscopy – mock test



$$V(r) = \sum_{i=1,2} a_i e^{-(r/b_i)^2} + a_3 m_\pi^4 f(r, b_3) \frac{e^{-2m_\pi r}}{r^2}$$

$$f(r, b_3) = \left(1 - e^{-(r/b_3)^2}\right)^2$$

There are six parameters  $a_i, b_i$  ( $i = 1, 2, 3$ ) for this potential !

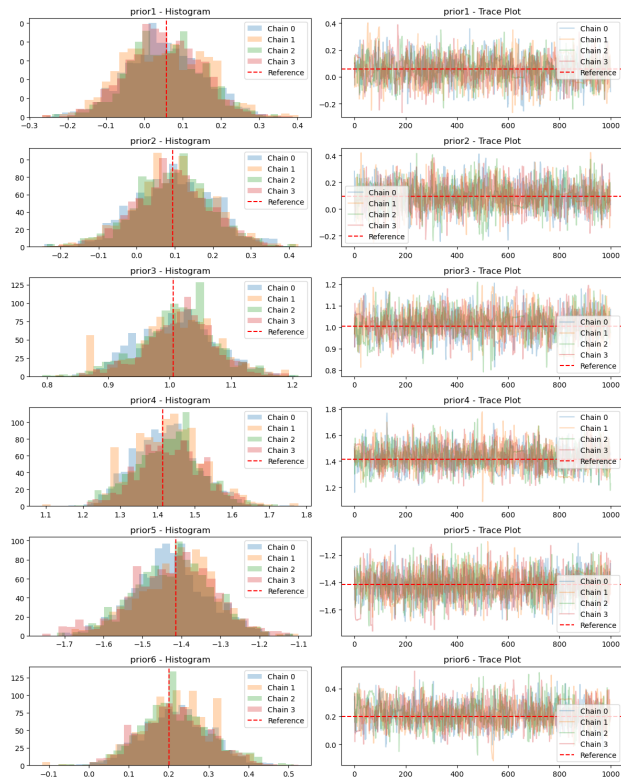
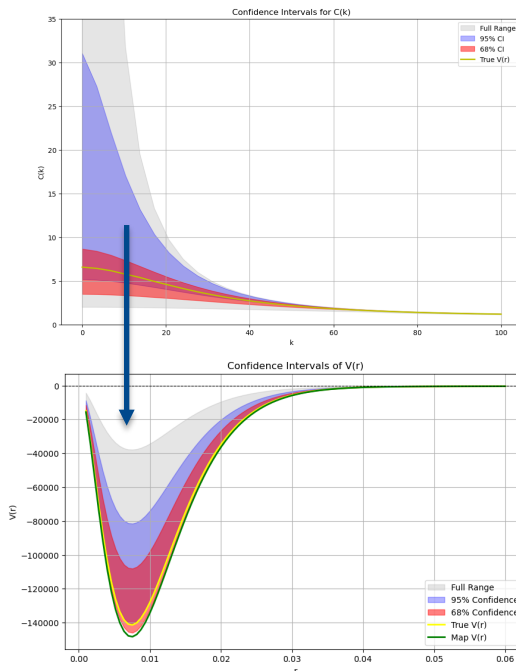
$$C(k) = \int S(\mathbf{r}) |\psi_k(\mathbf{r})|^2 d^3r,$$

$$-\frac{\hbar^2}{2\mu} \nabla^2 \psi + V\psi = E\psi,$$

$$S(r) = \frac{1}{(4\pi r_0^2)^{3/2}} \exp\left(-\frac{r^2}{4r_0^2}\right),$$

$$r_0 = 1.0 \text{ fm}$$

DNN emulator + PCA for correlation + PyMC  
With O. L, J. Z, X. C, etc., in preparation



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IOF Publishing

Journal of Physics G: Nuclear and Particle Physics

J. Phys. G: Nucl. Part. Phys. 51 (2024) 103001 (43pp)

<https://doi.org/10.1088/1361-6471/ad6a2b>

Topical Review

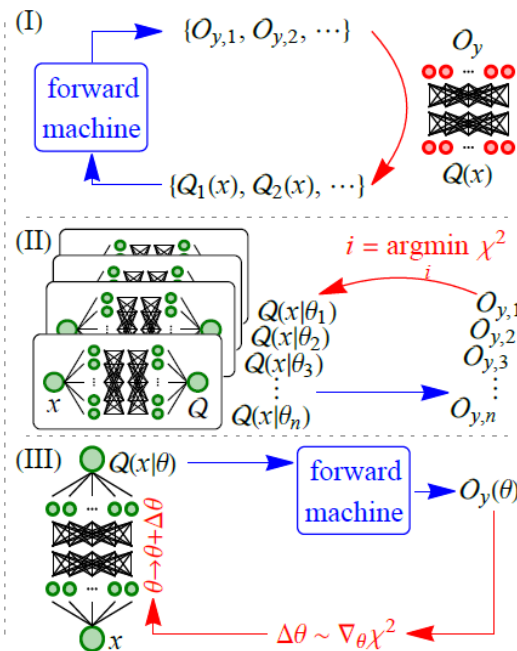
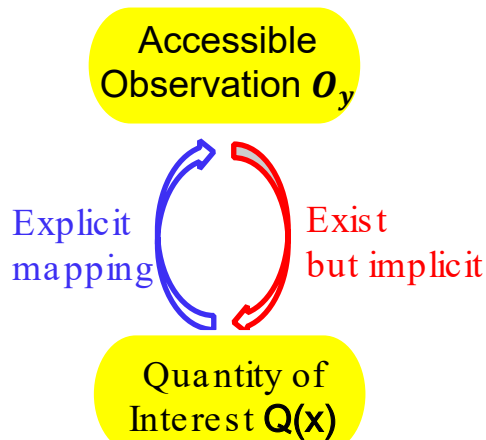
## Applications of emulation and Bayesian methods in heavy-ion physics

Jean-François Paquet

References	Pre-hydro	Hydro	Cooper-Frye	Data	Covariance
Bernhard <i>et al</i> [28]	Trento+f.s.	DNMR	P.T.B.; $\sigma$ meson production	Pb–Pb @ 2.76 TeV and 5.02 TeV	$\Sigma_{\text{emul}} + \text{diag. } \Sigma_{\text{expt}}^{\text{stat}}$ +non-diag. $\Sigma_{\text{expt}}^{\text{syst}}$ + $\Sigma_{\text{extra}}$
Moreland <i>et al</i> [89]	Trento w/ subnucleonic d.o. f.+f.s.	DNMR	P.T.B.; $\sigma$ meson production	p-Pb & Pb–Pb @ 5.02 TeV	$\Sigma_{\text{emul}} + \text{diag. } \Sigma_{\text{expt}}^{\text{stat}}$ +non-diag. $\Sigma_{\text{expt}}^{\text{syst}}$
JETSCAPE [15, 53]	Trento+f.s.	DNMR	Grad, Chapman–Enskog, P.T.B.	Au–Au @ 0.2 TeV and Pb–Pb @ 2.76 TeV	$\Sigma_{\text{emul}} + \text{diag. } \Sigma_{\text{expt}}^{\text{stat}}$ +diag. $\Sigma_{\text{expt}}^{\text{syst}}$
Nijs <i>et al</i> [111, 112]	Trento w/ subnucleonic d.o. f. + modified streaming	DNMR	P.T.B.; $\sigma$ meson production	Pb–Pb @ 2.76 TeV; p-Pb and Pb–Pb @ 5.02 TeV; added differential observables	$\Sigma_{\text{emul}} + \text{diag. } \Sigma_{\text{expt}}^{\text{stat}}$ +non-diag. $\Sigma_{\text{expt}}^{\text{syst}}$
Parkkila <i>et al</i> [52, 113]	Trento+f.s.	DNMR	P.T.B. $\sigma$ meson production	Pb–Pb @ 2.76 TeV and 5.02 TeV; added event-plane correlations	$\Sigma_{\text{emul}} + \text{diag. } \Sigma_{\text{expt}}^{\text{stat}}$ +non-diag. $\Sigma_{\text{expt}}^{\text{syst}}$ + $\Sigma_{\text{extra}}$
Liyanage <i>et al</i> [35]	Trento + anisotropic hydro parameters	Viscous anisotropic hydro	P.T.M.A.	Pb–Pb @ 2.76 TeV	$\Sigma_{\text{emul}} + \text{diag. } \Sigma_{\text{expt}}^{\text{stat}}$ +diag. $\Sigma_{\text{expt}}^{\text{syst}}$
Heffernan <i>et al</i> [24, 26]	IP-Glasma	DNMR	Grad, Chapman–Enskog	Pb–Pb @ 2.76 TeV; added event-plane correlations	$\Sigma_{\text{emul}} + \text{diag. } \Sigma_{\text{expt}}^{\text{stat}}$ +diag. $\Sigma_{\text{expt}}^{\text{syst}}$

## Different analyses = different constraints

- Use different data sets
- Different modelling assumptions:
  - Initial conditions
  - Cooper-Frye
  - Allowed parametrization of transport coefficient
- Treatment of correlations in experimental uncertainties

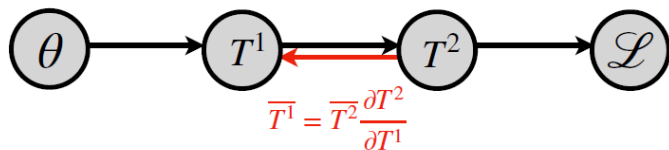


- Direct inverse mapping capturing :**  
with Supervised Learning
- Statistical approach to  $\chi^2$  fitting :**  
Bayesian Reconstruction for posterior or Heuristic (Generic) Algorithm to min.
- Automatic Differentiation :**  
fuse physical prior into reconstruction via differentiable programming strategy

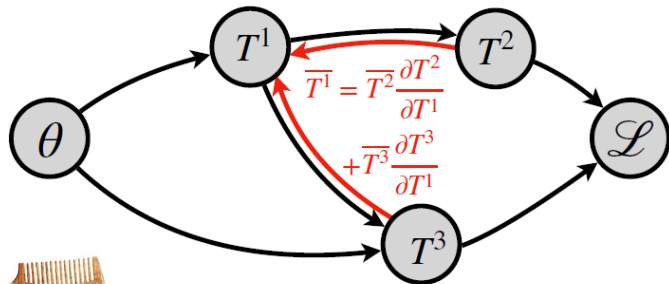
$$\frac{1}{2} \nabla_{\theta} \chi^2 = \sum_y \frac{\mathcal{F}_y[Q_{NN}(x|\theta)] - \mathcal{O}_y}{(\Delta \mathcal{O}_y)^2} \int dx \frac{\delta \mathcal{F}_y[Q(x)]}{\delta Q(x)} \bigg|_{Q(x)=Q_{NN}(x|\theta)} \nabla_{\theta} Q_{NN}(x|\theta)$$

**Deep Learning** composes differentiable components to a program, e.g. DNN, then optimizes it with **gradients**

(a)



(b)



“comb graph”

**Chain rule for gradients :**  $\frac{\partial \mathcal{L}}{\partial \theta} = \frac{\partial \mathcal{L}}{\partial T^n} \frac{\partial T^n}{\partial T^{n-1}} \cdots \frac{\partial T^2}{\partial T^1} \frac{\partial T^1}{\partial \theta}$

**Defining adjoint variables :**  $\bar{T} = \partial \mathcal{L} / \partial T$

$$\bar{T}^i = \bar{T}^{i+1} \frac{\partial T^{i+1}}{\partial T^i}$$

$$\bar{\theta} = \bar{T}^1 \frac{\partial T^1}{\partial \theta}$$

$$\bar{T}^i = \sum_{j: \text{child of } i} \bar{T}^j \frac{\partial T^j}{\partial T^i}$$

Differentiable programming tools

HIPS/autograd

TensorFlow

flux

PyTorch

MindSpore

SciML

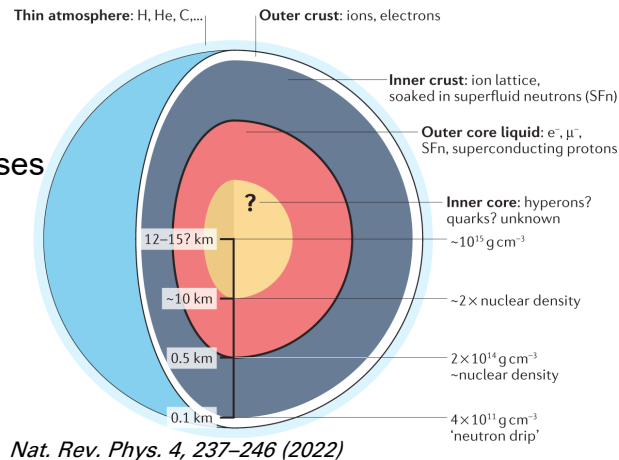
AX

dx

NiLang

# From EoS to NS Stellar Structure (MR)

- Mass ~ 2 solar masses
- Radii ~ 10 km
- Densities  $5-8\rho_0$



- Gravity  $\leftrightarrow$  Pressure

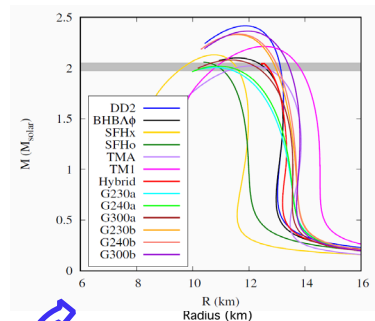
$$\frac{dP}{dr} = -\frac{G}{r^2} \left( \rho + \frac{P}{c^2} \right) \left( m + 4\pi r^3 \frac{P}{c^2} \right) \left( 1 - \frac{2Gm}{c^2 r} \right)^{-1}$$

$$M = m(R) = \int_0^R 4\pi r^2 \rho dr$$

- Dense matter Equation of State

$P(\rho) \leftarrow \rightarrow$

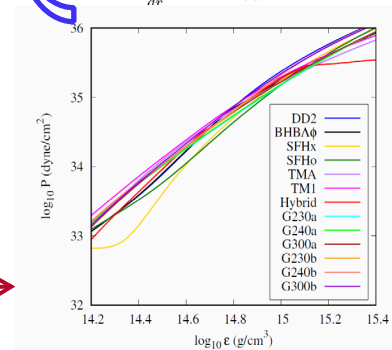
## ● Noisy/Limited NS Observables to EoS?



TOV equations

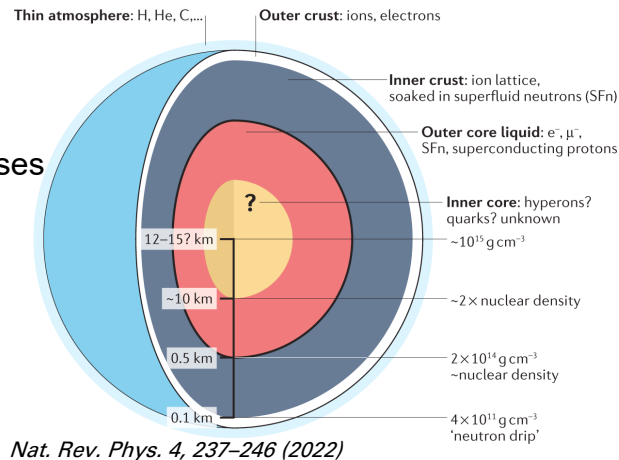
$$\frac{dP}{dr} = \frac{[\epsilon(r) + P(r)] [M(r) + 4\pi r^3 P(r)]}{r[r - 2M(r)]}$$

$$\frac{dM(r)}{dr} = 4\pi r^2 \epsilon(r),$$

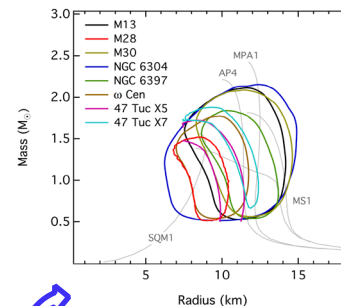




# From EoS to NS Stellar Structure (MR) -- Inverse ?



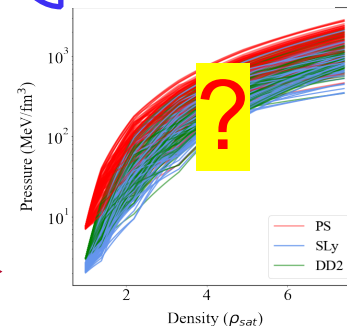
## Noisy/Limited NS Observables to EoS ?



