International Symposium Commemorating the 40th Anniversary of the Halo Nuclei

2025, Beijing

HALO-40

Ab initio study of Beryllium isotopes clustering, molecular orbital, and halo

Shihang Shen

Beihang University

European Research Council

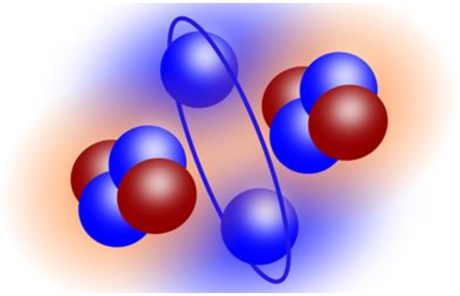
Established by the European Commission

Beryllium Isotopes

Diverse phenomena in beryllium Isotopes: clustering, molecular orbitals, halo, …

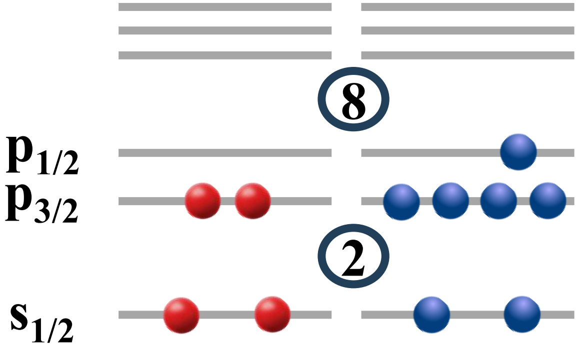
- ^{7}Be: significant role in nuclear astrophysics
  E. G. Adelberger et al., Rev. Mod. Phys. 70, 1265 (1998); Rev. Mod. Phys. 83, 195 (2011)

- ^{8}Be: unbound with decaying into two alpha particles



- ^{9}Be, ^{10}Be: clustering and molecular-like structures
  J. Chen, et al., Phys. Rev. Lett. 134, 012502 (2025)
  P. J. Li, et al., Phys. Rev. Lett. 131, 212501 (2023); Physics 16, s167 (2023)
  W. von Oertzen, M. Freer, and Y. Kanada-En'yo, Phys. Rep. 432, 43 (2006)

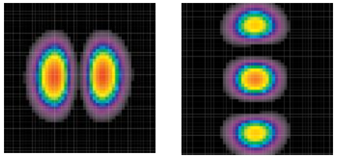
- ^{11}Be: ground-state parity inversion and **halo** structure
  A. Calci, et al., Phys. Rev. Lett. 117, 242501 (2016)
  W. Nörtershäuser, et al., Phys. Rev. Lett. 102, 062503 (2009)



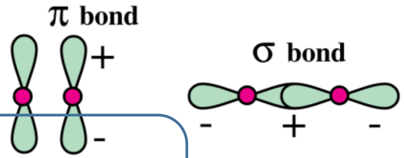
- ^{12}Be: breakdown of the $N = 8$ shell closure
  J. Chen, et al., Phys. Lett. B 781, 412 (2018)
  A. Navin, et al., Phys. Rev. Lett. 85, 266 (2000)

Theoretical Studies

- Antisymmetrized molecular dynamics Y. Kanada-En'yo, Phys. Rev. C 91, 014315 (2015)
- Fermionic molecular dynamics A. Krieger, et al., Phys. Rev. Lett. 108, 142501 (2012)



- Molecular-orbital models N. Itagaki and S. Okabe, Phys. Rev. C 61, 044306 (2000)
  W. Von Oertzen, Z. Phys. A 354, 37 (1996)
- Tohsaki-Horiuchi-Schuck-Röpke (THSR) wave function M. Lyu, et al., Eur. Phys. J. A 57, 51 (2021)
- Other cluster models Q. Zhao, M. Kimura, B. Zhou, and S. Shin, Phys. Rev. C 106, 054313 (2022)
  T. Fukui, J. Phys. G 49, 055102 (2022)
- Density functional theory J. Geng, P. W. Zhao, Y. F. Niu, and W. H. Long, Phys. Lett. B 858, 139036 (2024)
- Gamow shell model P. Linares Fernandez, et al., Phys. Rev. C 108, 044616 (2023)
- Green's function Monte Carlo R. B. Wiringa, et al., Phys. Rev. C 62, 014001 (2000)
- Monte Carlo shell model L. Liu, et al., Phys. Rev. C 86, 014302 (2012)
- No-core shell model A. E. McCoy, et al., Phys. Lett. B 856, 138870 (2024)
  A. Calci, P. Navrátil, R. Roth, D. Dohet-Eraly, S. Quaglioni, and G. Hupin, PRL 117, 242501 (2016)
- Resonating group method K. Kravvaris and A. Volya, Phys. Rev. Lett. 119, 062501 (2017)
- … … M. Kimura, T. Suhara, and Y. Kanada-En'yo, Eur. Phys. J. A 52, 373 (2016)
  Y. Funaki, H. Horiuchi, and A. Tohsaki, Prog. Part. Nucl. Phys. 82, 78 (2015)
  W. von Oertzen, M. Freer, and Y. Kanada-En'yo, Phys. Rep. 432, 43 (2006)



How about in ab initio framework with full many-body correlated wave function $\Psi(r_1, r_2, \ldots, r_A)$, which composed of millions of Slater determinant?

Nuclear lattice effective field theory

- First ab initio description of Hoyle state

PRL **106**, 192501 (2011) Selected for a Viewpoint in *Physics* PHYSICAL REVIEW LETTERS week ending 13 MAY 2011

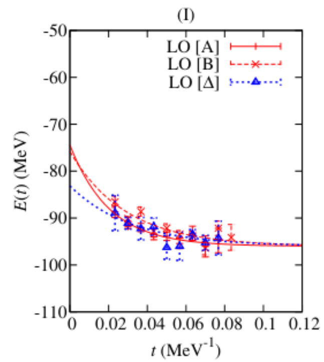
Ab Initio Calculation of the Hoyle State

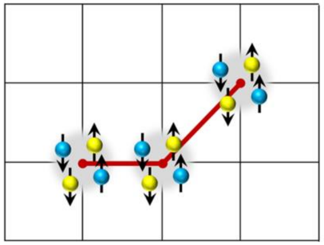
Evgeny Epelbaum,[1] Hermann Krebs,[1] Dean Lee,[2] and Ulf-G. Meißner[3,4]

PRL **109**, 252501 (2012) PHYSICAL REVIEW LETTERS week ending 21 DECEMBER 2012

Structure and Rotations of the Hoyle State

Evgeny Epelbaum,[1] Hermann Krebs,[1] Timo A. Lähde,[2] Dean Lee,[4] and Ulf-G. Meißner[5,2,3]





