Microscopic study of halo nuclei through (p,t) reactions

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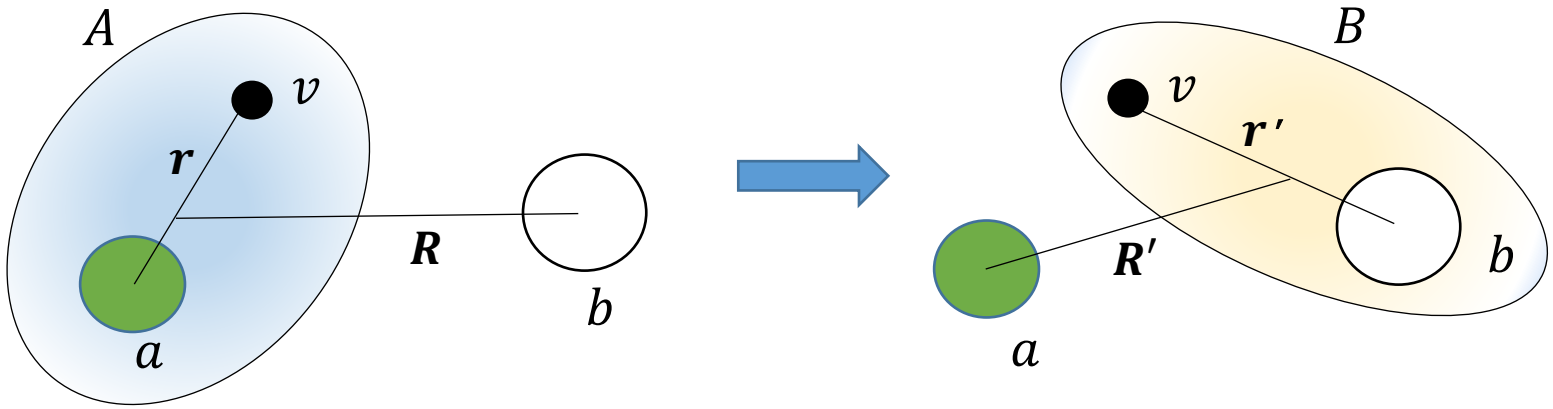
1. Introduction
2. Transfer cross sections
3. The Resonating Group Method (RGM)
   Overlap integrals for:
   - One-nucleon transfer: (d,p), (d,n): ***simpler***
   - Two-nucleon transfer: (p,t)
4. Application to ^{6}He(p,t)α
5. Application to ^{11}Li(p,t)^{9}Li (+ core excitations)
6. Conclusion

Ref.: P.D., Phys. Lett. B 862, 139356 (2025).

Transfer reactions are important in nuclear physics:

- (d,p) reactions used for a long time: provide information on the residual nucleus (spin, parity)
  A(d,p)B with B=A+n: the cross section is very sensitive to L between A and n



- In nuclear astrophysics:
  - Direct measurements, examples: $^{13}C(\alpha,n)^{16}O$, $^{15}N(p,\alpha)^{16}O$, $^{18}F(p,\alpha)^{15}O$, etc.
  - indirect tool to probe the nucleus near the threshold

    Example: $^{12}C(^{7}Li,t)^{16}O$ provides spectroscopic properties of ^{16}O levels

Two main topics

1. **Microscopic models**
   - Reactions (elastic, capture, transfer, etc.) at low energies
   - Spectroscopy of light nuclei

2. **Transfer reactions**
   - Important in nuclear astrophysics
   - Indirect method for spectroscopy
   - DWBA approximation

Can we combine both?

- One-particle transfer:
  Ref.: P.D., Eur. J. Phys. A58 (2022) 193
  Ex: ^{16}C(d,p)^{17}C

- **Two-particle transfer**
  More complicated
  Ex: ^{6}*He(p,t)*^{4}*He,* ^{11}*Li(p,t)*^{9}*Li*

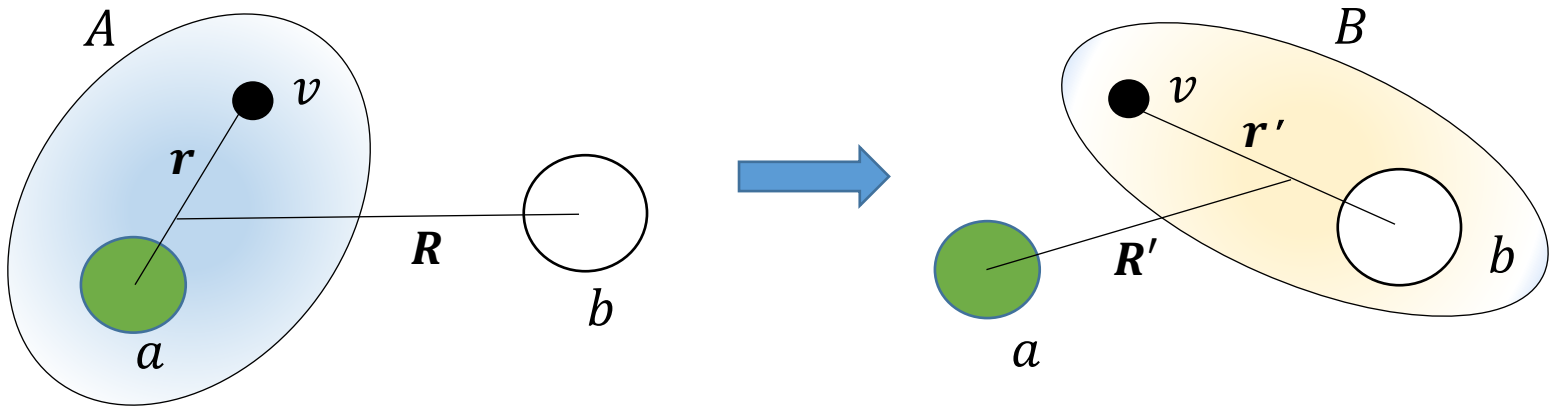
Common input: overlap integrals

- One neutron: ^{17}C=^{16}C+n (one coordinate)
- Two neutrons: ^{6}He=^{4}He+n+n
  ^{11}Li=^{9}Li+n+n
  (two coordinates)