TOV equations

$$\frac{dP}{dr} = \frac{[\epsilon(r) + P(r)] [M(r) + 4\pi r^3 P(r)]}{r[r - 2M(r)]}$$

$$\frac{dM(r)}{dr} = 4\pi r^2 \epsilon(r),$$



● Mass ~ 2 solar masses

● Radii ~ 10 km

● Densities 5-8  $\rho_0$

● Gravity  $\leftrightarrow$  Pressure

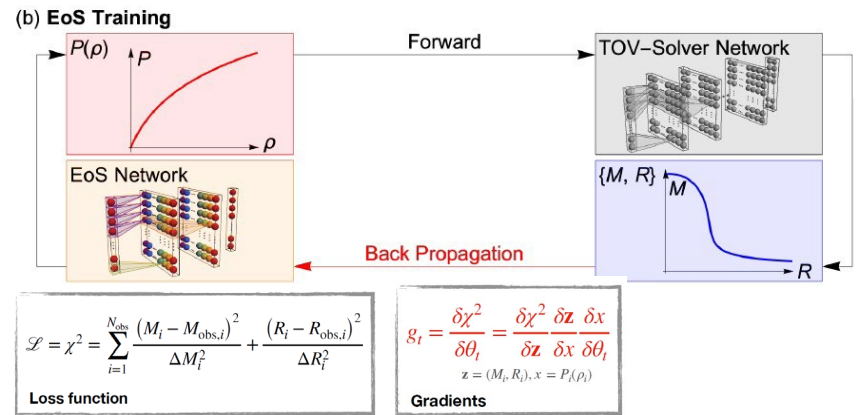
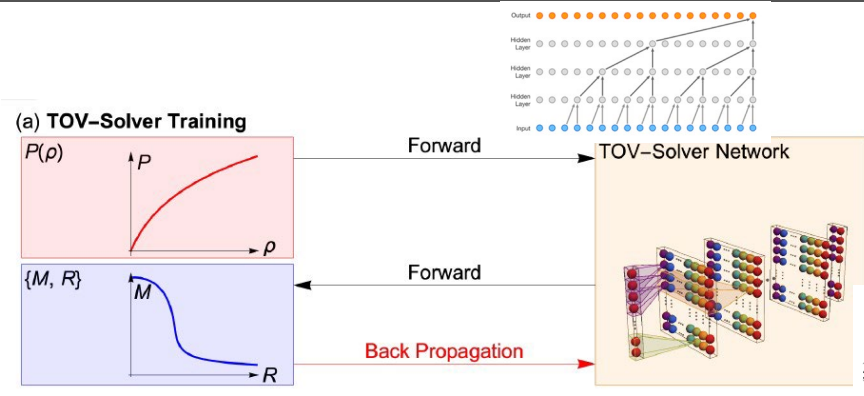
$$\frac{dP}{dr} = -\frac{G}{r^2} \left( \rho + \frac{P}{c^2} \right) \left( m + 4\pi r^3 \frac{P}{c^2} \right) \left( 1 - \frac{2Gm}{c^2 r} \right)^{-1}$$

$$M = m(R) = \int_0^R 4\pi r^2 \rho dr$$

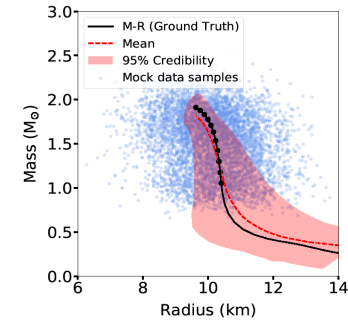
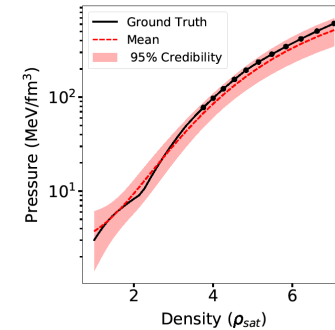
● Dense matter Equation of State

$P(\rho)$   $\leftarrow$   $\rightarrow$

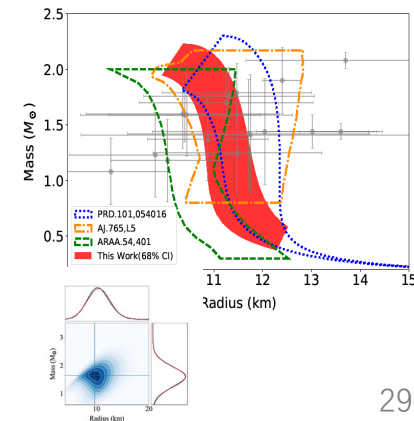
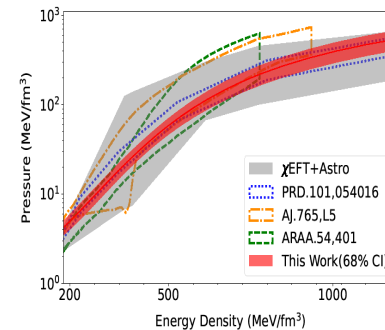
# Auto-diff framework and Results



## Well validated through Mock Tests



## With real observable we reconstruct the NS EoS also

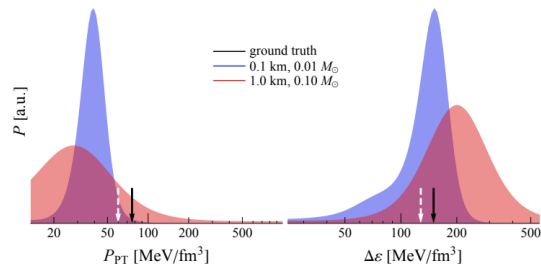
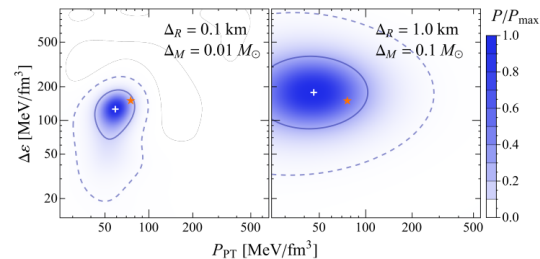
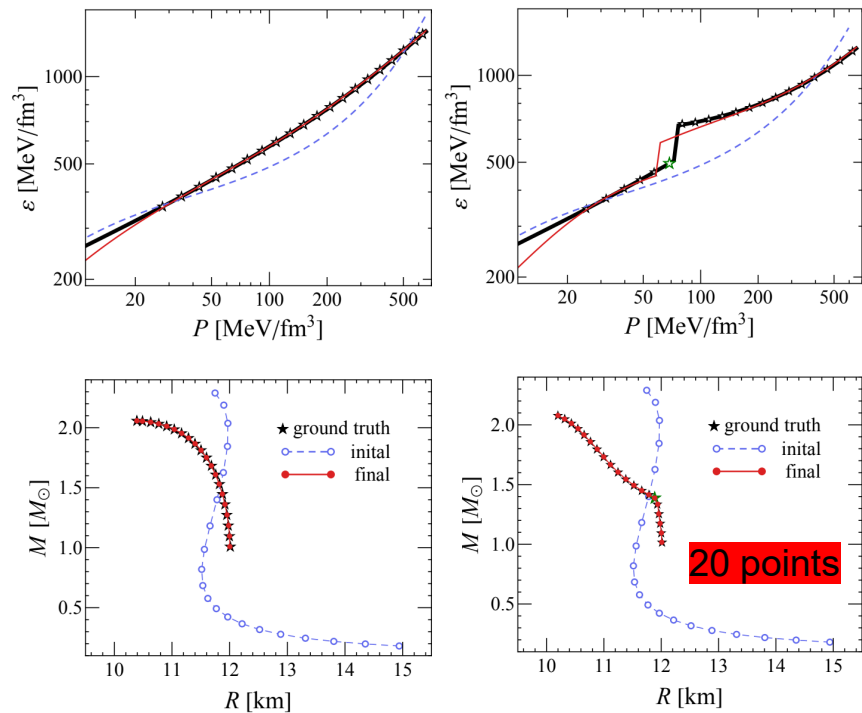


# Extend to First-order phase transition reconstruction with **AutoDiff**

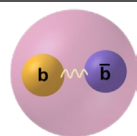
**Linear response analysis** get the gradients! Then use DNN :

We parameterize the inverse speed of sound squared containing both regular parts and Dirac- $\delta$  functions corresponding to possible first-order phase transitions,

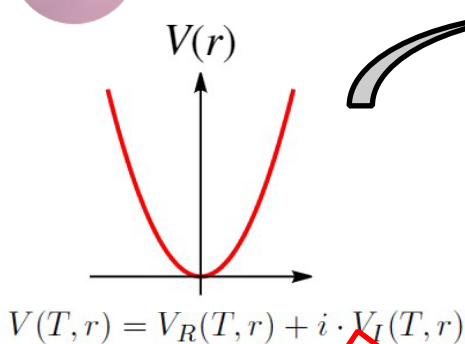
We adopt SFHo as the baseline EoS and introduce a PT with latent heat  $\Delta\varepsilon = 150 \text{ MeV/fm}^3$  at pressure  $P_{\text{PT}} = 76 \text{ MeV/fm}^3$ . Above the PT point, we take the stiffest (causal) limit that  $c_s = 1$ . We employ twenty



# HQ Potential Model, Inverse Schroedinger Eq.



$$\hat{H}\psi_n = -\frac{\nabla^2}{2m_\mu}\psi_n + V(r)\psi_n = E_n\psi_n \quad \{\psi_n(r)\}$$



How to extract **effective potential**  
given **limited spectroscopy** ? →

$$\chi^2 = \sum_{T,i} \frac{(m_{T,i} - m_{T,i}^{\text{lattice}})^2}{(\delta m_{T,i}^{\text{lattice}})^2} + \frac{(\Gamma_{T,i} - \Gamma_{T,i}^{\text{lattice}})^2}{(\delta \Gamma_{T,i}^{\text{lattice}})^2}$$

$T \in \{0, 151, 173, 199, 251, 334\} \text{ MeV}$

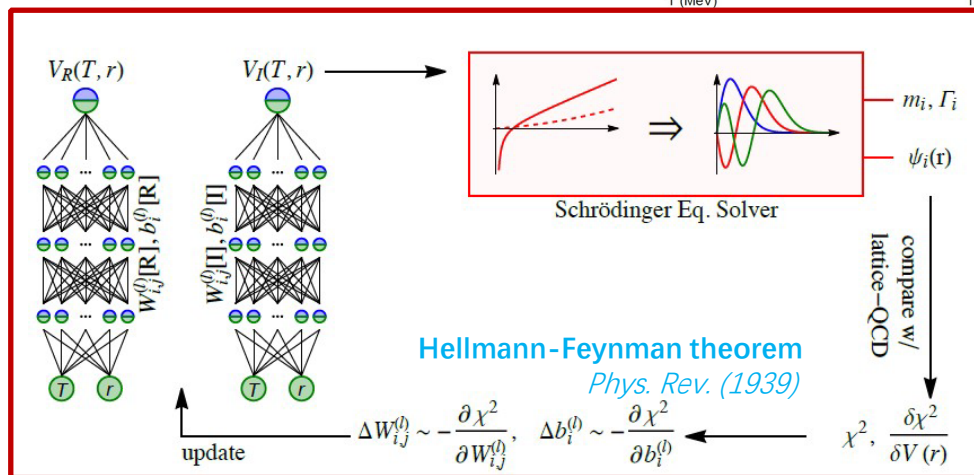
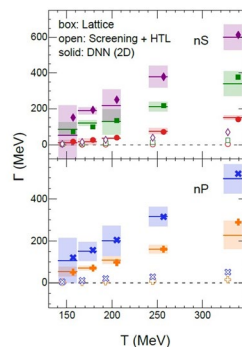
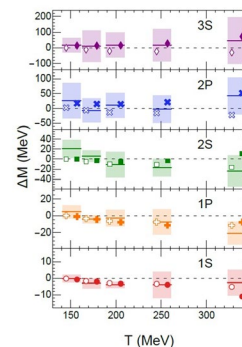
$i \in \{1S, 2S, 3S, 1P, 2P\}$

**S.S, K. Z, J.Z, S.M., P. Z, Phys. Rev. D 105 (2022) 1, 1**

*R. Larsen, et.al, PRD(2019),  
PLB(2020), PRD(2020)*

$$\begin{cases} \text{Re}[E_n] = m - 2m_b \\ \text{Im}[E_n] = -\Gamma \end{cases}$$

**New IQCD results cannot be explained by  
Perturbative HTL-inspired potentials !**

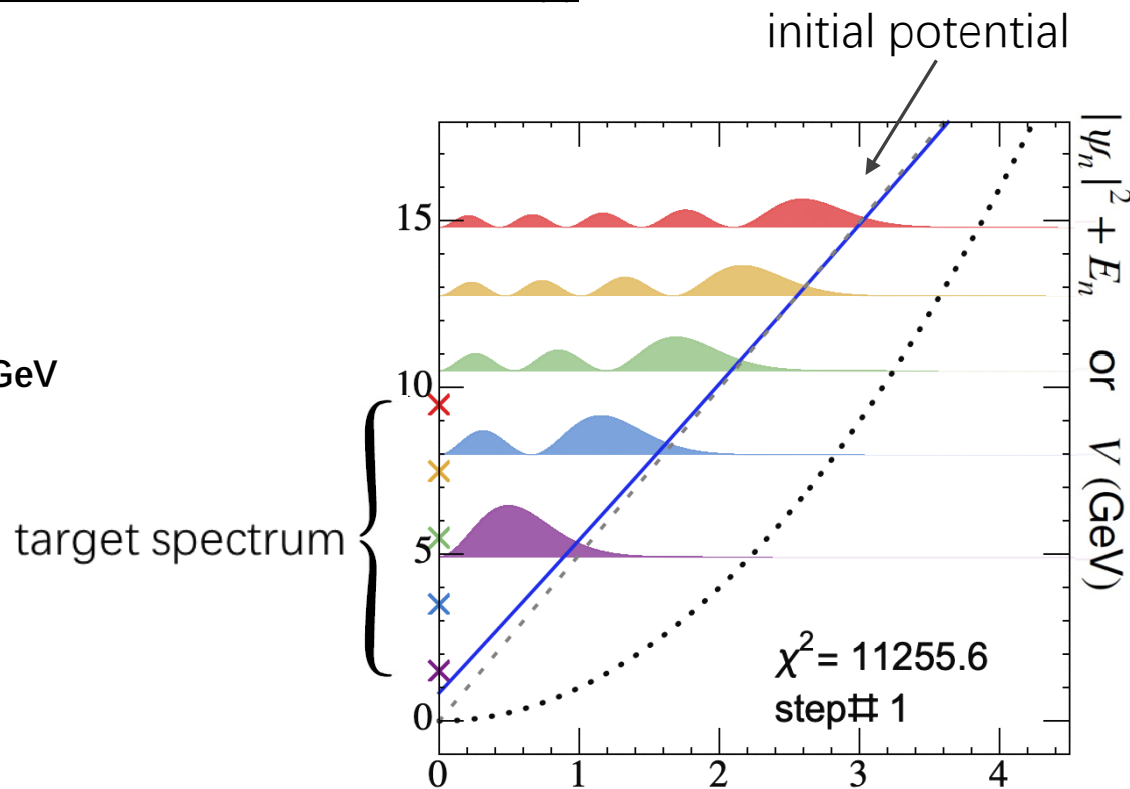


# Proof of Concept

limited spectrum  $\{E_n\}$  to continuous interaction  $V(r)$  ?

Learn  $V(r)$  from 5 eigenvalues :

$\{E_n\} = \{3/2, 7/2, 11/2, 15/12, 19/2\}$  GeV



# Proof of Concept

limited spectrum  $\{E_n\}$  to continuous interaction  $V(r)$  ?

-- Yes! But to some range decided by the used states.

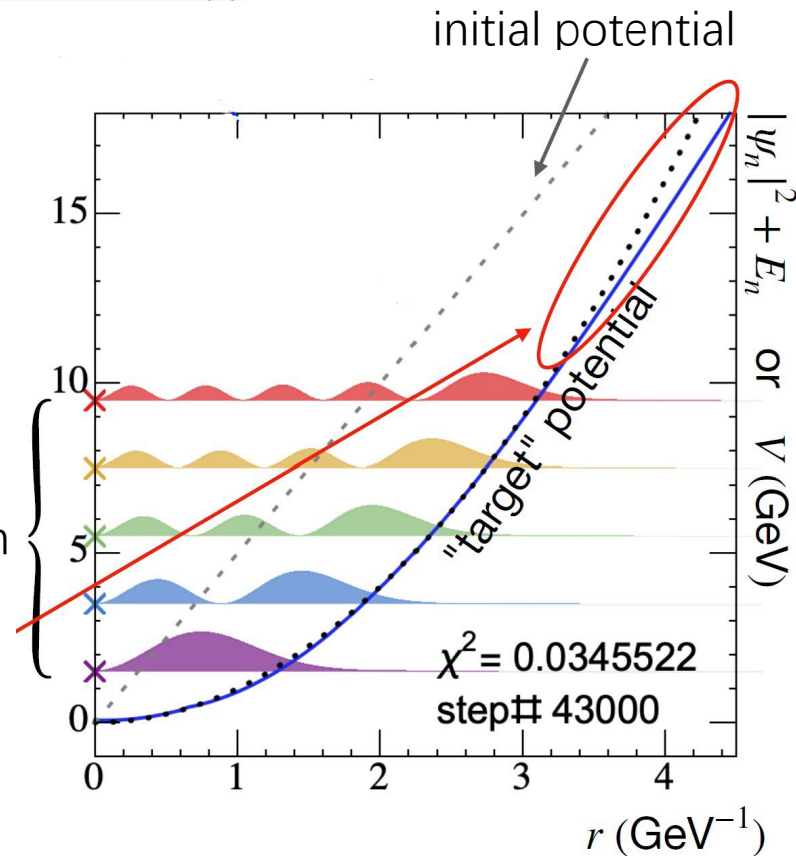
Learn  $V(r)$  from 5 eigenvalues :

$\{E_n\} = \{3/2, 7/2, 11/2, 15/12, 19/2\}$  GeV

Deviation @ given states' wavefunction vanishes

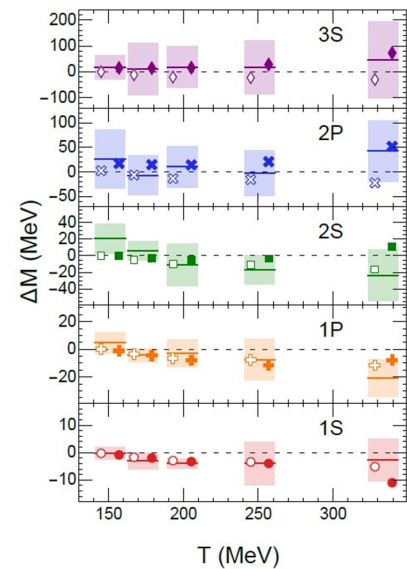
$$\delta E_n = \langle \psi_n | \delta V(r) | \psi_n \rangle$$

target spectrum

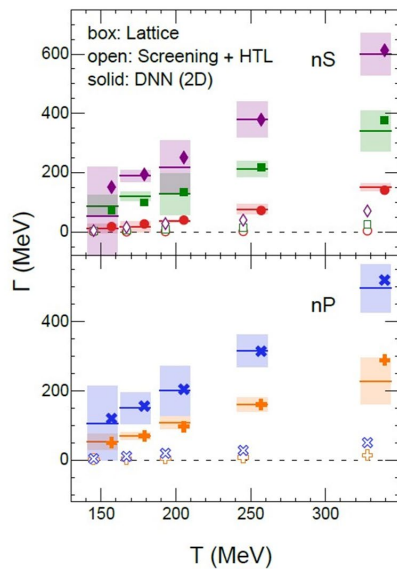




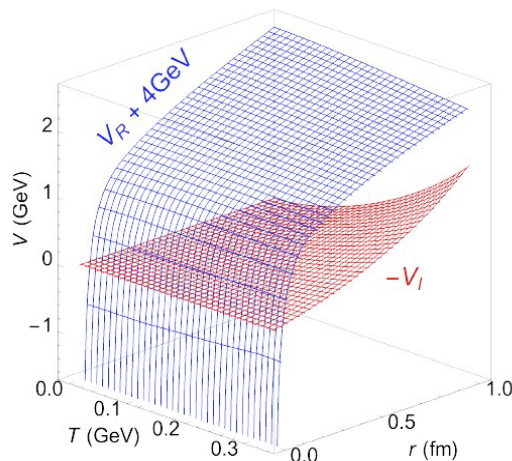
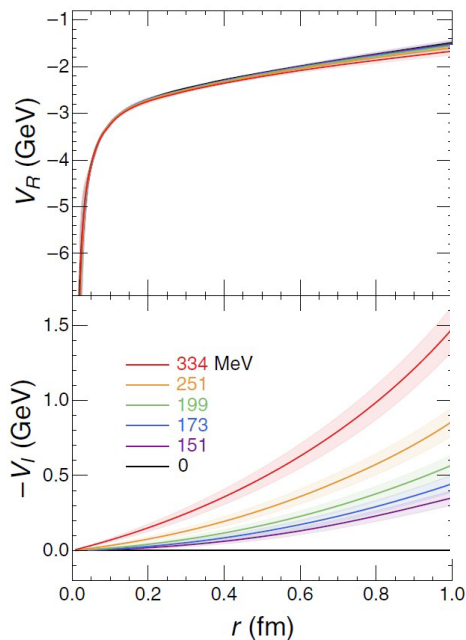
# Results with lattice data for mass/width and the reconstructed HQ Potential



