A coherent microscopic picture for the exotic structure of ^{11}Be

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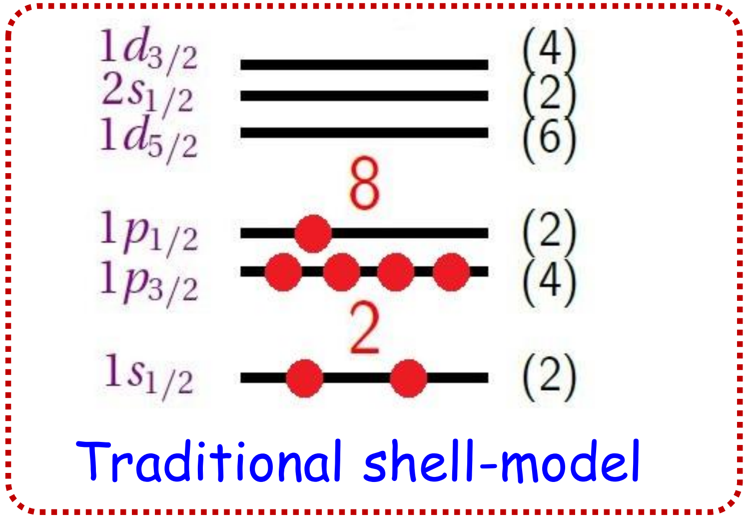
Outline

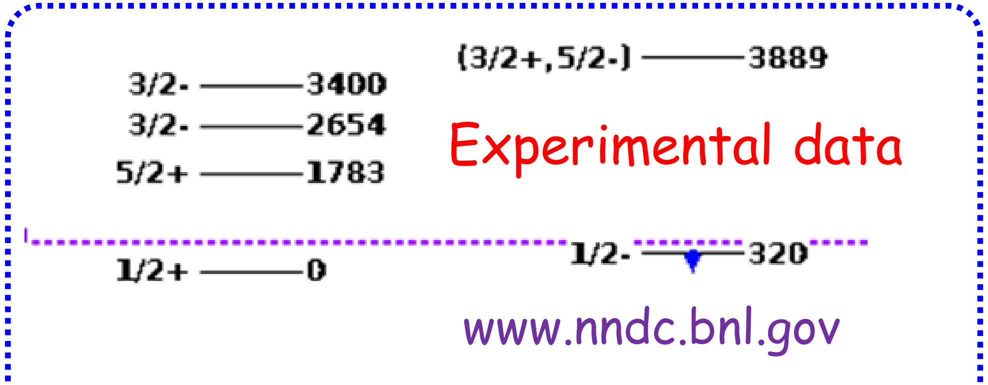
- Introduction
- Axially Deformed RHFB model
- The novel phenomena of ^{11}Be
- Summary

Novel Phenomena in ^{11}Be

- Parity Inversion: p-shell nucleus with even-parity ground state

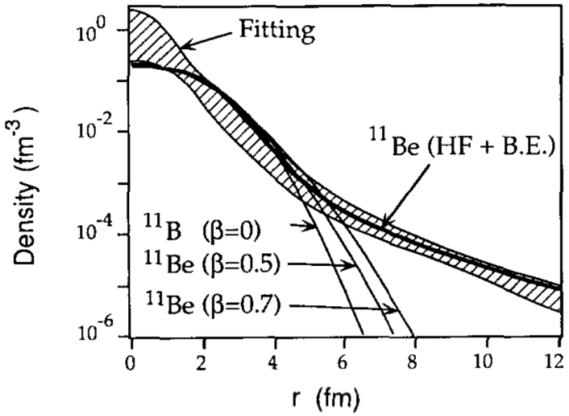


VS



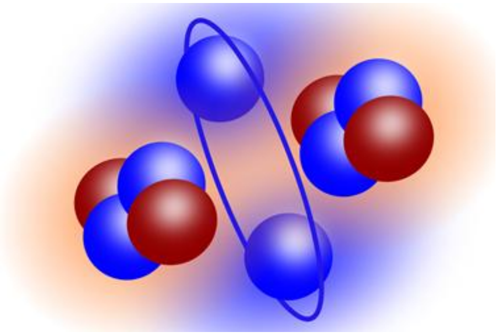
D.H. Wilkinson, et. al., PR **113** (1959) 563
P.F. Donovan, et. al., PR **123** (1961) 589
J.P. Deutsch, et. al., Phys. Lett. **28B** (1968) 178

- Halo Structure



PLB **268** (1991)339;PRL **84** (2000) 35;PRL **108** (2012) 192701.