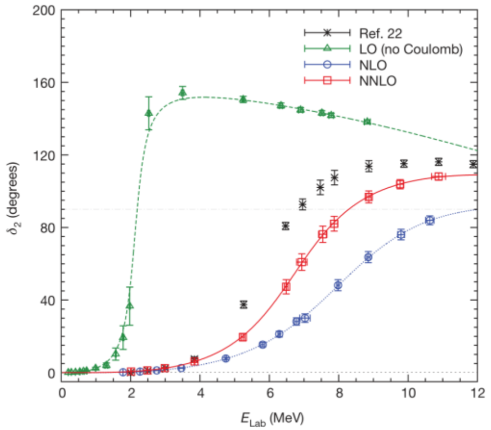
- First ab initio description of α-α scattering

Letter | Published: 02 December 2015

***Ab initio* alpha–alpha scattering**

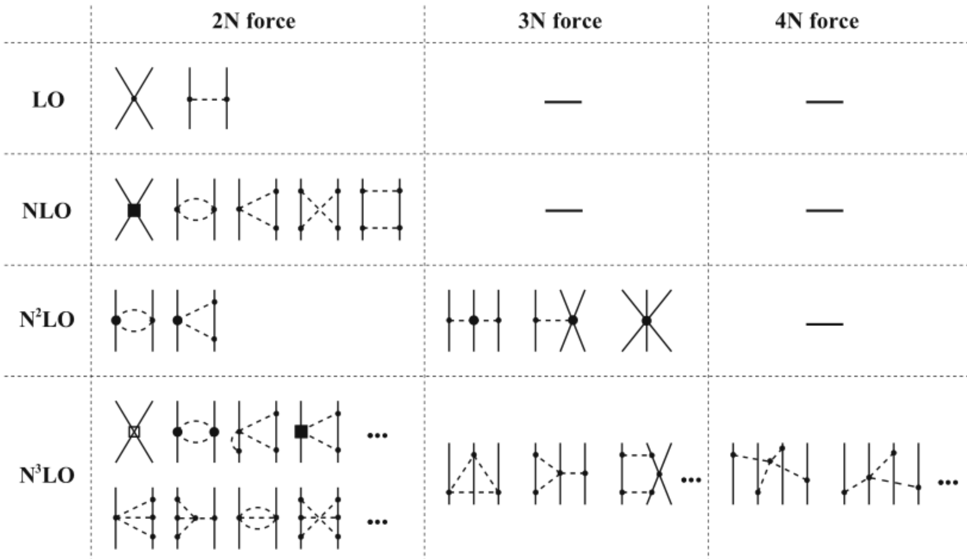
Serdar Elhatisari, Dean Lee, Gautam Rupak, Evgeny Epelbaum, Hermann Krebs, Timo A. Lähde, Thomas Luu & Ulf-G. Meißner

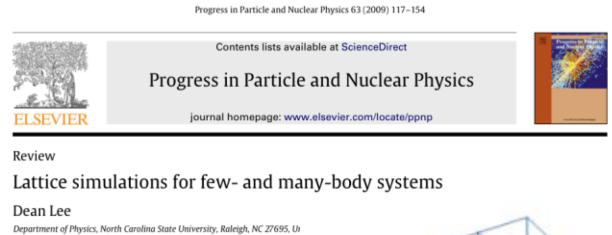
Nature **528**, 111–114 (2015) | Cite this article



- Nuclear thermodynamics B. N. Lu, et al., Phys. Rev. Lett. 125, 192502 (2020)
- Emergent geometry and duality in ^{12}C SS, et al., Nat. Commun. 14, 2777 (2023)
- Clustering in hot dilute matter Z. Ren, et al., Phys. Lett. B 850, 138463 (2024)
- Hyperneutron matter H. Tong, S. Elhatisari, and U. G. Meißner, Sci. Bull. 70, 825 (2025)
- … …

Theoretical Framework




Progress in Particle and Nuclear Physics 63 (2009) 117–154

Contents lists available at ScienceDirect

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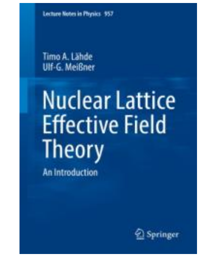
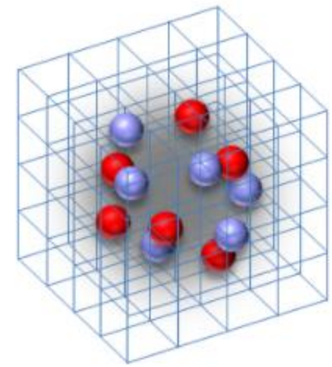
Review

Lattice simulations for few- and many-body systems

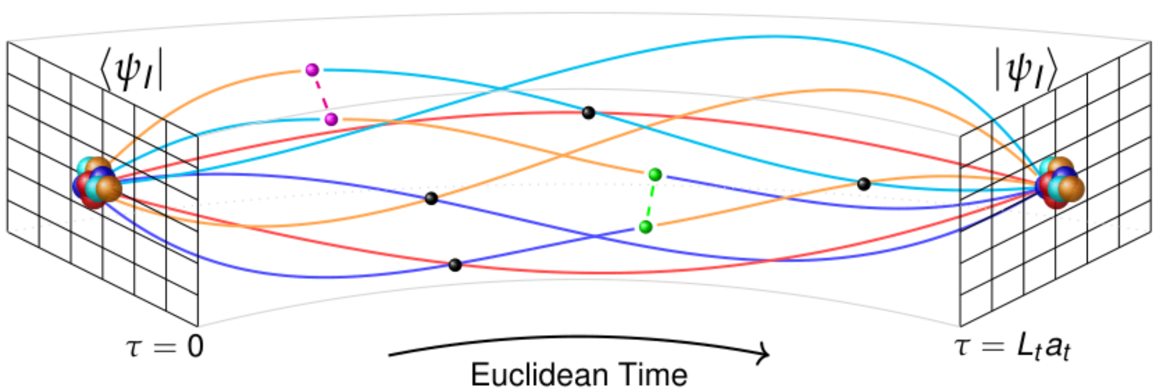
Dean Lee

Department of Physics, North Carolina State University, Raleigh, NC 27695, U





$$M = \exp(-a_t H) \qquad \langle \Psi_I | M M \ldots M O M \ldots M M | \Psi_I \rangle$$



Upper left figure is in courtesy of E. Epelbaum
lattice figure from https://www.physics.ncsu.edu/ntg/leegroup/research.html
Lower figure is in courtesy of S. Elhatisari

