HALO-40

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A subtractive renormalization scheme on Chiral EFT and Halo EFT

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RUHR UNIVERSITÄT BOCHUM

OUTLINE

- Introduction
- Theoretical framework
- Results and discussion
- Summary

Nuclear force — Weinberg's seminal work

Nuclear forces from chiral lagrangians

Steven Weinberg [1]
Theory Group, Department of Physics, University of Texas, Austin, TX 78712, USA

Received 14 August 1990 PLB251(1990)288-292

EFFECTIVE CHIRAL LAGRANGIANS FOR NUCLEON–PION INTERACTIONS AND NUCLEAR FORCES

Steven WEINBERG*
Theory Group, Department of Physics, University of Texas, Austin, TX 78712, USA

Received 2 April 1991 NPB363(1991)3-18

- **Self-consistently** include many-body forces

$$V = V_{2N} + V_{3N} + V_{4N} + \cdots$$

- **Systematically improve** order by order (heavy baryon ChEFT)

$$V_{iN} = V_{iN}^{\rm LO} + V_{iN}^{\rm NLO} + V_{iN}^{\rm NNLO} + \cdots$$

- Scattering amplitude: **Schrödinger / Lippmann-Schwinger Eq.**

$$\left[\left(\sum_{i=1}^{A} -\frac{\nabla_i^2}{2m_N}\right) + V_{2N} + V_{3N} + V_{4N} + \ldots\right] |\Psi\rangle = E|\Psi\rangle$$

☐ Provide **a systematic and solid theoretical approach** to study the few-nucleon scattering

Renormalization issue of chiral force

- Renormalizability: important feature of an EFT

  - Iteration of the chiral NN potential within LSE

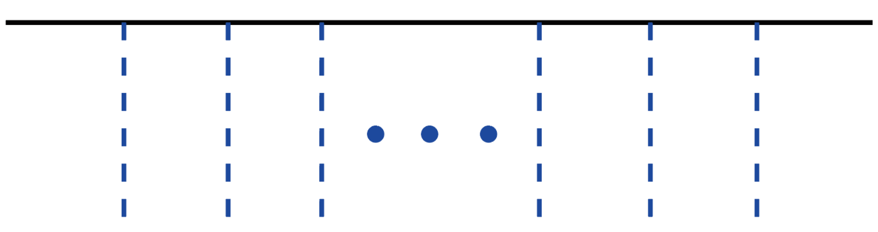
$$T\left(\boldsymbol{p}', \boldsymbol{p}\right) = V\left(\boldsymbol{p}', \boldsymbol{p}\right) + \int \frac{d^3 k}{(2\pi)^3} V\left(\boldsymbol{p}', \boldsymbol{k}\right) \frac{m_N}{\boldsymbol{p}^2 - \boldsymbol{k}^2 + i\epsilon} T(\boldsymbol{k}, \boldsymbol{p})$$

➡ **UV divergencies cannot be absorbed by contact terms!**

  - Leading order NN potential

$$V_{\mathrm{LO}} = C_S + C_T \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 - \frac{g_A^2}{4 f_\pi^2} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \frac{\boldsymbol{\sigma}_1 \cdot \boldsymbol{q}\, \boldsymbol{\sigma}_2 \cdot \boldsymbol{q}}{\boldsymbol{q}^2 + m_\pi^2}$$

  - Iterated one-pion exchange potential (ladder diagrams)



M. Savage, arXiv:nucl-th/9804034

$$\xrightarrow[\text{Spin-triplet}]{k \to \infty}$$

Logarithmic Divergence

$$\sim (Q m_N)^n$$

cannot be absorbed by C_S, C_T

Weinberg's proposal is inconsistent with renormalization, even at LO!

Deal with the renormalization issue

Possible solutions

- **Weinberg power counting**

  ✓ Chiral potential $V = V_{\rm LO} + V_{\rm NLO} + \ldots$ iterated in LSE

  ✓ Keep finite cutoff lower than hard scale: $\Lambda \leq \Lambda_{\chi PT} \sim 1\,\mathrm{GeV}$

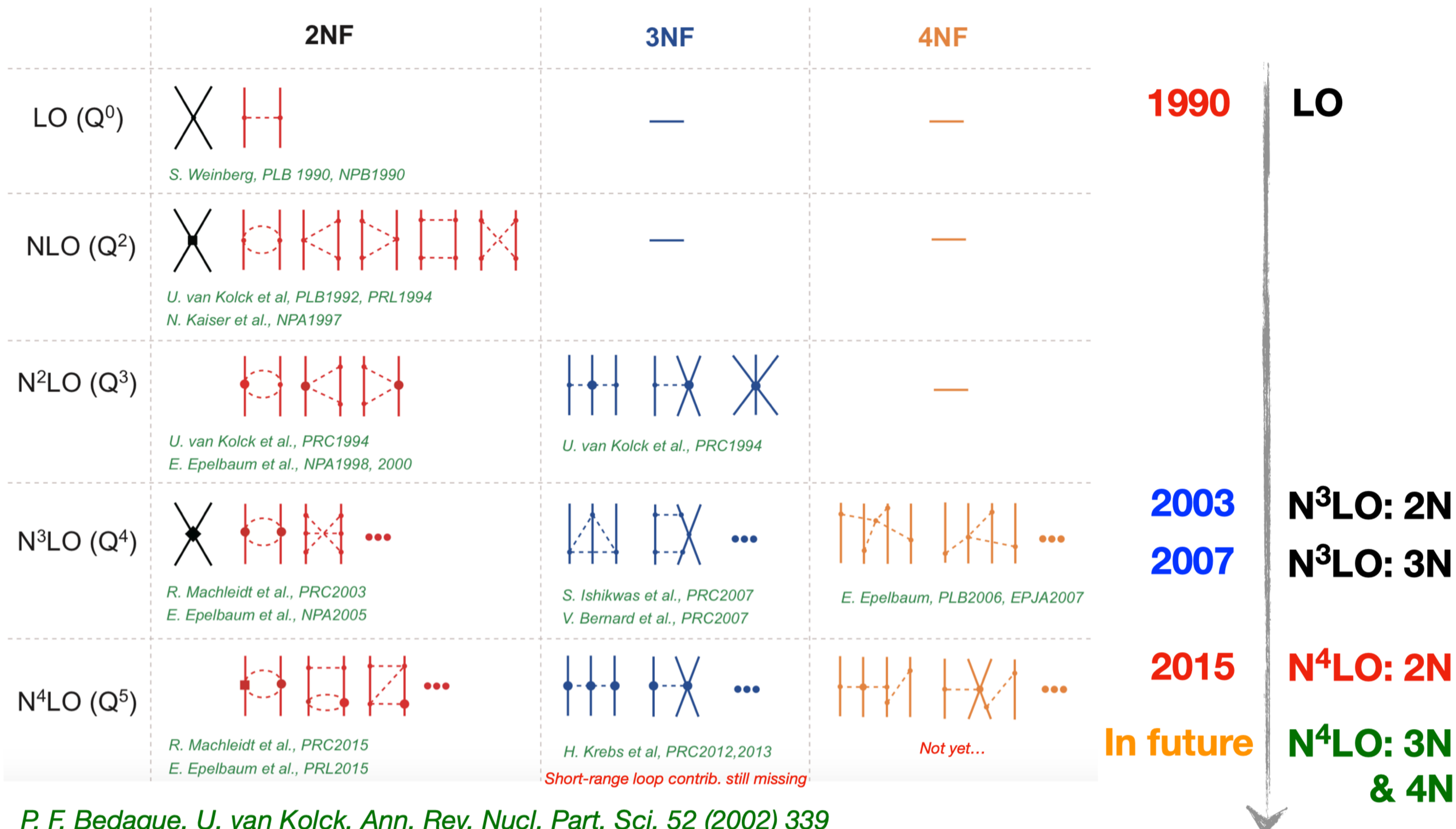
$$T\left(\boldsymbol{p}', \boldsymbol{p}\right) = V\left(\boldsymbol{p}', \boldsymbol{p}\right) + \int^{\Lambda} \frac{d^3 k}{(2\pi)^3} V\left(\boldsymbol{p}', \boldsymbol{k}\right) \frac{m_N}{\boldsymbol{p}^2 - \boldsymbol{k}^2 + i\epsilon} T(\boldsymbol{k}, \boldsymbol{p})$$