- Underlying Cluster structure



$^{11}\mathrm{Be} = {}^{10}\mathrm{Be} + 1n$

Does the ^{11}Be ground state exhibit a cluster structure?

Li, et.al., PRL **131**(2023) 212501

Theoretical Research for novel phenomena

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- **Shell model** Parity inversion and Halo structure

  Otsuka, et. al., PRL **70** (1993) 1385; Sagawa, et. al., PLB **309** (1993) 1

  The direct picture of cluster structure?

- **Antisymmetrized molecular dynamics method** Parity inversion and Cluster structure

  Y. Kanada-En'yo, et. al., PRC 66 (2002) 024305

  It is not easy to describe the halo structure due to the limitation of Gaussian function

- **Ab initio** Parity inversion, Halo and Cluster structure

  no-core shell model with continuum: continuum effects and three-nucleon interaction

  A. Calci, et. al., PRL 117 (2016) 242501, Atkinson, et.al., PRC 105, (2022) 054316

  nuclear lattice effective field theory with the N^3LO interaction

  S. H. Shen, et. al., PRL 134 (2025) 162503

- **Energy Density Functional Theory** Halo structure

  Skyrme-Hartree-Fock model Relativistic Mean Field model

  J. C. Pei, et. al., NPA 765 (2006) 29; X. Li, et. al., PRC 54 (1996) 1617

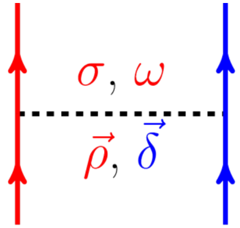
  Can not describe the parity inversion correctly

Relativistic Hartree-Fock theory

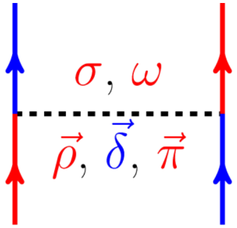
- Relativistic Hartree-Fock (RHF) theory: contains the Fock term explicitly

  Bouyssy (1987), Bernardos (1993), Shi (1995), Marcos (2004), Long (2004-now), ……

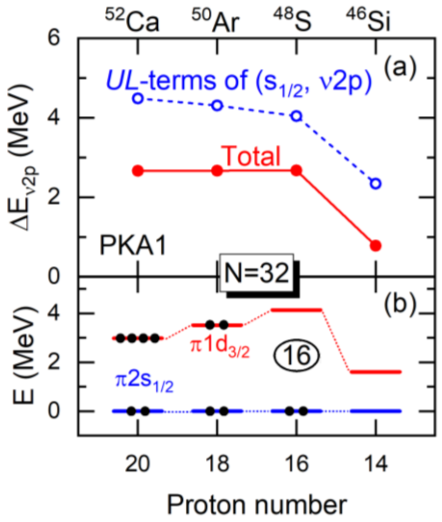
  - Fock terms play an important role in the ground state and excited state for nucleus: tensor force, Lorentz tensor vertex