Theoretical Framework

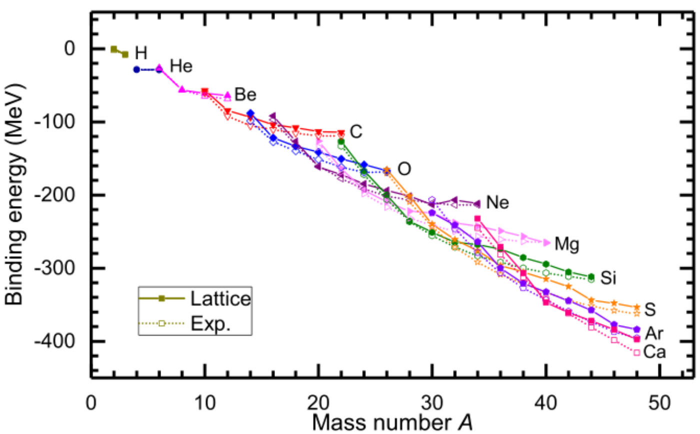
- Wigner SU(4) symmetric interaction (spin and isospin independent):

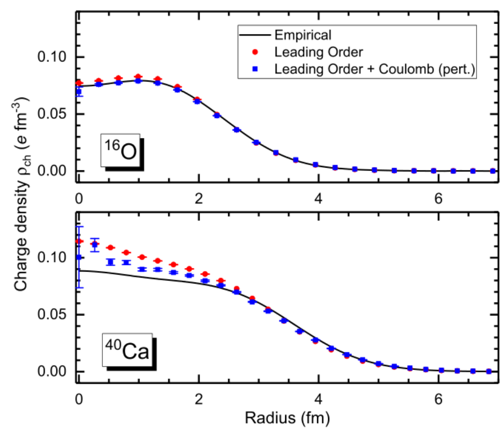
S. Elhatisari et al., PRL 119, 222505 (2017)

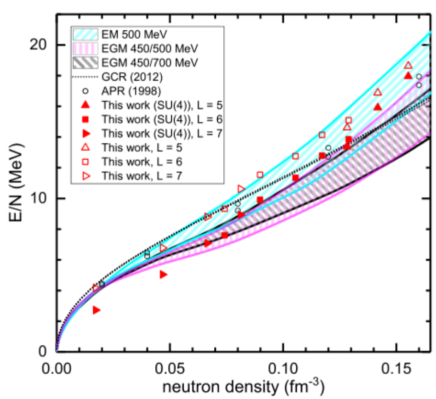
$$V = \frac{C_2}{2!}\sum_{\mathbf{n}} \tilde{\rho}(\mathbf{n})^2 + \frac{C_3}{3!}\sum_{\mathbf{n}} \tilde{\rho}(\mathbf{n})^3,$$

$$\tilde{\rho}(\mathbf{n}) = \sum_{i=1}^{A} \tilde{a}_i^\dagger(\mathbf{n})\tilde{a}_i(\mathbf{n}) + s_L \sum_{|\mathbf{n}'-\mathbf{n}|=1} \sum_{i=1}^{A} \tilde{a}_i^\dagger(\mathbf{n}')\tilde{a}_i(\mathbf{n}'), \qquad \tilde{a}_i(\mathbf{n}) = a_i(\mathbf{n}) + s_{\mathrm{NL}} \sum_{|\mathbf{n}'-\mathbf{n}|=1} a_i(\mathbf{n}').$$

  - Sign problem is largely suppressed J.W. Chen, D. Lee, T. Schäfer, PRL, 93, 242302 (2004)
  - Ground state properties of light and medium mass nuclei, and neutron matter can be well reproduced B.-N. Luu et al., PLB 797 (2019) 134863







  - Higher order (e.g. N3LO) can be built on with perturbation

Elhatisari et al., Nature 630, 59 (2024)