  ✓ WPC is consistent *G.P. Lepage, nucl-th/9706029; E.Epelbaum, J.Gegelia, Ulf-G. Meißner, NPB925(2017)161*

  - Renormalization achieved only at infinite chiral order
  - Towards a formal proof *A.M. Gasparyan, E. Epelbaum PRC105(2022)024001; 107 (2023) 044002*

Deal with the renormalization issue

- Possible solutions
  - **Weinberg power counting**



P. F. Bedaque, U. van Kolck, Ann. Rev. Nucl. Part. Sci. 52 (2002) 339
E. Epelbaum, H.-W. Hammer, Ulf-G. Meißner, Rev. Mod. Phys. 81 (2009) 1773
R. Machleidt, D. R. Entem, Phys. Rept. 503 (2011) 1

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✓ High precision chiral nuclear force

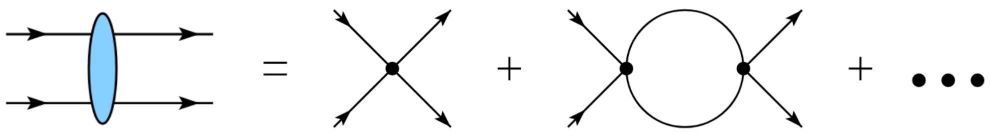
		Phenomenological forces			Non-Rel. Chiral nuclear force					
		Reid93	**AV18**	**CD-Bonn**	LO	NLO	NNLO	**N^{3}LO**	**N^{4}LO$^+$** include 4 ct. in F-waves	
	No. of para.	**50**	**40**	**38**	2+2	9+2	9+2	24+2 (3 redundant)	**24+3+4** (3 redundant)	
np	χ^2/datum *np 0-300 MeV*	**1.03**	**1.04**	**1.02**	94	36.7	5.28	1.27	**1.10**	**Idaho** *D. Entem, et al., PRC96(2017)024004*
					75	14	4.2	2.01	**1.06**	**Bochum/Juelich** *P. Reinert, et al., EPJA54(2018)86*

Deal with the renormalization issue

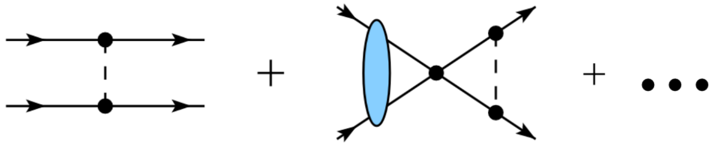
Possible solutions

- **Weinberg power counting**
- **Kaplan, Savage, and Wise (KSW) power counting**
  - ✓ Treat the exchange of pions perturbatively *D.B. Kaplan, M.J. Savage, M.B. Wise, PLB424(1998)390*