2. Transfer cross sections

Transfer reactions are **difficult**:

- Two-body projectile : at least a 3-body model (+coupled-channel problem)



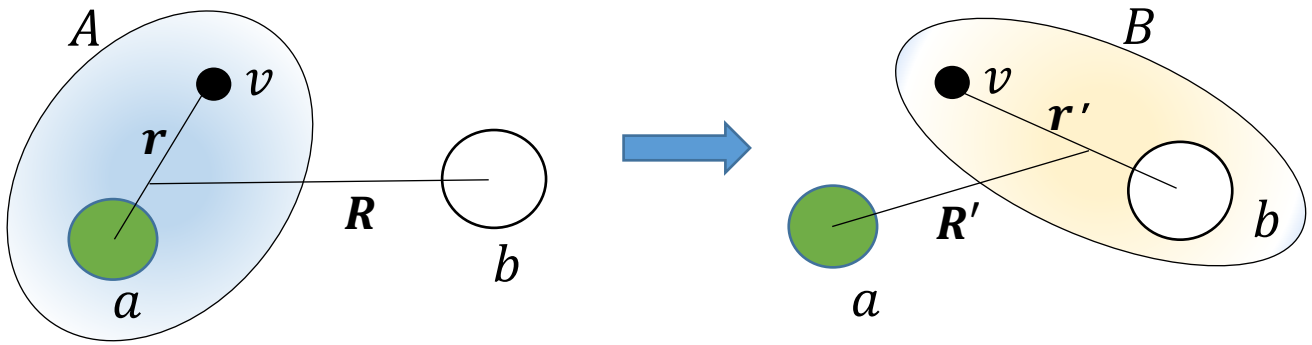
- Interactions are different in the entrance and exit channels
- 4 coordinates (r,r',R,R') but only 2 are independent → r and r' expressed as a function of R and R'
- Needs many inputs
  - optical potentials for b+A and a+B
  - Bound-state potentials (→overlap integrals) for A=a+v and B=b+v
  - Spectroscopic factors for A and B
- The theoretical formalism is well established

  G. R. Satchler, Direct Nuclear Reactions, Oxford (1983) (+many others).

  Theory not "simple" but does not require long computer times (several public codes)

Transfer cross section

- Initial state $i = A + b$
- Final state $f = a + B$



Integrated cross section

$$\sigma_{if}(E) = \frac{\pi}{k^2} \sum_{J\pi} \frac{2J+1}{(2I_1+1)(2I_2+1)} \left| U_{if}^{J\pi}(E) \right|^2$$

- Definition common to all models
- Differential cross section

  $\frac{d\sigma_{if}}{d\Omega}$: more complicated [also involves the scattering matrices $U_{if}^{J\pi}(E)$]

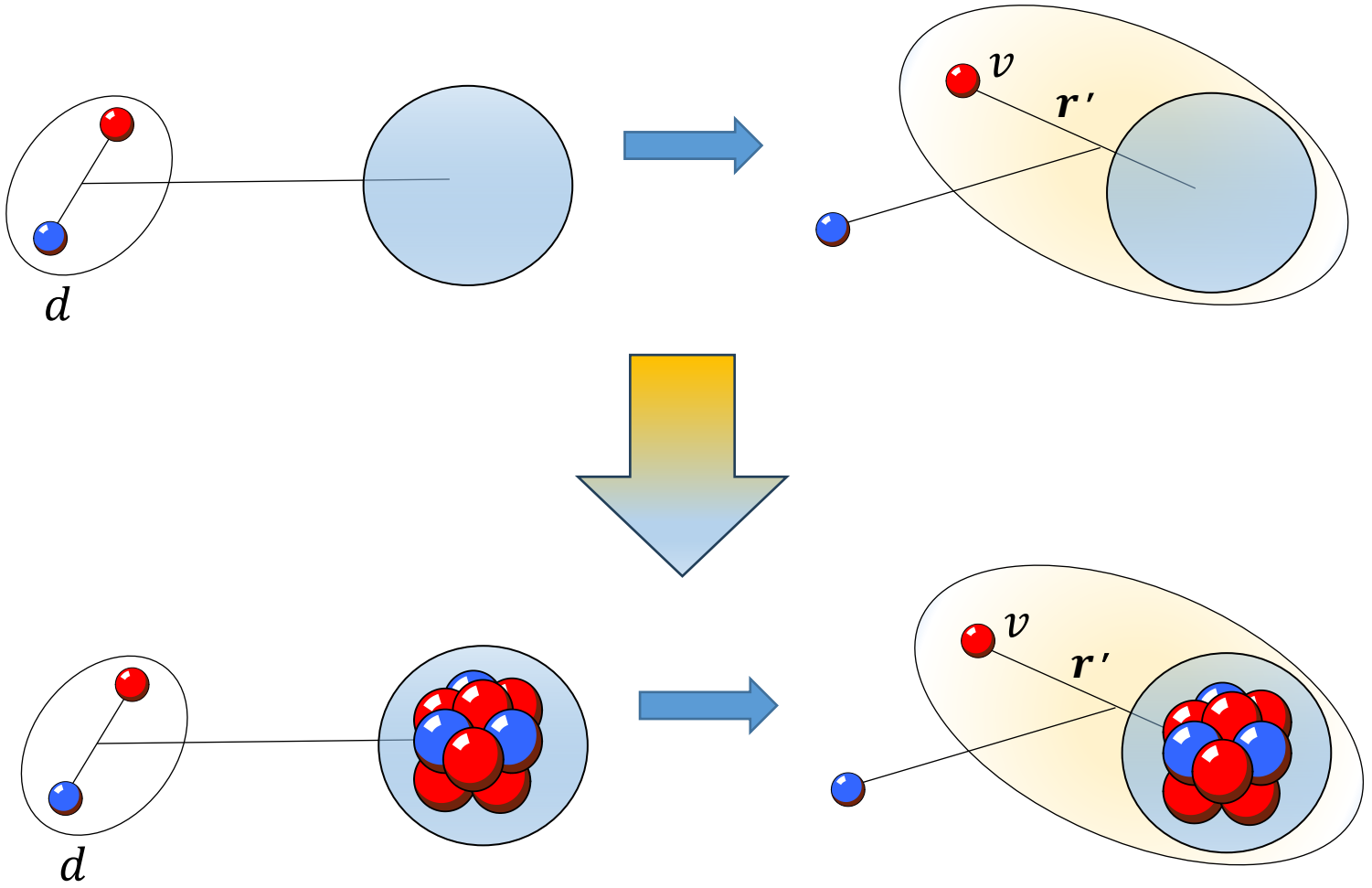
scattering matrices $U_{if}^{J\pi}(E)$ (spin neglected)

