Enrico Vigezzi INFN Milano

A dynamical model of light neutron halos

In collaboration with
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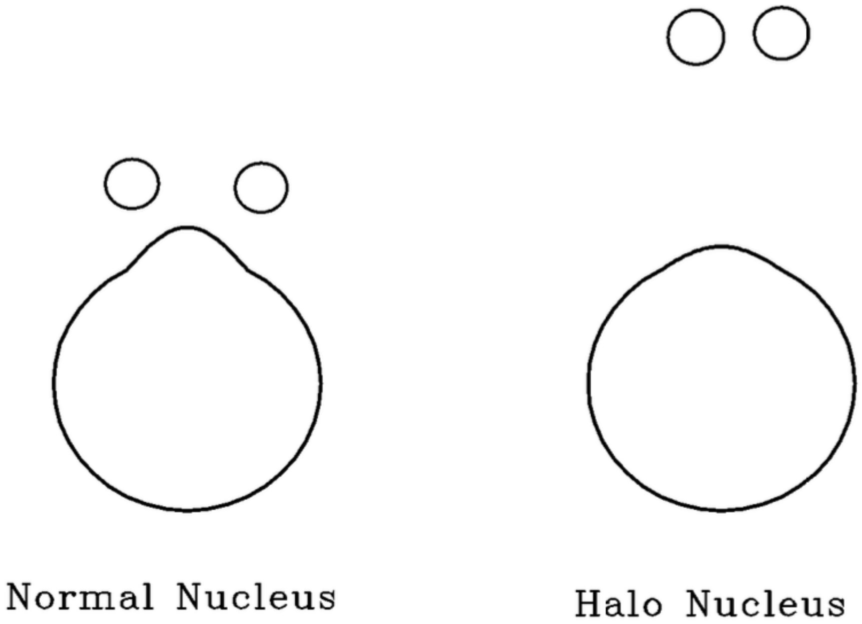
Gregory Potel
University of Sevilla

Halo40 Symposium, Bejing, 13-17 October 2025

INFN

MOdeling Nuclear STructure and REactions

In 1997 Kuo et al predicted that in halo nuclei core polarization would be suppressed, and that the fundamental nucleon-nucleon interaction could be probed in a clearer and more direct way in halo nuclei than in ordinary nuclei….

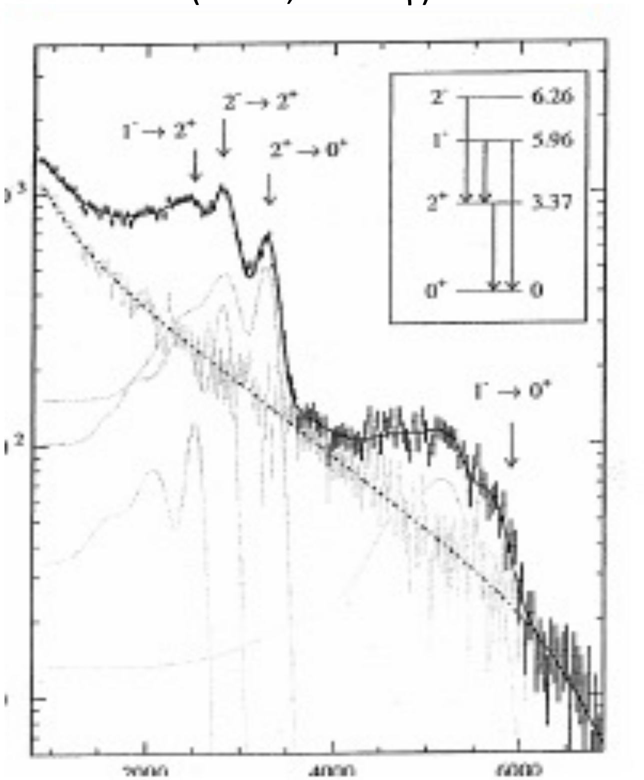


T.T.S. Kuo et al,
PRL 78 (1997) 2708

… But experiments demonstrated that the core dynamics plays an important role…

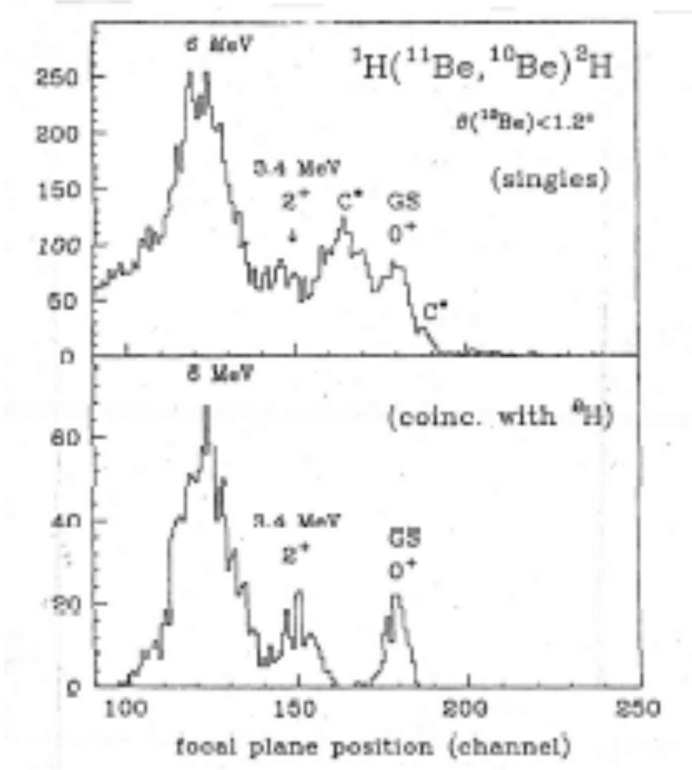
The admixture of $d_{5/2}$ x 2^+ configuration in the $1/2^+$ g.s. of ^{11}Be is about 15%

9Be(11Be,10Be+γ)X

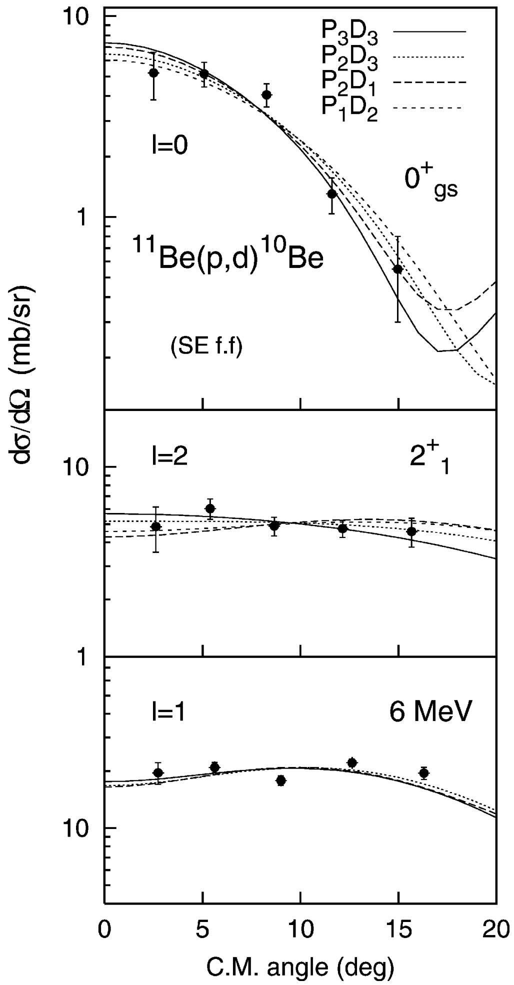


T. Aumann et al. PRL 84 (2000) 35

p(11Be,10Be)d



S. Fortier et al. Phys. Lett. B461 (1999)22
J.S. Winfield et al., Nucl.Phys. A683 (2001)48



J.S. Winfield et al.,
Nucl.Phys. A683 (2001)48

A careful analysis of transfer reactions is needed to estimate phonon admixtures in the wavefunctions

$$\left|{}^{11}\mathrm{Be}_{gs}\right\rangle = \alpha\left|{}^{10}\mathrm{Be}(0^+)\otimes 2s\right\rangle + \beta\left|{}^{10}\mathrm{Be}(2^+)\otimes 1\mathrm{d}\right\rangle$$

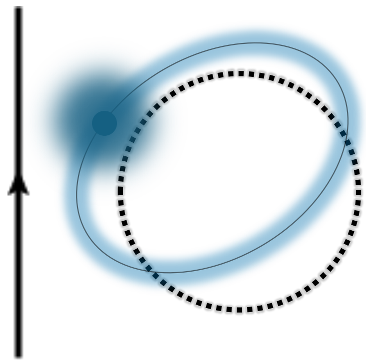
Good agreement with 2+ cross sections is obtained in DWBA with $\beta^2 = 0.17$ considering the coupling effects on the transfer form factor; using β as a simple spectroscopic factor one finds $\beta^2 = 0.28$

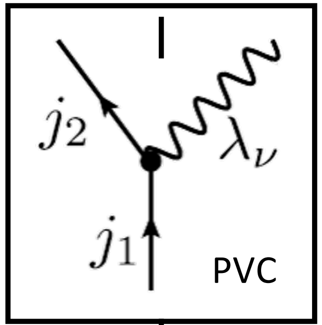
Can we obtain a consistent description of halos in terms of elementary modes of excitation including some core degrees of freedom ?