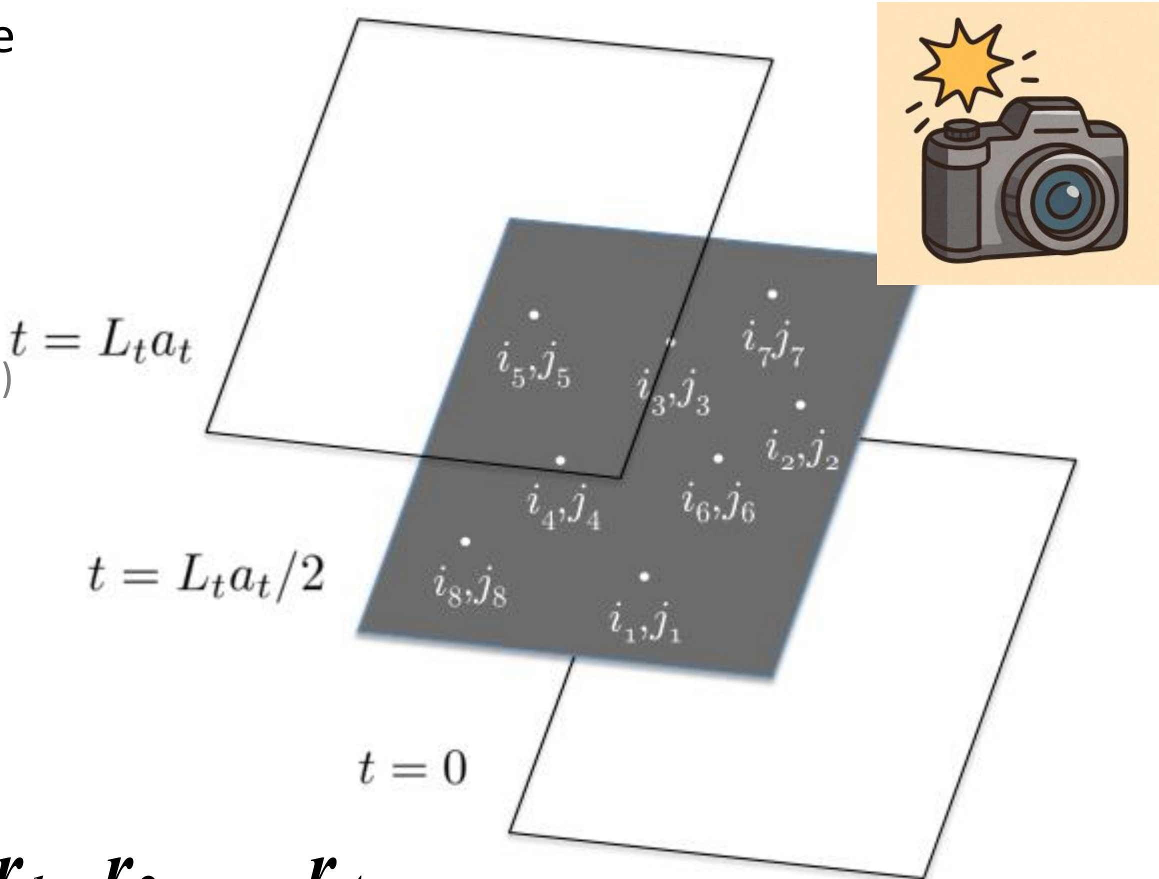
Theoretical Framework

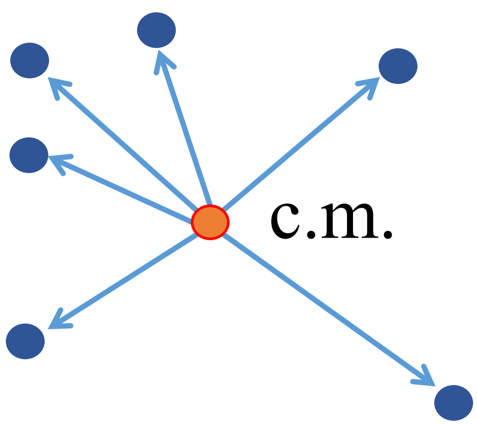
- Pinhole algorithm

$$\langle \Phi_0 | e^{-H\tau/2} \rho(\mathbf{n}_1, \mathbf{n}_2, \ldots, \mathbf{n}_A) e^{-H\tau/2} | \Phi_0 \rangle$$

A time slice is inserted to sample the positions and spin-isospin indices in the middle time step.

S. Elhatisari et al., PRL 119, 222505 (2017)





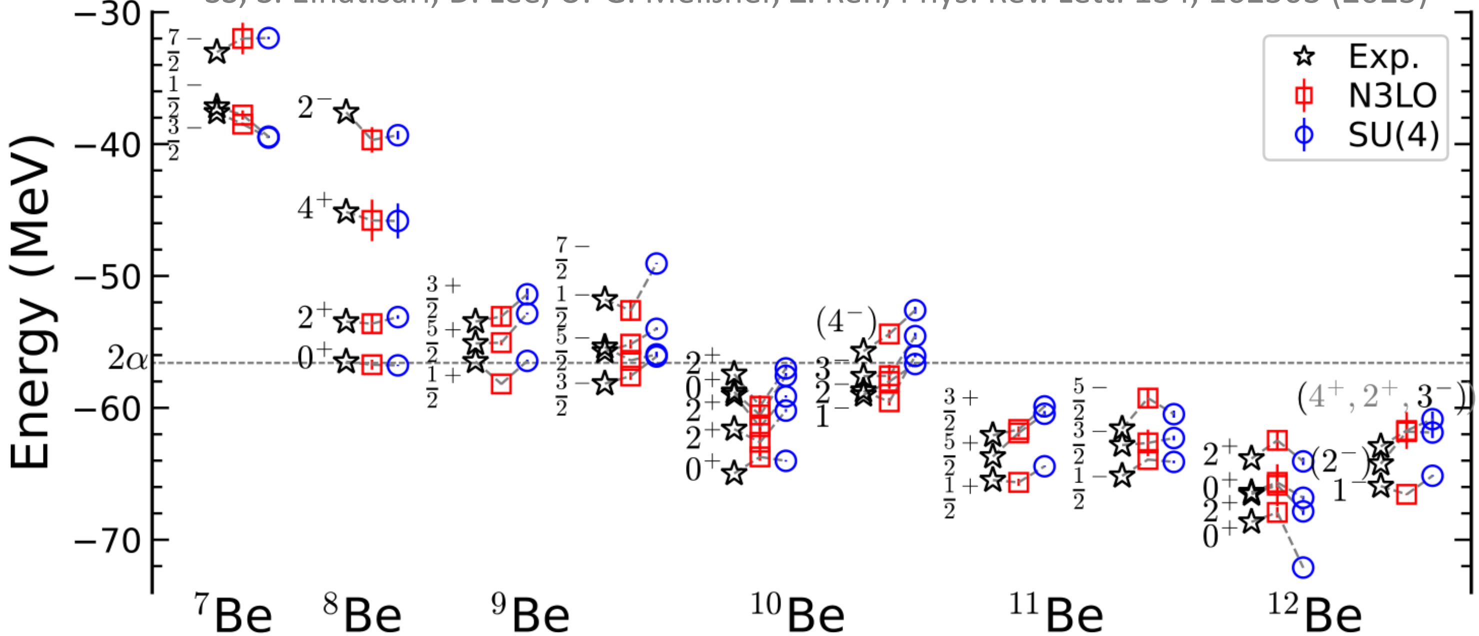
$\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_A$

Calculation by NLEFT - Spectrum

- N^3LO interaction S. Elhatisari, et al., Nature 630, 59 (2024)
  lattice spacing $a = 1.32$ fm, box size $L = 13.2$ fm

- SU(4) interaction SS, S. Elhatisari, T. Lähde, D. Lee, B. Lu, and U.-G. Meißner, Nat. Commun. 14, 2777 (2023)
  lattice spacing $a = 1.64$ fm, box size $L = 14.8$ fm

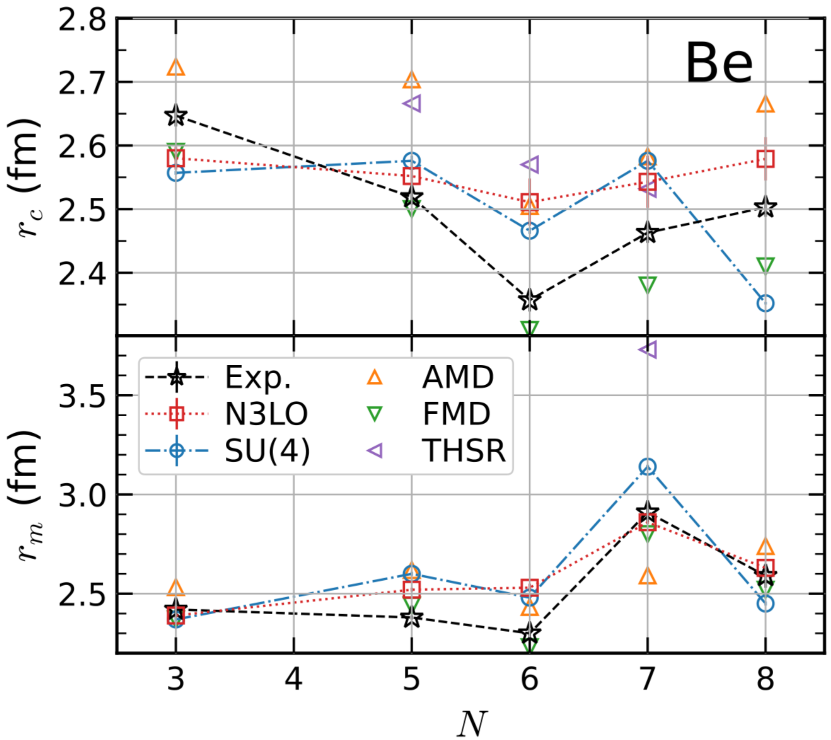
SS, S. Elhatisari, D. Lee, U.-G. Meißner, Z. Ren, Phys. Rev. Lett. 134, 162503 (2025)