$$U^{J\pi} = \begin{pmatrix} U_{ii} & U_{if} \\ U_{fi} & U_{ff} \end{pmatrix}$$

elastic scattering i→i
transfer i→ f
elastic scattering f→f

➔ The calculation of the scattering matrices is the main issue in scattering problems

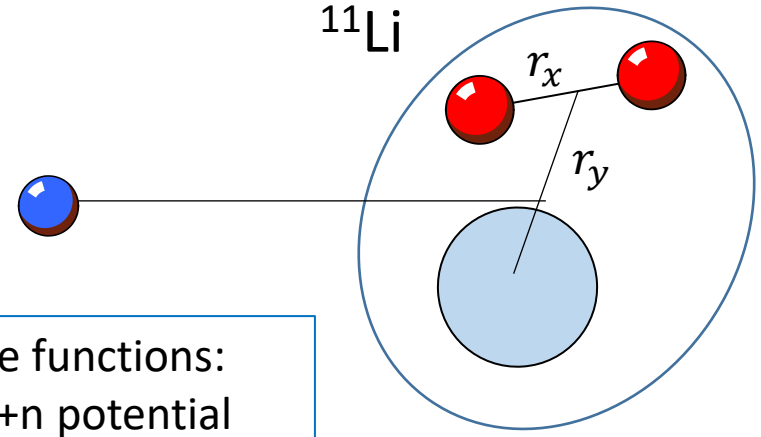
Various approaches
One-particle transfer



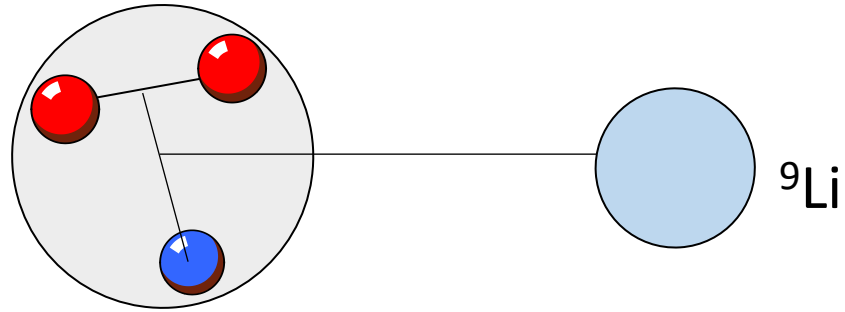
- Standard DWBA method
- The structure of a,b,v is neglected

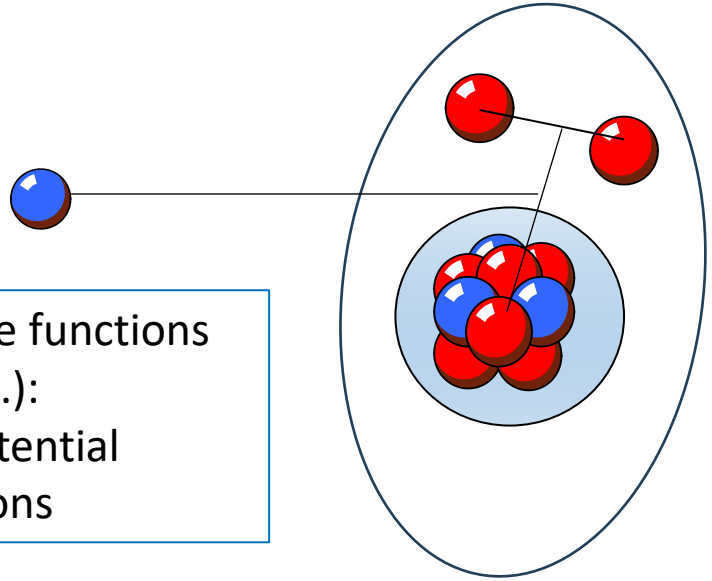
- Microscopic wave functions of the residual nucleus are needed
- RGM method (based on NN interaction)

Two-particle transfer: much more complicated!