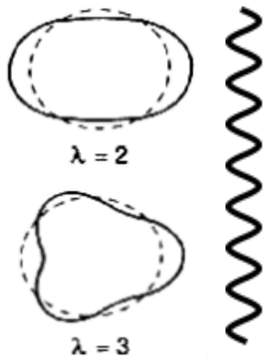
Independent Particles

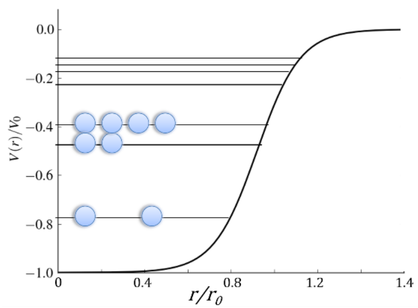
Collective Phonons

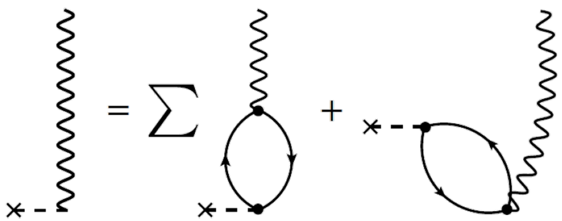
Particle-vibration coupling







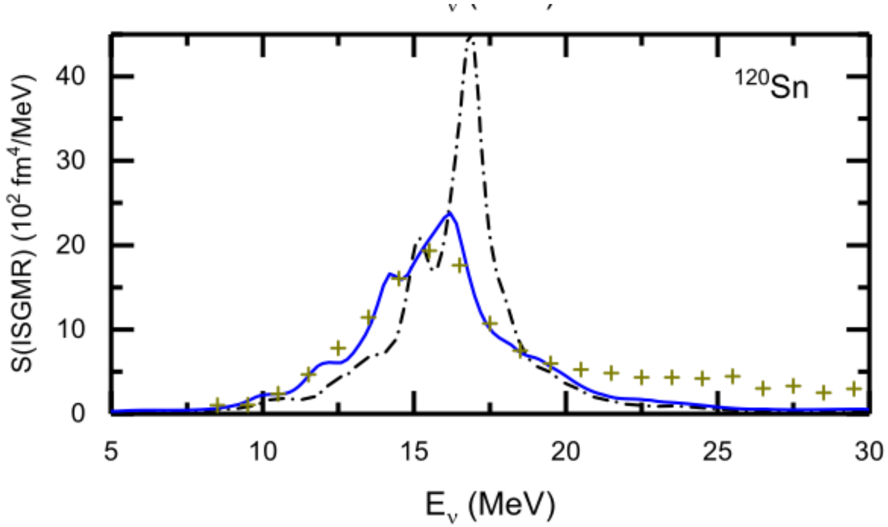




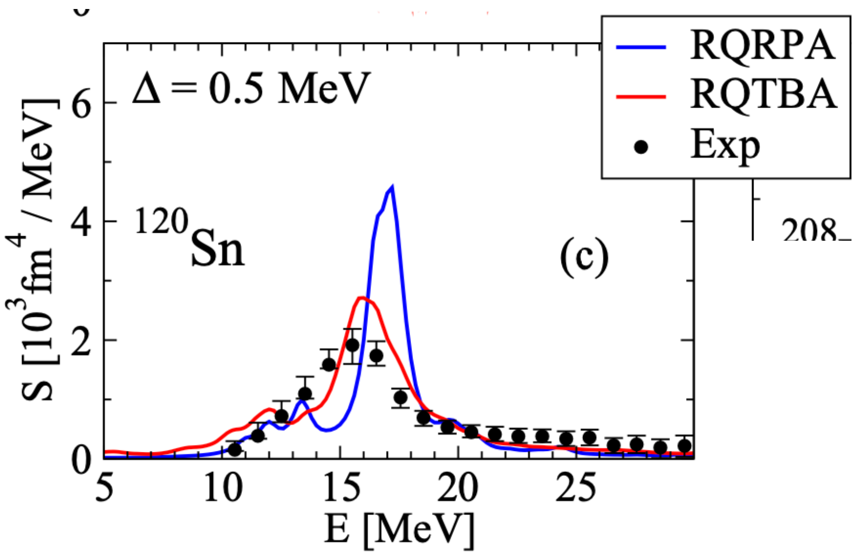
Hartree-Fock mean Field

Random Phase Approximation

Particle-vibration coupling on top of self-consistent density functional calculations has been mostly applied to heavy nuclei near closed shells. It provides a successful reproduction of the width of giant resonance modes ....



Z.Z. Li, Y.F. Niu, G. Colò. PRL 131 (2023) 082501



E. Litvinova, PRL 107 (2023) L041302

... although the situation is less clear concerning the centroids and the renormalization of single particle states

The Particle Vibration Coupling Hamiltonian:

$$H = H_{vib} + H_p + H_{PVC}$$

$H_{Vib} = \sum_{\lambda\mu} \hbar\omega_\lambda [\Gamma^+_{\lambda\mu} \Gamma_{\lambda\mu} + 1/2]$ → Collective ($\mathbf{\Gamma_{\lambda\mu}}^+$ creates a phonon)

$H_p = -\hbar^2/2m\ d^2/dr^2 + V(r) + V_{ls}(r)$ → Single-particle

$H_{PVC} = \sum_{\lambda\mu} -r dV/dr \beta_\lambda Y_{\lambda\mu} [\Gamma^+_{\lambda\mu} + (-1)^\mu \Gamma_{\lambda\mu}]$ → Linear interaction

↓ (β_λ)

Def. parameter

$$H\Psi_a = \tilde{E}\Psi_a$$

$\Psi_a = [\psi^x_a + (\psi^C_b \cdot \Phi_\lambda)_{j_a} + \ldots]\Phi^A_{GS}$ → Odd dressed neutron

$\psi^x_a = (R^x_a(r)/r)\Theta_{j_a m_a}$ → Single-particle part

$(\psi^C_b \cdot \Phi_\lambda)_{j_a} = (R^C_b(r)/r)(\Theta_{j_b} \cdot \Phi_\lambda)_{j_a}$ → Phonon admixture

An early interpretation of parity inversion: dynamical coupling between the core and the loosely bound neutron

VOLUME 70, NUMBER 10 PHYSICAL REVIEW LETTERS 8 MARCH 1993

Structure of Exotic Neutron-Rich Nuclei

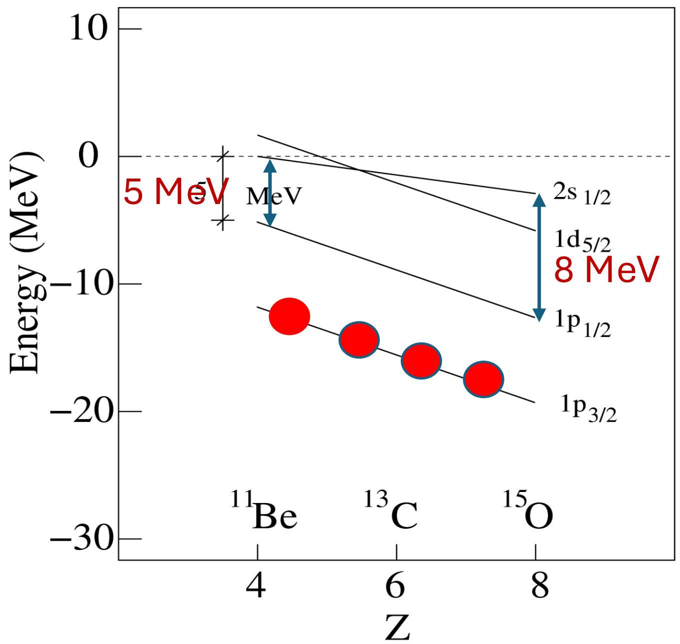
Takaharu Otsuka, Nobuhisa Fukunishi, and Hiroyuki Sagawa

Department of Physics, University of Tokyo, Hongo, Bunkyo-ku, Tokyo, 113, Japan

(Received 13 November 1992)

A new framework, the variational shell model, is proposed to describe the structure of neutron-rich unstable nuclei. An application to ^{11}Be is presented. Contrary to the failure of the spherical Hartree-Fock model, the anomalous $\frac{1}{2}^+$ ground state and its neutron halo are reproduced with the Skyrme (SIII) interaction. This state is bound due to dynamical coupling between the core and the loosely bound neutron, which oscillates between the $2s_{1/2}$ and the $1d_{5/2}$ orbits.

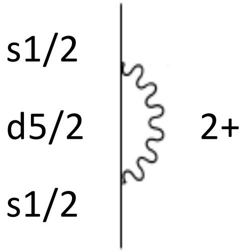
Parity inversion in N=7 isotones is not reproduced by spherical mean field obtained from non relativistic energy density functionals.

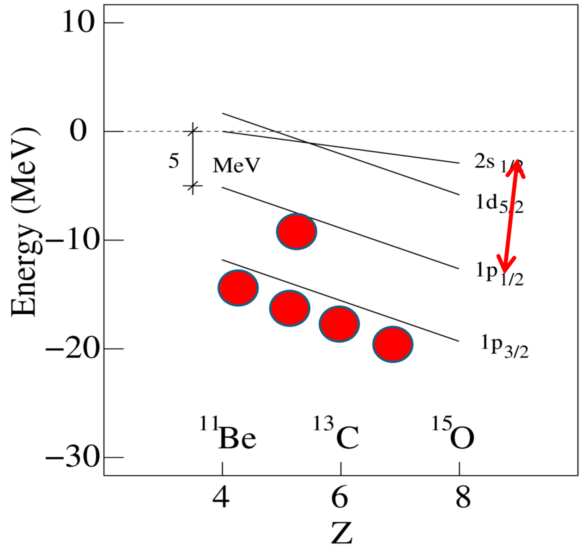


Typical spherical mean-field results with Skyrme forces (Sagawa,Brown,Esbensen PLB 309(93)1)