	Exp.	N^3LO	SU(4)
^{11}Be (1/2+)	−65.5	−65.6(3)	−64.6(1)
^{11}Be (1/2-)	−65.2	−63.9(1)	−64.1(1)

Radii and Transitions

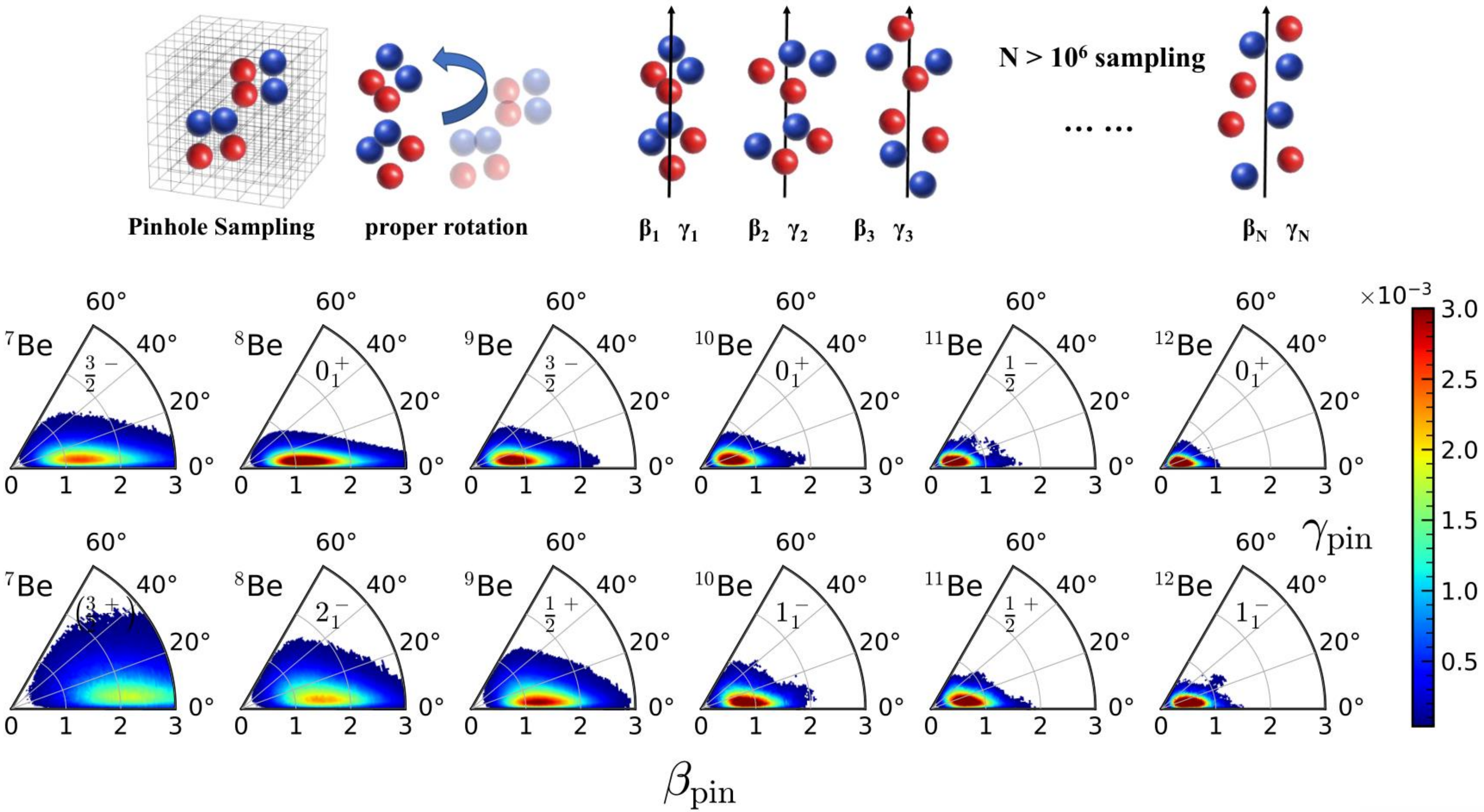
Radii and electromagnetic properties can also be well described



		SU(4)	N^3LO	Exp.
^{7}Be	$E2, \frac{3}{2}^- \to \frac{1}{2}^-$	16.0(2)	15.2(5)	26(6)(3) [91]
^{9}Be	$Q(\frac{3}{2}^-)$	7.3(1)	7.4(1.0)	5.29(4) [92]
	$E1, \frac{1}{2}^+ \to \frac{3}{2}^-$	0.131(3)	0.060(15)	0.136(2) [93]
	$E1, \frac{5}{2}^+ \to \frac{3}{2}^-$	0.045(14)	0.049(5)	0.010(8) [67]
	$E2, \frac{5}{2}^- \to \frac{3}{2}^-$	35.7(1.8)	27.8(1.9)	27.1(2.0) [67]
	$E2, \frac{7}{2}^- \to \frac{3}{2}^-$	11.6(2.5)	5.3(8)	9.5(4.1) [67]
^{10}Be	$E1, 3_1^- \to 2_1^+$	0.026(2)	0.004(3)	0.009(1) [67]
	$E2, 2_1^+ \to 0_1^+$	10.6(4)	8.5(9)	9.2(3) [31]
^{11}Be	$E1, \frac{1}{2}^- \to \frac{1}{2}^+$	0.023(3)	0.038(3)	0.102(2) [94]
^{12}Be	$E1, 0_1^+ \to 1_1^-$	0.049(2)	0.056(26)	0.051(13) [95]
	$E2, 2_1^+ \to 0_1^+$	7.8(1.1)	9.0(3.1)	14.2(1.0)(2.0) [6]

SS, S. Elhatisari, D. Lee, U.-G. Meißner, Z. Ren, Phys. Rev. Lett. 134, 162503 (2025)