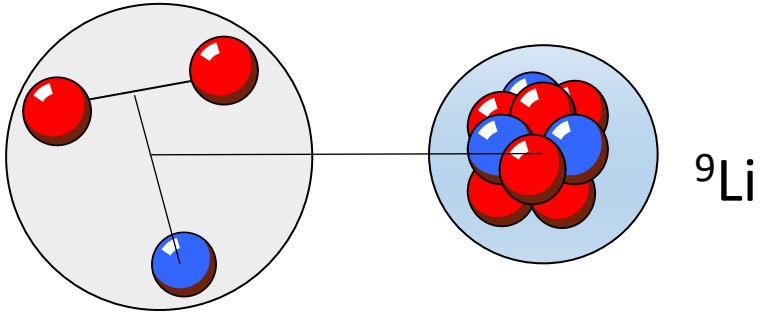


^{9}Li+n+n wave functions:
→ needs ^{9}Li+n potential





11-nucleon wave functions
(+cluster approx.):
→ needs NN potential
→ Core excitations



2. Transfer cross sections

Cross section computed from the scattering matrices (general formula at the **DWBA approximation**)

$$U_{if} = -\frac{i}{\hbar}\iint \chi_i(R) K_{i,f}(R,R') \chi_f(R') dR\, dR'$$

scattering wave functions in the **entrance channel:** optical potential

transfer kernel: computed from the BS wave functions and the potentials

Scattering wave functions in **the exit channel:** optical potential

- One-nucleon transfer: b=1, B=A+1:
  $K_{i,f}(R,R') = \phi_d(r) \times \Delta V(R,r) \times I(r')$
  - with $I(r') =< \phi_A | \phi_{A+1} >$ =overlap integral (reduced width amplitude)
  - ΔV=transition potential (p+n potential in (d,p) reactions, α+t potential in (^{7}Li,t) reactions)

- Two-nucleon transfer: Similar form with $I(r',s') =< \phi_A | \phi_{A+2} >$

→ Main problem: to determine the overlap integrals $I(r)$ or $I(r',s')$

→ Spectroscopic factor $S = \int |I(r)|^2 dr$ or $S = \int |I(r,s)|^2 dr ds$

Standard DWBA: the target A has no structure

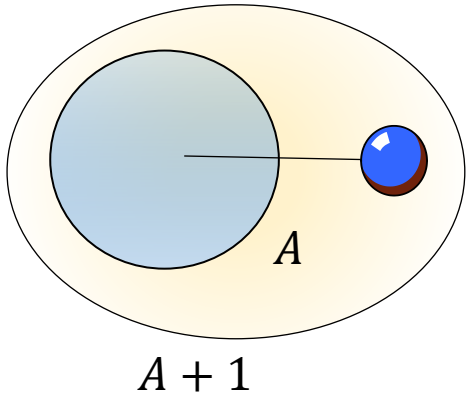
$\phi_{A+1} = \phi_A u(r)$

With $u(s)$ determined from $\left[-\frac{\hbar^2}{2\mu}\left(\frac{d^2}{dr^2} - \frac{l(l+1)}{r^2}\right) + V_f(r)\right] u_f(r) = E_f u_f(r)$, $V_f(r) = A + n$ potential with energy E_f

→ $I(r) =< \phi_{A+1} |\phi_A > = u_f(r)$

→ A spectroscopic factor is needed: $I(r) = \sqrt{S_f}\, u_f(r)$

(additional parameter)



Present method: Resonating Group Method (RGM)

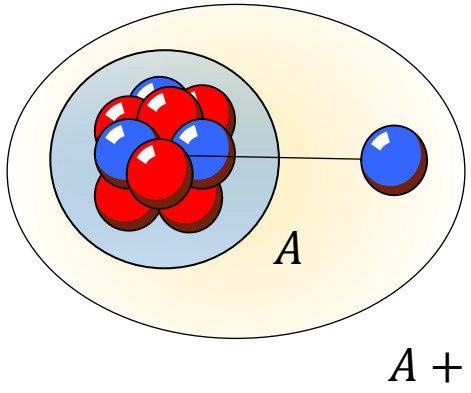
The nucleon structure is included

$\phi_{A+1} = \mathcal{A}\phi_A\phi_n u(r)$

→ $I(r) =< \phi_{A+1} |\phi_A > \neq u(r)$

Spectroscopic factor: $S = \int |I(r)|^2 dr$ is $\neq 1$

=output of the calculation (not fitted)

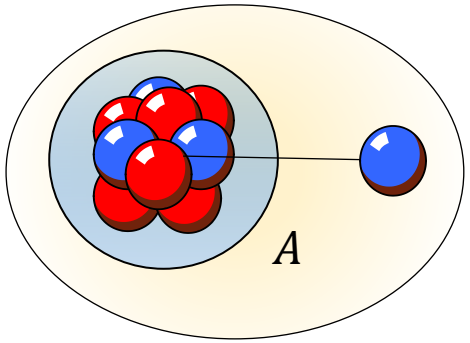


3. The Resonating Group Method (RGM)

Microscopic cluster model

Hamiltonian: $H = \sum_i T_i + \sum_{i<j} V_{ij}$

T_i = kinetic energy of nucleon *i*
V_{ij} = nucleon-nucleon **effective** interaction (Minnesota, Volkov)