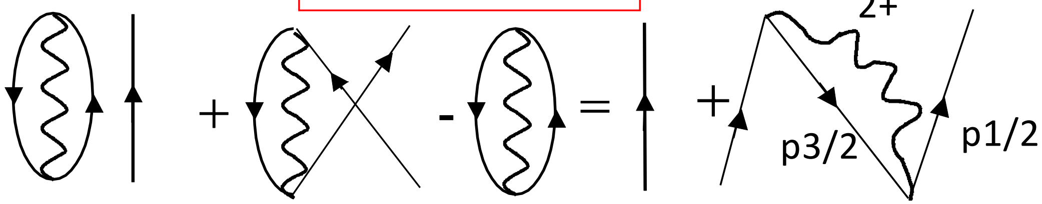
^{11}Be

Eshift = - 2.5 MeV





Eshift = + 2.5 MeV



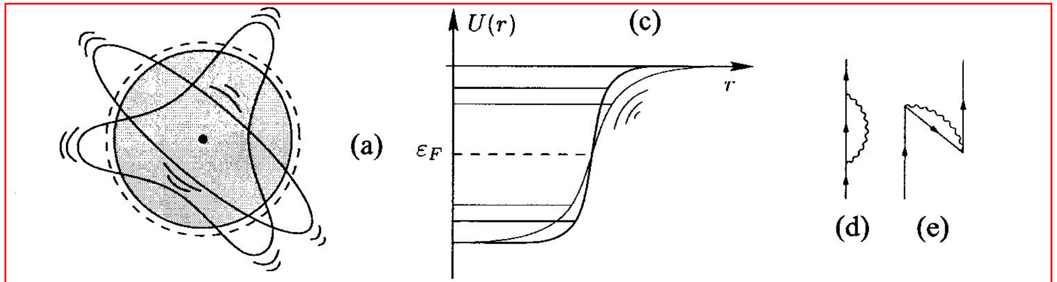
Self-energy

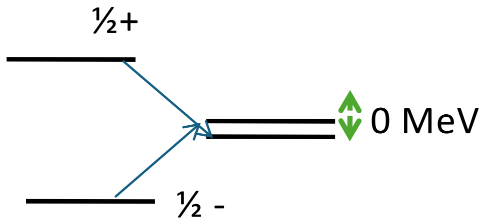
+

Pauli blocking of core ground state correlations

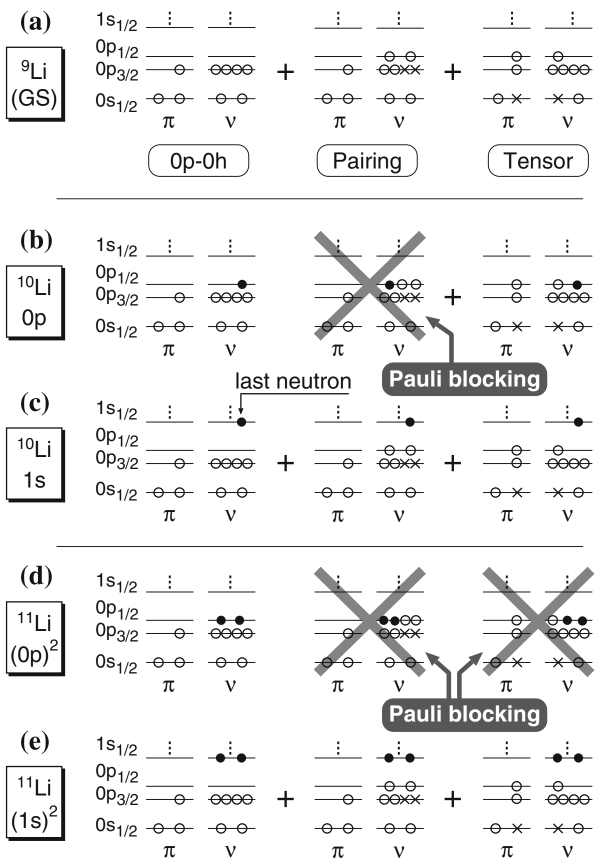
↓

Level inversion





Comparison with the model by Ikeda, Myo et al.



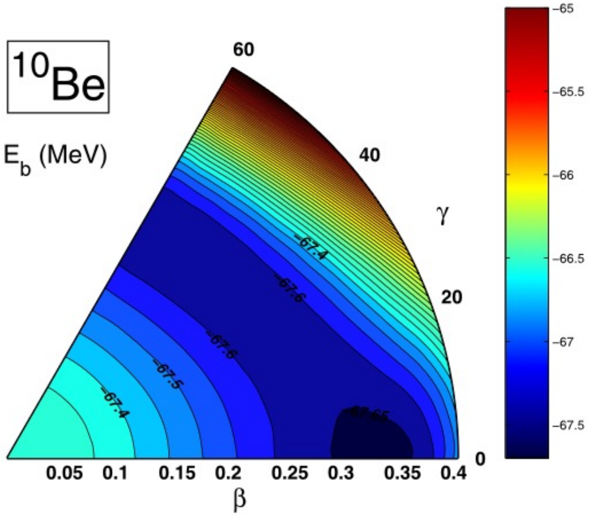
The $p_{1/2}$ orbit is pushed up by pairing correlations and tensor force. Only 3/2- configurations are included: coupling to core vibrations (1/2-) is not considered. Binding energy is given as input. 50%(s^2)-50%(p^2) wavefunction is obtained

K. Ikeda et al,
Lect. Notes in Physics 818 (2010)

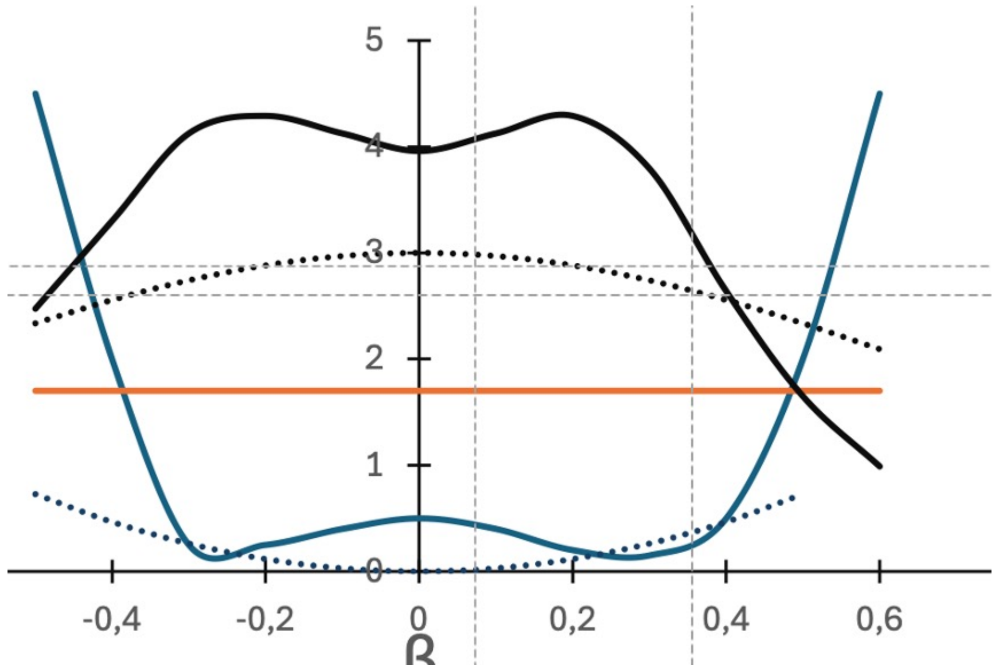
The core: spherical or deformed?
Important role of fluctuations expected in light nuclei
We propose a dynamical description based on Nuclear Field Theory

N. Vinh Mau, Nucl. Phys. A 592 (1995) 43
G.F. Esbensen and H. Sagawa, Phys. Rev C 51 (1995)1274
P. Descouvemont, Nucl. Phys. A 626 (1997) 647
F.M. Nunes and I. Thompson, Nucl. Phys. A 703 (2002) 593
I. Hamamoto and S. Shimoura, J. Phys. G 34 (2007) 2715
G. Blanchon et al., Phys Rev. C 82 (2010) 034313
T. Myo et al, PRC 86 (2012) 024318

We assume a rather shallow energy surface, in which the microscopic collective wavefunction has a reasonable overlap with a wavefunction centered at β =0 , with a value of $<\beta^2>$ derived from the experimental B(E2), according to the collective model .