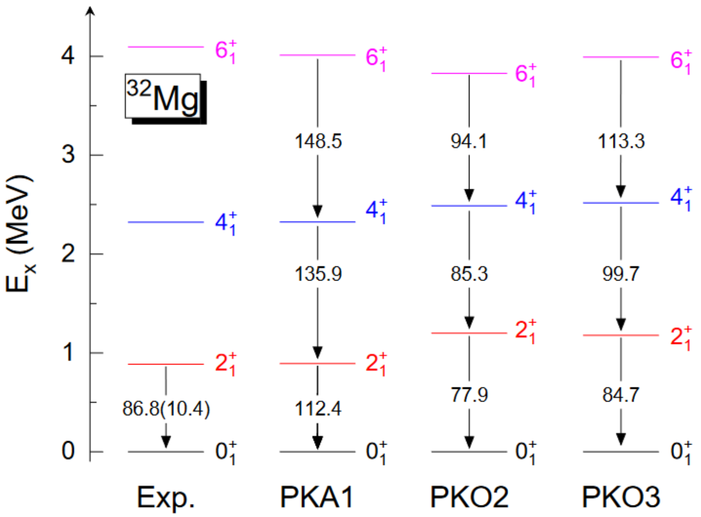
Hartree terms



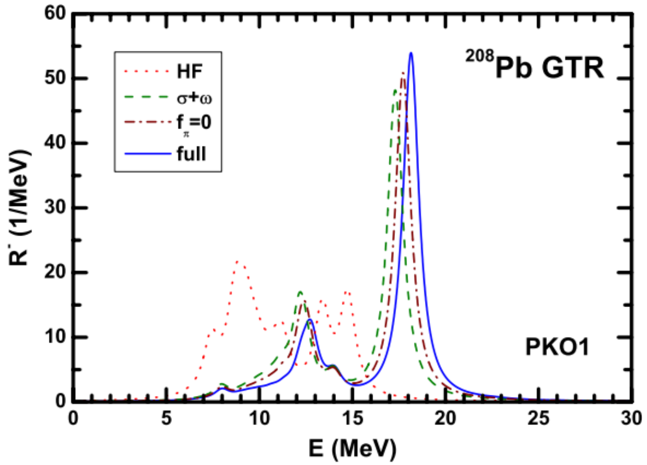
Fock terms



J.Liu, et,al., PLB **806** (2020) 135524



Y. Peng, et.al., CPC **49**, 064112 (2025)



H. Z. Liang, et,al., PRL **101** 122502 (2008)

- Axially deformed Relativistic Hartree-Fock-Bogoliubov (D-RHFB) model PRC **105**, 034329 (2022)

unified treatment for tensor force, deformation, pairing correlations and continuum effects → Find the unified description for novel phenomena of ^{11}Be in mean field level

D-RHFB model

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- Hamiltonian of systems: $\phi = \sigma$-S, ω-V, ρ-V, **ρ-T, ρ-VT, π-PV**, A-V

$$H = \int d\boldsymbol{x}\bar{\psi}(\boldsymbol{x})\left(-i\gamma\cdot\boldsymbol{\nabla}+M\right)\psi(\boldsymbol{x}) + \frac{1}{2}\sum_{\phi}\int d\boldsymbol{x}d\boldsymbol{x}'\bar{\psi}(\boldsymbol{x})\bar{\psi}(\boldsymbol{x}')\Gamma_{\phi}D_{\phi}\psi(\boldsymbol{x}')\psi(\boldsymbol{x}).$$

Kinetic term

Two-body interaction term

- Quantization in Bogoliubov quasi-particle space $\psi(x) = \sum_k\left[\psi_k^U(x)\beta_k + \psi_{\tilde{k}}^V(x)\beta_k^\dagger\right]$

- Two-body interaction term

$$\widehat{V}_{\phi} = \frac{1}{2}\iint d\boldsymbol{x}_1 d\boldsymbol{x}_2 \sum_{k_1k_1'}\sum_{k_2k_2'}\left[\bar{\psi}_{\tilde{k}_1}^V(x_1)\bar{\psi}_{\tilde{k}_2}^V(x_2)\Gamma_\phi D_\phi \psi_{\tilde{k}_2'}^V(x_2)\psi_{\tilde{k}_1'}^V(x_1)\beta_{k_1}\beta_{k_2}\beta_{k_2'}^\dagger\beta_{k_1'}^\dagger\right]$$

$$+\frac{1}{2}\iint d\boldsymbol{x}_1 d\boldsymbol{x}_2 \sum_{k_1k_1'}\sum_{k_2k_2'}\left[\bar{\psi}_{\tilde{k}_1}^V(x_1)\bar{\psi}_{k_2}^U(x_2)\Gamma_\phi D_\phi \psi_{k_2'}^U(x_2)\psi_{\tilde{k}_1'}^V(x_1)\beta_{k_1}\beta_{k_2}^\dagger\beta_{k_2'}\beta_{k_1'}^\dagger\right]$$