Deformation

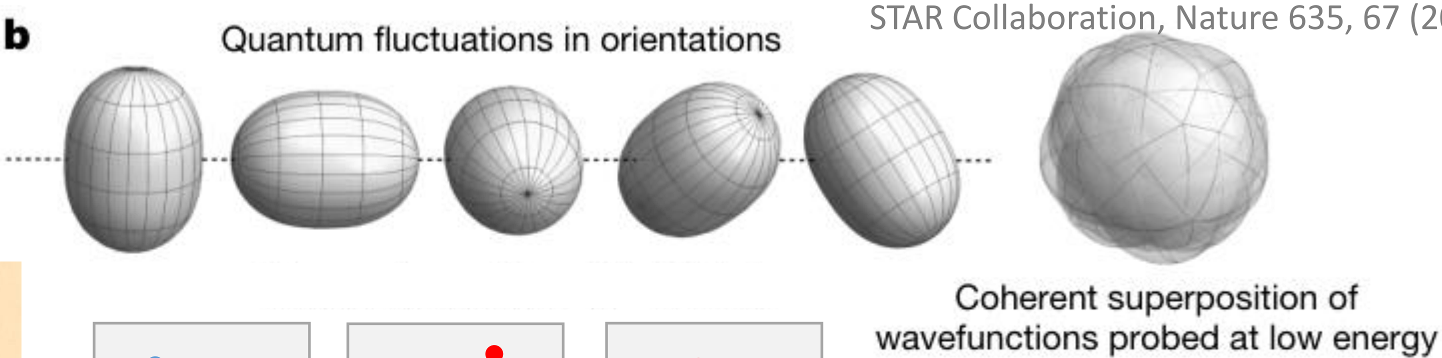


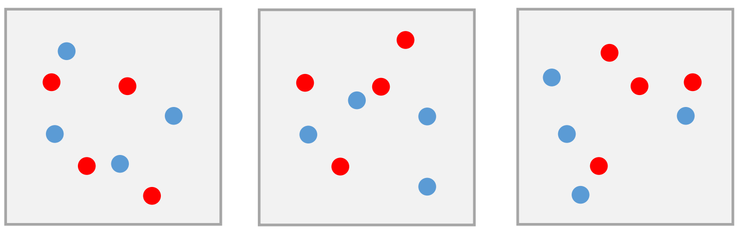
SS, S. Elhatisari, D. Lee, U.-G. Meißner, Z. Ren, Phys. Rev. Lett. 134, 162503 (2025)

Intrinsic density

What is intrinsic density?

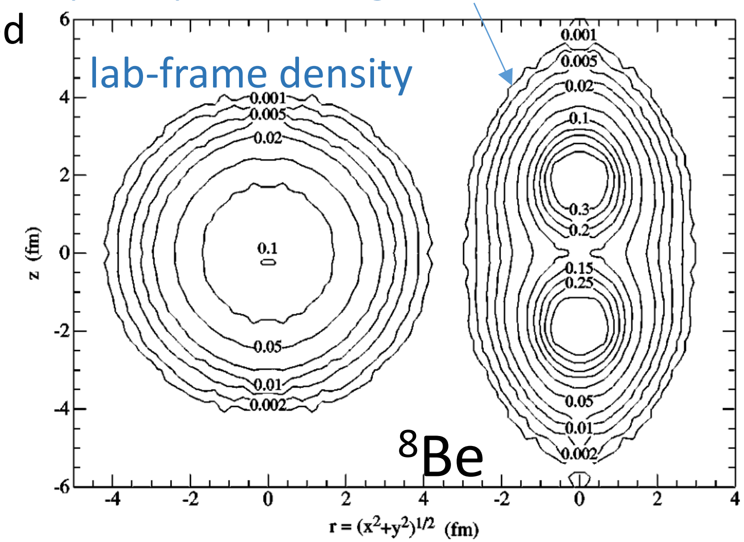
STAR Collaboration, Nature 635, 67 (2024)





… …

principal-axis-aligned intrinsic density



In QMC study, nucleon coordinates (r_1, r_2, …, r_A) are rotated to the (long) principal axis of moment of inertia matrix

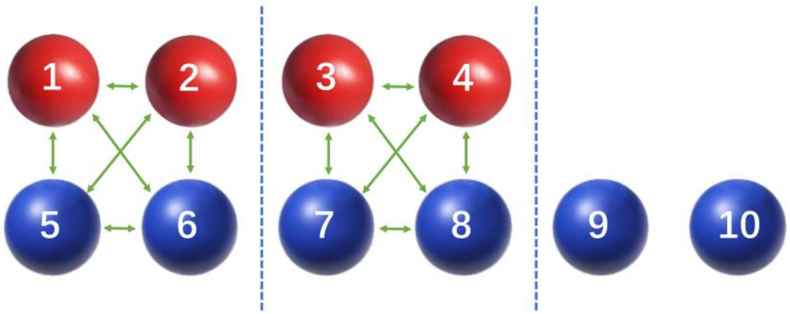
$$\mathcal{M}=\sum_{i=1}^{A}\begin{pmatrix} x_i^2 & x_i y_i & x_i z_i \\ y_i x_i & y_i^2 & y_i z_i \\ z_i x_i & z_i y_i & z_i^2 \end{pmatrix}$$