$A+1$

Total wave function: $\phi_{A+1} = \mathcal{A}\phi_A\phi_n u(r)$ (=**Resonating Group Method** definition – RGM)

Overlap integral in the RGM

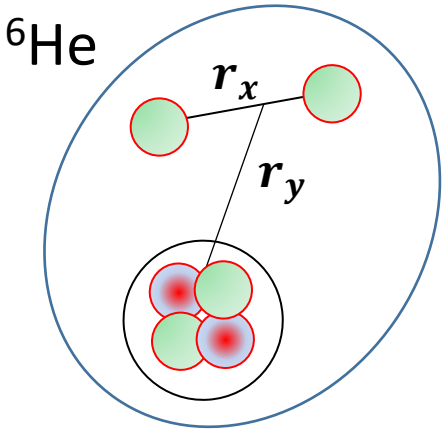
$I(r) =< \phi_A|\phi_{A+1} >$ =integral over the A internal coordinates
difficult owing to the antisymmetrization

$\boldsymbol{I(r) = \mathcal{N}u(r) = \int \mathcal{N}(r,r')u(r')dr'}$ with $\mathcal{N}(r,r')$=overlap kernel
Technique proposed by K. Varga and R. G. Lovas, Phys. Rev. C **37**, 2906 (1988)

Can be generalized to 3-cluster overlap integrals $I(r,s) =< \phi_A|\phi_{A+2} >$

Two-neutron transfer reaction

- 3-cluster RGM approximation : $\Psi_A^{j\pi} = \mathcal{A}\varphi_{A-2}\varphi_n\varphi_n g^{j\pi}(\boldsymbol{r_x}, \boldsymbol{r_y})$
- Use of hyperspherical coordinates

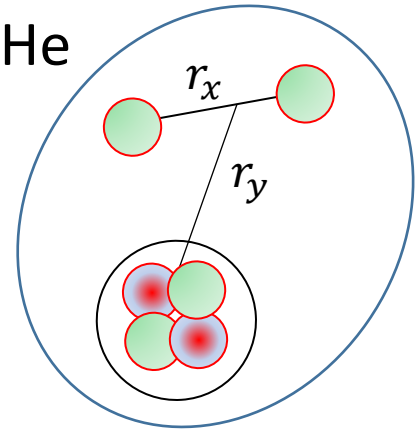


- RGM ^{6}He: S. Korennov and P.D., Nucl. Phys. A740 (2004) 249
- RGM ^{11}Li: P. D.. *Phys. Rev. C99* (2019), 064308

- Calculation of the 3-cluster overlap integrals [P. D., *Phys. Rev. C 107, 014312 (2023)]*
  - From $\Psi_A^{j\pi}$: → $I_A(\boldsymbol{r_x}, \boldsymbol{r_y}) = \langle\varphi_{A-2}|\Psi_A^{j\pi}\rangle$
  - For ^{11}Li: calculation of $\langle\varphi_{9Li}|\Psi_{11}^{j\pi}\rangle$ and $\langle\varphi_{9Li*}|\Psi_{11}^{j\pi}\rangle$ → acces to ^{11}Li(p,t)^{9}Li*