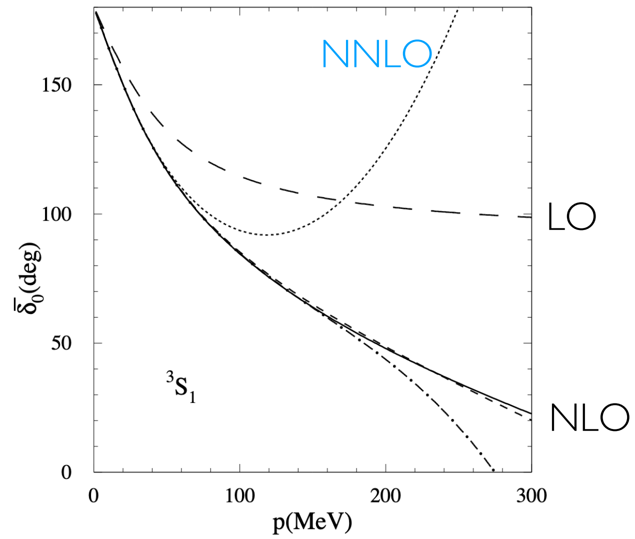
LO: only contacts

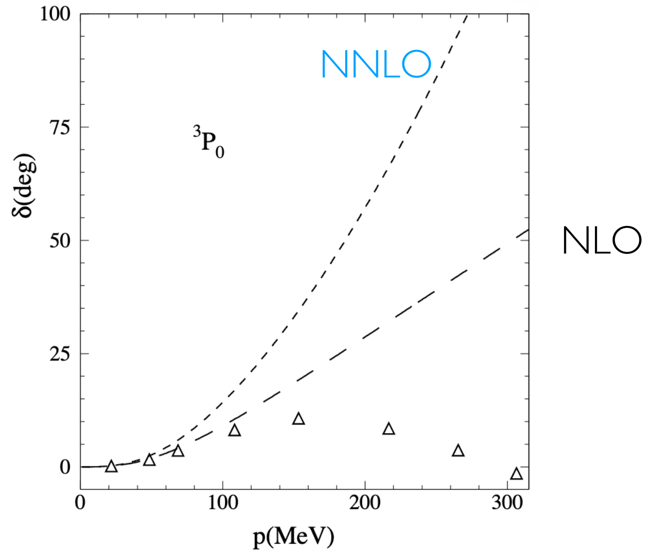


NLO: perturbative OPE



  - ✓ Fail to converge in certain spin-triplet channels *S. Fleming, et al., Nucl.Phys. A677 (2000) 313* *D.B. Kaplan, PRC102(2020)034004*





  - ✓ Perturbative pion scheme with re-organized contacts *Bingwei Long et al. CD2024, in progress*

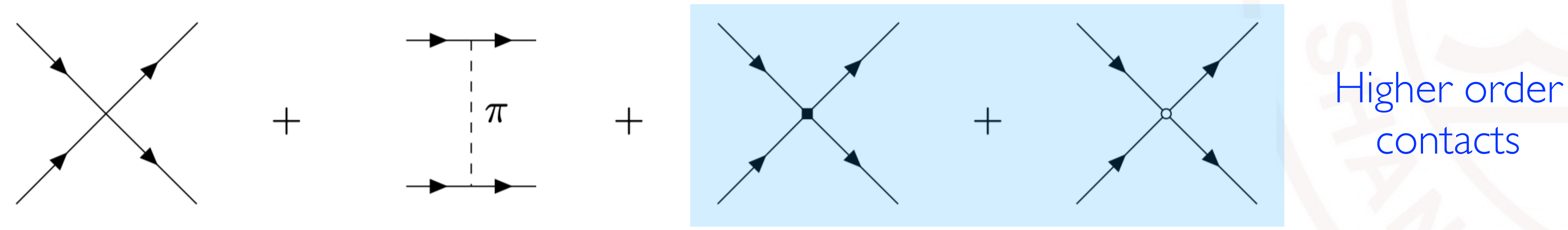
Deal with the renormalization issue

Possible solutions

- **Weinberg power counting**
- **Kaplan, Savage, and Wise (KSW) power counting**
- **Modified Weinberg power counting**
  - ✓ Promote the higher order contact terms to the lower chiral order

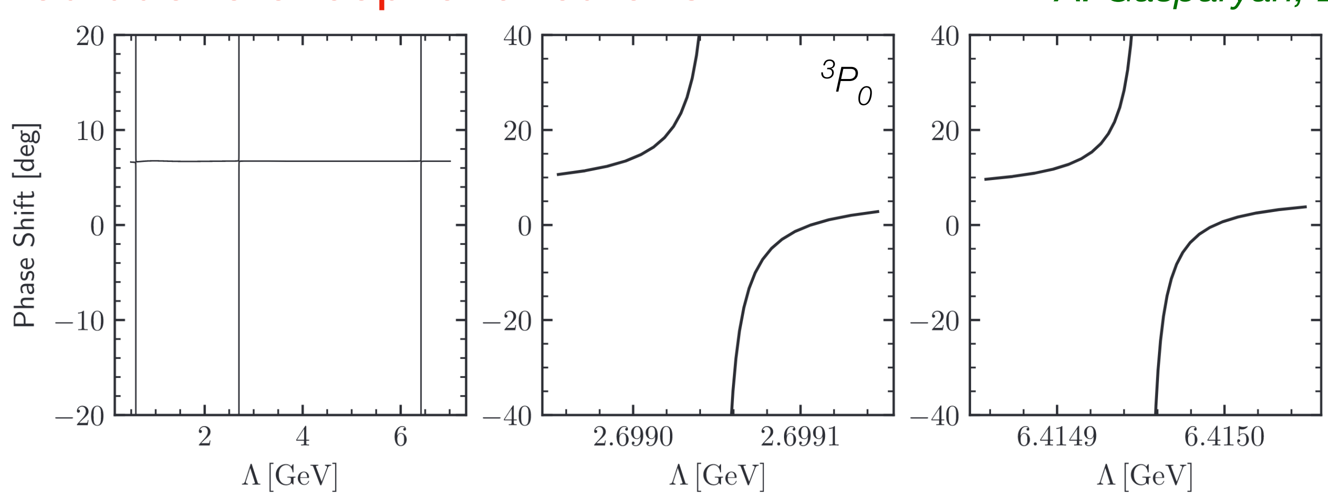
    A. Nogga, et al., PRC72(2005)054006 M. C. Birse, PRC74(2006)014003 M. Pavon Valderrama,PRC72(2005) 054002.
    B. Long and C.-J. Yang, PRC84(2011)057001 …
    H. W. Hammer, S. König, U. van Kolck, Rev. Mod. Phys. 92(2), 025004 (2020)
  - ✓ Renormalization achieved at every chiral order with $\Lambda \gg \Lambda_{\chi PT}$

$V^{[\mathrm{LO}]}_{\mathrm{MWPC}} =$



- ‣ cannot be fulfilled due to exceptional cutoffs

A. Gasparyan, E.Epelbaum, PRC107, 034001 (2023)



Elab=130 MeV

Deal with the renormalization issue

- Possible solutions
  - **Weinberg power counting**
  - **Kaplan, Savage, and Wise (KSW) power counting**
  - **Modified Weinberg power counting**

Still under debate !!!