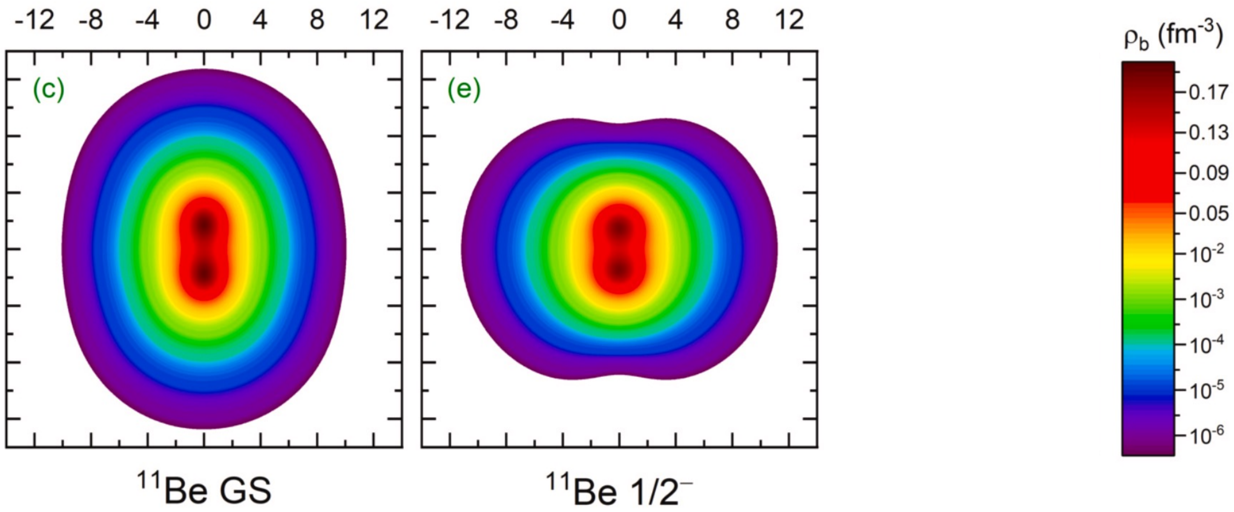


Y. Zhang, H. Sagawa et al.,
PTP 120 (2008) 129

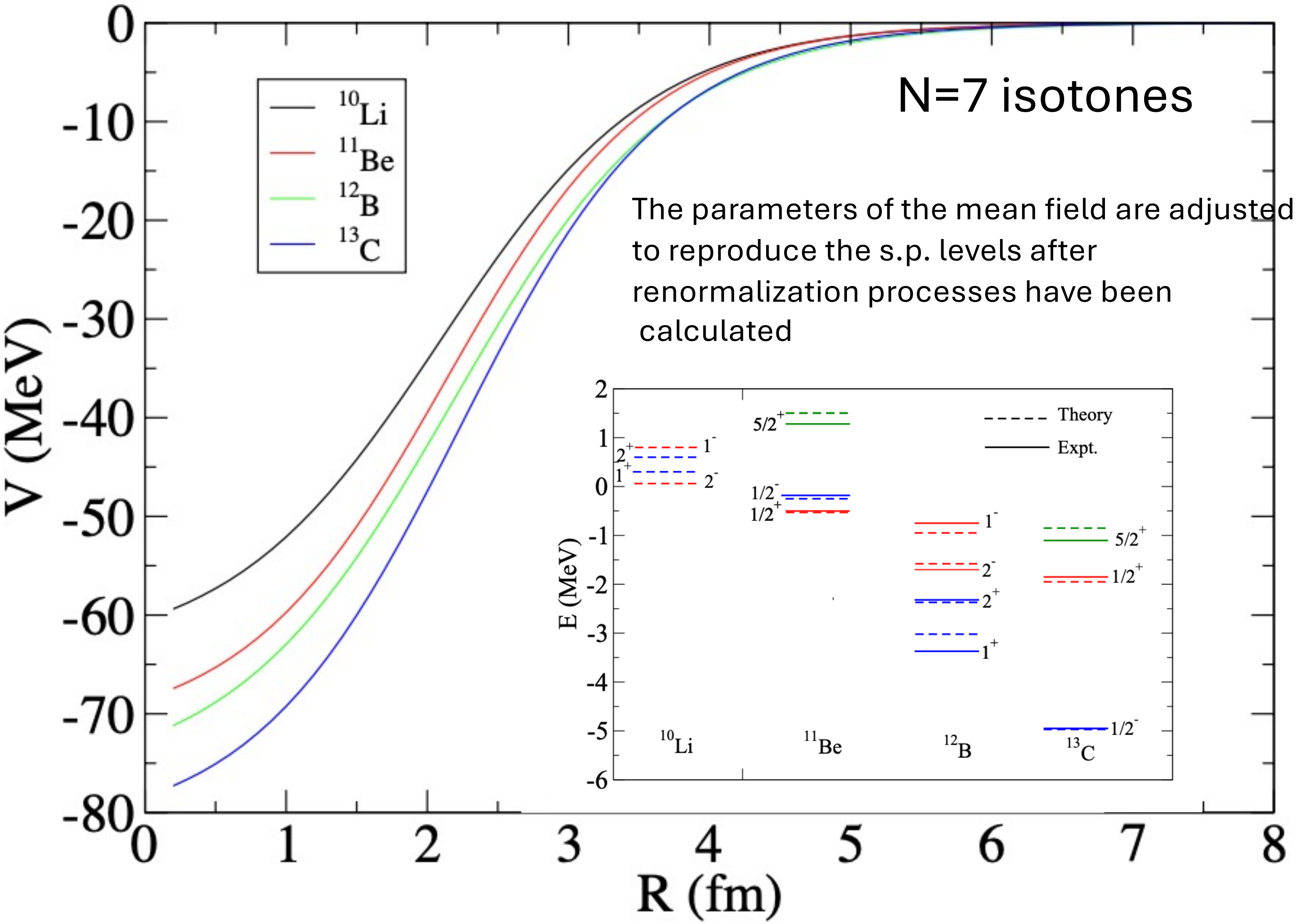


Dynamical picture:
Quadrupole fluctuations in the excited state are blocked by Pauli principle

Static picture:
The excited state is associated with a much smaller deformation



J. Geng et al., PLB 858 (2024) 139036

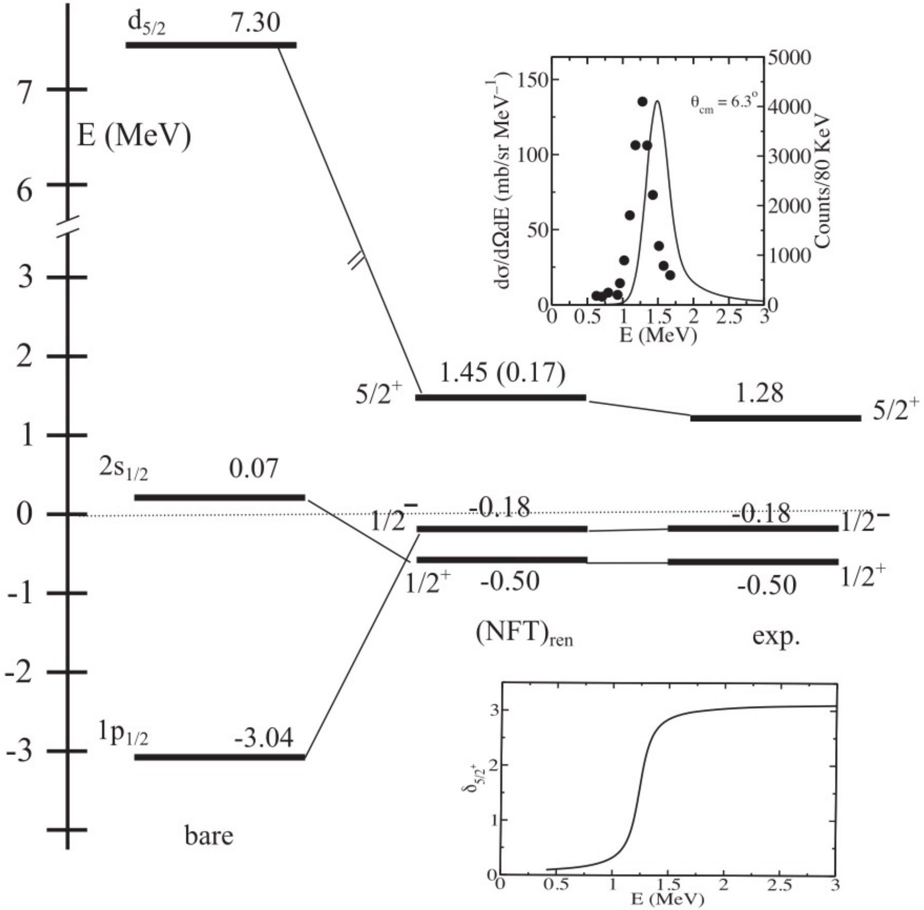


F. Barranco et al.,
PRL 119 (2017) 082501
PRC 101 (2020) 031305(R)

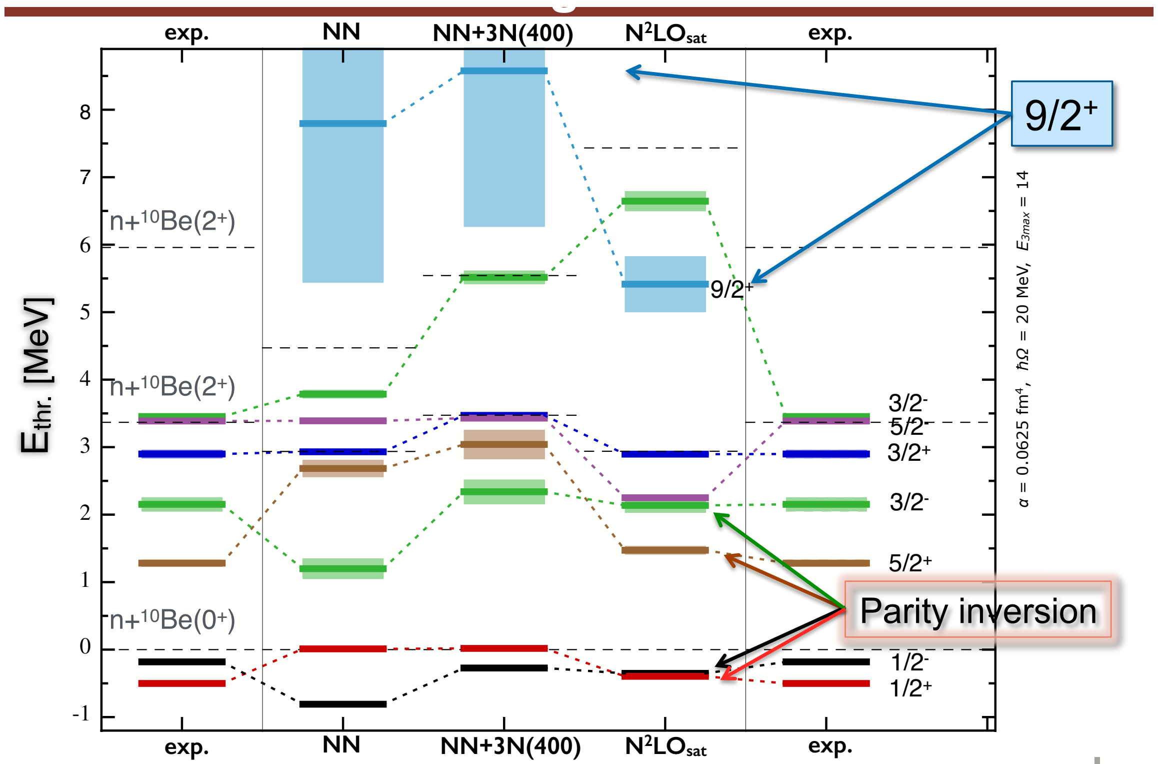
The description of the experimental results from complementary approaches can be of great interest

^{11}Be

PVC

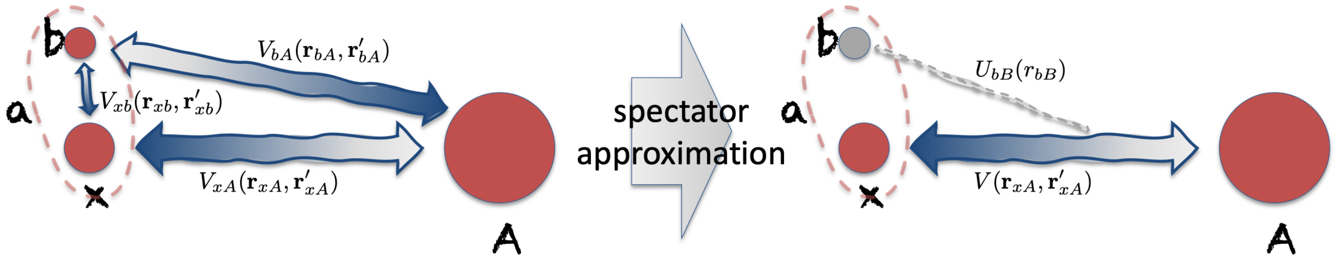


Ab Initio



A. Calci, P. Navratil, R. Roth, J. Dohet-Eraly, S. Quaglioni, G. Hupin, PRL 117, 242501 (2016)

Transfer reaction populating states of the continuum



Green Function Transfer: allows to obtain the cross section as a function of the energy of the “spectator” particle b using the x-A microscopic self-energy