4. Application to ^{6}He(p,t)α

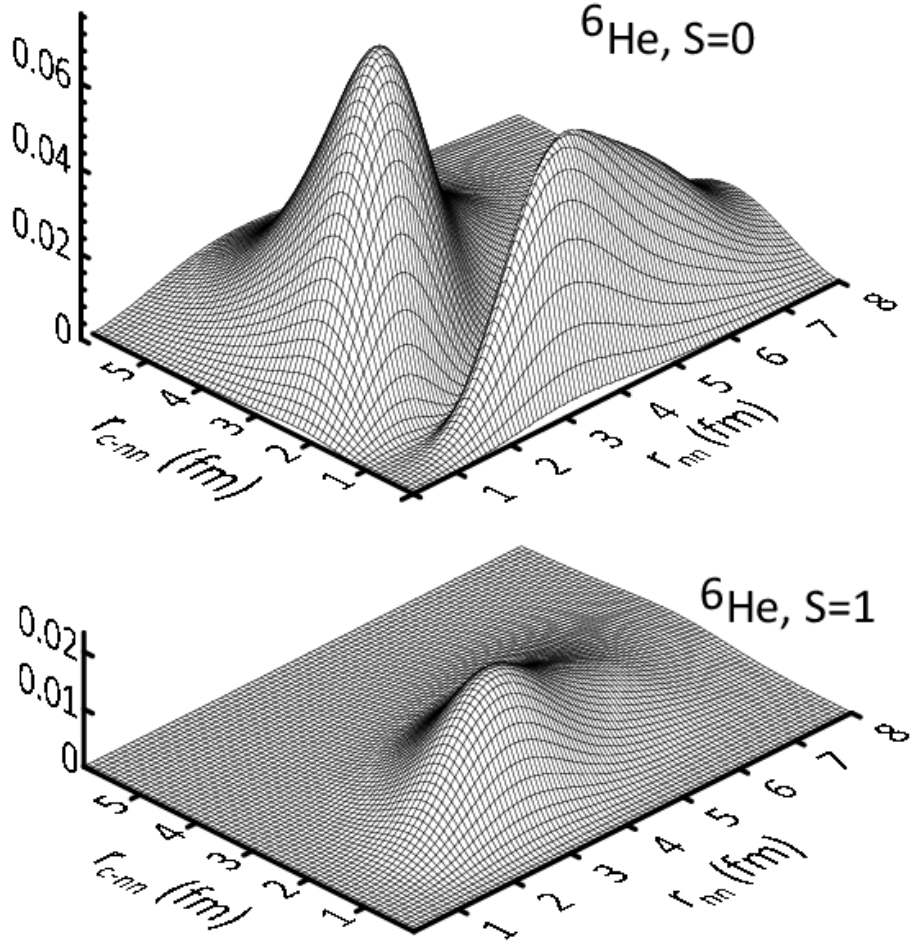
Description of ^{6}He



- NN interaction: Minnesota + spin-orbit
- Admixture parameter adjusted to reproduce S_{2n}=0.97 MeV: u=1.0047
- 2 coordinates $P_c(r_x, r_y) = \int |I_c(\boldsymbol{r_x}, \boldsymbol{r_y})|^2 d\Omega_x d\Omega_y$
- Spectroscopic factor $S = \int P_c(r_x, r_y) r_x^2 r_y^2 dr_x dr_y$

  S=1.41 (good agreement with the literature)

^{6}He overlap integral



Scattering matrix for a given $J\pi$

$$U_{if} = -\frac{i}{\hbar}\iint \chi_i(R) K_{i,f}(R,R') \chi_f(R') dR\, dR'$$

- Entrance channel ^{6}He+p: $\chi_i(R)$
  2 choices for the optical potential

  1. Koning-Delaroche compilation *Nucl. Phys. A*, *713*, 231 : solid lines
  2. Two potentials fitted on elastic scattering (Wolski et al.): dashed lines

- Exit channel α+t: $\chi_f(R')$
  Potential from Giot et al., Phys. Rev. C **71**, 064311 (2005).

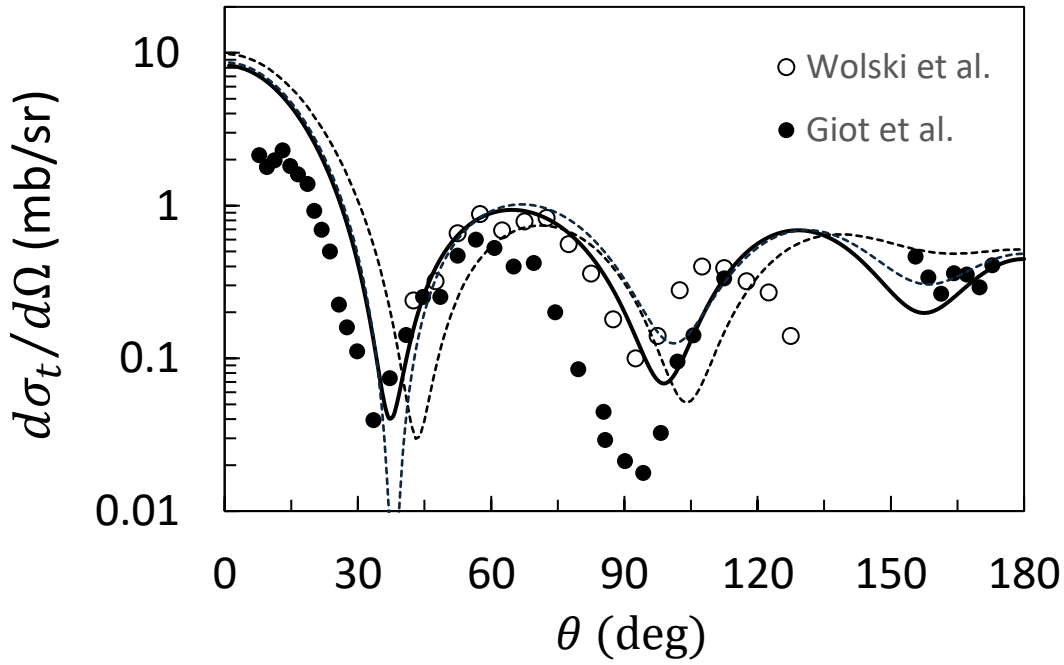
- Transfer kernel $K_{i,f}(R,R')$
  Computed from the overlap integrals and the potentials (see PRC104 (2021) 024613)

^{6}He(p,t)α cross section

Data from

- R. Wolski et al.,, Phys. Lett. B **467**, 8 (1999).
- L. Giot et al., Phys. Rev. C **71**, 064311 (2005).