Nuclear Forces for Precision Nuclear Physics: A Collection of Perspectives

Few-Body Syst (2022) 63:67

The collection represents the reflections of a vibrant and engaged community of researchers on the status of theoretical research in low-energy nuclear physics, …

- We tentatively propose a subtractive renormalization scheme for short range interaction in ChEFT and also extend to deal with P-wave halo state

Eur. Phys. J. A (2020) 56:152 **Few-Body Syst. (2021) 62:51**

Theoretical framework

Renormalization of EFTs

- **Effective field theories (EFTs) contain <u>all possible operators</u> compatible with underlying symmetries**


S.Weinberg


K.Wilson

- Effective Lagrangian with **bare** couplings

$$\mathcal{L} = K(\psi) + \sum_{i=1}^{\infty} g_i O_i(\psi)$$

- K : the kinetic part
- O_i: interaction terms

- Physical quantities in terms of bare couplings are always UV divergent!

- Renormalization: introducing counter terms

✓ Lagrangian with **renormalized** couplings and **counter terms**

$$\mathcal{L} = K(\psi) + \sum_{i=1}^{\infty} \left[g_i^R(\mu) + \sum_{j=1}^{\infty} \delta g_i^j \left(g_1^R(\mu), g_2^R(\mu), \ldots, g_N^R(\mu), \mu_1, \ldots, \mu_N, \Lambda \right) \right] O_i(\psi)$$

renormalization scales μ : $\{\mu_1, \mu_2, \cdots, \mu_N\}$ dependent on the regularization scheme

Renormalization of EFTs

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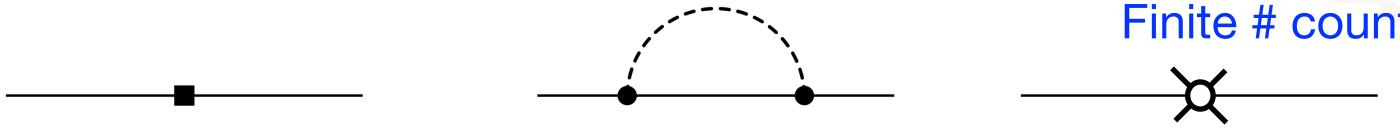
renormalization scales $\mu : \ \{\mu_1, \mu_2, \cdots, \mu_N\}$ dependent on the regularization scheme

- Works very well & clear in the perturbative EFT calculations

✓ e.g. nucleon mass in ChPT



- <u>Difficulties with Few-Nucleon System in chiral EFT</u>

✓ Sum up an infinite number of diagrams via LSE → Infinite # counter terms in Lag.

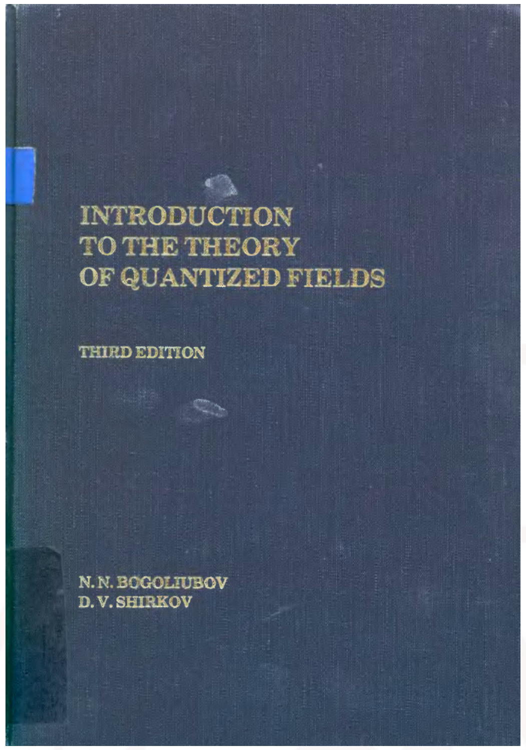
BPHZ-inspired subtractive renormalization

- **B**ogoliubov, **P**arasiuk, **H**epp, **Z**immermann renormalization

  - **A subtraction scheme:** renormalizes the Feynman diagrams by subtracting **sub-divergences and overall divergences** and treats all coupling constants as renormalized finite parameters

    ✓ via Zimmermann's forest formula

  - **These subtractions correspond to the counter terms in Lagrangian**



N. Bogoliubov and O. Parasiuk, Acta Math. 97 (1957) 227–266
K. Hepp, Comm. Math. Phys. 2, 301 (1966)
W. Zimmerman, Comm. Math. Phys. 15, 208 (1969)

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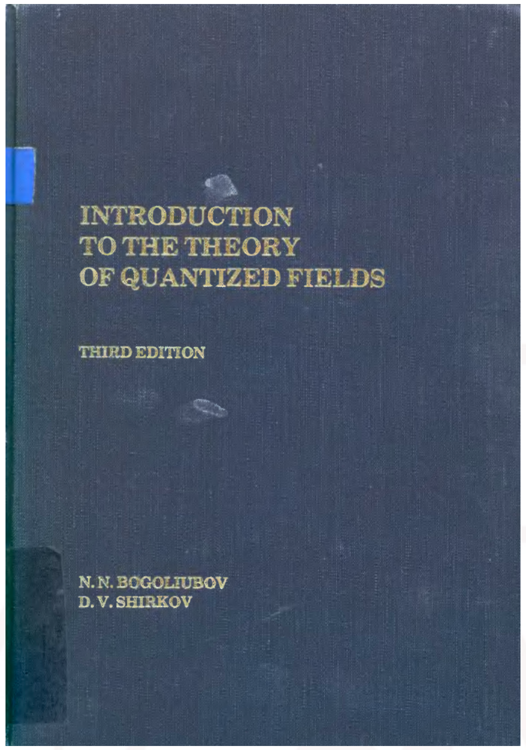

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K. Hepp, Comm. Math. Phys. 2, 301 (1966)
W. Zimmerman, Comm. Math. Phys. 15, 208 (1969)

- Inspired by BPHZ method, we tentatively propose a subtraction method to deal with the iteration part for **singular potential**

$$T_S = V_S + \hbar V_S G T_S$$

Expand Green function:
E = –E$_\mu$

$$\boxed{G_e = \sum_{n=0}^{N} \frac{1}{n!} (E+E_\mu)^n \frac{d^n G(E)}{(dE)^n}\bigg|_{E=-E_\mu}}$$

$$T_S^r = V_S + \hbar V_S \left(G - G_e\right) T_S^r$$

BPHZ-inspired subtractive renormalization

Renormalized T-matrix:

$$T_S^r = V_S + \hbar\, V_S\, (G - G_e)\, T_S^r$$

Equivalently rewritten as:

$$T_S^r = \tilde{V} + \hbar\, \tilde{V}\, G\, T_S^r$$

$$\tilde{V} = V_S - \hbar\, V_S\, G_e\, \tilde{V}$$

Modified effective potential $\tilde{V}$:

all necessary counter terms absorbing overall and sub-divergences!