Hartree term

Fork term

Pairing

PKO3 and PKO1

PKA1

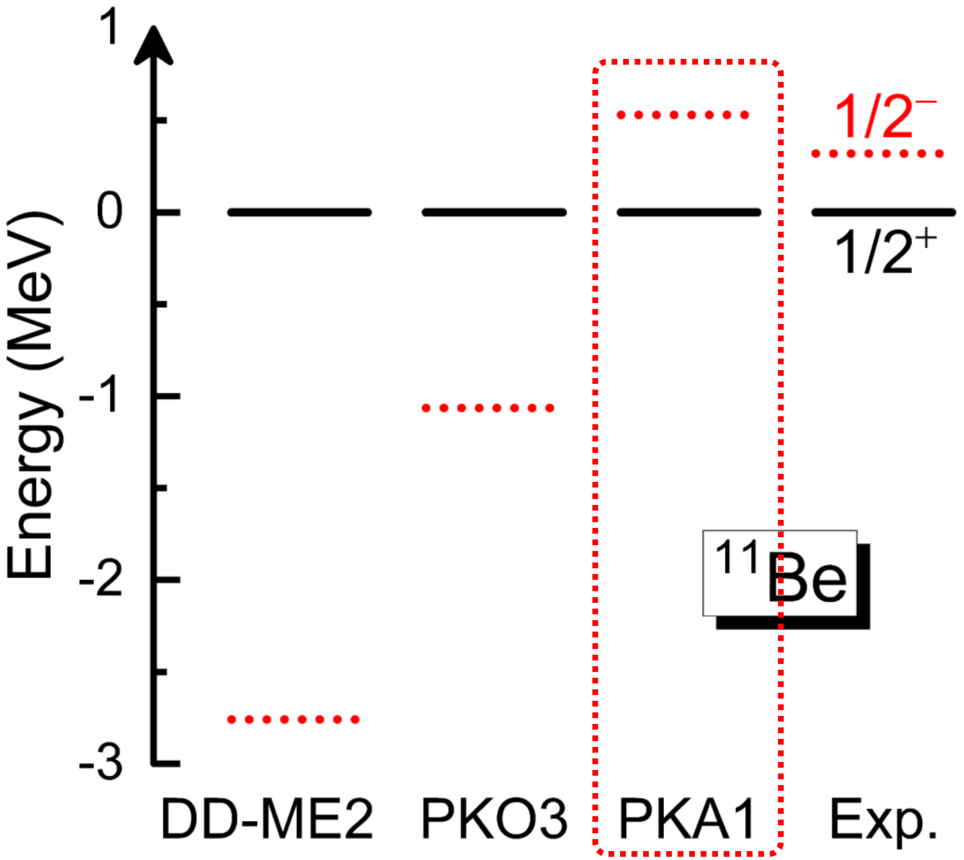
σ-S, ω-V, ρ-V, A-V, π-PV and ρ-T

RMF model and PKO2

$$\Gamma_{\rho\text{-T}} = \frac{1}{4M^2}\left(f_\rho\sigma_{\nu k}\vec{\tau}\partial^k\right)_x\left(f_\rho\sigma^{\nu l}\vec{\tau}\partial_l\right)_{x'} \propto Y_{20}$$

Enhance the deformation effects

Parity inversion



PLB. **858**, 139036 (2024)

PKA1

σ-S, ω-V, ρ-V

π-PV, ρ-T

PKO3

σ-S, ω-V, ρ-V

π-PV

DD-ME2

σ-S, ω-V, ρ-V

- Only PKA1 correctly describes the positive-parity g.s. for ^{11}Be, as well as the neighboring excited negative-parity ones