E_{lab}=150 MeV, E_{cm}=21.4 MeV



→ Weak influence of the ^{6}He+p potential

5. Application to ^{11}Li(p,t)^{9}Li

Minnesota NN interaction adjusted to S_{2n}

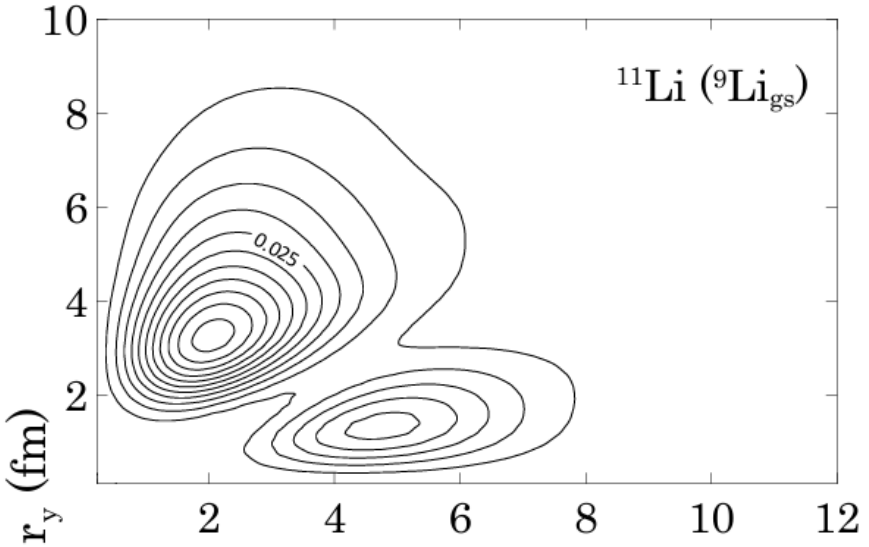
^{9}Li core : 6 neutrons, 3 protons :not a closed shell
15x6=90 Slater determinants

^{6}He: $\Psi_6^{j\pi} = \mathcal{A}\varphi_\alpha\varphi_n\varphi_n g^{j\pi}(\boldsymbol{r_x}, \boldsymbol{r_y})$

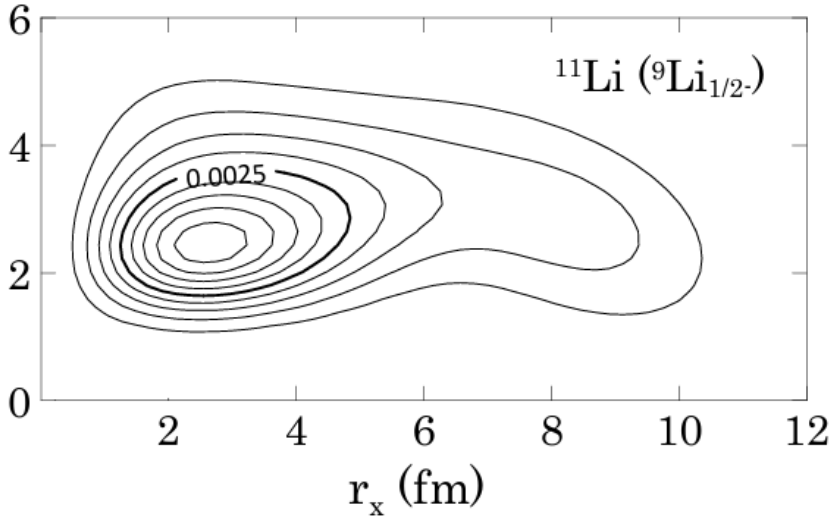
^{11}Li: $\Psi_{11}^{j\pi} = \sum_c \mathcal{A}\varphi_{9Li}^c\varphi_n\varphi_n g_c^{j\pi}(\boldsymbol{r_x}, \boldsymbol{r_y})$

With φ_{9Li}^c=shell-model ^{9}Li wave functions
(combinations of 90 Slater determinants)
c=3/2- gs, 1/2- excited state + others (Pseudostates)

^{9}Li core	**Spectroscopic factor**
$3/2^-$	0.78 (S_{12} = 0) 0.13 (S_{12} = 1)
$1/2^-$	0.054 (S_{12} = 0), 0.050 (S_{12} = 1)



➔ ^{11}Li(p,t)^{9}Li(gs)



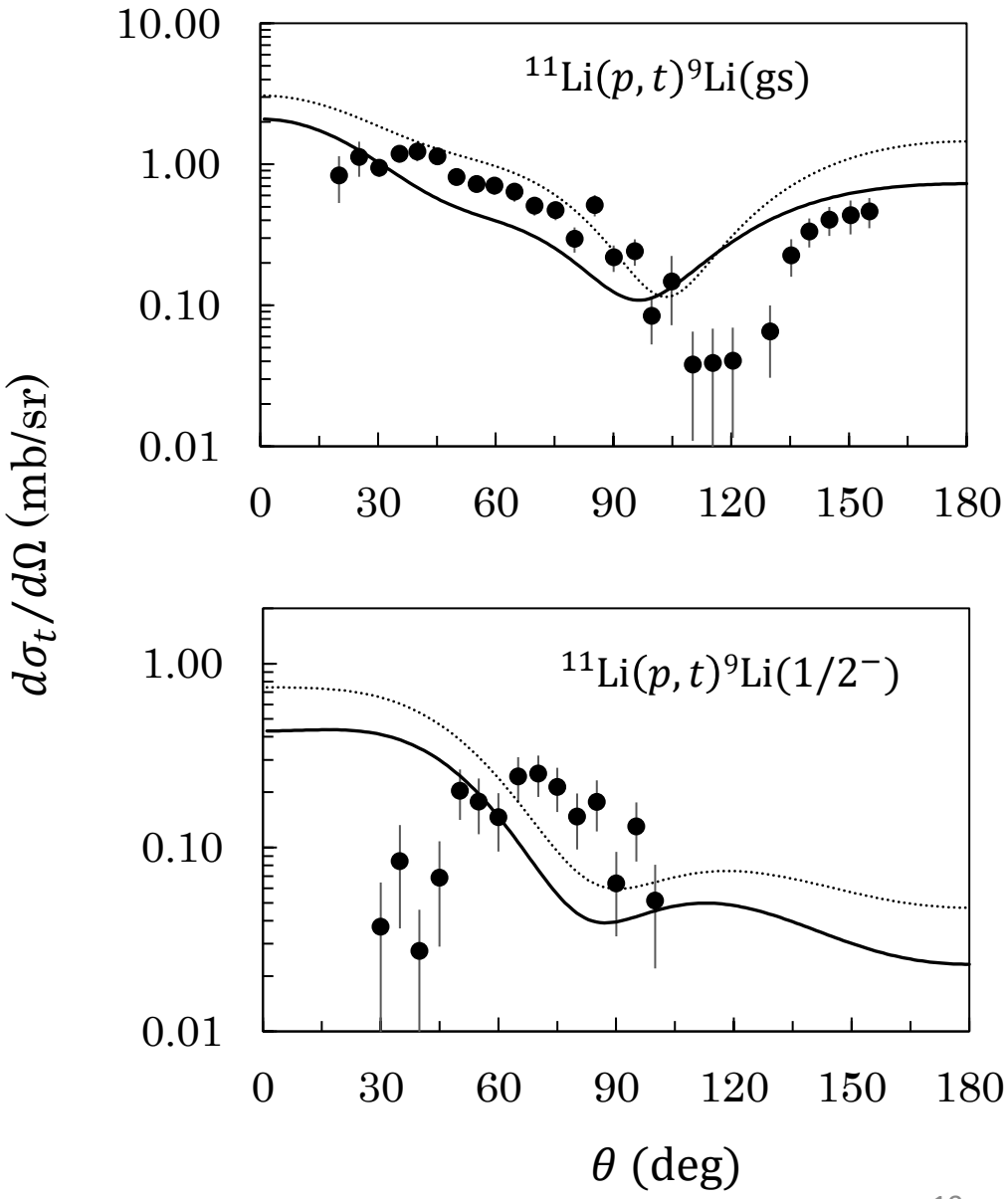
➔ ^{11}Li(p,t)^{9}Li(1/2$^-$)