We subtract all UV divergences, and obtain a renormalizable T matrix, which allow us to take the limit $\Lambda \to \infty$

Results and discussion

Toy model for two-body scattering

- "Underlying" potential of two-body scattering

$$V(r)=\frac{\alpha\left(e^{-m_1 r}-e^{-Mr}\right)}{r^3}+\frac{\alpha\left(m_1-M\right)e^{-m_1 r}}{r^2}+\frac{\alpha\left(M-m_1\right)^2 e^{-m_2 r}}{2r} -\frac{1}{6}\alpha\left(2m_1-3m_2+M\right)\left(M-m_1\right)^2 e^{-m_1 r}$$

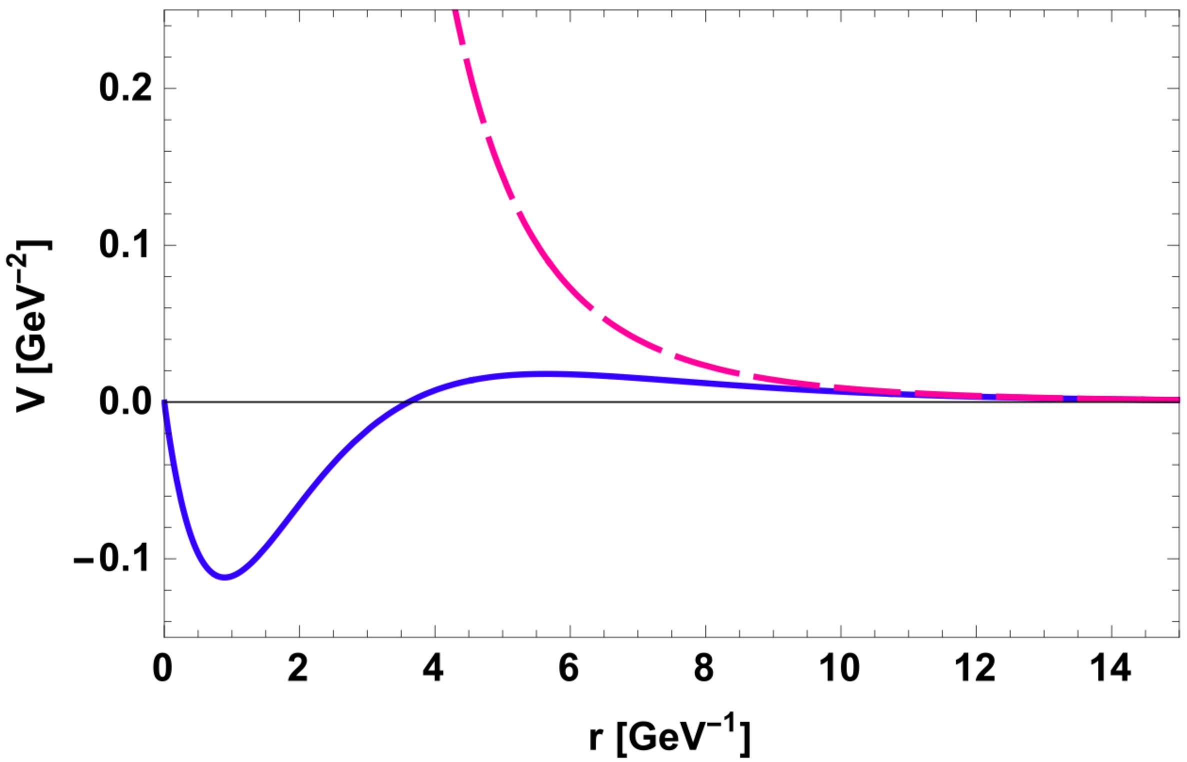
with $\alpha = 50\,\mathrm{GeV}^{-2}$, $M = 0.1385$ GeV, $m_1 = 0.75$ GeV and $m_2 = 1.15$ GeV

- $V(r)$ vanishes for $r \to 0$, and behaves as $-\alpha e^{-Mr}/r^3$ for large r

- Leading order approximation in EFT

$$V_{\mathrm{LO}} = C\,\delta^3(\boldsymbol{r}) - \frac{\alpha\, e^{-M\,r}}{r^3}$$