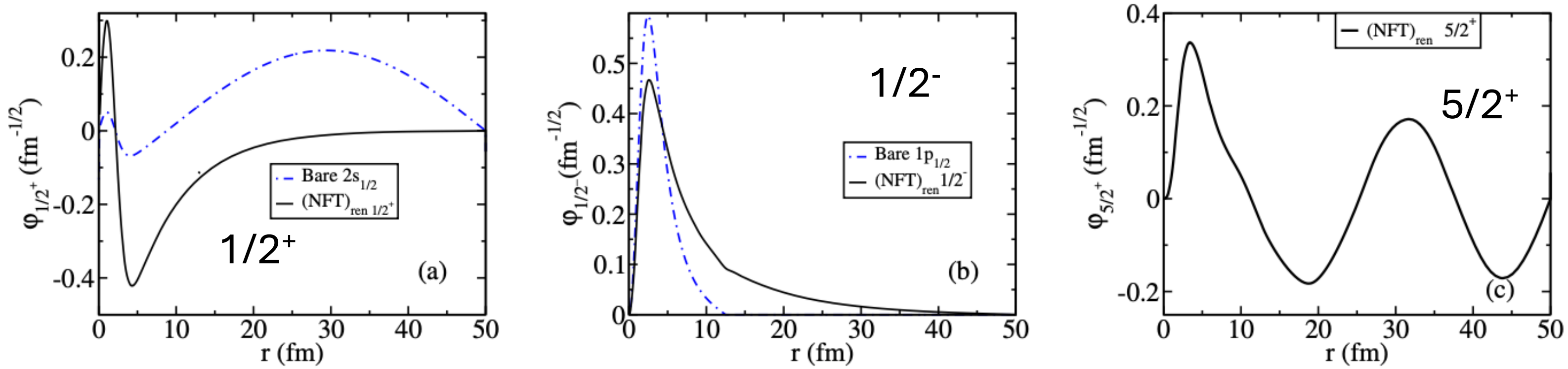
$$\frac{d\sigma_R^J(E, E_b)}{dE_b} = \frac{2\mu}{\hbar^2 k_x} \langle \psi_0^J | \mathrm{Im}\mathcal{V}(E - E_b) | \psi_0^J \rangle \rho(E_b),$$

G. Potel, F. M. Nunes, and I. J. Thompson, Phys. Rev. C 92, 034611 (2015)
G. Potel et al, EPJ A 53 (2017) 178

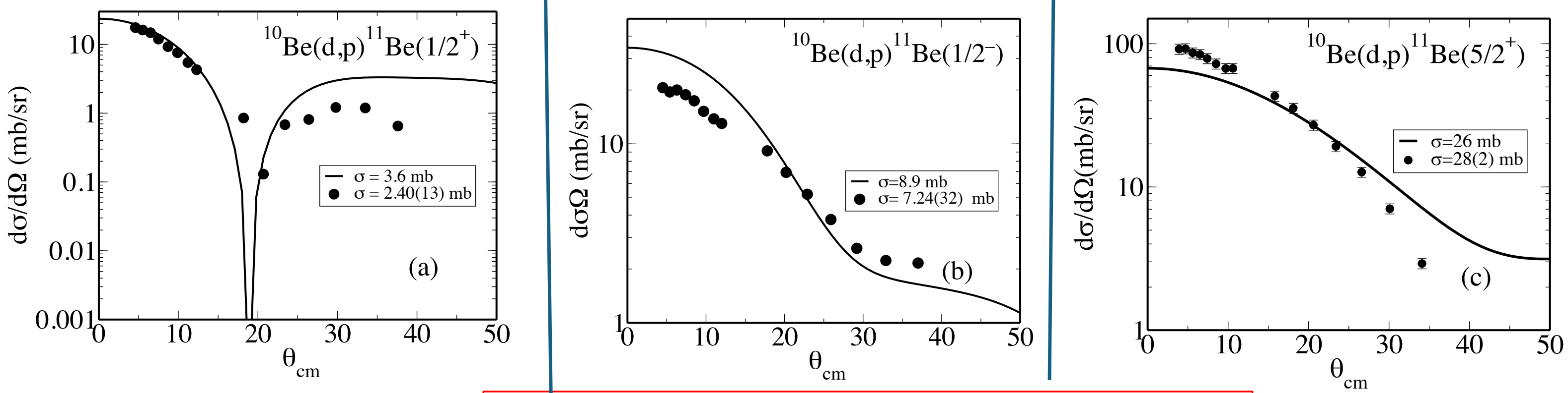
$^{10}Be(d,p)^{11}Be$ at E_d = 21.4 MeV

Test of the single-particle component of the many-body wavefunction

Form factors



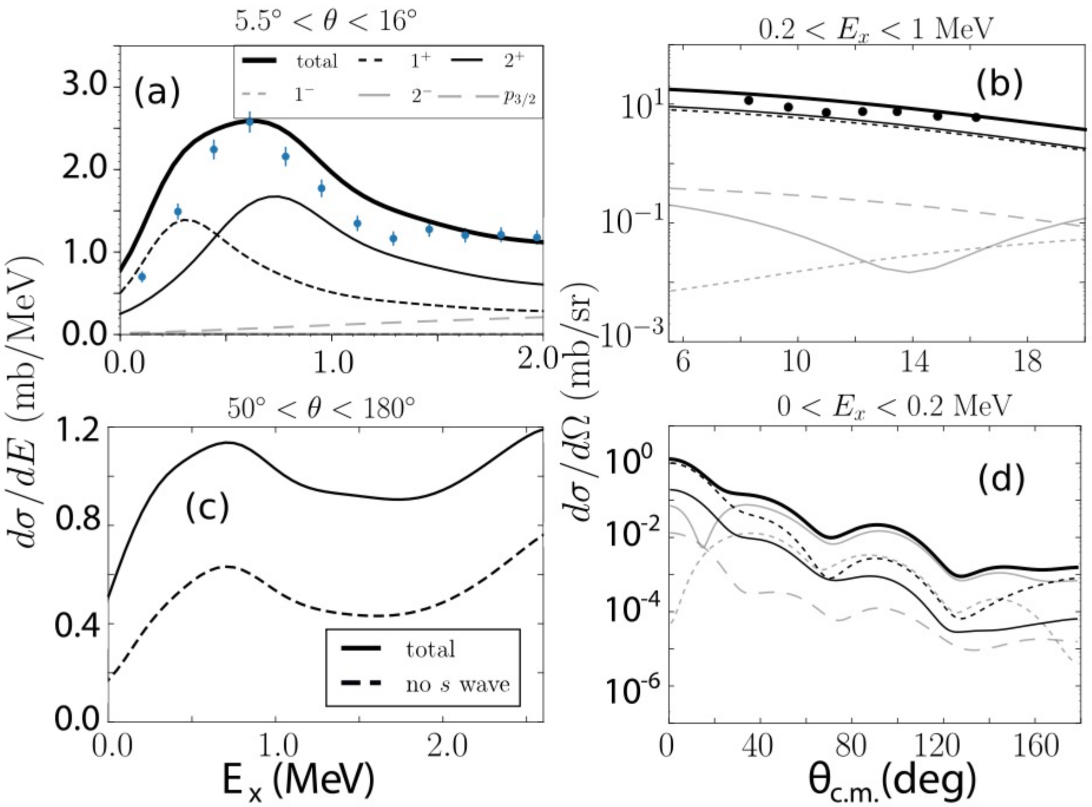
Cross sections

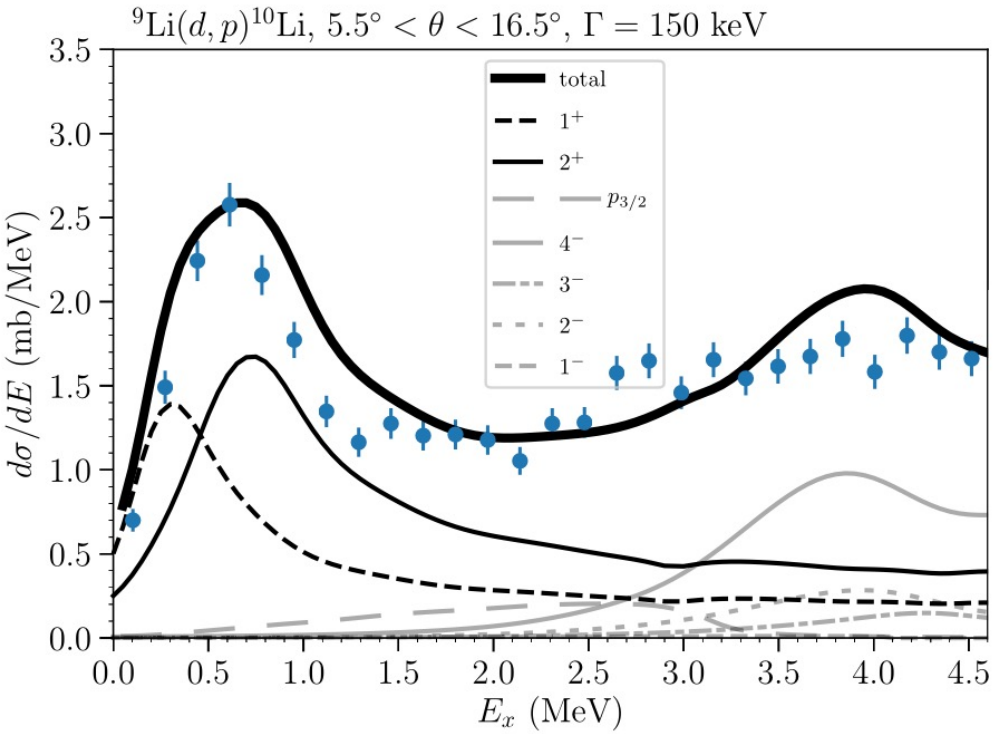


K.T. Schmitt et al., PRC88 (2012) 064612

^{10}Li and the ^{9}Li(d,p)^{10}Li reaction

F. Barranco et al,
Phys. Rev. C 101 (2020) 031305(R)



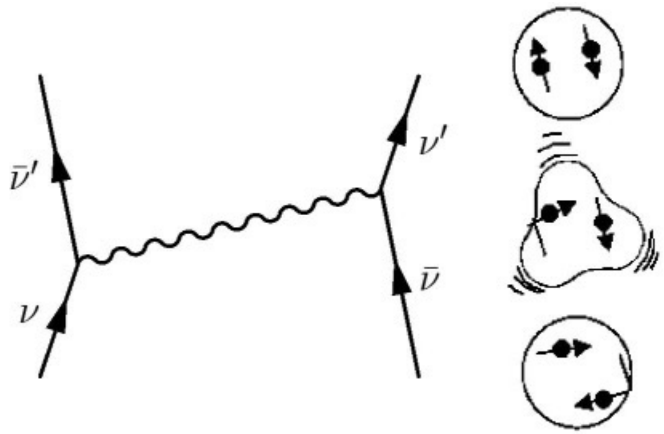


^{11}Li structure

Two new elements in ^{11}Li:

a) The existence of a collective low-lying
1- vibration

b) The pairing interaction receives an important
contribution from the interaction induced
by the exchange of collective vibrations,
including the 1-