Microcosmic mechanism of parity inversion

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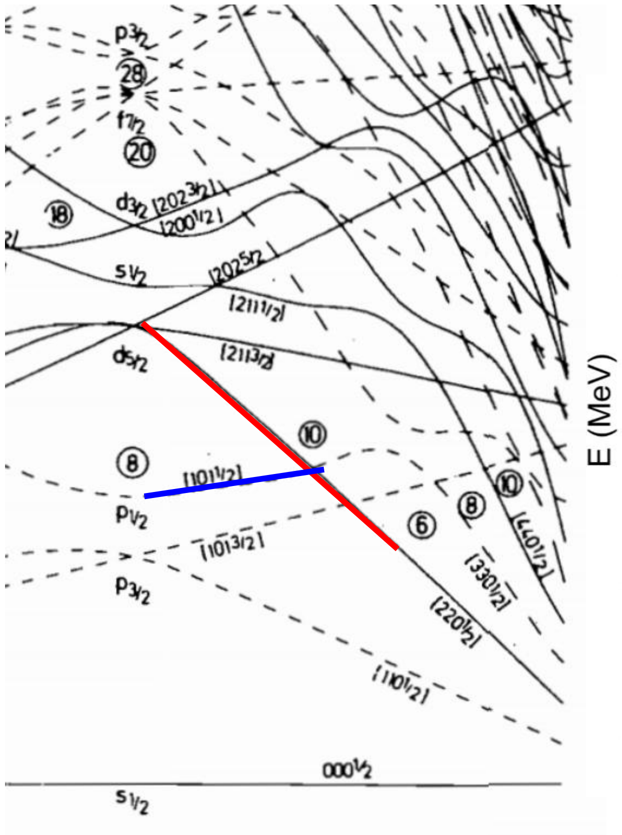
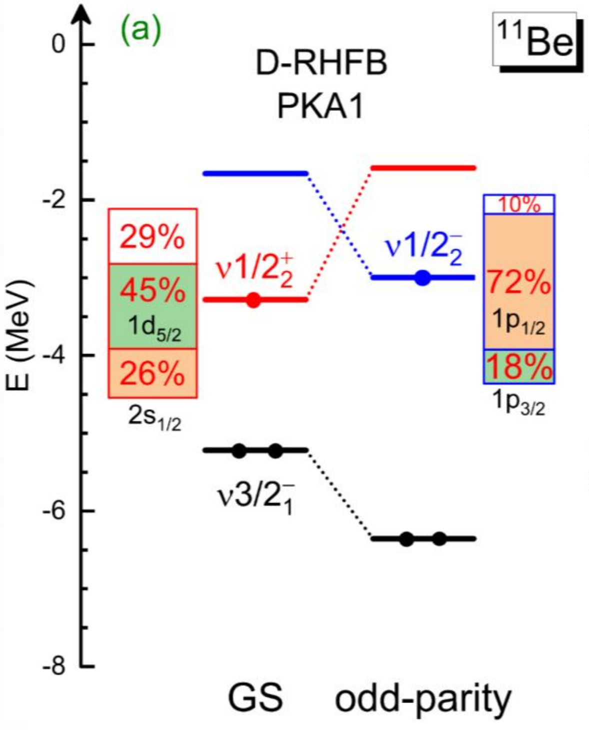
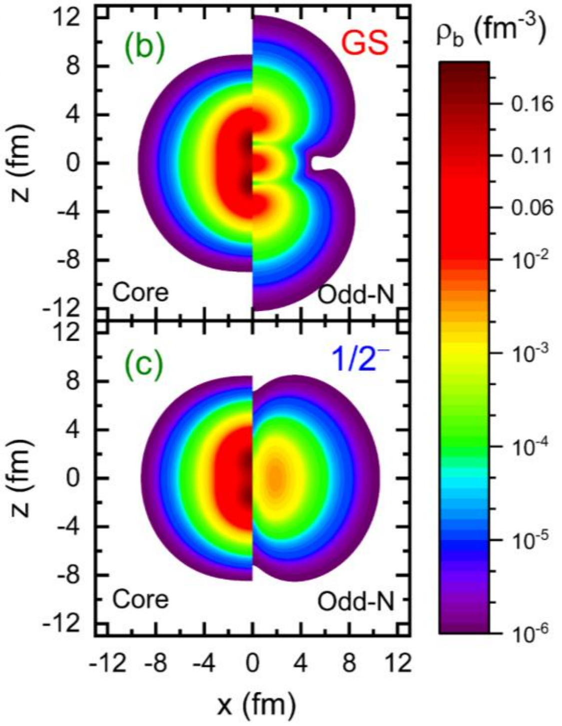