Singular potential



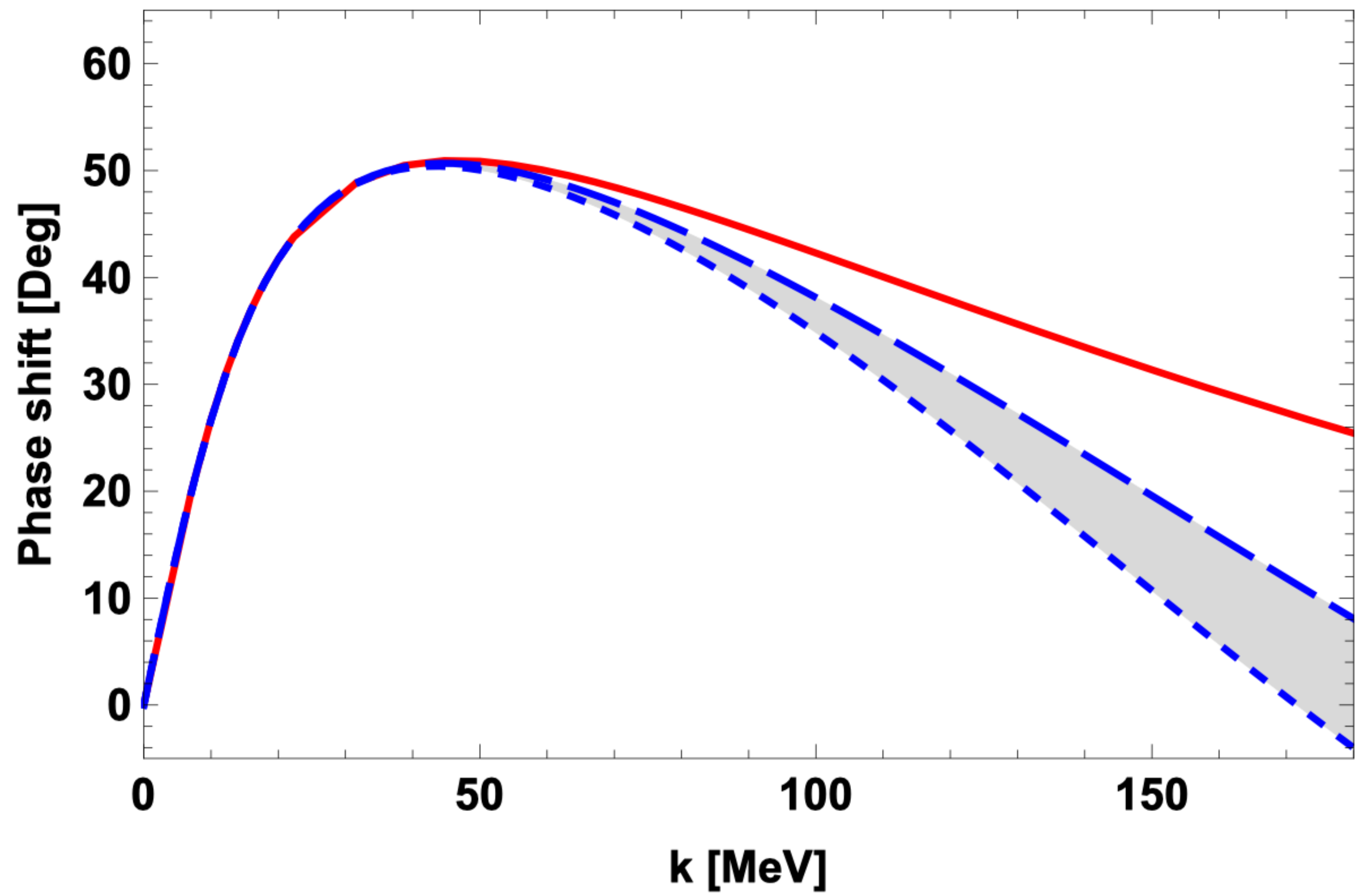
Eur. Phys. J. A (2020) 56:152

Toy model: S-wave phase shift

- S-wave T-matrix via the subtractive renormalization
  - Lippmann-Schwinger equation

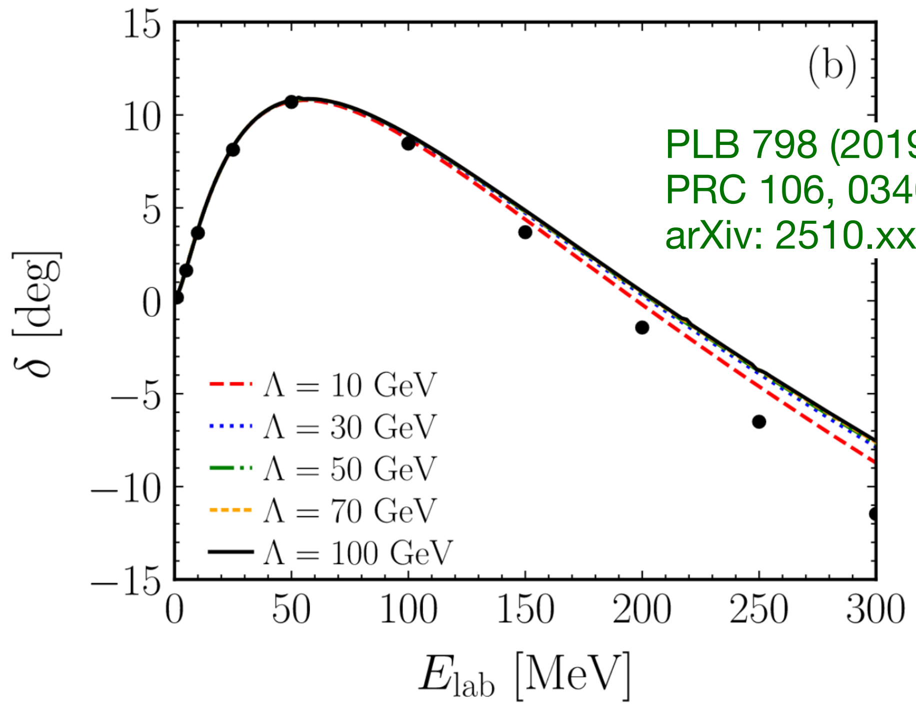
$$T_{\mathrm{LO}}(p',p) = V_{\mathrm{LO}}(p',p) + \int_0^\infty \frac{l^2 dl}{(2\pi)^3} V_{\mathrm{LO}}(p,l) \left[\frac{1}{q^2 - l^2 + i\epsilon} - \frac{1}{-\mu^2 - l^2 + i\epsilon}\right] T_{\mathrm{LO}}(l,p')$$

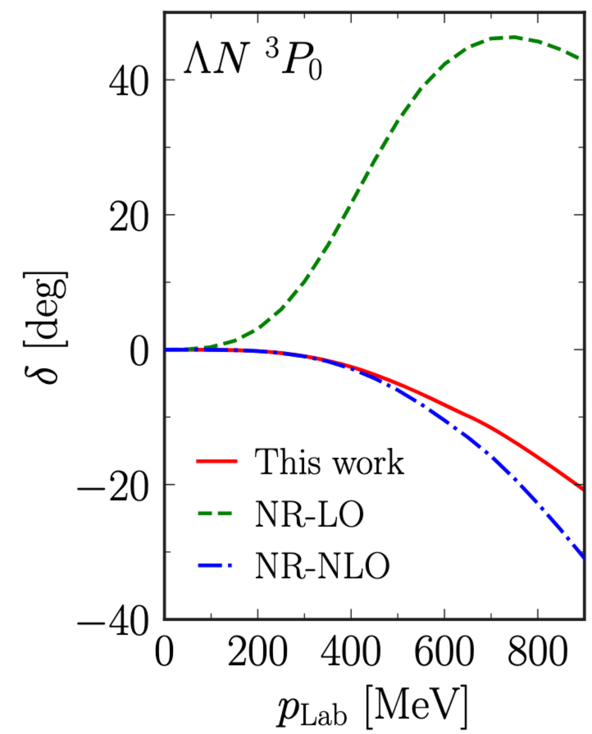
  - Subtractions of loop integrals are performed at $E = -\mu^2/(1\,\mathrm{GeV})$
  - We fix **the renormalized coupling C_R** by the low-energy phase shift of underlying toy-model potential $\mu \sim 300 - 700$ MeV, $\Lambda \to \infty$