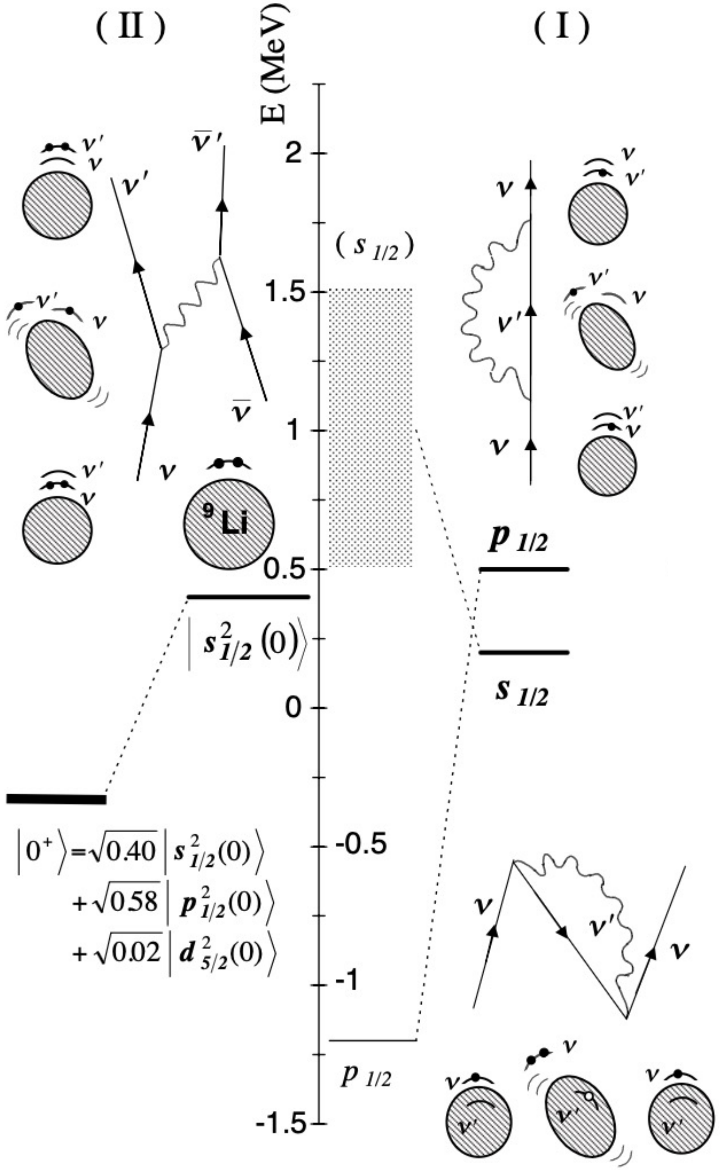


1^- wavefunction /low-lying strength

	$1p_{1/2}^{-1}2s_{1/2}$	$1p_{1/2}^{-1}3s_{1/2}$	$1p_{1/2}^{-1}4s_{1/2}$	$1p_{1/2}^{-1}1d_{3/2}$	$1p_{3/2}^{-1}5d_{5/2}$	$1p_{3/2}^{-1}6d_{5/2}$	$1p_{3/2}^{-1}7d_{5/2}$
X_{ph}	0.847	−0.335	0.244	0.165	0.197	0.201	0.157
Y_{ph}	0.088	0.060	0.088	0.008	0.165	0.173	0.138

Valence transitions

Transitions involving core states



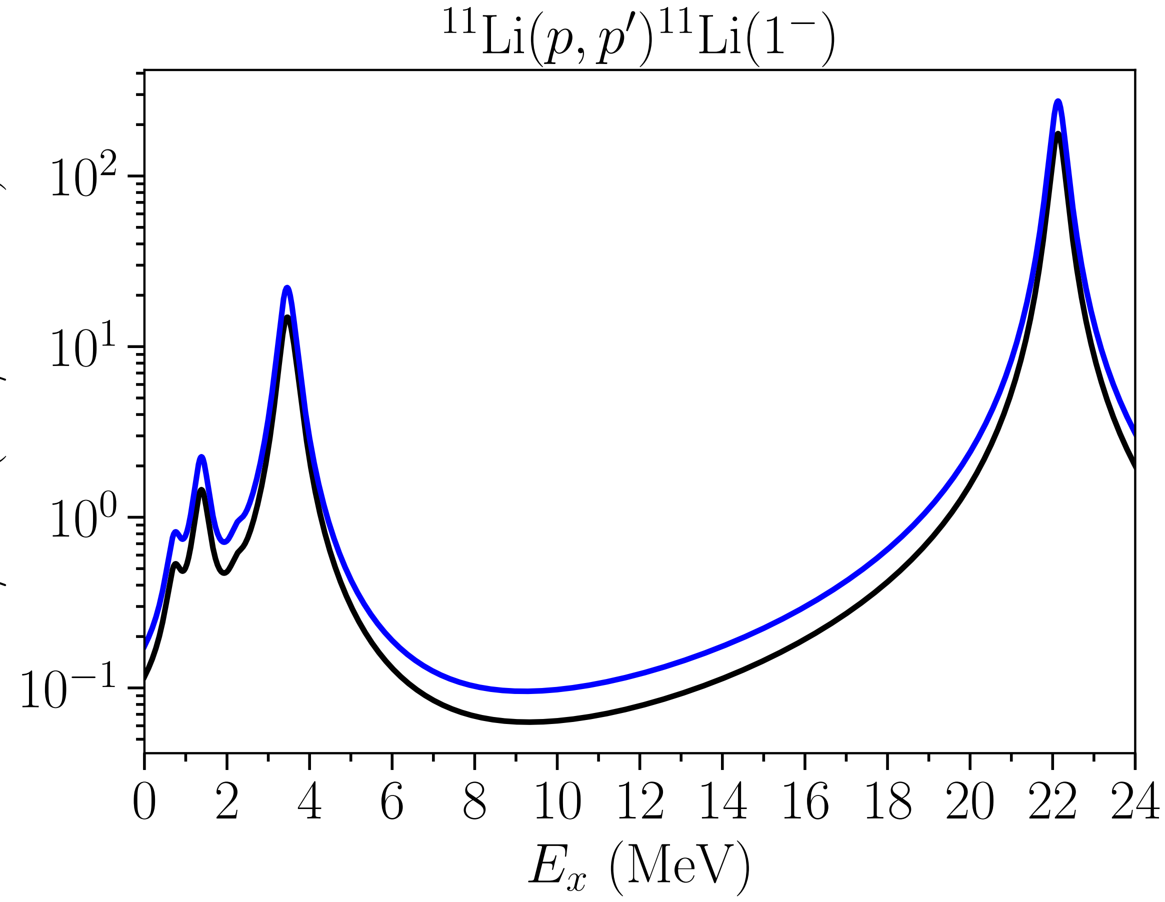
11Li correlated wave function

$$|\tilde{0}\rangle = |0\rangle + 0.7|(ps)_{1^-} \otimes 1^-;0\rangle + 0.1|(sd)_{2^+} \otimes 2^+;0\rangle$$

$$|0\rangle = 0.45|s^2_{1/2}(0)\rangle + 0.55|p^2_{1/2}(0)\rangle + 0.04|d^2_{5/2}(0)\rangle$$

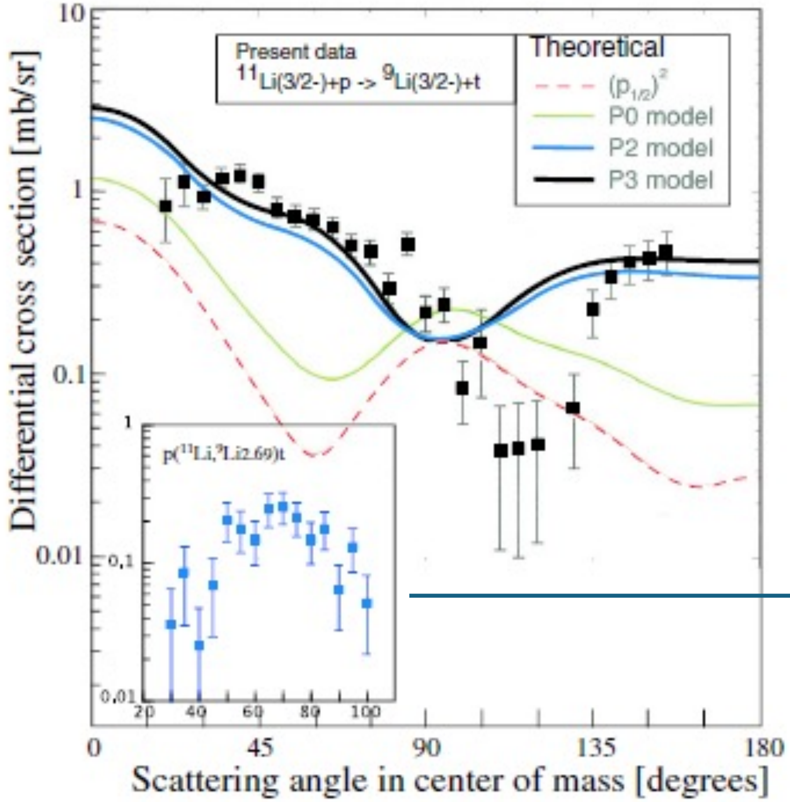
^{11}Li
F. Barranco et al. EPJ A11 (2001) 385

Calculation of the inelastic scattering from the soft mode to GDR:
Analysis of the corresponding experiment underway (Y. Ayyad et al.)