R. Wiringa, S. Pieper, J. Carlson, and V. Pandharipande, Phys. Rev. C 62, 014001 (2000)

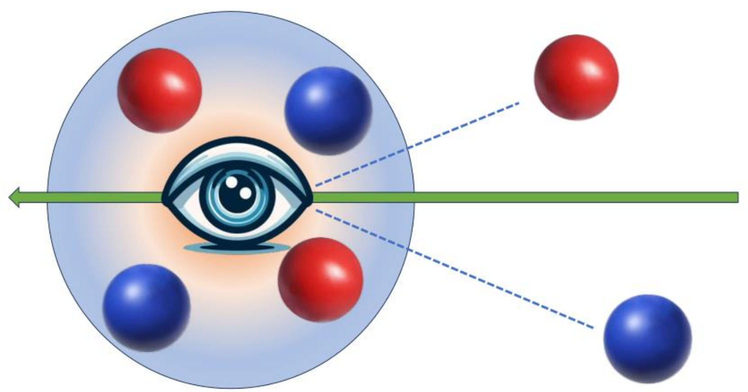
Intrinsic density

α-view intrinsic density

1. Grouping closest 2 protons and 2 neutrons together



2. Rotate the nucleon coordinates (r_1, r_2, …, r_A) such that the cluster is aligned along the symmetry axis

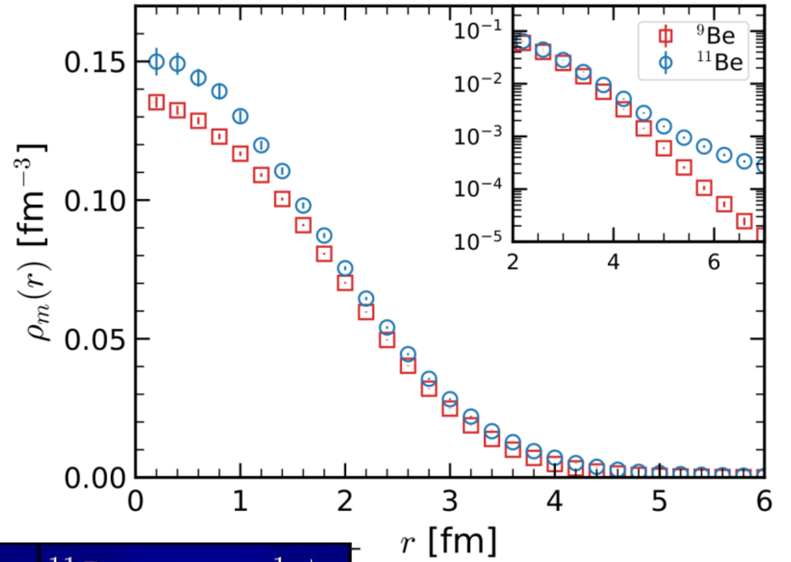


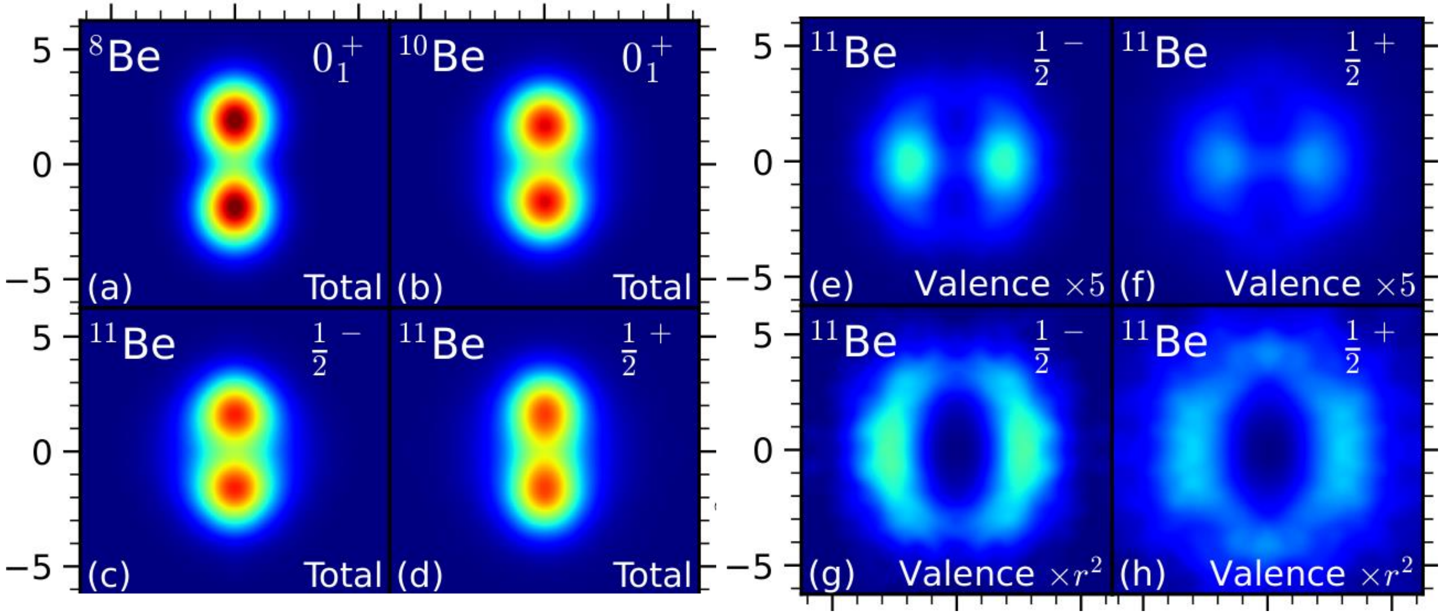
SS, S. Elhatisari, D. Lee, U.-G. Meißner, Z. Ren, Phys. Rev. Lett. 134, 162503 (2025)

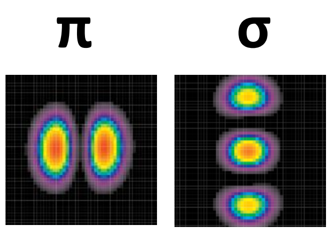
α-view intrinsic density

Intrinsic density calculated by NLEFT:

- (left) total density
- (right) valence neutron density





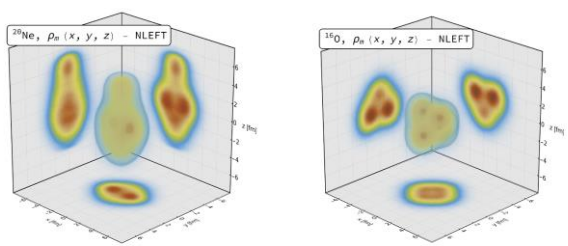


Y. Kanada-En'yo, H. Horiuchi, and A. Dote, J. Phys. G 24, 1499 (1998)

SS, S. Elhatisari, D. Lee, U.-G. Meißner, Z. Ren, Phys. Rev. Lett. 134, 162503 (2025)

Summary

- Berrylium isotopes have been studied by nuclear lattice effective field theory
- Observables such as low-lying spectrum, density profiles, electromagnetic properties can be well described
- Molecular dynamics and halo structure are revealed



? Cluster in heavier system

? ANC

? … …

PHYSICAL REVIEW LETTERS **135,** 012302 (2025)

Exploiting ^{20}Ne Isotopes for Precision Characterizations of Collectivity in Small Systems

Giuliano Giacalone,[1,*] Benjamin Bally,[2] Govert Nijs,[3] Shihang Shen,[4] Thomas Duguet,[5,6] Jean-Paul Ebran,[7,8] Serdar Elhatisari,[9,10] Mikael Frosini,[11] Timo A. Lähde,[12,13] Dean Lee,[14] Bing-Nan Lu,[15] Yuan-Zhuo Ma,[14] Ulf-G. Meißner,[10,16,17] Jacquelyn Noronha-Hostler,[18] Christopher Plumberg,[19] Tomás R. Rodríguez,[20] Robert Roth,[21,22] Wilke van der Schee,[3,23,24] and Vittorio Somà[5]

PHYSICAL REVIEW LETTERS **134,** 082301 (2025)

Anisotropic Flow in Fixed-Target ^{208}Pb + ^{20}Ne Collisions as a Probe of Quark-Gluon Plasma

Giuliano Giacalone,[1,*] Wenbin Zhao,[2,3,†] Benjamin Bally,[4] Shihang Shen,[5] Thomas Duguet,[6,7] Jean-Paul Ebran,[8,9] Serdar Elhatisari,[10] Mikael Frosini,[11] Timo A. Lähde,[12,13] Dean Lee,[14] Bing-Nan Lu,[15] Yuan-Zhuo Ma,[14] Ulf-G. Meißner,[16,17,5,18] Govert Nijs,[19] Jacquelyn Noronha-Hostler,[20] Christopher Plumberg,[21] Tomás R. Rodríguez,[22] Robert Roth,[23,24] Wilke van der Schee,[19,25,26] Björn Schenke,[27,‡] Chun Shen,[28,29,§] and Vittorio Somà[6]