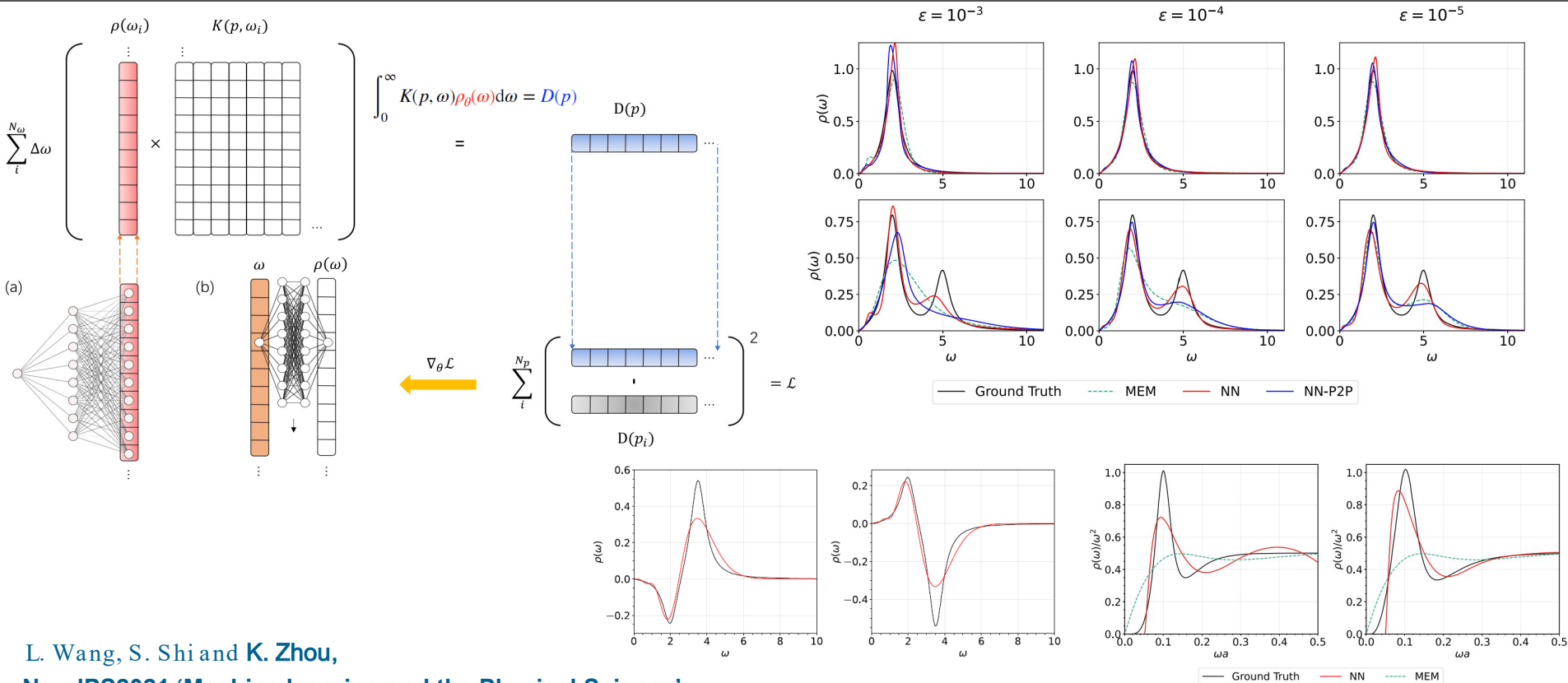
Chi2-per-data=16.5/30



## The reconstructed $T, r$ dependent potential



# Spectral function reconstruction from Euclidean correlator



L. Wang, S. Shi and K. Zhou,

NeurIPS2021 'Machine learning and the Physical Science',

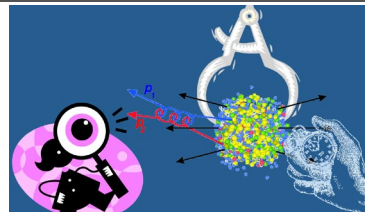
Phys. Rev. D 106, L051502 (Letter),

Computer Physics Communications (2022) 108547,

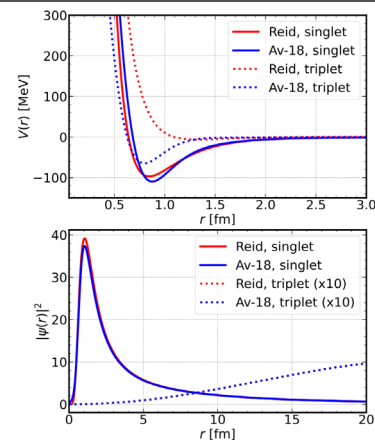
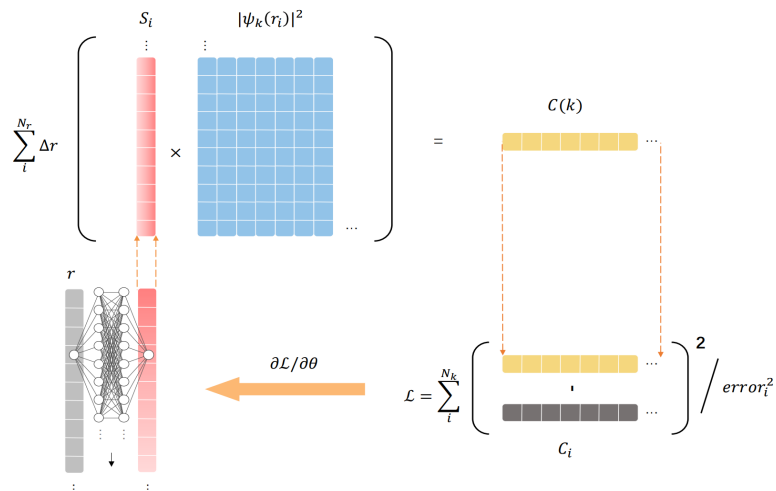
$$G(\tau, T) = \int_0^\infty \frac{d\omega}{2\pi} K(\omega, \tau, T) \rho(\omega, T)$$

$$K(\omega, \tau, T) = \frac{\cosh \omega(\tau - \frac{T}{2})}{\sinh \frac{\omega T}{2}}$$

# Hadron emission source reconstruction via femtoscopy



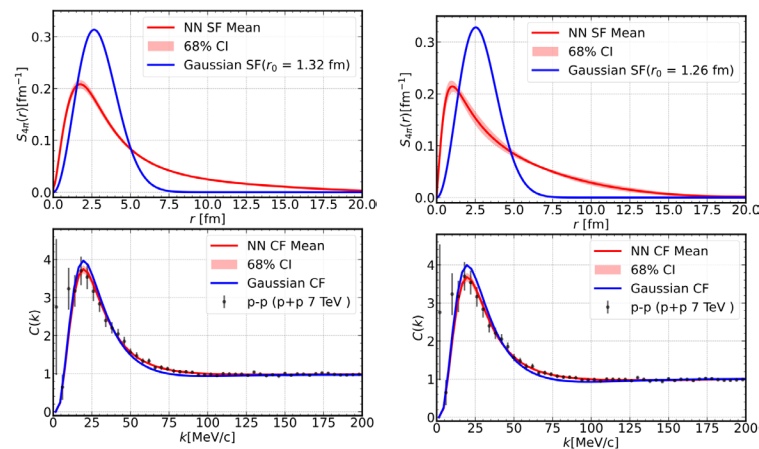
$$C(k) = \int S(\mathbf{r}) |\psi_k(\mathbf{r})|^2 d^3r,$$



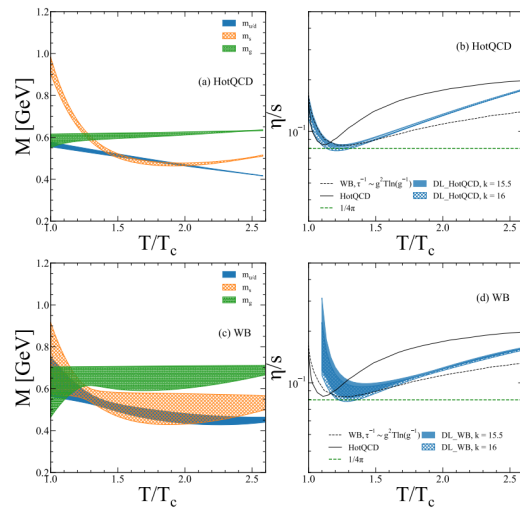
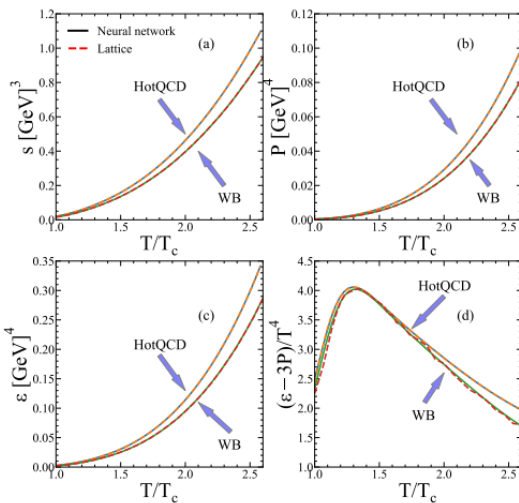
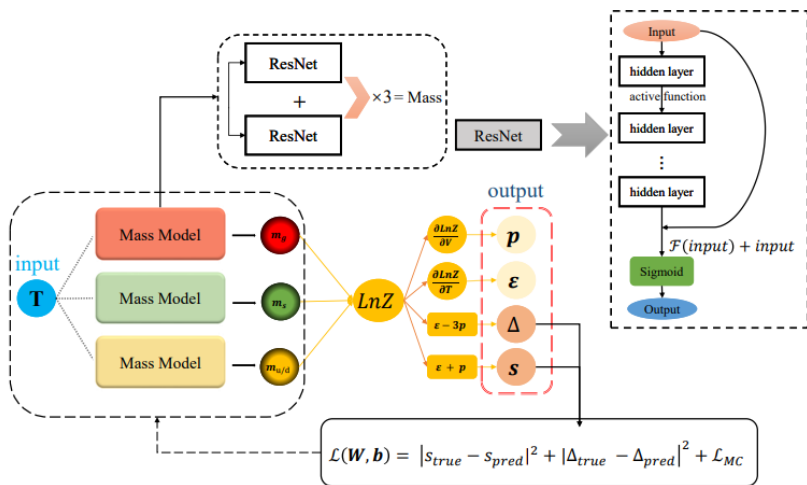
## Learning Hadron Emitting Sources with Deep Neural Networks

Lingxiao Wang<sup>1</sup> and Jiaxing Zhao<sup>2,3,\*</sup>

arXiv:2411.16343



# Quasi-particle analysis of IQCD thermodynamics



$$\ln Z_g(T) = -\frac{16V}{2\pi^2} \int_0^\infty p^2 dp \ln \left[ 1 - \exp \left( -\frac{1}{T} \sqrt{p^2 + m_g^2(T)} \right) \right], \quad (2)$$

$$\ln Z_{q_i}(T) = +\frac{12V}{2\pi^2} \int_0^\infty p^2 dp \ln \left[ 1 + \exp \left( -\frac{1}{T} \sqrt{p^2 + m_{q_i}^2(T)} \right) \right], \quad (3)$$

$$P(T) = T \left( \frac{\partial \ln Z(T)}{\partial V} \right)_T, \quad (5)$$

$$\epsilon(T) = \frac{T^2}{V} \left( \frac{\partial \ln Z(T)}{\partial T} \right)_V, \quad (6)$$

See talk of **Fu-peng Li**  
tomorrow morning 10:20am

$$\chi_i^B = \frac{\partial P(T, \hat{\mu}_B)/T^4}{\partial \hat{\mu}_B^i} \Big|_{\hat{\mu}_B=0}, \quad \hat{\mu}_B = \mu_B/T.$$

$$\mathcal{L}(\theta_1, \theta_2, \theta_3) = |s_{NN} - s_{input}| + \left| \frac{\Delta_{NN} - \Delta_{input}}{T} \right| + |\chi_{2,NN}^B - \chi_{2,input}^B| + |\chi_{4,NN}^B - \chi_{4,input}^B| + \mathcal{L}_{MC}$$

# Neural-net Quantum State + VMC for Nuclear Many-Body Problem

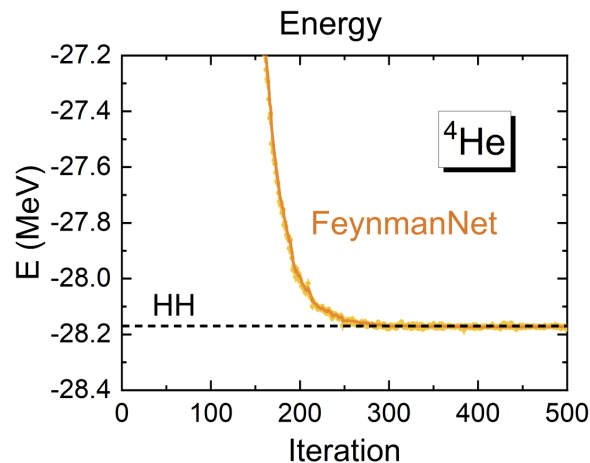
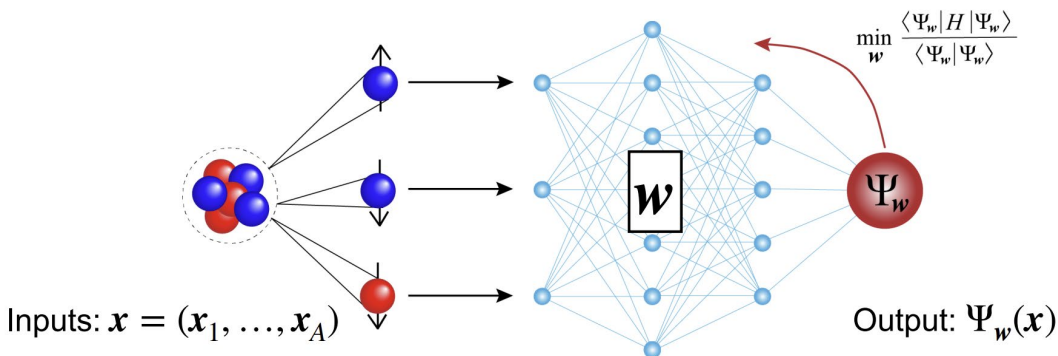
$$H = \sum_{i=1}^A \frac{p_i^2}{2M_i} + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk}$$

$$H\Psi(x_1, x_2, \dots, x_A) = E\Psi(x_1, x_2, \dots, x_A)$$

See talk of **Yi-Long Yang**  
this afternoon, 15:40pm

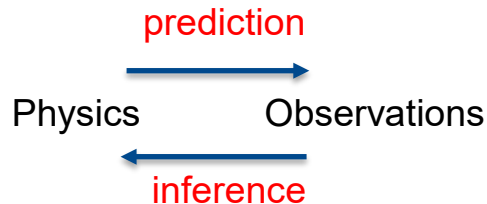
$$\psi_T = \psi_T(R, \vec{\theta})$$

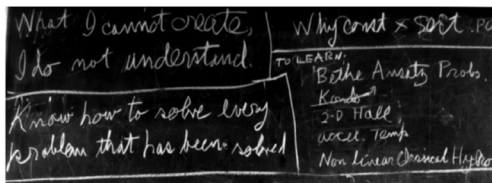
$$E_T = \frac{\langle \psi_T | H | \psi_T \rangle}{\langle \psi_T | \psi_T \rangle} \geq E_0$$



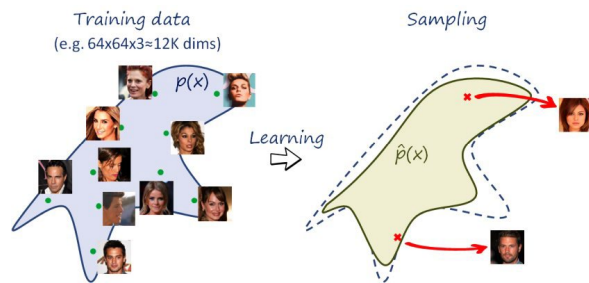
**FeynmanNet:** YL Yang, PWZ\*, PRC 107, 034320 (2023)

- Merge ML/DL with accumulated Physics Knowledge via **AD**
- **Differentiable** Physics Programming to directly invert the simulation  
Key words: inverse control, inverse design
- **Physics priors** still important in DNN representation constraints
- Uncertainties can be obtained via Bayesian perspective training : **Langevin**





“What I can not create, I do not understand”



Want to **model** the observed data's underlying but unknown **distribution**, to further :

- Understand/Inference the data (inherent structure, properties, features...)
- Sample according to the distribution

Suppose observation dataset :

$$\mathbf{X} = \{x^{(1)}, x^{(2)}, \dots, x^{(N)}\} \stackrel{i.i.d}{\sim} p_{data}(x)$$

We use parametric model to approach the data distribution :

$$p_{\theta}(x) \rightarrow p_{data}(x)$$

Often use NN to parametrize transformation  $\log p_{\theta}(\mathbf{x}) = \log p_{\theta}(f_{\theta}(\mathbf{z}_0))$

- Maximize Likelihood Estimation : (given training samples)

$$\theta^* = \arg \max_{\theta} \log p_{\theta}(\mathbf{X}) = \arg \max_{\theta} \frac{1}{N} \sum_{i=1}^N \log p_{\theta}(x^{(i)})$$

Reverse KL Divergence : Sample many  $\mathbf{z}_0 \sim p_0(\mathbf{z}_0)$   
(given unnormalized target distribution, e.g., Action)

$$\theta^* = \arg \min_{\theta} \frac{1}{N} \sum_{i=1}^N [\log p_{\theta}(f_{\theta}(\mathbf{z}_0)) - \log \tilde{p}_{target}(f_{\theta}(\mathbf{z}_0))]$$