Probing ^{11}Li halo-neutrons correlations via (p,t) reaction

I. Tanihata et al, PRL 100 (2008) 192502

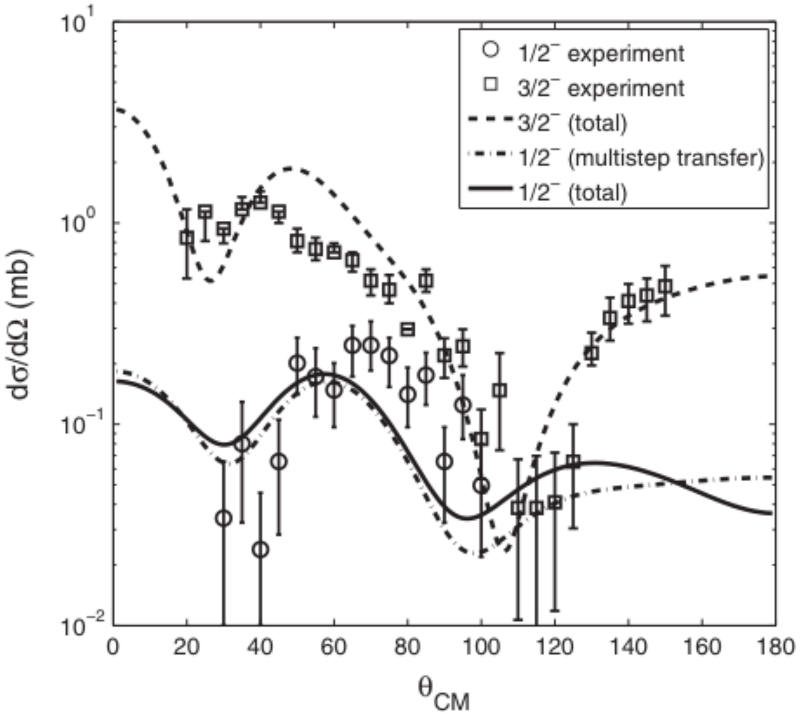


Unique information:
angular distribution associated with the excited state of the ^{9}Li core

TABLE I. Optical potential parameters used for the present calculations.

	V MeV	r_V fm	a_V fm	W MeV	W_D MeV	r_W fm	a_W fm	V_{so} MeV	r_{so} fm	a_{so} fm
$p + {}^{11}$Li [10]	54.06	1.17	0.75	2.37	16.87	1.32	0.82	6.2	1.01	0.75
$d + {}^{10}$Li [11]	85.8	1.17	0.76	1.117	11.863	1.325	0.731	0		
$t + {}^{9}$Li [12]	1.42	1.16	0.78	28.2	0	1.88	0.61	0		

Calculation of absolute two-nucleon transfer cross section by finite-range DWBA calculation



$$|\tilde{0}\rangle = |0\rangle + 0.7|(ps)_{1^-} \otimes 1^-;0\rangle + 0.1|(sd)_{2^+} \otimes 2^+;0\rangle$$

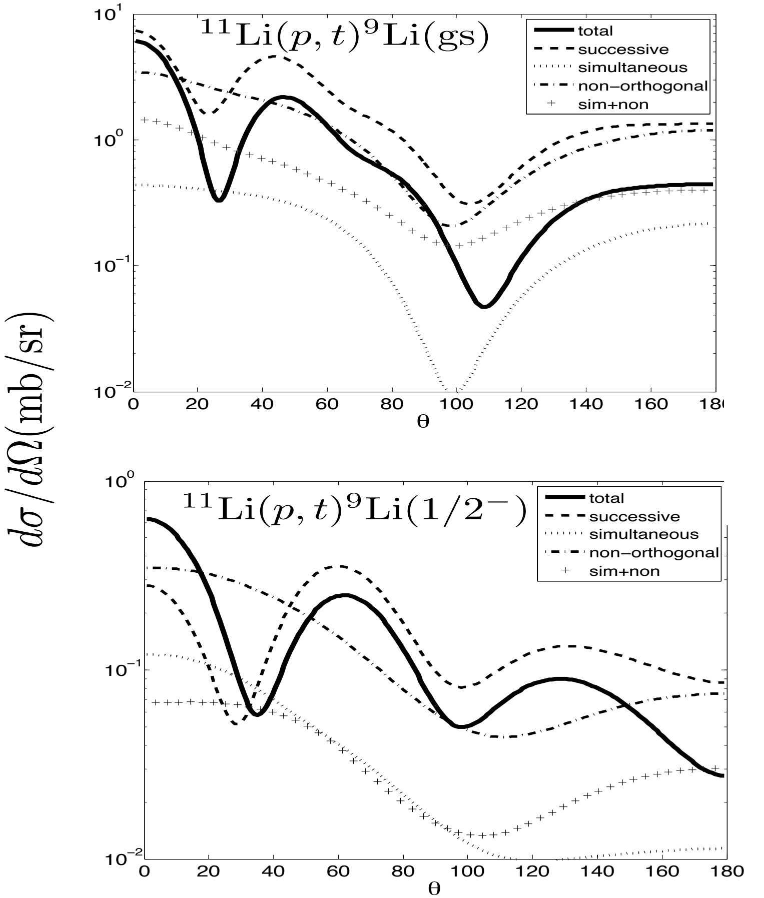
Essential component to populate the excited state of the core

	$\sigma(^{11}\text{Li(gs)} \to {}^{9}\text{Li (i)})$ (mb)		
i	ΔL	Theory	Experiment
gs ($3/2^-$)	0	6.1	5.7 ± 0.9
2.69 MeV ($1/2^-$)	2	0.5	1.0 ± 0.36

This calculation should be repeated by other groups!

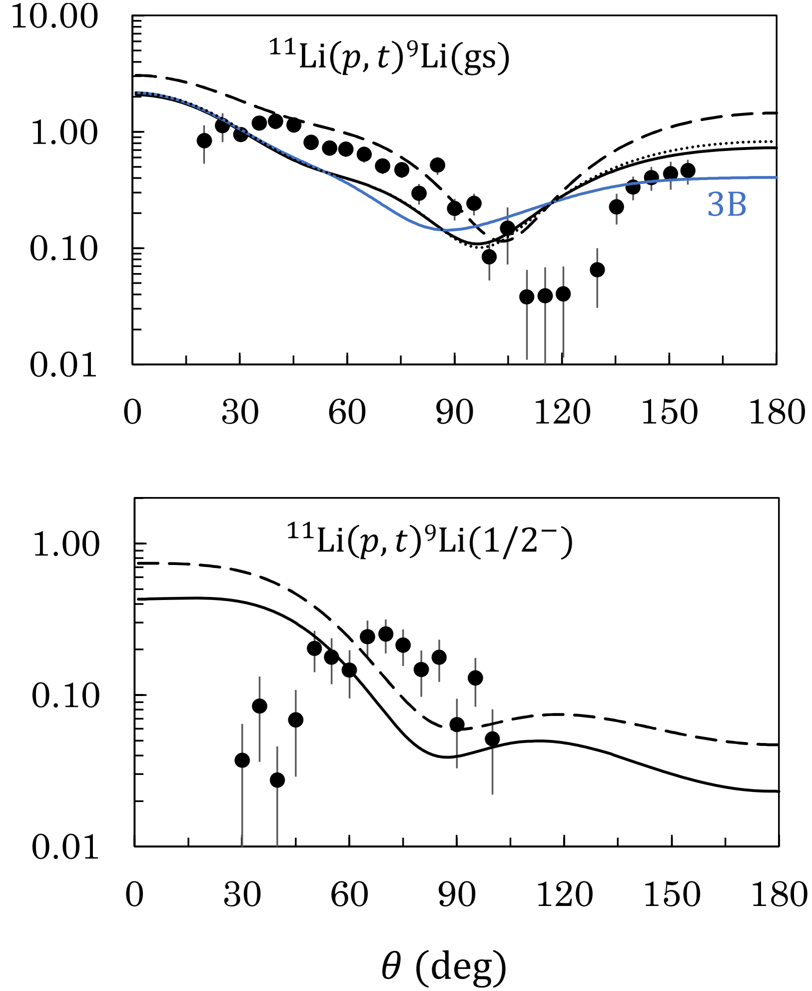
G. Potel et al., PRL 105 (2010) 172502

11Li w.f.: 50% 1- admixture
1% 2+ admixture

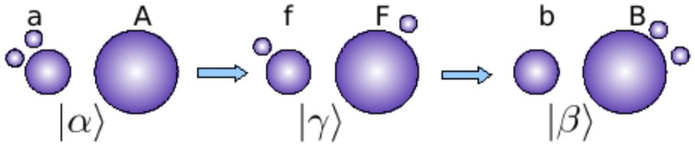


11Li w.f. : 10% 2+ admixture

P. Descouvemont, PLB **862** (2025) 139356



Second order DWBA



Potel *et al.*, PRL **107** 092501 (2011)
Potel *et al.*, PRL **105** 172502 (2010)

$$\sigma \sim |T_{2NT}|^2 = \left| \sum_{j_f, j_i} B_{j_f} B_{j_i} \left(T^{(1)} + T^{(2)}_{succ} + T^{(2)}_{NO} \right) \right|^2$$

→ coherent sum of many terms

structure input:
coefficients of the initial and final wavefunctions (e.g., from a BCS calculation)