- Data from I. Tanihata et al, Phys. Rev. Lett. **100**, 192502 (2008):
  ^{9}Li ground state and 1st excited state

- E_{lab}=33 MeV → E_{cm}=2.75 MeV

- ^{11}Li+p optical potential unknown at 2.75 MeV→ 2 potentials
  - Equivalent potential from a CDCC 4-body calculation [P. D., Phys. Rev. C **101**, 64611 (2020)]
  - Koning-Delaroche global potential (KD03)

- ^{9}Li+t: global potential of Y. Pang et al., Phys. Rev. C **79**, 24615 (2009).

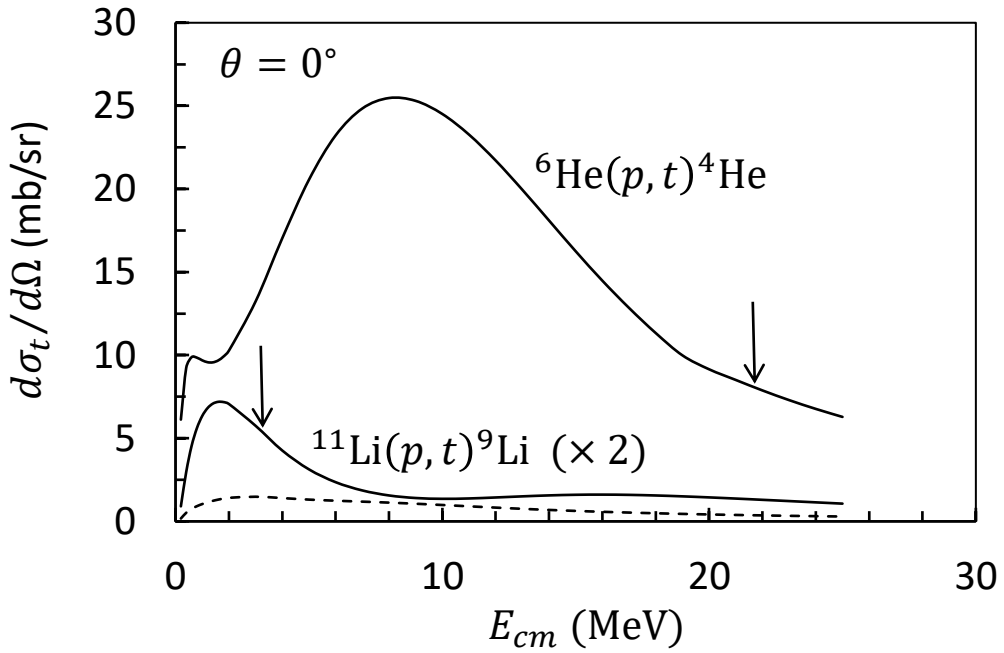


Energy dependence of the cross section

- Data are available at a single energy
- Calculations at several energies ($\theta = 0°$)
- ^{11}Li(p,t)^{9}Li: 2.75 MeV is near the predicted maximum
- ^{6}He(p,t)^{4}He: 22 MeV is far from the maximum

→ **Needs for theory**

- Other energies
- Simultaneous measurement of transfer and elastic scattering
- Other nuclei? ^{14}Be seems to be a good candidate



6. Conclusion

- Overlap integrals are simple to obtain in cluster models (very difficult in ab initio models)
- No need of spectroscopic factors, no need of (bound-state) potentials
- Core excitations are easily included
- Extended to three-cluster systems:
  - ^{6}He(p,t)α
  - ^{11}Li(p,t)^{9}Li with microscopic ^{11}Li wave functions (^{9}Li core more difficult than α)
- Reasonable agreement with experiment (no parameter)
- Elastic scattering data are important (preferably at the same energy)!
  - Excellent tests for theory
  - Determination of optical potentials (if necessary)