- Variational free energy minimization - Reverse KL divergence

$$D_{\text{KL}}(q_{\theta} \parallel p) = \sum_{\mathbf{s}} q_{\theta}(\mathbf{s}) \ln \left( \frac{q_{\theta}(\mathbf{s})}{p(\mathbf{s})} \right) = \beta(F_q - F) \quad F_q = \frac{1}{\beta} \sum_{\mathbf{s}} q_{\theta}(\mathbf{s}) [\beta E(\mathbf{s}) + \ln q_{\theta}(\mathbf{s})]$$

$$\mathbb{E}_{X \sim p(X)} [E(X) + k_B T \ln p(X)]$$

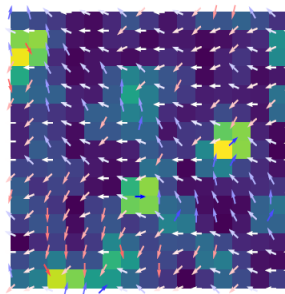
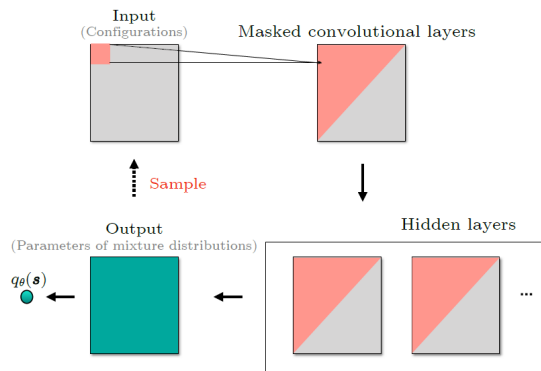
$$p(\mathbf{s}) = \frac{e^{-\beta E(\mathbf{s})}}{Z}$$

- **Autoregressive**  $q_{\theta}(\mathbf{s}) = \prod_{i=1}^N q_{\theta}(s_i \mid s_1, \dots, s_{i-1})$

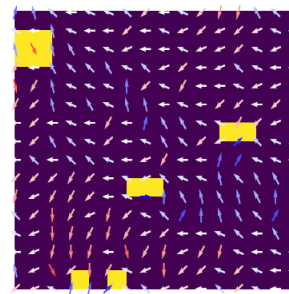
D. Wu, Lei Wang and P. Zhang, **PRL** 122, 080602 (2019)

- **Continuous** Autoregressive Net for XY model

L. Wang, Y. Jiang, L. He, K. Zhou, **CPL** 39, 120502 (2022)



Probability distributions from CANs

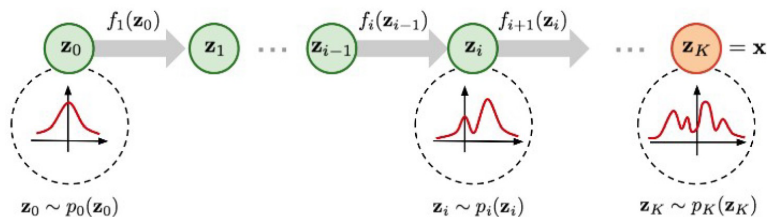


Vortices

# Flow based generative model given unnormalized distribution

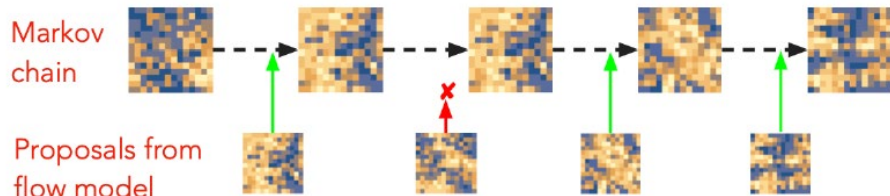
A series (**Flow**) of invertible/bijective transformations for  $p(\mathbf{z})$

compose several invertible transformations to form the flow :



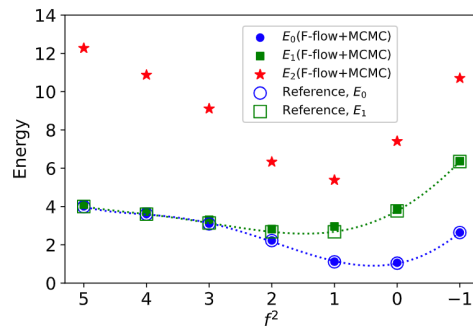
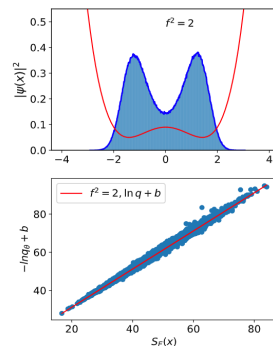
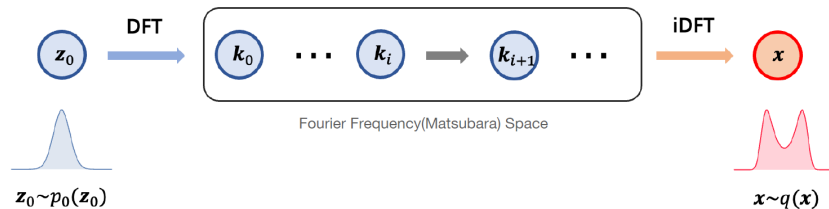
$$p_i(\mathbf{z}_i) = p_{i-1}(f_i^{-1}(\mathbf{z}_i)) |\det J_{f_i^{-1}}| = p_{i-1}(\mathbf{z}_{i-1}) |\det J_{f_i}|^{-1}$$

$$\rightarrow \log p(\mathbf{x}) = \log p_0(f^{-1}(\mathbf{x})) + \sum_{i=1}^K \log |\det J_{f_i^{-1}}| = \log p_0(\mathbf{z}_0) - \sum_{i=1}^K \log |\det J_{f_i}|$$



Albergo +, 1904.12072; Boyda +, 2008.05456; Favoni +, 2012.12901; Abbott +, 2208.03832; Abbott +, 2211.07541; Abbott +, 2305.02402; Bulgarelli+ 2412.00200 (SU(3)); Abbott +, arXiv:2502.00263  
K.C., G. K., S. R., D. R., P. S., **Nature Reviews Physics** 5, 526-535 (2023)

## Fourier Flow Model

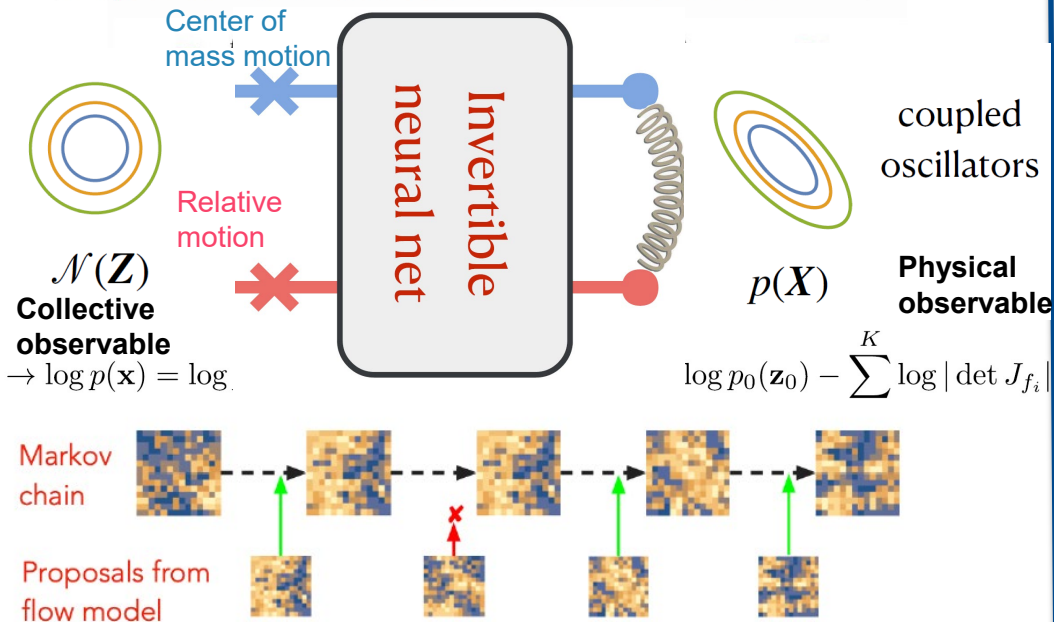


S.C, O. S, S. Z, B. C, H. S, L. W, **K. Zhou**,  
**Phys. Rev. D** 107, 056001(2023)

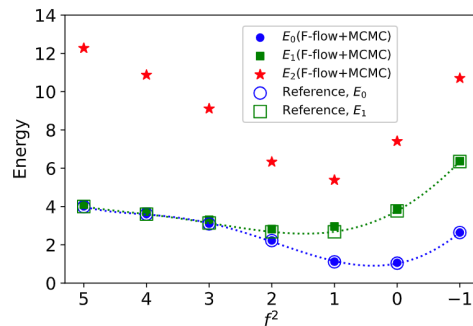
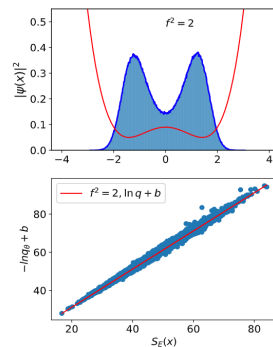
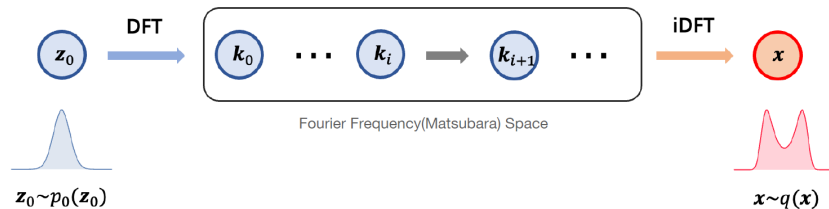
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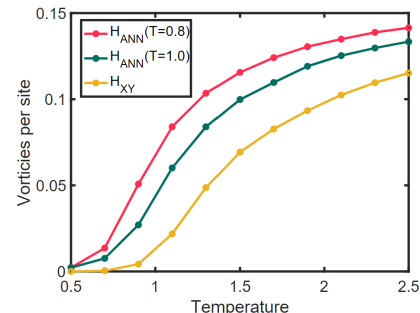
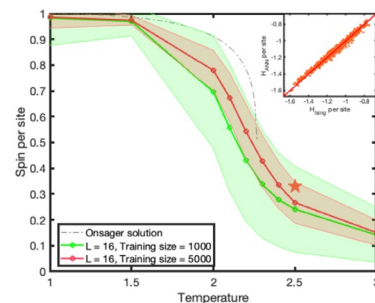
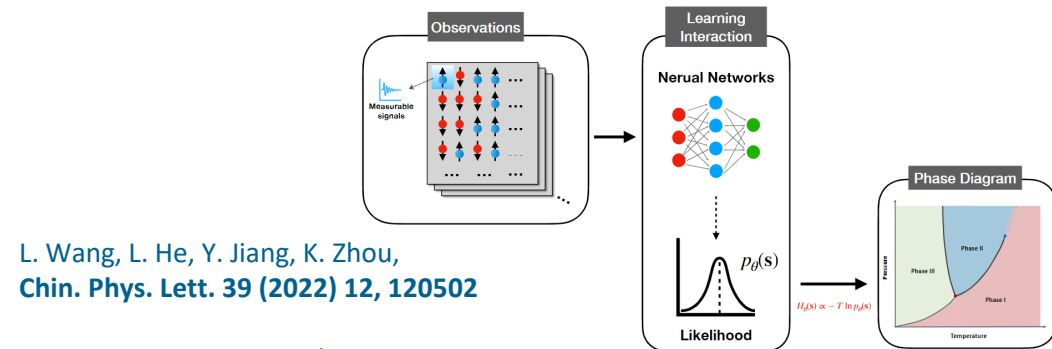
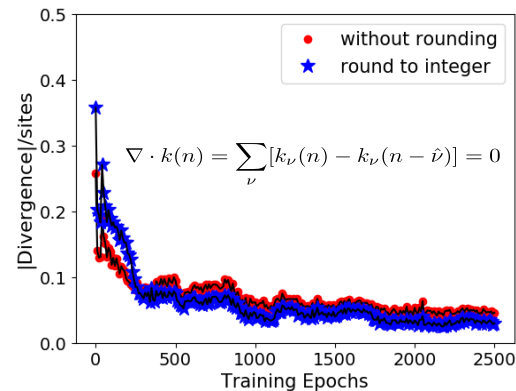
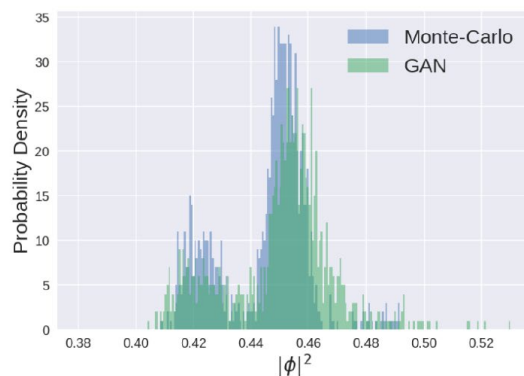
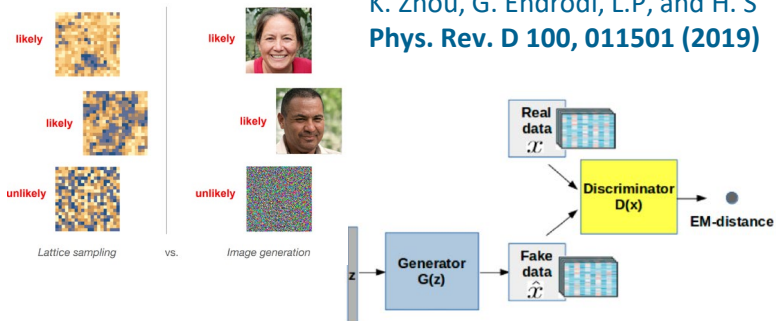
## Fourier Flow Model



Albergo +, 1904.12072; Boyda +, 2008.05456; Favoni +, 2012.12901; Abbott +, 2208.03832; Abbott +, 2211.07541; Abbott +, 2305.02402; Bulgarelli+ 2412.00200 (SU(3)); Abbott +, arXiv:2502.00263  
K.C., G. K., S. R., D. R., P. S., **Nature Reviews Physics** 5, 526-535 (2023)

S.C, O. S, S. Z, B. C, H. S, L. W, **K. Zhou**,  
**Phys. Rev. D** 107, 056001(2023)

# Given an ensemble of data from the target distribution



L. Wang, L. He, Y. Jiang, K. Zhou,  
**Chin. Phys. Lett. 39 (2022) 12, 120502**

T. Xu, L. Wang, L. He, K. Zhou, Y. Jiang,  
**Chin. Phys. C 48 (2024) 10, 103101**

# Diffusion Model



香港中文大學(深圳)  
The Chinese University of Hong Kong, Shenzhen

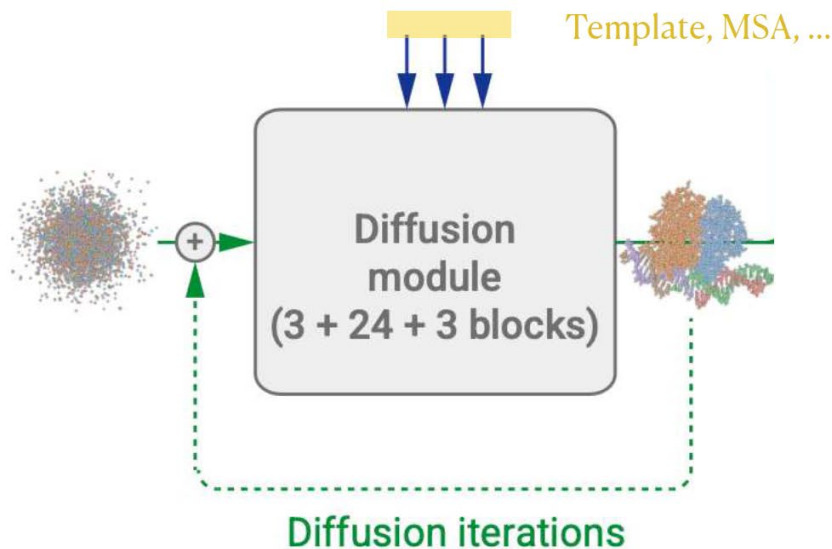
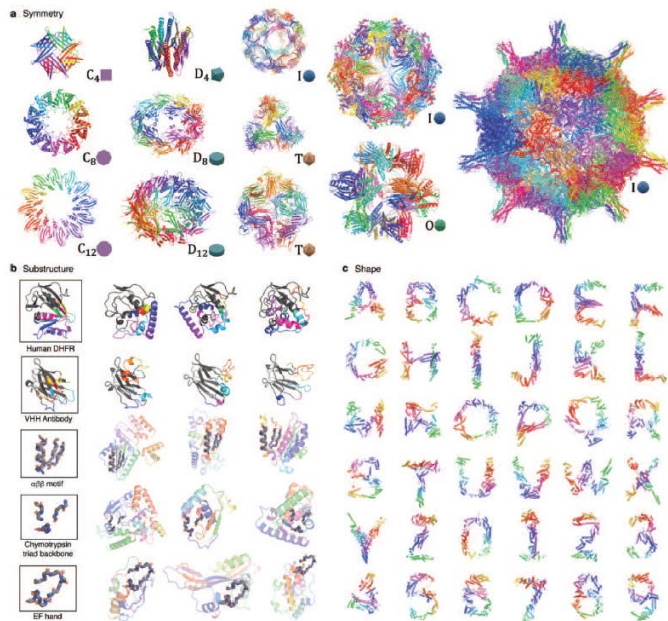


“A heavy quark move inside quark-gluon plasma”





## protein structure prediction and design

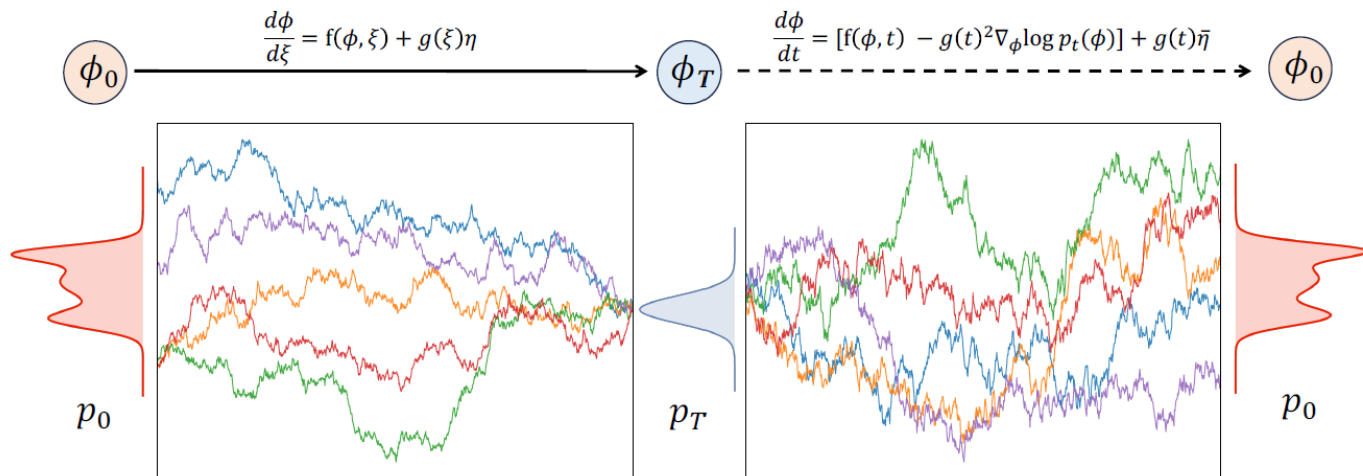
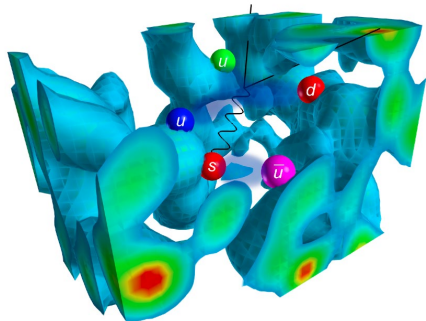