Successive transfer

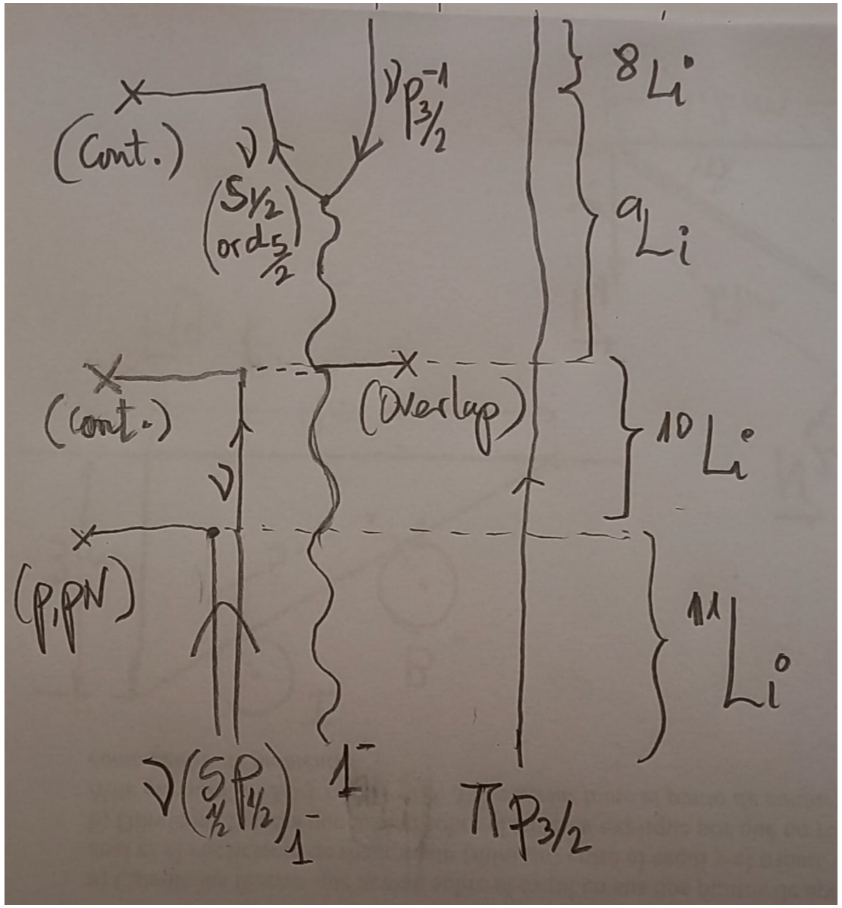
$$T^{(2)}_{succ}(j_i, j_f) = 2 \sum_{K,M} \sum_{\substack{\sigma_1 \sigma_2 \\ \sigma_1' \sigma_2'}} \int d\mathbf{r}_{fF} d\mathbf{r}_{b1} d\mathbf{r}_{A2} [\Psi^{j_f}(\mathbf{r}_{A1}, \sigma_1) \Psi^{j_f}(\mathbf{r}_{A2}, \sigma_2)]^{0*}_0$$
$$\times \chi^{(-)*}_{bB}(\mathbf{r}_{bB}) v(\mathbf{r}_{b1}) [\Psi^{j_f}(\mathbf{r}_{A2}, \sigma_2) \Psi^{j_i}(\mathbf{r}_{b1}, \sigma_1)]^K_M$$
$$\times \int d\mathbf{r}'_{fF} d\mathbf{r}'_{b1} d\mathbf{r}'_{A2} G(\mathbf{r}_{fF}, \mathbf{r}'_{fF}) [\Psi^{j_f}(\mathbf{r}'_{A2}, \sigma'_2) \Psi^{j_i}(\mathbf{r}'_{b1}, \sigma'_1)]^K_M$$
$$\times \frac{2\mu_{fF}}{\hbar^2} v(\mathbf{r}'_{f2}) [\Psi^{j_i}(\mathbf{r}'_{b2}, \sigma'_2) \Psi^{j_i}(\mathbf{r}'_{b1}, \sigma'_1)]^0_0 \chi^{(+)}_{aA}(\mathbf{r}'_{aA})$$

How to probe the dipole component?

$$|\tilde{0}\rangle = |0\rangle + 0.7|(ps)_{1^-} \otimes 1^-;0\rangle + 0.1|(sd)_{2^+} \otimes 2^+;0\rangle$$

A bold guess:

The (p,pN) reaction should leave the residual core ^{9}Li in an excited state ($1^- \otimes \pi p_{3/2}$) that could decay to ^{8}Li by neutron emission



Conclusions

A description based on the PVC including some key fenomenological parameters can give a rather accurate description of several structure and reaction data in nuclei with A $\sim$ 10-15 with one and two valence nucleons.

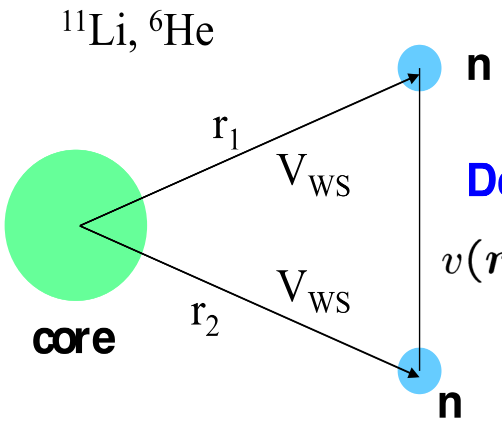
This helps to develop a more unified treatment of structure and reactions and to identify some of the main many-body mechanisms involving the dynamics of the core.

A quantitative check of the predicted core admixtures requires challenging experiments.

THE INERT CORE MODEL

Three-body model with density-dependent delta force

G.F. Bertsch and H. Esbensen,
Ann. of Phys. 209('91)327
H. Esbensen, G.F. Bertsch, K. Hencken,
Phys. Rev. C56('99)3054



Density-dependent delta-force

$$v(\boldsymbol{r}_1, \boldsymbol{r}_2) = v_0(1 + \alpha\rho(r)) \times \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2)$$

$$H = \frac{\boldsymbol{p}_1^2}{2m} + \frac{\boldsymbol{p}_2^2}{2m} + V_{nC}(r_1) + V_{nC}(r_2) + V_{nn} + \frac{(\boldsymbol{p}_1 + \boldsymbol{p}_2)^2}{2A_c m}$$

Comparison with the model by Bertsch and Esbensen

OUR MODEL

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Single-particle potential

Standard Bohr-Mottelson

Depth adjusted to experimental $p_{1/2}$ single particle energy

2-body interaction

Bare Argonne interaction+ particle-vibration coupling with phenomenological parameters (low-lying vibrations)

Strength fitted to S_{2n} in ^{12}Be

$$v_{\rm eff}(\mathbf{r}_1,\mathbf{r}_2)=\delta(\mathbf{r}_1-\mathbf{r}_2)\left(v_0+v_\rho\left(\frac{\rho_c((\mathbf{r}_1+\mathbf{r}_2)/2)}{\rho_0}\right)^P\right).$$

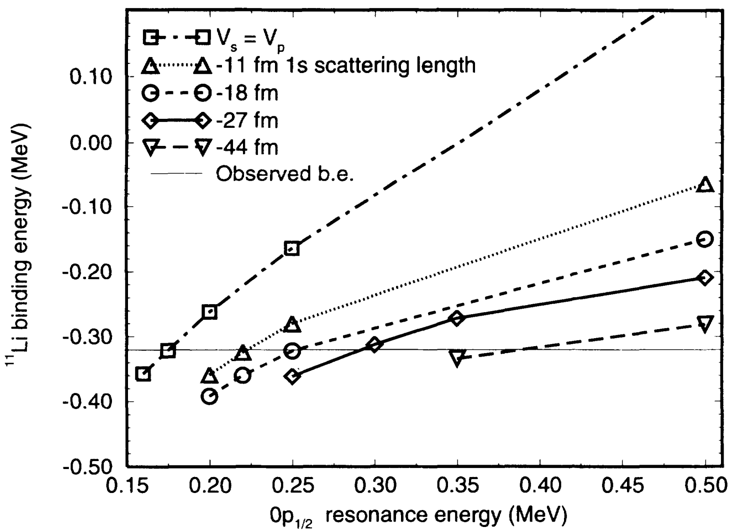
Results

Good reproduction of binding energies in ^{12}Be and ^{11}Li
50% $(s_{1/2})^2$

Good reproduction of binding energy
Low $(s_{1/2})^2$ admixture unless two different s.p. potentials are used

We obtain a 2n separation energy S_{2n} = 0.33 MeV, close to the exp. value.
Neglecting the Argonne n-n interaction decreases this value by less than 100 keV. The bare interaction is not efficient in coupling low-angular momentum extended wavefunctions.
Most of the binding comes from the coupling to vibrations.

It is difficult to compare with other calculations, which start from single-particle states fitted to the exp. values (different potentials for s- and p-waves) and use different n-core potentials and/or density dependent interactions.



I.J. Thompson and M.V. Zhukov, PRC 49 (1994) 1904

Charge radius : an indication of core polarization

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The charge radius in Be isotopes has been precisely measured. In particular, $R_c(^{10}$ Be) = 2.361 fm and $R_c(^{11}$ Be) = 2.466 fm,

In the independent particle limit, adding a neutron to ^{11}Be in an orbital of radius $r_n \approx$ 3 fm, should lead to a very small modification of the charge radius of ^{10}Be due to the recoil of the center of mass:

$$< r^2 >_{11Be} = < r^2 >_{10} Be + (r_n/11)^2 \approx 2.361^2 + 0.07 fm^2 \qquad (13)$$

The coupling to phonons brings an extra correction which is proportional to the admixture α of the 2^+ phonon in the ground state. One obtains

$$< r^2 >_{11Be} = < r^2 >_{10Be} + (3/11)^2 + \alpha\beta^2 < r^2 >_{10Be} \qquad (14)$$

$$= 2.361^2 + 0.07 + 0.2 * 0.7^2 * 2.361^2 = 2.361^2 + 0.07 + 0.546 = 6.19 = (2.49)^2 \text{fm}^2, \qquad (15)$$

where we have used the value $\beta_2 = 0.7$ for the deformation of the ^{10}Be, and which is in good agreement wth the experimental value. Actually, the charge radius is probably the best test of the admixture factor.