Collaborators

- Serdar Elhatisari, KFUPM
- Timo A. Lähde, Jülich
- Dean Lee, MSU
- Bing-Nan Lu, GSCAPE
- Ulf-G. Meißner, Bonn/Jülich/Beihang
- Zhengxue Ren, Jülich
- NLEFT Collaboration

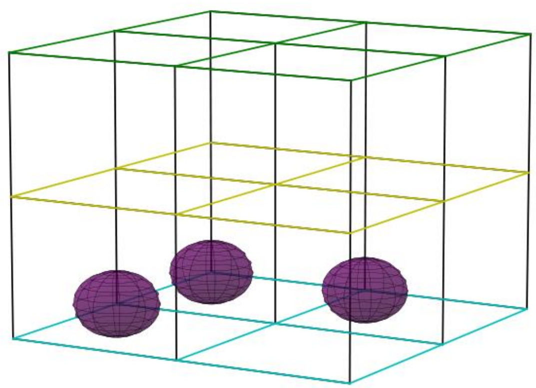
Thank you for your attention!

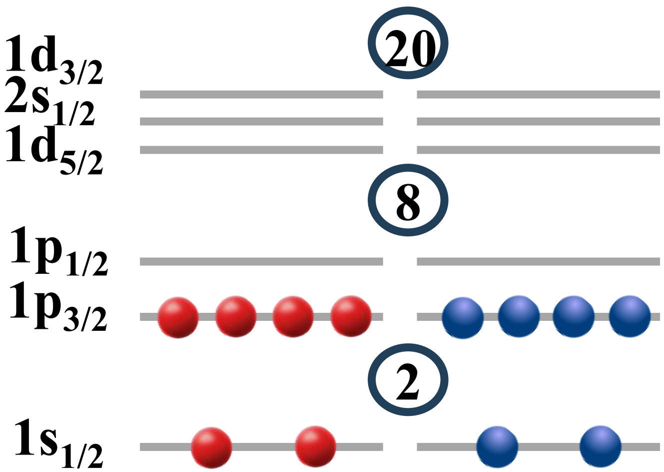
Theoretical Framework

- Starting from an initial many-body wave function:

$$|\Phi_0\rangle = \mathscr{A}\left[\phi_1(\mathbf{r}_1)\phi_2(\mathbf{r}_2)\ldots\phi_A(\mathbf{r}_A)\right]$$

$$\phi(\mathbf{r}) = \exp\left(-(\mathbf{r}-\mathbf{r}_0)^2/2w^2\right)$$



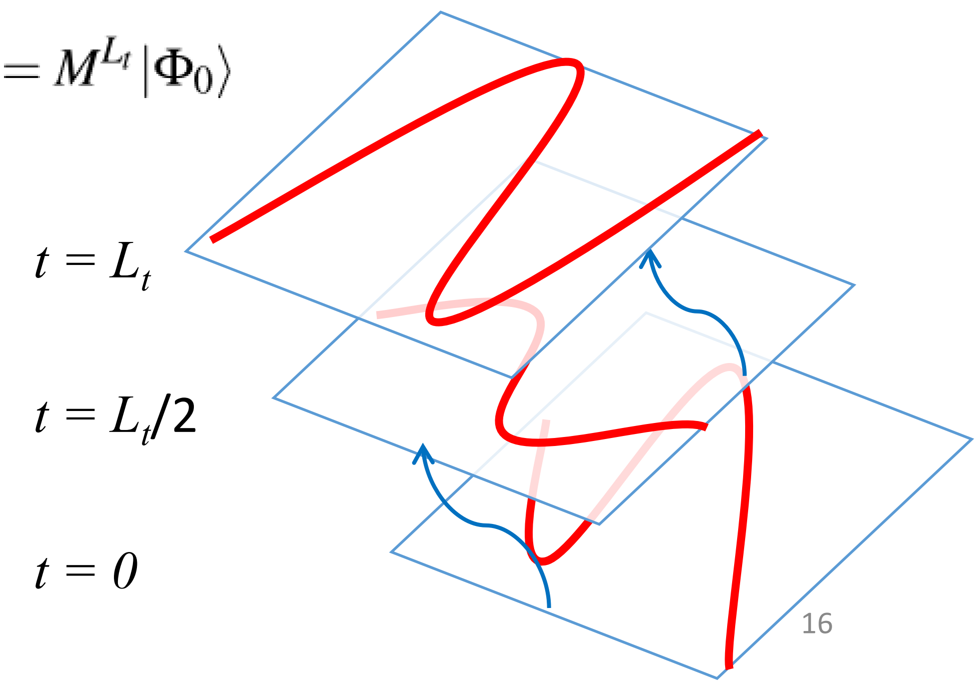


- Euclidean time projection with transfer matrix:

$$M = \exp(-a_t H)$$

with H the many-body Hamiltonian, a_t and a the temporal and spatial lattice spacing.

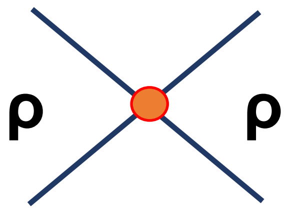
$$|\Phi_{L_t}\rangle = M^{L_t}|\Phi_0\rangle$$

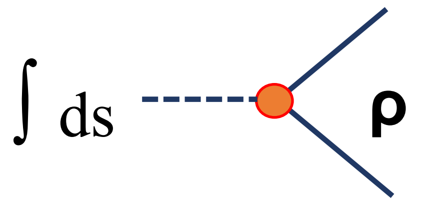


Theoretical Framework

- Auxilary field with Monte-Carlo sampling

$$\exp\left(-\frac{C\alpha_t}{2}\rho^2\right) := \sqrt{\frac{1}{2\pi}}\int_{-\infty}^{\infty} ds : \exp\left(-\frac{1}{2}s^2 + \sqrt{-C\alpha_t}s\rho\right):$$





- Final state is composed of millions of Slater determinants (configurations)

$$|\Phi_{L_t}\rangle = \sum_{s_i} |\Phi_{s_i,L_t}\rangle$$

$$|\Phi_{s_i,L_t}\rangle = M_{s_i}^{L_t}|\Phi_0\rangle = \mathscr{A}[\phi_{s_i,1}(\mathbf{r}_1)\phi_{s_i,2}(\mathbf{r}_2)\ldots\phi_{s_i,A}(\mathbf{r}_A)]$$

Dealing with strong nuclear forces

- Wave function matching   S. Elhatisari, et al., Nature 630, 59 (2024)