Ingraham et al, Chroma, Nature 2023  
<https://generatebiomedicines.com/chroma>

Abramson et al, AlphaFold3, Nature 2024  
<https://deepmind.google/technologies/alphafold/>

$$p(\phi) = e^{-S(\phi)}/Z$$

$$\langle O \rangle \approx \frac{1}{N} \sum_i O_i$$



L. Wang, G. Aarts, K. Zhou, **JHEP 05 (2024) 060**

L. Wang, G. Aarts, K. Zhou, **arXiv:2311.03578** (NeurIPS 2023 workshop “ML&Physical Sciences”)

G. A., D. E. H., L. W., K. Zhou, **arXiv:2410.21212** (NeurIPS 2024 workshop “ML&Physical Sciences”) → **“Best Physics for AI Paper” Award**

Q. Zhu, G. Aarts, W. Wang, K. Zhou, L. Wang, **arXiv:2410.19602** (NeurIPS 2024 workshop “ML&Physical Sciences”)

G. Aarts, D. E. H., L. W., K. Zhou, **arXiv:2510.01328**



# Diffusion Model for field configurations

- Forward diffusion SDE

$$\frac{d\phi}{d\xi} = f(\phi, \xi) + g(\xi)\eta(\xi) \quad \langle \eta(\xi)\eta(\xi') \rangle = 2\alpha\delta(\xi - \xi')$$

- Backward diffusion SDE

$$\frac{d\phi}{dt} = [f(\phi, t) - g^2(t)\nabla_{\phi} \log p_t(\phi)] + g(t)\bar{\eta}(t) \quad \underline{t \equiv T - \xi}$$

- Score matching Training  $\mathcal{L}_{\theta} = \sum_{i=1}^N \sigma_i^2 \mathbb{E}_{p_0(\phi_0)} \mathbb{E}_{p_i(\phi_i|\phi_0)} \left[ \|s_{\theta}(\phi_i, \xi) - \nabla_{\phi_i} \log p_i(\phi_i|\phi_0)\|_2^2 \right]$

- Sample generation SDE in **variance exploding scheme** :  $\frac{d\phi}{d\tau} = g_{\tau}^2 \nabla_{\phi} \log q_{\tau}(\phi) + g_{\tau} \bar{\eta}(\tau) \quad \tau \equiv T - t$

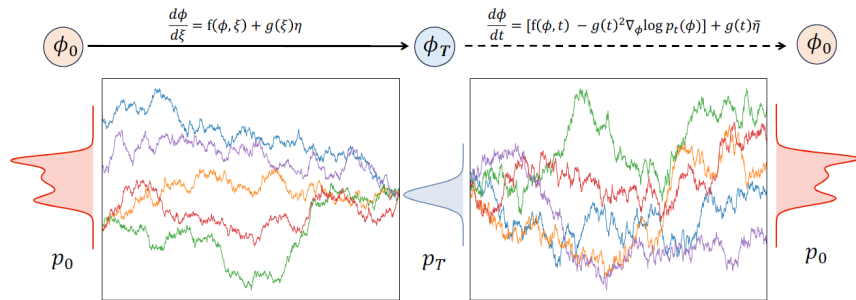
$$\frac{\partial p_{\tau}(\phi)}{\partial \tau} = \int d^n x \left\{ g_{\tau}^2 \frac{\delta}{\delta \phi} \left( \bar{\alpha} \frac{\delta}{\delta \phi} + \nabla_{\phi} S_{\text{DM}} \right) \right\} p_{\tau}(\phi),$$

$$p_{\text{eq}}(\phi) \propto e^{-S_{\text{DM}}/\bar{\alpha}}$$

$$\nabla_{\phi} S_{\text{DM}} \equiv -\nabla_{\phi} \log q_{\tau}(\phi)$$

- A flow of **effective action** will be learned in DMs

sampling from a DM is equivalent to optimizing a stochastic trajectory to approach the “equilibrium state”



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$$p_{\text{eq}}(\phi) \propto e^{-S_{\text{DM}}/\bar{\alpha}}$$

$$\nabla_{\phi} S_{\text{DM}} \equiv -\nabla_{\phi} \log q_{\tau}(\phi)$$

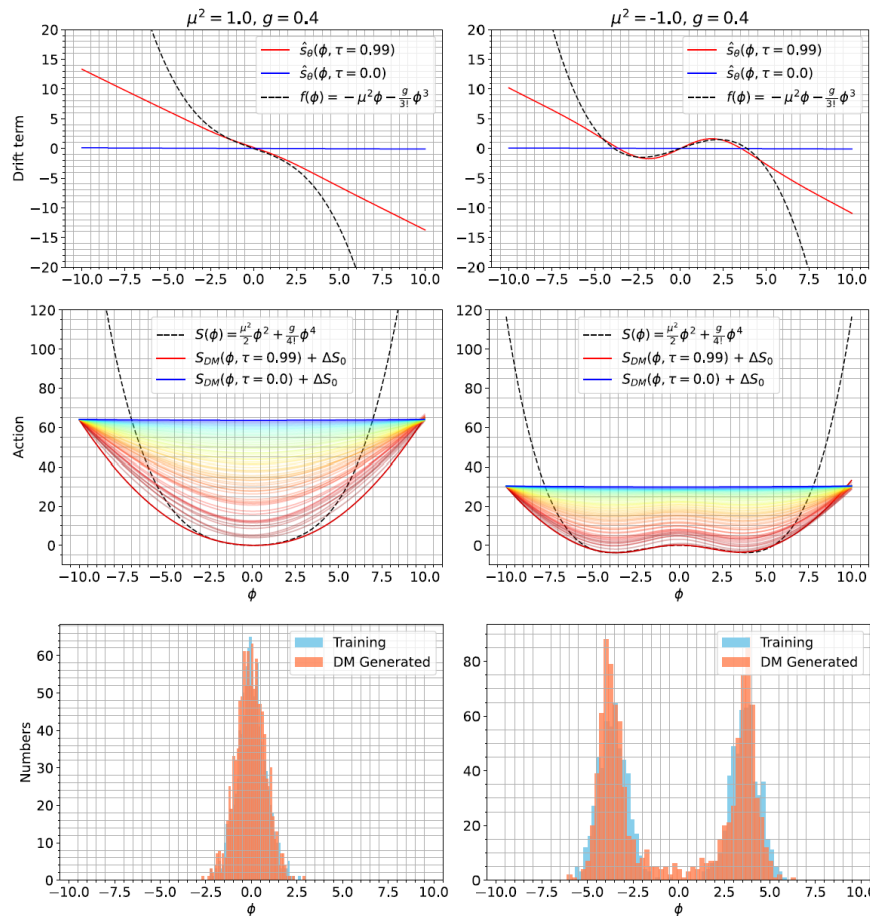
## A flow of **effective action** will be learned in DMs

sampling from a DM is equivalent to optimizing a stochastic trajectory to approach the “equilibrium state”

## Physical motivation

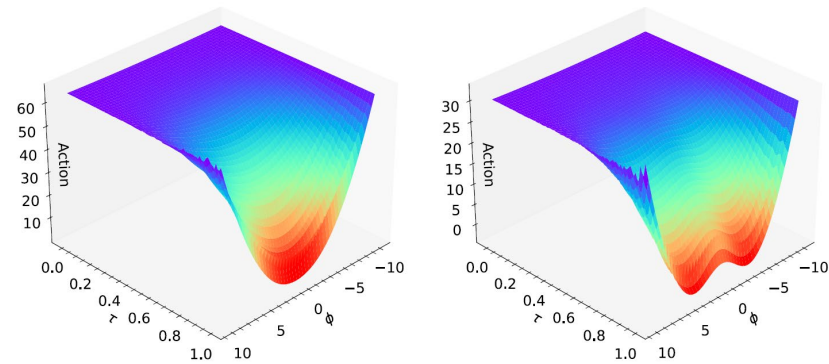
- Destroy structure in data through a diffusive process.
- Carefully record the destruction.
- Use deep networks to reverse time and create structure from noise.

# Effective Action on A Toy model



○ Flow of the effective action

$$S(\phi) = \frac{\mu^2}{2}\phi^2 + \frac{g}{4!}\phi^4, \quad f(\phi) = -\frac{\partial S(\phi)}{\partial \phi} = -\mu^2\phi - \frac{g}{3!}\phi^3$$



$$S_{DM}(\phi, \tau) = \int^\phi \hat{s}_\theta(\tilde{\phi}, \tau) d\tilde{\phi}$$

$$\frac{\partial \phi(x, \tau)}{\partial \tau} = - \frac{\delta S_E[\phi]}{\delta \phi(x, \tau)} + \eta(x, \tau)$$

$$\langle \eta(x, \tau) \rangle = 0, \quad \langle \eta(x, \tau) \eta(x', \tau') \rangle = 2\alpha \delta(x - x') \delta(\tau - \tau')$$

$\tau$ : fictitious time,  $\alpha$ : diffusion constant

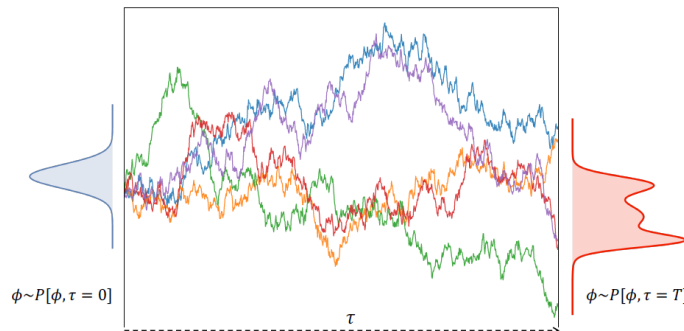
- Fokker-Planck equation**

$$\frac{\partial P[\phi, \tau]}{\partial \tau} = \alpha \int d^n x \left\{ \frac{\delta}{\delta \phi} \left( \frac{\delta}{\delta \phi} + \frac{\delta S_E}{\delta \phi} \right) \right\} P[\phi, \tau]$$

Equilibrium solution (long-time limit),

$$P_{\text{eq}}[\phi] \propto e^{-\frac{1}{\alpha} S_E[\phi]}$$

One can construct stochastic process to reproduce the quantum path integral with its equilibrium:



Thermal equilibrium limit  $\rightarrow$  Quantum distribution

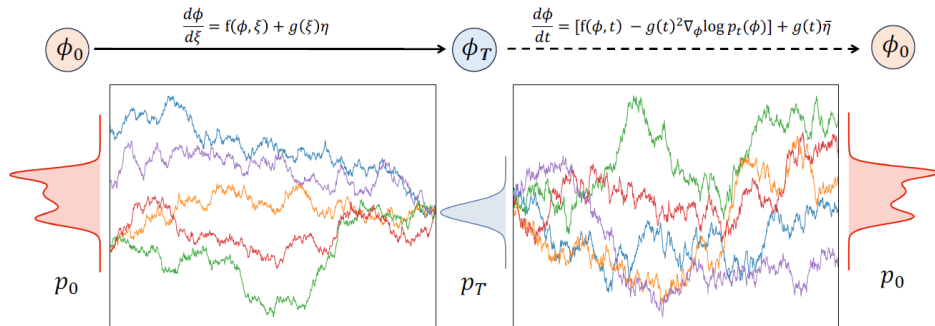
- Set the diffusion constant as  $\alpha = \hbar$

$$P_{\text{eq}}[\phi] \sim e^{-\frac{1}{\hbar} S_E[\phi]} = P_{\text{quantum}}[\phi]$$

## DM generation SDE and Stochastic Quantization :

$$\frac{\partial \phi(x, \tau)}{\partial \tau} = g^2(\tau) \nabla_{\phi} \log P(\phi; \tau) + g(\tau) \eta(x, \tau)$$

$$\frac{\partial \phi(x, \tau)}{\partial \tau} = -\nabla_{\phi} S(\phi) + \sqrt{2} \eta(x, \tau)$$



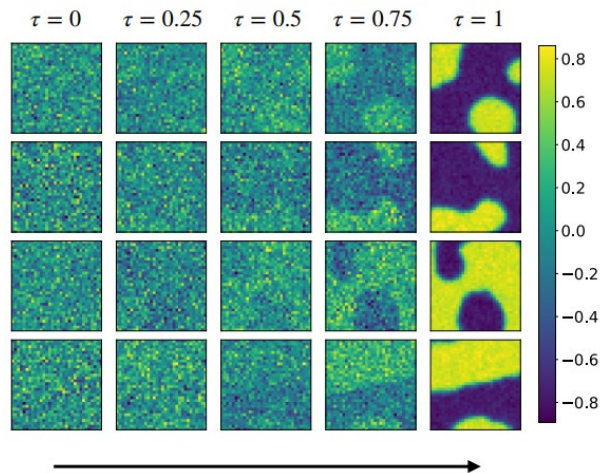
## Similarities and differences:

- ✓ SQ: fixed drift, determined from known action  
constant noise variance (but can be generalised using kernels)  
thermalisation followed by long-term evolution in equilibrium
- ✓ DM: drift and noise variance time-dependent, learn from data  
evolution between  $0 \leq \tau \leq T = 1$  , many short runs

- 32x32 lattice, HMC generated 5120 configurations as training set

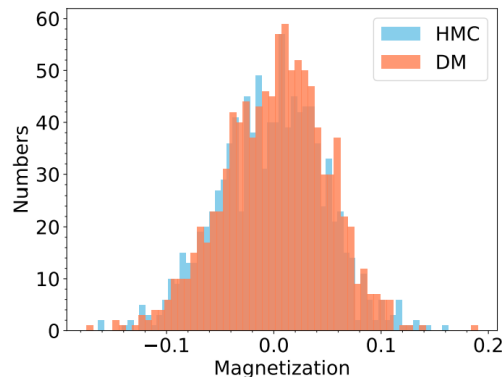
$$S_E = \sum_x [-2\kappa \sum_{\mu=1}^d \phi(x)\phi(x + \hat{\mu}) + (1 - 2\lambda)\phi(x)^2 + \lambda\phi(x)^4].$$

**Broken phase :**



numerous “bulk” patterns emerge

**symmetric phase :**



data-set	$\langle M \rangle$	$\chi_2$	$U_L$
Training (HMC)	$0.0012 \pm 0.0007$	$2.5160 \pm 0.0457$	$0.1042 \pm 0.0367$
Testing (HMC)	$0.0018 \pm 0.0015$	$2.4463 \pm 0.1099$	$-0.0198 \pm 0.1035$
Generated (DM)	$0.0017 \pm 0.0015$	$2.4227 \pm 0.1035$	$0.0484 \pm 0.0959$

- Forward diffusion kernel: **gaussian smoothing**

$$p_{\xi}(\phi_{\xi}|\phi_0) = \mathcal{N}\left(\phi_{\xi}; \phi_0, \frac{1}{2\log\sigma}(\sigma^{2\xi} - 1)\mathbf{I}\right)$$

$$\phi_{\tau}(\mathbf{x}) = \phi_0(\mathbf{x}) + \sqrt{\frac{\sigma^{2\tau}-1}{2\log\sigma}}\epsilon(\mathbf{x}) \text{ with } \epsilon \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$$

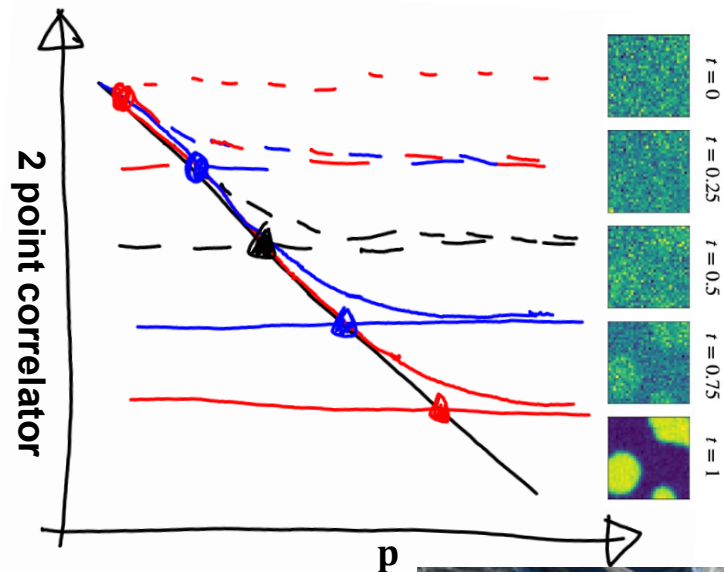
- In **Fourier space**:

$$\phi_{\tau}(p) = \phi_0(p) + \sqrt{\frac{\sigma^{2\tau}-1}{2\log\sigma}}\epsilon(p).$$

- ! the above evolution will perturb (smear) higher momentum modes first,
- With decreasing cut scale because of the gradually increasing noise level!

!

In **FRG**, the high frequency (short-distance) degrees of freedom is progressively integrated out !