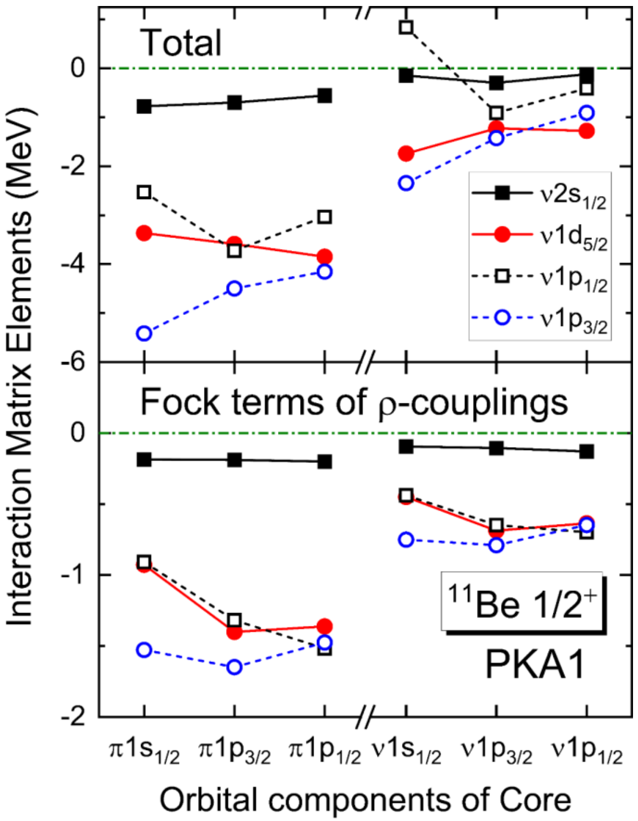
sd orbits mixing
prolate deformation



p orbits mixing
oblate deformation

PLB. **858**, 139036 (2024)

Two body interaction matrix

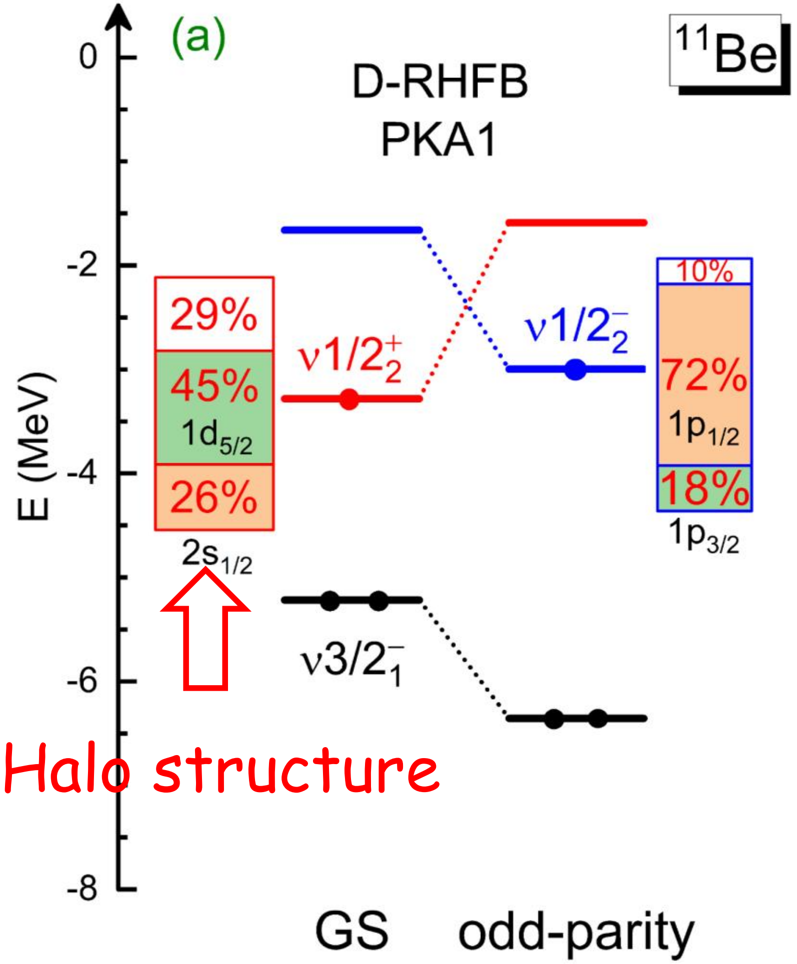


CPC. **47**, 044102 (2023)