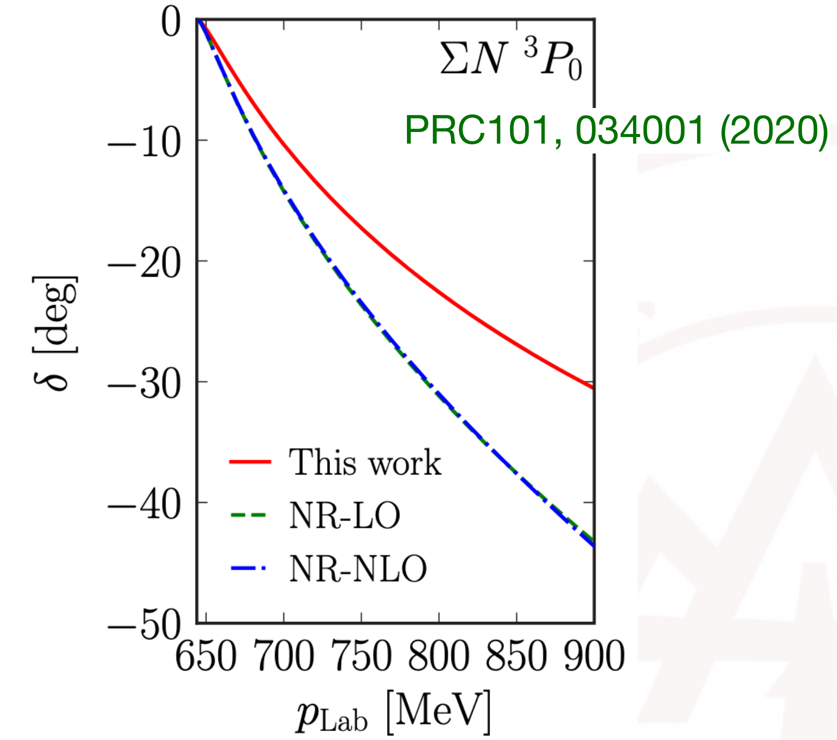


Extend the BPHZ-inspired subtractive renormalization

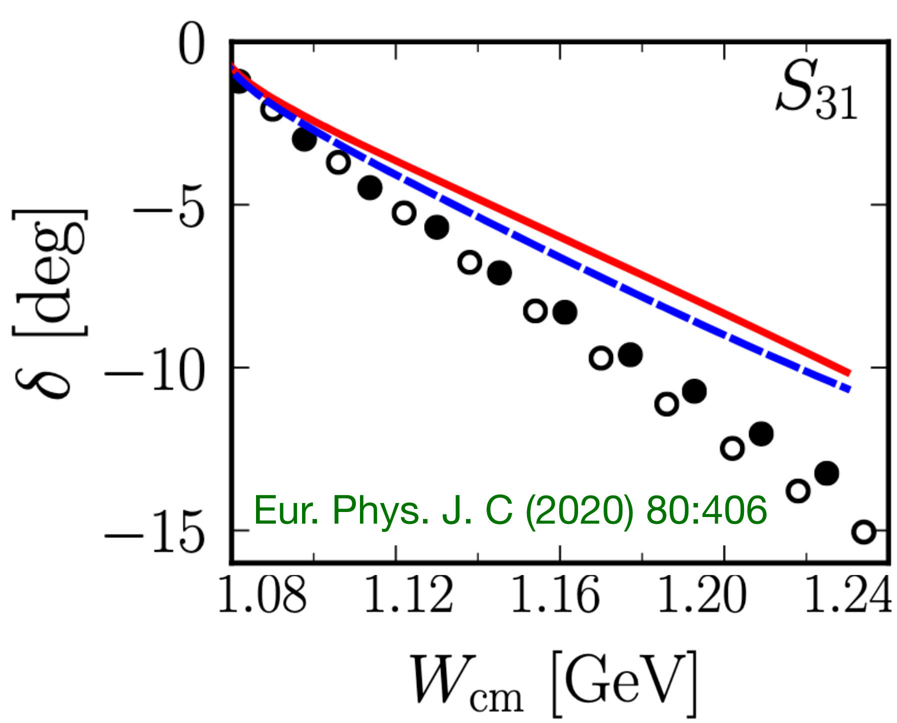
- NN and YN interactions

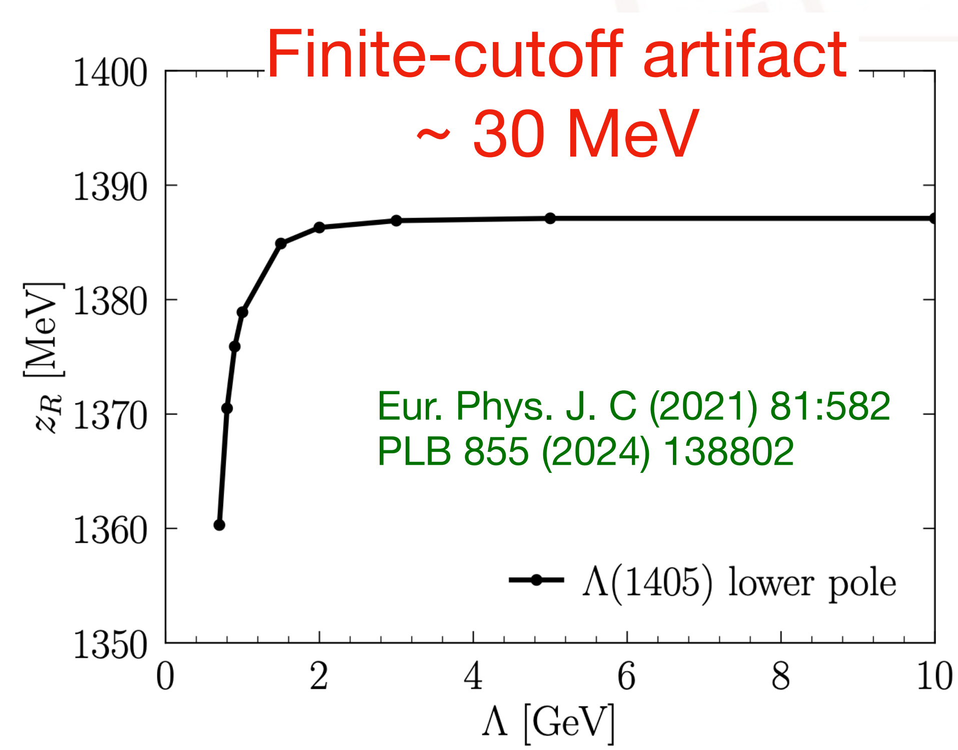






- πN and $\bar{K}N$ interactions and $\Lambda(1405)$

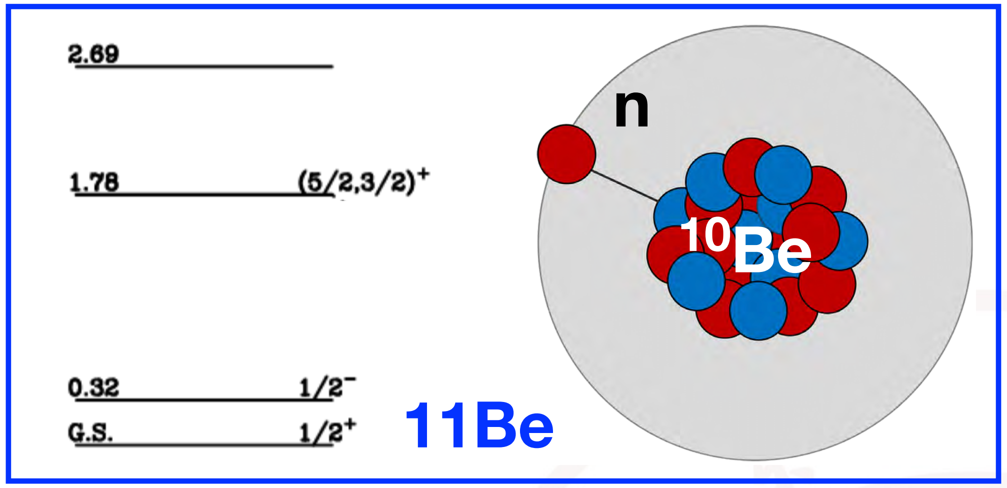




P-wave halo state

One-nucleon Halo Nuclei

- S-wave weakly bound state
  - ✓ ^{11}Be g.s. $1/2^+$: **s-wave halo** with $s_n \approx 500\ \mathrm{keV}$
- P-wave weakly bound state
  - ✓ ^{11}Be l.e.s. $1/2^-$: **p-wave halo** with $s_n \approx 180\ \mathrm{keV}$



e.g. N. Talmi and I. Unna, Phys. Rev. Lett. 4, 469 (1960)
K. Riisager, Rev.Mod.Phys. 66 (1994) 1105-1116
H.-W. Hammer, D.R. Phillips, NPA865 (2011) 17–42 …

Fine-tuning ERE parameters for shallow bound states

- S-wave amplitude

$$f_0(k) = \frac{1}{-1/a_1 + \frac{1}{2} r_1 k^2 - ik \quad \sim 0} \longrightarrow 1/a_0 \sim M_{\rm lo}$$

Large value

- P-wave amplitude

$$f_1(k) = \frac{k^2}{-1/a_1 + \frac{1}{2} r_1 k^2 - ik^3 \quad \sim 0} \longrightarrow 1/a_1 \sim M_{\rm lo}^3, \quad r_1 \sim M_{\rm lo}$$

unnaturally large   nature size

P-wave halo state needs more fine-tuning

Renormalized P-wave amplitude with BPHZ

- In low-energy EFT with (only) contact terms
  - P-wave potential $V(p',p) = C_2\, p'p + C_4\, p'p\left(p'^2 + p^2\right) + \dots$