# Generative model to speed up HIC simulation

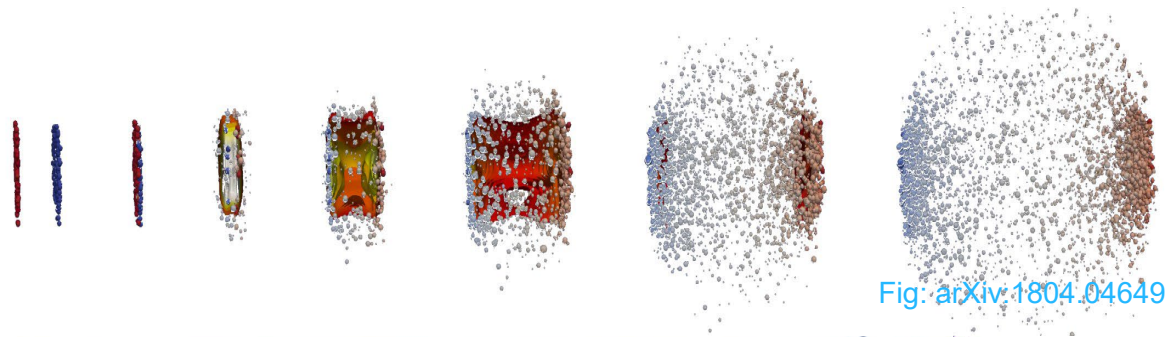


Fig: arXiv:1804.04649

**Initial Stage**

Impact parameter  
Nuclear structure

$$\nabla_\mu T^{\mu\nu} = 0$$
$$\nabla_\mu N^\mu = 0$$

Bulk Matter properties:  
Phase Transition  
EOS  
Shear/Bulk viscosities  
Hard Probes

$$p^\mu \partial_\mu f + F \cdot \partial_p f = C$$

**Final State**

Event Generation  
Final observables

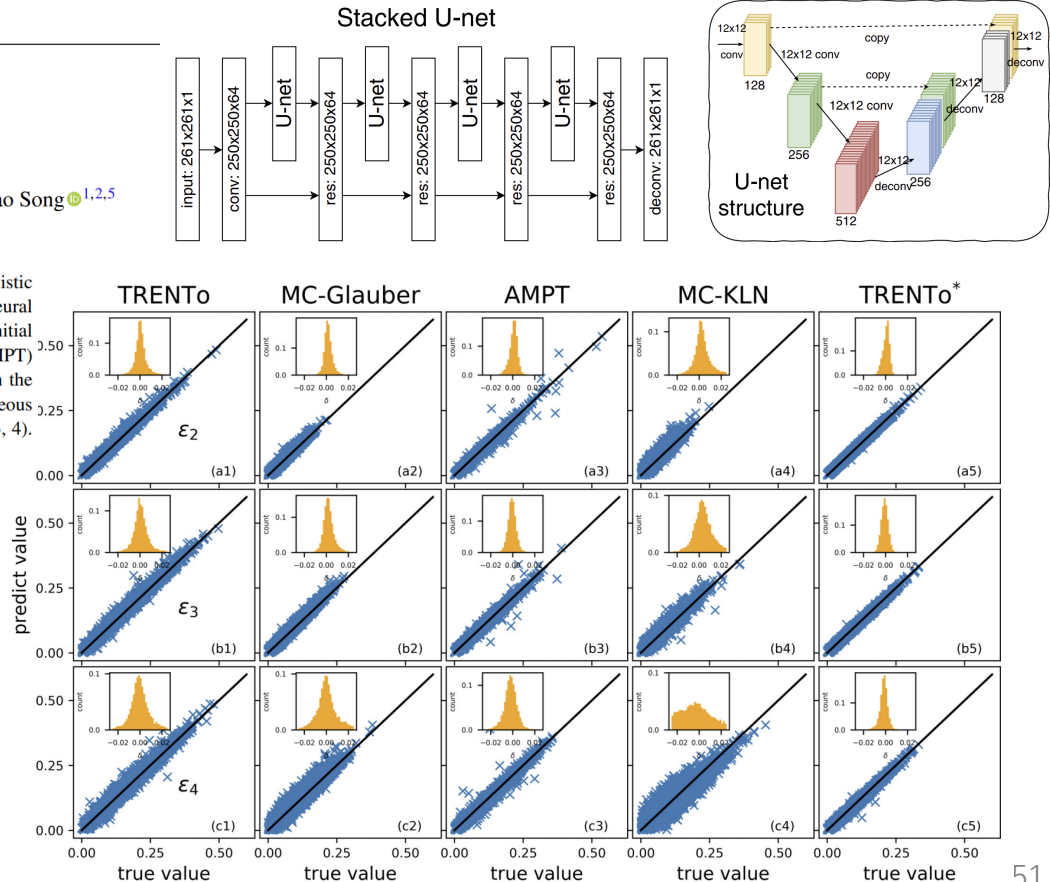
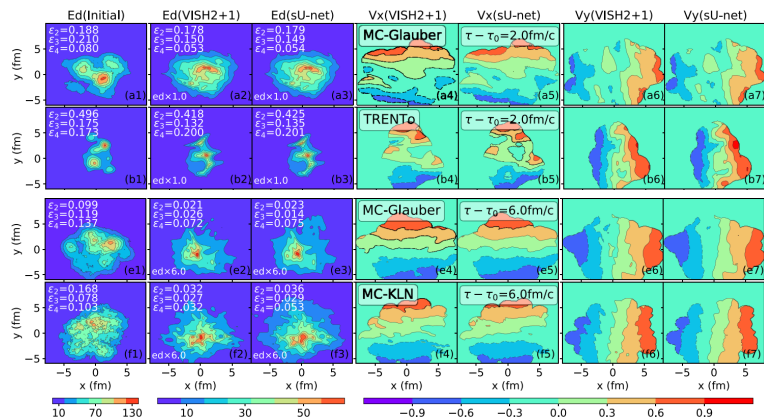
Applications to Lattice QFT

PHYSICAL REVIEW RESEARCH **3**, 023256 (2021)

## Applications of deep learning to relativistic hydrodynamics




Hengfeng Huang,<sup>1,2</sup> Bowen Xiao,<sup>3</sup> Ziming Liu,<sup>1</sup> Zeming Wu,<sup>1,2</sup> Yadong Mu,<sup>3,4</sup> and Huichao Song<sup>1,2,5</sup>

tic heavy-ion collisions. Using 10 000 initial and final profiles generated from (2+1)-dimensional relativistic hydrodynamics vISH2+1 with Monte Carlo Glauber (MC-Glauber) initial conditions, we train a deep neural network based on the stacked U-net, and use it to predict the final profiles associated with various initial conditions, including MC-Glauber, MC Kharzeev-Levin-Nardi (MC-KLN), a multiphase transport (AMPT) model, and the reduced thickness event-by-event nuclear topology (TRENTTo) model. A comparison with the vISH2+1 results shows that the network predictions can nicely capture the magnitude and inhomogeneous structures of the final profiles, and creditably describe the related eccentricity distributions  $P(\epsilon_n)$  ( $n = 2, 3, 4$ ).



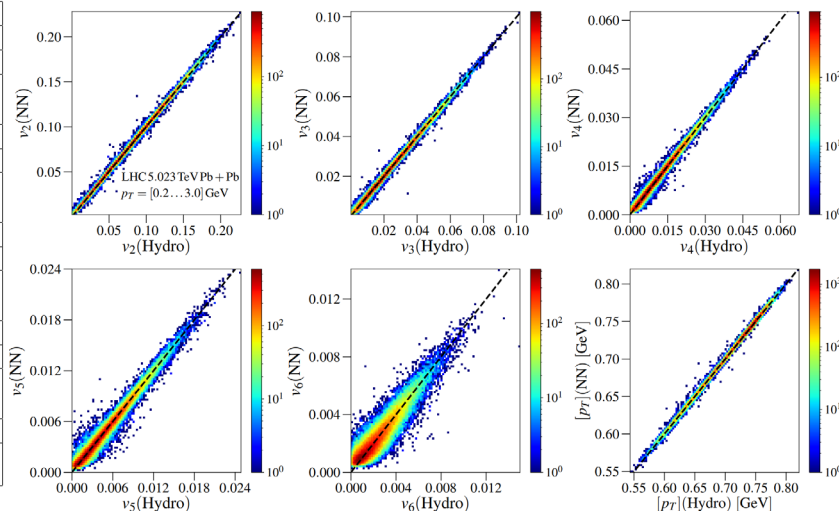
PHYSICAL REVIEW C **108**, 034905 (2023)

## Deep learning for flow observables in ultrarelativistic heavy-ion collisions

H. Hirvonen , K. J. Eskola , and H. Niemi 

As an input, the DenseNet model uses discretized initial energy density in the transverse-coordinate  $(x, y)$  plane calculated from the EKRT model with a grid size  $269 \times 269$  and a resolution of 0.07 fm. The DenseNet model is trained to reproduce a set of final state  $p_T$  integrated observables  $v_n$ , average transverse momentum  $[p_T]$ , and charged particle multiplicity  $dN_{ch}/d\eta$  for each event. The input energy density is normalized in such a

Block	Output size	Layers
Convolution	134x134x64	7x7 conv, stride 2
Pooling	67x67x64	3x3 max pool, stride 2
Dense Block	67x67x256	1x1 conv 3x3 conv x 6
Transition Layer	67x67x128	1x1 conv
	33x33x128	2x2 average pooling, stride 2
Dense Block	33x33x512	1x1 conv 3x3 conv x 12
Transition Layer	33x33x256	1x1 conv
	16x16x256	2x2 average pooling, stride 2
Dense Block	16x16x896	1x1 conv 3x3 conv x 20
Transition Layer	16x16x448	1x1 conv
	8x8x448	2x2 average pooling, stride 2
Dense Block	8x8x1216	1x1 conv 3x3 conv x 24
Output Layer	1x1x1216	8x8 global average pooling
	$N_{out}$	Fully connected layer with ReLU activation



$10^7$  events using the neural network, which takes around 20 h with the GPU. This is a very substantial difference compared to doing full hydrodynamic simulations using CPU, which would take about  $5 \times 10^6$  CPU hours.

# Generative diffusion model to heavy-ion collisions

See the talk of  
**Jing-An Sun**  
tomorrow 10:40am

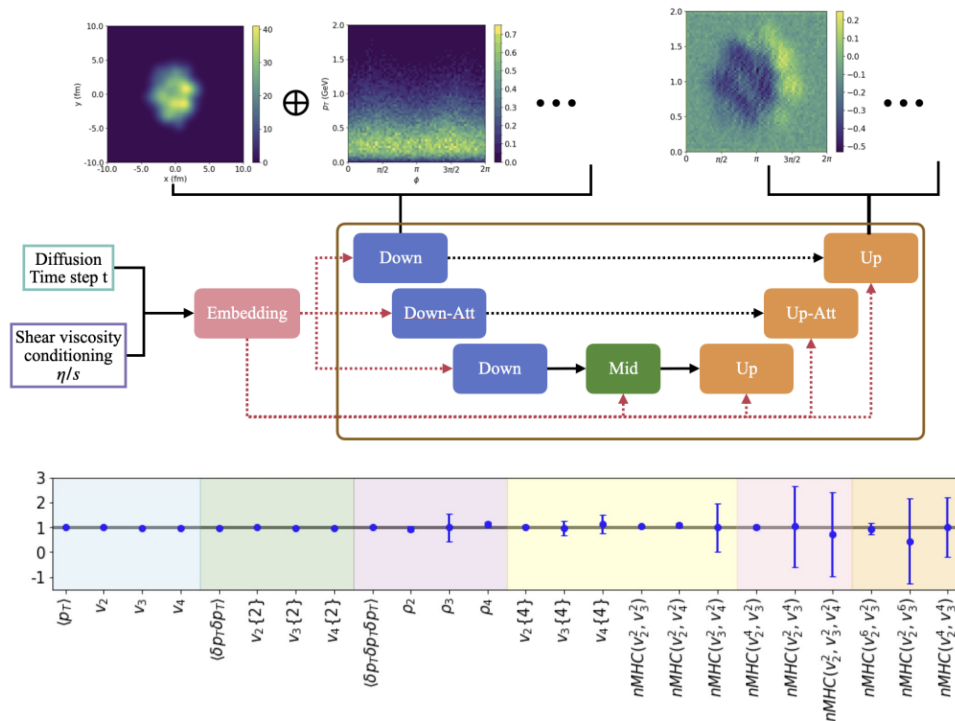
An end-to-end generative diffusion model for heavy-ion collisions

Jing-An Sun,<sup>1,2</sup> Li Yan,<sup>1,3</sup> Charles Gale,<sup>2</sup> and Sangyong Jeon<sup>2</sup>

**Phys. Rev. C (Letter) 2025**  
**arXiv:2410.13069**

Initial density profile  $\oplus$  charged particle spectra

The predicted noise



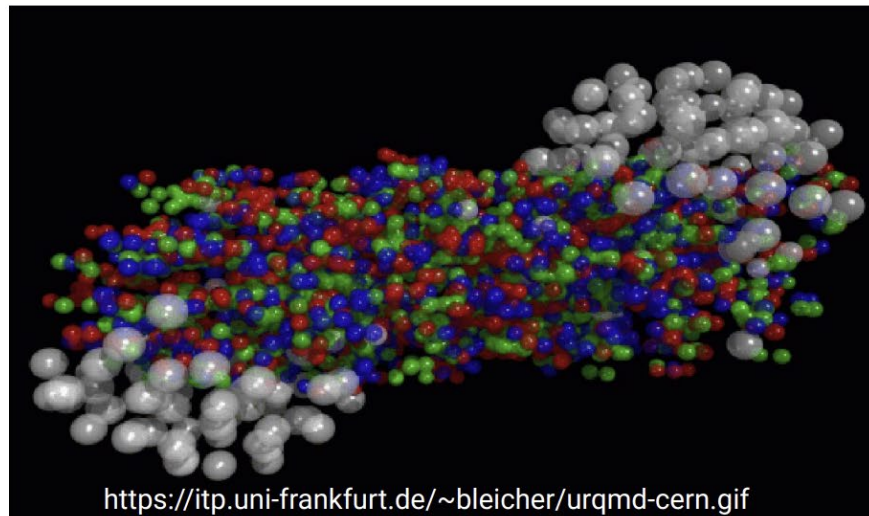
tor. We carried out (2+1)D minimum bias simulations of Pb-Pb collisions at 5.02 TeV, choosing the shear viscosity  $\eta/s$  to be one of three distinct values: 0.0, 0.1, and 0.2. For each value of  $\eta/s$ , we generate 12,000 pairs of initial entropy density profiles and final particle spectra, corresponding to 12,000 simulated events, as the training dataset. 70% of the total events are used for training and the rest are used for validation.

Considering that the spectra  $\mathcal{S}_0$  depend on the initial entropy density profiles  $\mathbf{I}$  and the shear viscosity  $\eta/s$ , we train a conditional reverse diffusion process  $p(\mathcal{S}_0|\mathbf{I}, \eta/s)$  without modifying the forward process.

one single central collision event in just  $10^{-1}$  seconds on a GeForce GTX 4090 GPU.

ble precision, as the traditional numerical simulation of hydrodynamics for one central event typically takes approximately 120 minutes ( $10^4$  seconds) on a single CPU.

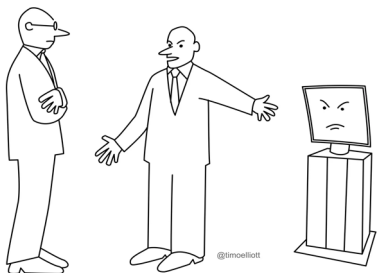
- Event-by-event collision output
- Microscopic non-equilibrium description
- hadrons on classical trajectories
  - stochastic binary scatterings
  - color string formation
  - resonance excitation and decays
- interactions based on scattering cross sections
- default setup effective EoS: Hadron Resonance Gas
- Non-trivial interactions can be added through QMD approach



Can we emulate UrQMD with DL?



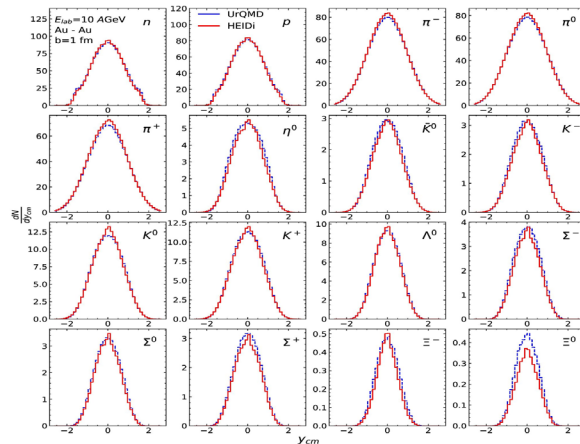
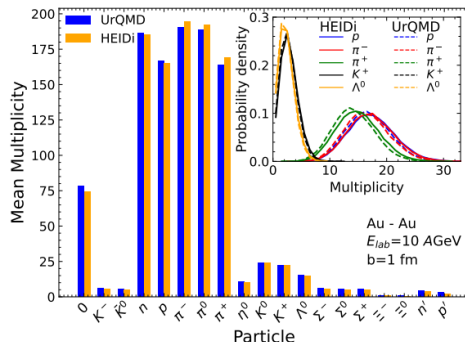
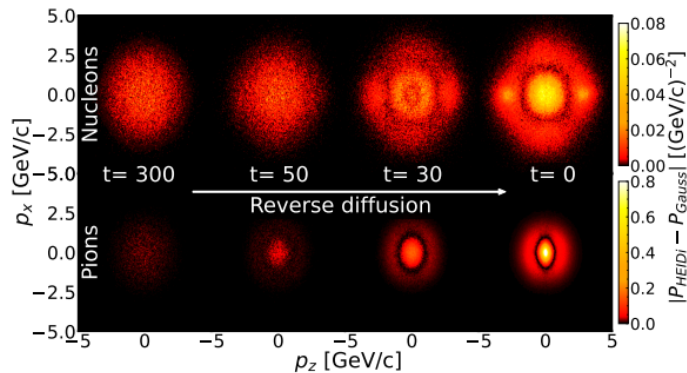
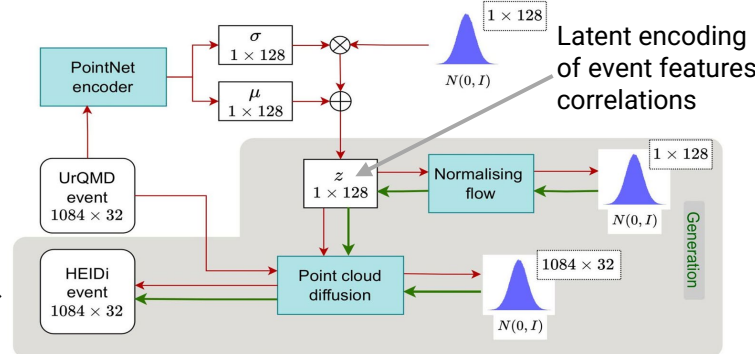
# Point Cloud Diffusion Model for HICs – AI clone of simulation



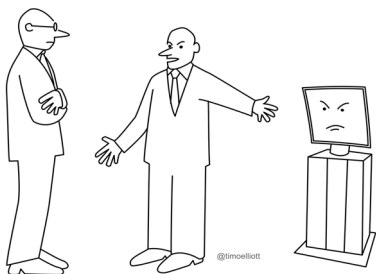
His decisions aren't any better than yours  
— but they're WAY faster...