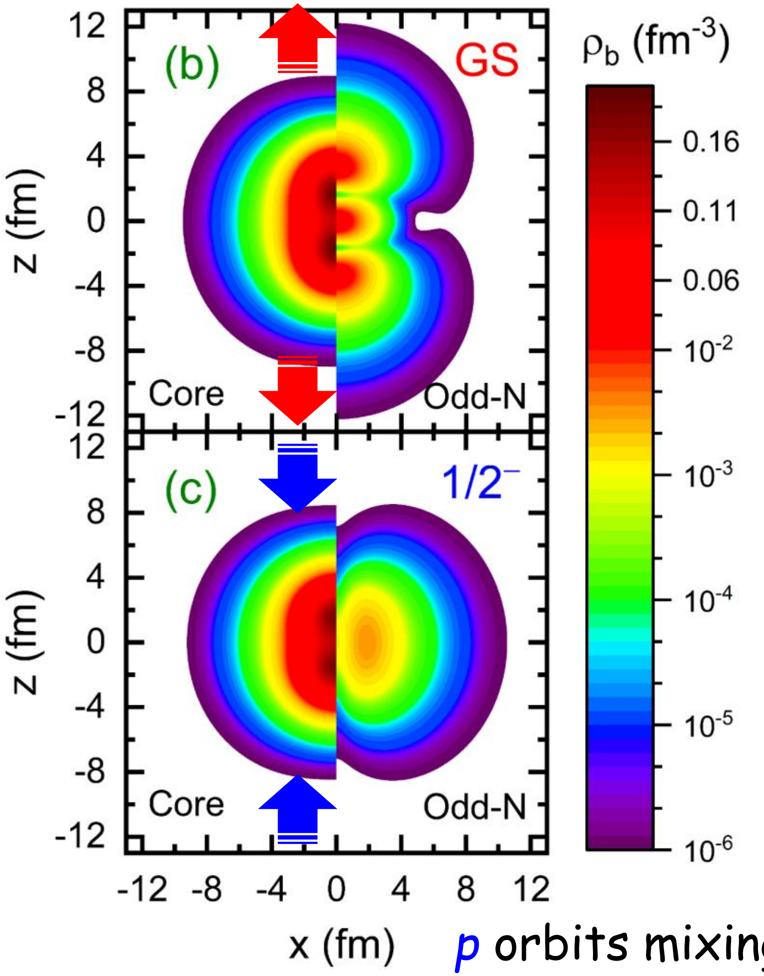
- ☐ *sd*-shell intrude to the *p*-shell ➔ large deformation (mean field level)
- ☐ ρ-T coupling enhance the coupling between the valence orbital and core

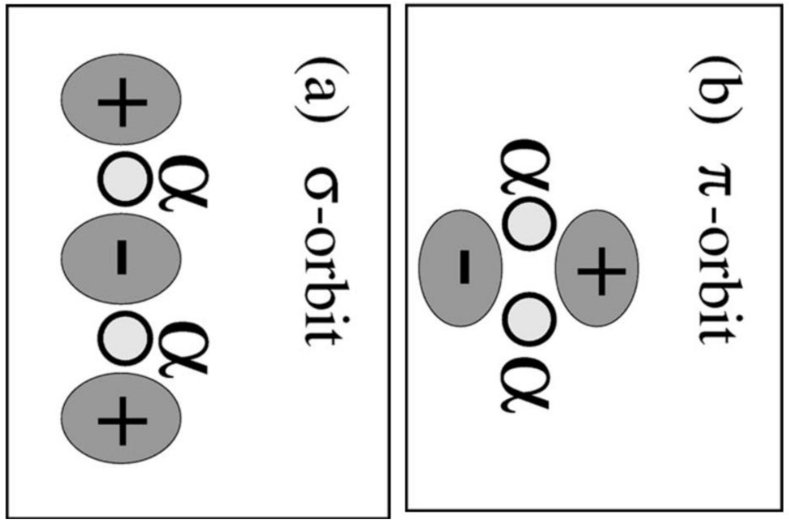
Parity Inversion and Halo structure

sd orbits mixing ➔ prolate deformation ➔enhance the distance between the cluster region