- 18k UrQMD simulation events for central Au-Au@10 AGeV collisions
- HEIDI:**  
Heavy-ion Events through Intelligent Diffusion

PointNet encoder + Normalizing flow decoder + Pointcloud diffusion →



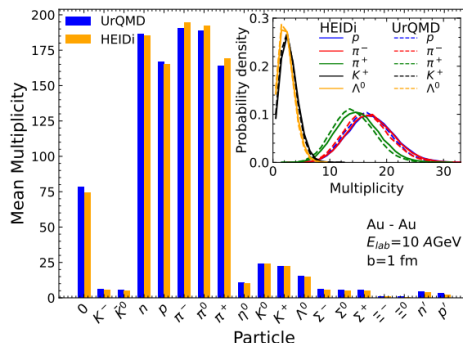
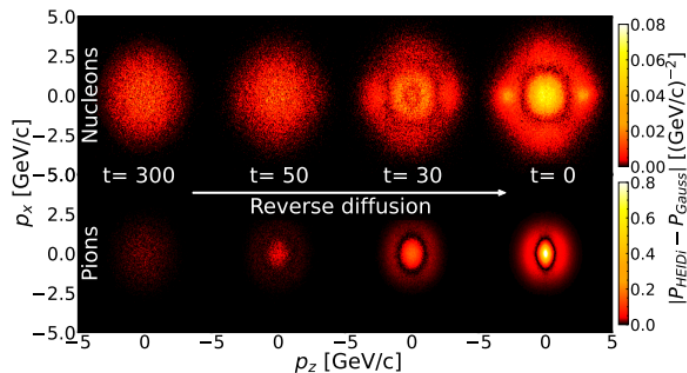
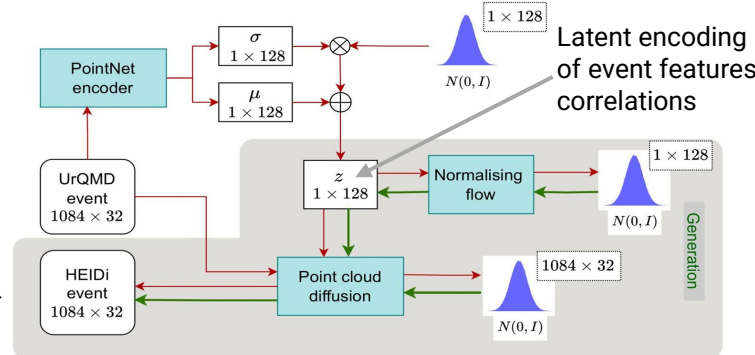
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Heavy-ion Events through Intelligent Diffusion

PointNet encoder + Normalizing flow decoder + Pointcloud diffusion →



- Running time of** UrQMD simulation  
cascade : ~ 3 sec/event;  
with potential : ~ 3 min/event;  
hybrid : ~ 1 hour/event
- HEIDI on A100: ~ 30 ms/event
- Speedup **2 ~ 5 orders** of magnitude

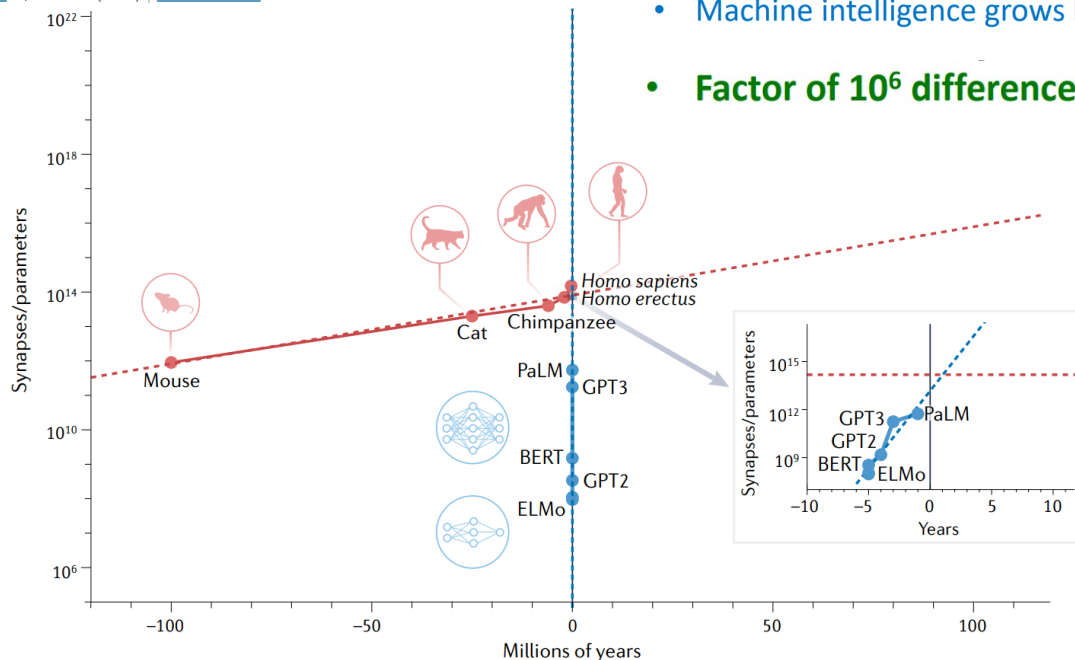


# What's the future? – Generative AI and LLM maybe the future

## Should artificial intelligence be interpretable to humans?

Matthew D. Schwartz

*Nature Reviews Physics* 4, 741–742 (2022) | [Cite this article](#)



- Biological intelligence grows by a factor of 2 in one million years
- Machine intelligence grows by a factor of 10 in 1 year
- **Factor of  $10^6$  difference in exponent**

Chat-GPT



Chat-GPh.T?  
Chat-GPh.D?

**Fig. 1 | The evolution of biological and artificial intelligence takes place on dramatically different timescales.** Any hope of interpreting and understanding AI will exponentially fade. Some example data points are highlighted in the evolution of biological (red) and artificial (blue) intelligence. The dashed lines represent the linear regression to these points. The acronyms in the figure are: Pathways Language Model (PaLM), Embeddings from Language Model (ELMo), Bidirectional Encoder Representations from Transformers (BERT), Generative Pre-trained Transformer (GPT).

- **Deep Learning** help bridging HIC experiment with theory/model for physics exploration/inversion     **caveat: model dependency**
  - **Bayesian Inference** for EoS from different beam energy experiment data (v2 and mT) perform well - consistent with  $dv_1/dy$  measurements and BNSM constraint  
**sensitivity check reveals tension: measurement uncertainty or model limitation**
  - **Auto-diff** help physics extraction taking advantage of GPU and DNN
- Combined global fit of EoS from HIC and NS obs. ? (**need to take care of isospin dependence**)
- **More Physics Priors, and**, Generative AI with Discriminative AI **together** as the fifth paradigm for nuclear physics

Nature Review Physics (2025)

Prog. Part. Nucl. Phys. 135 (2024) 104084

Nucl. Sci. Tech. 34 (2023) 6, 88

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Nature Review Physics (2025)

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Nucl. Sci. Tech. 34 (2023) 6, 88

# Thanks !

# Formal solution for forward process in DM

- forward process:  $\dot{x}(t) = K(x(t), t) + g(t)\eta(t)$

- backward process:

$$x'(\tau) = -K(x(\tau), T - \tau) + g^2(T - \tau)\partial_x \log P(x, T - \tau) + g(T - \tau)\eta(\tau)$$

score

two main schemes:

- variance-expanding (VE): no drift  $K(x, t) = 0$
- variance-preserving (VP) or denoising diffusion probabilistic models (DDPMs):

$$\text{linear drift } K(x(t), t) = -\frac{1}{2}k(t)x(t)$$

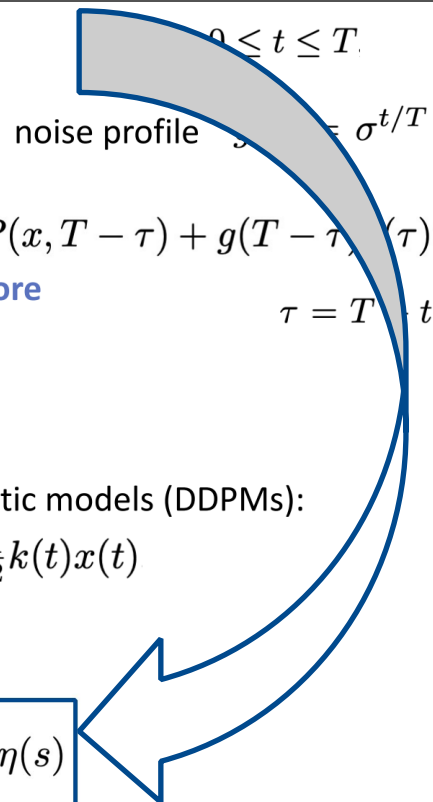
- initial data from target ensemble  $x_0 \sim P_0(x_0)$

- solution:

$$x(t) = x_0 f(t, 0) + \int_0^t ds f(t, s) g(s) \eta(s)$$

- with

$$f(t, s) = e^{-\frac{1}{2} \int_s^t ds' k(s')}$$



# Correlations evolution in forward process in DM

- moments  $\mu_n(t) = \mathbb{E}[x^n(t)]$  and cumulants or connected  $n$ -point functions  $\kappa_n(t)$   $\kappa_n = \mu_n - \sum_{m=2}^{n-2} \binom{n-1}{m-1} \kappa_m \mu_{n-m}$

- second moment/cumulant: (assume: first moment vanishes:  $x_0 \rightarrow x_0 - \mathbb{E}_{P_0}[x_0]$ )

$$\kappa_2(t) = \mu_2(t) = \mu_2(0)f^2(t, 0) + \Xi(t)$$

$$\Xi(t) = \int_0^t ds \int_0^t ds' f(t, s)f(t, s')g(s)g(s')\mathbb{E}_\eta[\eta(s)\eta(s')] = \int_0^t ds f^2(t, s)g^2(s)$$

- higher-order moment and cumulants:

$$\kappa_3(t) = \mu_3(t) = \kappa_3(0)f^3(t, 0)$$

$$\mu_4(t) = \mu_4(0)f^4(t, 0) + 6\mu_2(0)f^2(t, 0)\Xi(t) + 3\Xi^2(t)$$

$$\kappa_4(t) = \mu_4(t) - 3\mu_2^2(t)$$

$$\kappa_4(t) = [\mu_4(0) - 3\mu_2^2(0)] f^4(t, 0) = \kappa_4(0)f^4(t, 0)$$

variance-expanding  
scheme: no drift

$$f(t, 0) = 1$$

higher cumulants  
conserved!

→  $\kappa_{n>2}(t) = \kappa_n(0)f^n(t, 0)$

- in variance-expanding scheme (  $f(t, 0) = 1$  , no drift): distribution at end of forward process as correlated as target distribution

# Generating functions – simple structures

- proof to all orders: generating functionals  $Z[J] = \mathbb{E}[e^{J(t)x(t)}]$   $W[J] = \log Z[J]$

- average over both noise and target distribution

$$Z_\eta[J] = \mathbb{E}_\eta[e^{J(t)x(t)}] = \frac{\int D\eta e^{-\frac{1}{2} \int_0^t ds \eta^2(s) + J(t)[x_0 f(t,0) + \int_0^t ds f(t,s)g(s)\eta(s)]}}{\int D\eta e^{-\frac{1}{2} \int_0^t ds \eta^2(s)}}$$

- noise average:  $Z_\eta[J] = e^{J(t)x_0 f(t,0) + \frac{1}{2} J^2(t)\Xi(t)}$

- total average:  $Z[J] = \mathbb{E}[e^{J(t)x(t)}] = e^{\frac{1}{2} J^2(t)\Xi(t)} \int dx_0 P_0(x_0) e^{J(t)x_0 f(t,0)}$

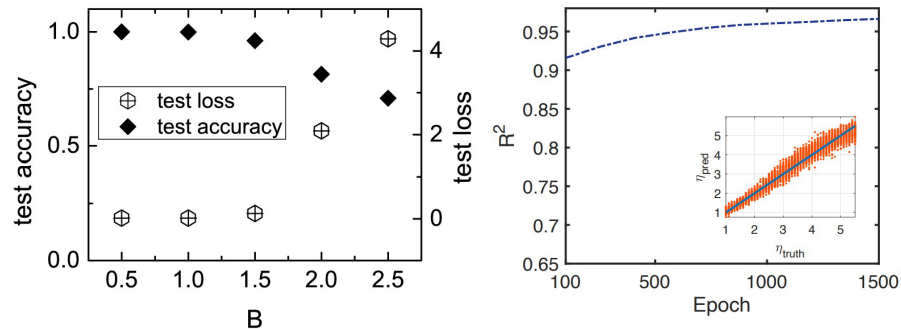
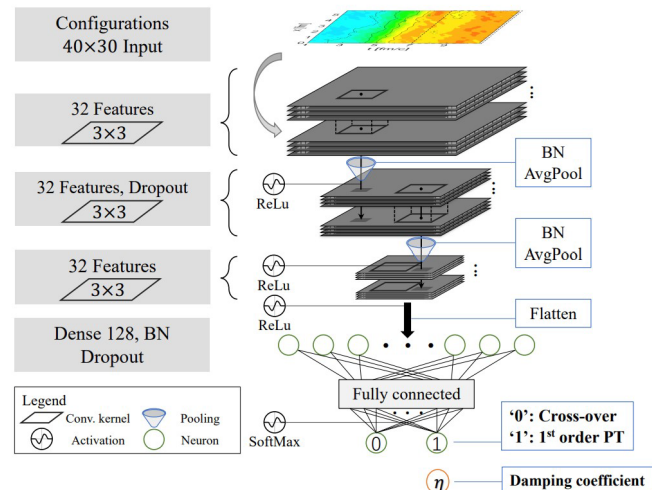
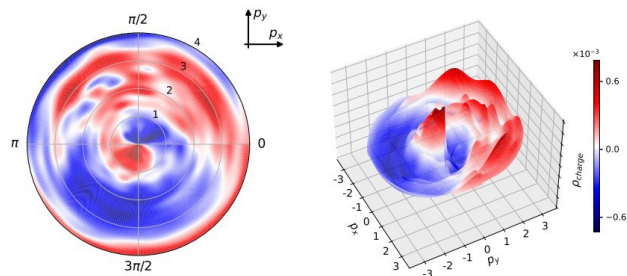
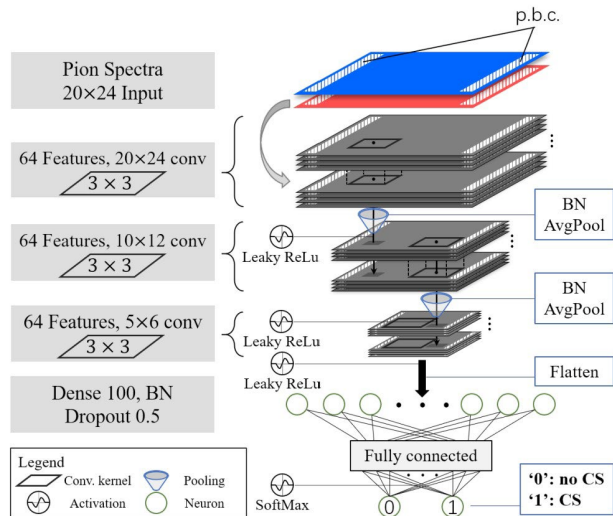
- cumulants:  $W[J] = \log Z[J] = \frac{1}{2} J^2(t)\Xi(t) + \log \int dx_0 P_0(x_0) e^{J(t)x_0 f(t,0)}$

- 2<sup>nd</sup> cumulant:  $\kappa_2(t) = \left. \frac{d^2 W[J]}{dJ(t)^2} \right|_{J=0} = \Xi(t) + \mathbb{E}_{P_0}[x_0^2] f^2(t,0)$  ✓

G. Aarts, D. E.H, L. W, and K. Zhou,  
**Mach. Learn.: Sci. Technol.** 6(2025)025004

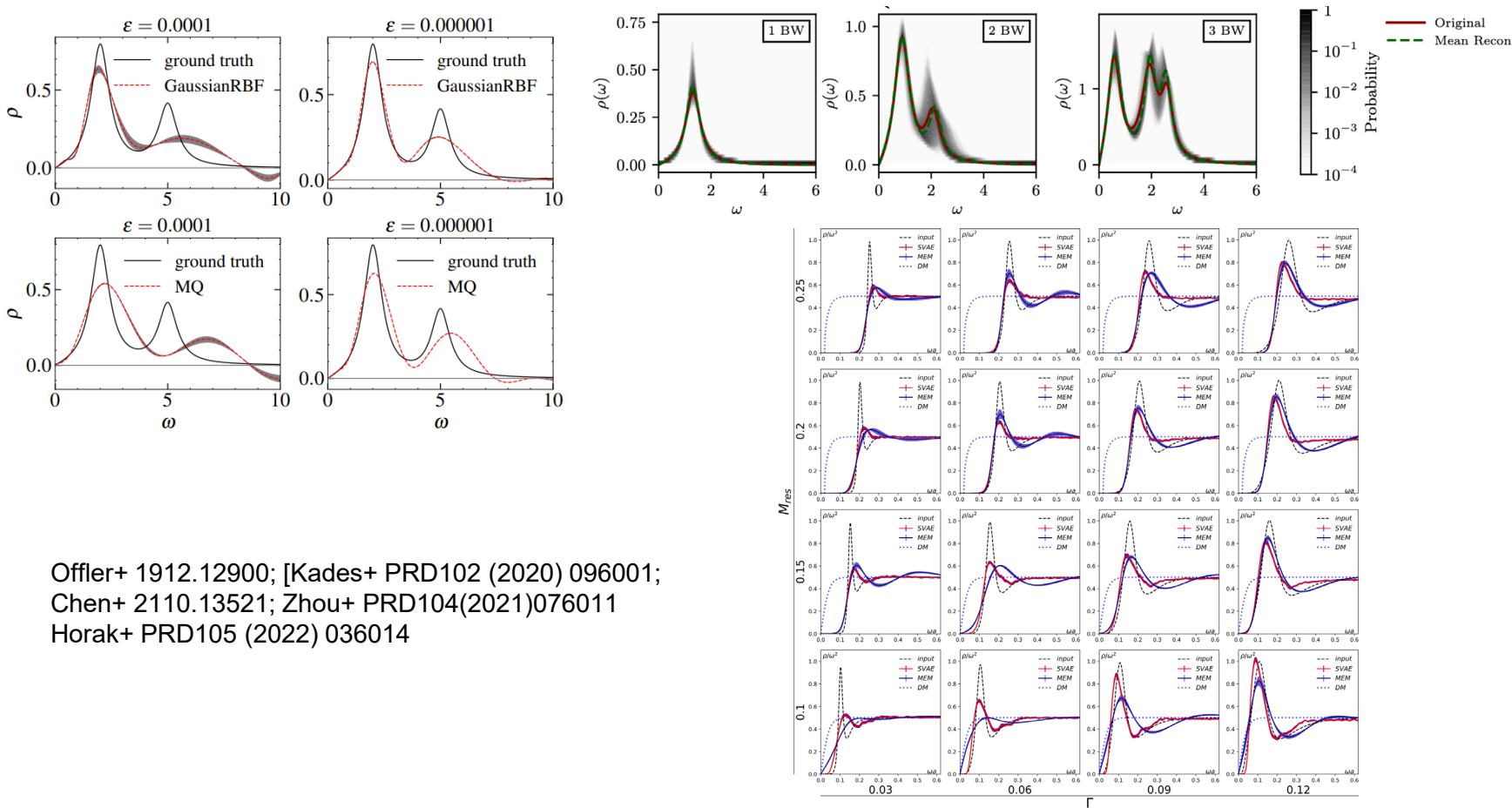
- higher-order cumulants:  $\kappa_{n>2}(t) = \left. \frac{d^n W[J]}{dJ(t)^n} \right|_{J=0} = \frac{d^n}{dJ(t)^n} \log \mathbb{E}_{P_0}[e^{J(t)x_0 f(t,0)}] \Big|_{J=0} = \kappa_n(0) f^n(t,0)$  ✓

# CNN to detect CME, and regress stochastic dynamics in HICs



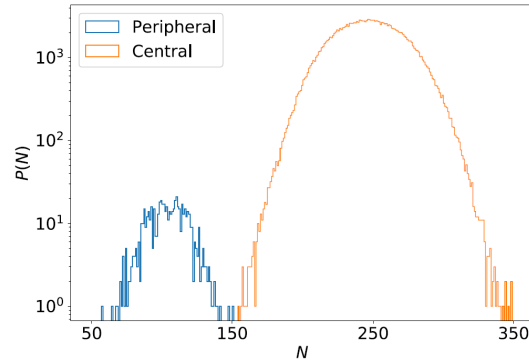


# Spectral function reconstruction from Euclidean correlator

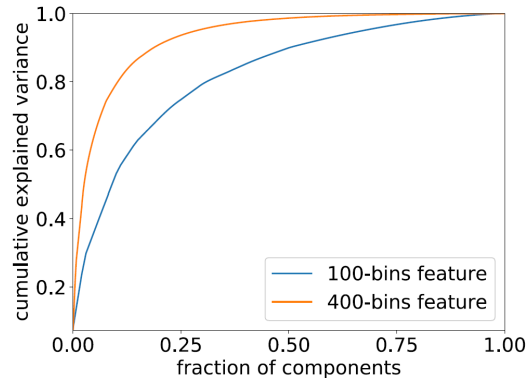


Offler+ 1912.12900; [Kades+ PRD102 (2020) 096001;  
Chen+ 2110.13521; Zhou+ PRD104(2021)076011  
Horak+ PRD105 (2022) 036014

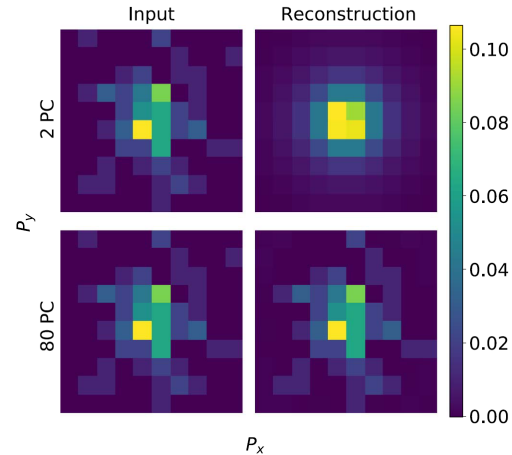
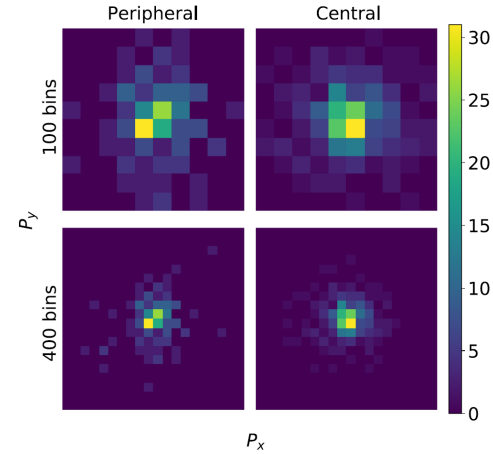
- Use **centrality misclassification** as example



- PCA to reduce dim while keep some **reconstruction** →



Histogram of the charged particle number



A series (**Flow**) of invertible/bijective transformations for  $p(z)$

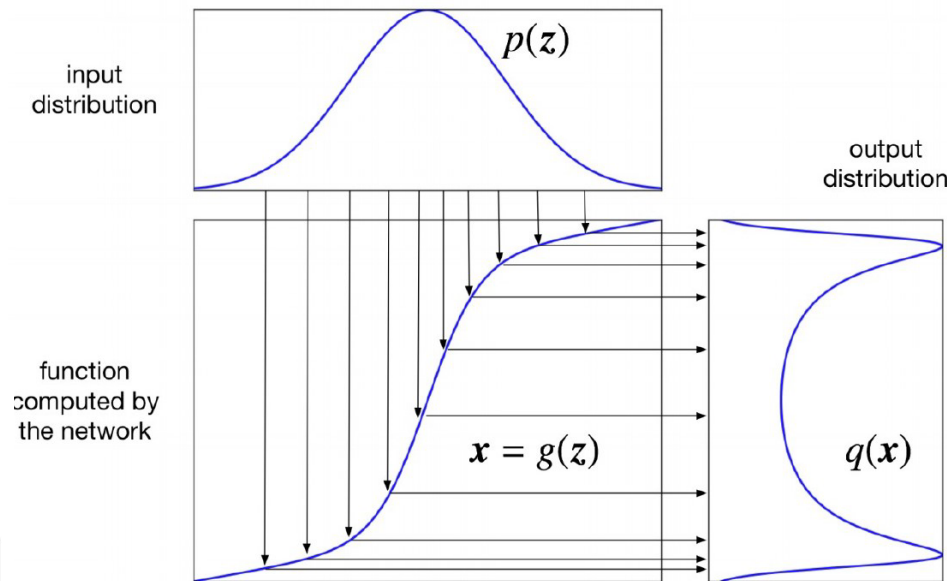
**Normalizing** : keep the probability to be normalized  $\rightarrow$

- Change of variable theorem

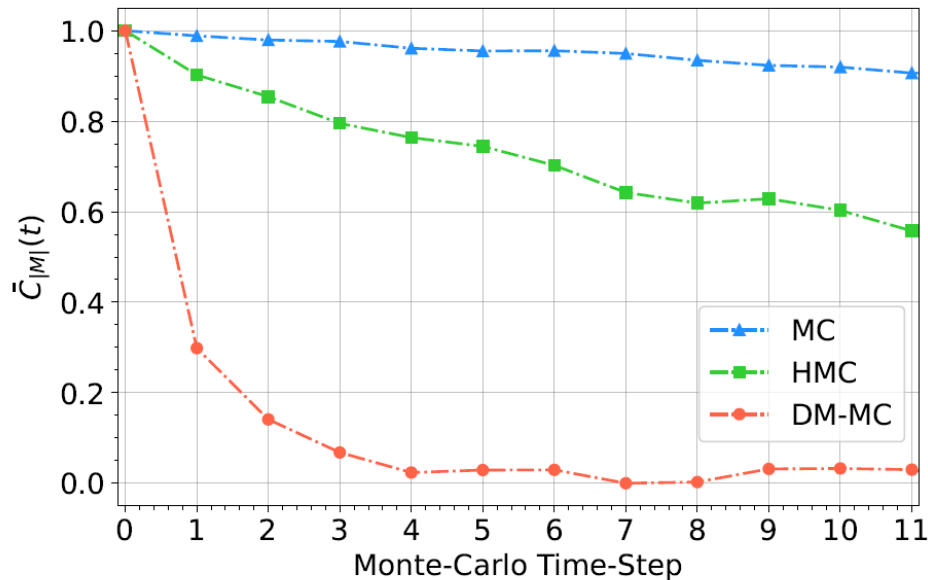
$$z \sim p(z)$$

$$x = g(z)$$

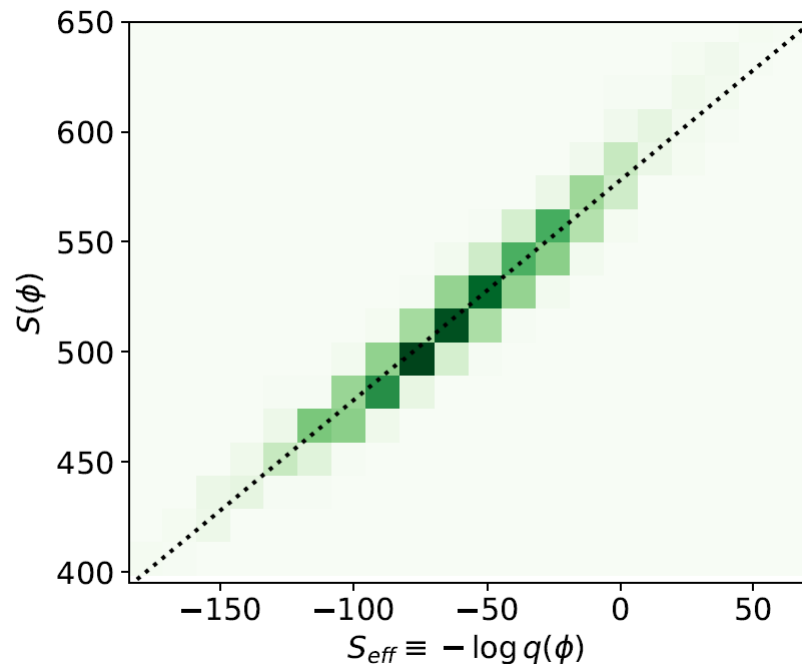
$$\begin{aligned} q(x) &= p(z) \left| \frac{dz}{dx} \right| \\ &= p(g^{-1}(x)) \left| \frac{dg^{-1}(x)}{dx} \right| \end{aligned}$$



# Autocorrelation time and finally captured effective Action



validation R2 ~0.96



$$C_O(t) = \langle (O_{t_0} - \langle O_{t_0} \rangle)(O_{t_0+t} - \langle O_{t_0+t} \rangle) \rangle = \langle O_{t_0} O_{t_0+t} \rangle - \langle O_{t_0} \rangle \langle O_{t_0+t} \rangle$$

- UrQMD outputs a list of final state hadrons along their momentum info
- Pointclouds: ideal representation

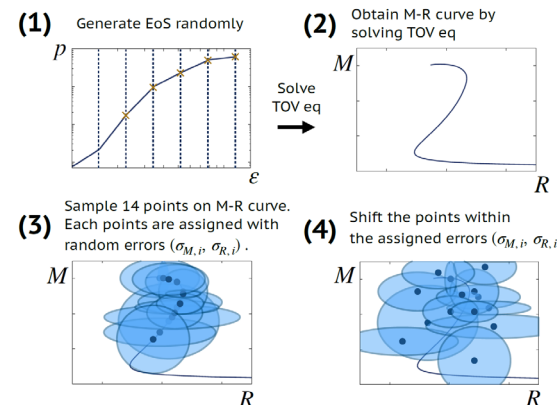
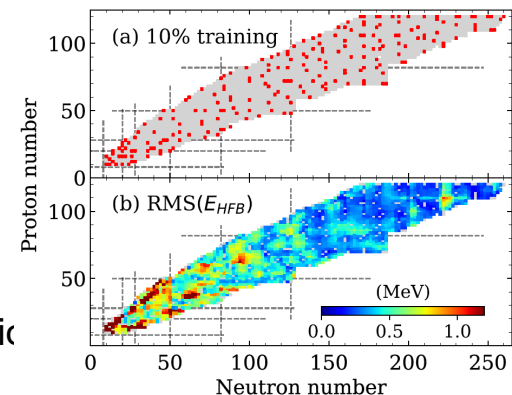
- Consider Au-Au 10 AGeV, impact parameter  $b=1$  fm
  - An event= 1084 X 32
  - Empty rows=0,0,0,0,...
  - $p_x, p_y, p_z$ , One hot encoded PID
  - 26 hadron species, spectator nucleons, empty particles

$$\mathbf{X}^{(0)} = \{\mathbf{x}_i^{(0)}\}_{i=1}^{1084}$$

$$\mathbf{x}_i^{(0)} = \{\mathbf{p}_i^{(0)}, \text{ID}_i^{(0)}\},$$

$$\mathbf{p}_i^{(0)} = (p_{x_i}^{(0)}, p_{y_i}^{(0)}, p_{z_i}^{(0)})$$

- Nuclear properties prediction
  - Dripline locations, atomic masses, separation energies, superheavy nuclei location...
  - ANN application since 1992, later to beta-decay etc., → BNN
  - Different ML methods, SVM, Gradient boost, BDT,...
- Interpo-/extrapolation of nuclear data, augment nuclear model
  - Nuclear masses, nuclear charge radii, alpha-decay rate,
  - Fission yield constrain, fusion cross-section estimation, isotopic cross-section prediction
  - Within nuclear DFT, Energy density functional (EDF) need to be adjusted to exp data – with ML
- Nuclear matter equation of state and Neutron Star properties
  - Inverse problems in heavy ion collisions and EoS extraction
  - Experimental global analysis, for QGP properties and PDF
  - Neutron Star analysis with Bayesian, DNN, Auto-diff...
  - Fast Simulation (Emulator) for HICs
- In lattice QCD
  - Inverse problem: spectral function or interaction or PDF reconstruction
  - Bayesian inference and DNN, and also auto-diff
  - Configurations Generation via Generative models



# (particle phy) Supervised Learning – Regressive Tasks

- **Jet Tagging**, PID, - BDT, CNN, GNN, PCN (CMS-DeepJet, ATLAS)

- B-tagging (identifying jets originating from b-quarks)
- Tau-jet : Lepton/photon vs. hadron separation
- Heavy-flavor jets (specific particle decays)
- Pion, kaon, proton identification, medium-like/vacuum-like jets

- **Reconstruction** – GNN, CNN, Self-Supervised (ML4ParticleFlow)

- Convert raw detector signals to physical variables (4 mom, vertex)
- Calibrating reconstructed energies in calorimeter
- Correcting measured momenta from tracking detectors

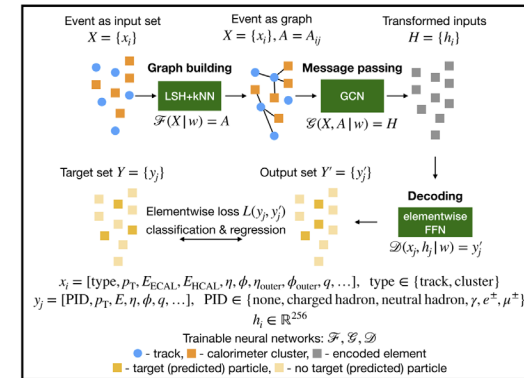
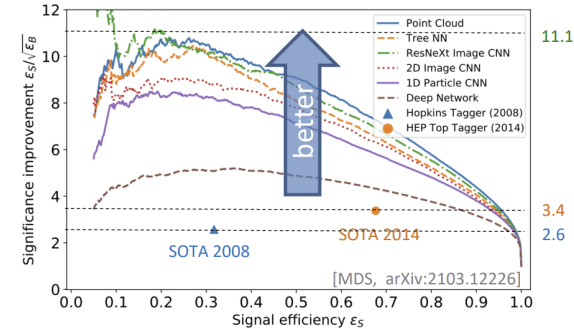
- **Real-Time Trigger / Filtering** system – CNN, RL, Q-learning

- Ultra-low latency classification of collision event / signals
- Implementing ML inference on specialized hardware (FPGAs)
- Online distillation reducing raw data flow from Terabytes/second to manageable levels

- **Inference** – cINN, flows

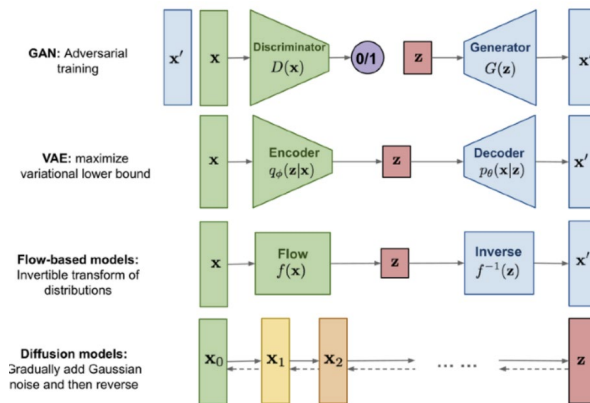
- Learn param of theory from high-d exp data – simulation based inference
- Inverse problem solving

## Top Tagging (2008 – 2022)





- **Simulation** – GAN, VAE, flow, Diffusion (CaloGAN)
  - Fast simulation of collision events and detector responses.
  - Use classical simulation or collider data as input, train surrogate
  - 3D voxel image or Point Cloud
  - Replace time-consuming full GEANT4 simulations to accelerate experimental analyses.
- **Unfolding** – GNN, CNN, Self-Supervised (ML4ParticleFlow)
  - Recover theoretical-level physical distributions (e.g., transverse momentum) from detector-level data.
  - Calibrating reconstructed energies in calorimeter
  - Correct for detector effects such as resolution and efficiency.
  - NN based direct unfolding, or Generative based probabilistic unfolding
- **Anomaly Detection** – Classifier, PCA, AutoEncoder,
  - Detect physics phenomena beyond standard models.
  - Search for rare events, such as dark matter signals or new resonances.



## Fast Calorimeter Simulation Challenge 2022

[View on GitHub](#)

Welcome to the home of the first-ever Fast Calorimeter Simulation Challenge!

The purpose of this challenge is to spur the development and benchmarking of fast and high-fidelity calorimeter shower generation using deep learning methods. Currently, generating calorimeter showers of interacting particles (electrons, photons, pions, ...) using GEANT4 is a major computational bottleneck at the LHC, and it is forecast to overwhelm the computing budget of the LHC experiments in the near future. Therefore there is an urgent need to develop GEANT4 emulators that are both fast (computationally lightweight) and accurate. The LHC collaborations have been developing fast simulation methods for some time, and the hope of this challenge is to directly compare new deep learning approaches on common benchmarks. It is expected that participants will make use of cutting-edge techniques in generative modeling with deep learning, e.g. GANs, VAEs and normalizing flows.

This challenge is modeled after two previous, highly successful data challenges in HEP – the top tagging community challenge and the LHC Olympics 2020 anomaly detection challenge.