Halo structure



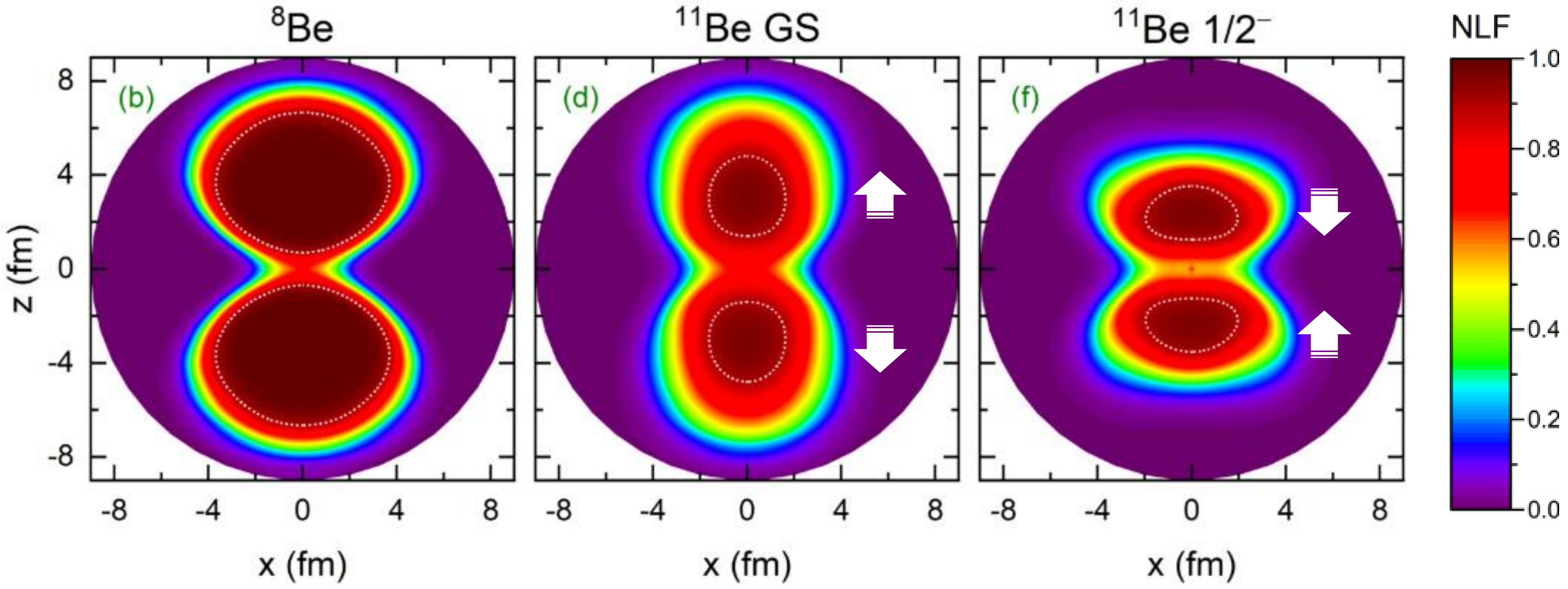


Y. Kanada-En'Yo *et al.* PRC. **66**, 024305 (2002)

p orbits mixing ➔ oblate deformation ➔reduce the distance between the cluster region

- Much more notable halo is illustrated in even-parity g.s., in contrast to odd-parity one

Parity Inversion and Cluster structure



PLB. **858**, 139036 (2024)

- Nucleon Localization Function (NLF) Reinhard, et.al., PRC **83**, 034312 (2011)
- Much more notable cluster structure is illustrated in positive-parity ground state, in contrast to negative-parity one

Summary

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- the coherence of the parity inversion, the halo, and the underlying cluster structure of ^{11}Be has been verified by using the D-RHFB model.
- There exists a significant mixing of the $2s_{1/2}$ and $1d_{5/2}$ waves in the positive parity valence orbit, as caused by the parity inversion.
- The mixing of wave function lead to the formation of the halo from the predominant $2s_{1/2}$ wave, and enhances the clustering signal in the even-parity ground state of ^{11}Be.

Thanks!