  - Amplitude $$T(p',p) = V(p',p) + m\int_0^\Lambda \frac{l^2 dl}{(2\pi)^3}\frac{V(p,l)\,T(l,p')}{q^2 - l^2 + i\epsilon}$$

The on-shell amplitude $T(q) \equiv T(q,q)$:

$$\frac{q^2}{T(q)} = -I(q)\,q^2 - I_3 + \frac{(C_4\,I_5 - 1)^2}{C_4\left(k^2\left(2 - C_4 I_5\right) + C_4\,I_7\right) + C_2}$$

where the integrals I_n and $I(k)$ are defined via

Power-like UV Divergences!

$$I_n = -m\int_0^\Lambda \frac{l^2 dl}{(2\pi)^3} l^{n-3} = -\frac{m\Lambda^n}{(2\pi)^3 n}, \quad n = 1, 3, 5, \dots$$

$$I(q) = \int_0^\Lambda \frac{l^2 dl}{(2\pi)^3}\frac{m}{q^2 - l^2 + i\epsilon} = I_1 - i\frac{m\,q}{(4\pi)^2} - \frac{m\,q}{2(2\pi)^3}\ln\frac{\Lambda - q}{\Lambda + q}$$

Renormalized P-wave amplitude with BPHZ

- Separate out power-like UV divergences

$$I_n^S = -m\int_0^{\mu_n}\frac{l^2 dl}{(2\pi)^3} l^{n-3} - m\int_{\mu_n}^{\Lambda}\frac{l^2 dl}{(2\pi)^3} l^{n-3} \equiv I_n^R(\mu_n) + \Delta_n(\mu_n)$$

μ_n: subtraction scales

$$I^S(q) \equiv I_1^R(q,\mu_1) - \Delta_1(\mu_1)$$

  - Replacing (integrals) I_n, $I(q)$ with $I_n^R(\mu_n), I_1^R(q,\mu_1)$
  - Replacing (bare) C_2, C_4 with renormalized C_2^R, C_4^R
  - Safely take the limit $\Lambda \to \infty$

Renormalized T-matrix

- Effective range expansion
  - C_2^R, C_4^R fixed by the scattering length and effective range

$$k^3\cot\delta = -\frac{1}{a} + \frac{1}{2}rk^2 - \frac{k^4}{2\pi}\frac{3(4\mu_1+\pi r)^2}{6\pi a^{-1} - 4\mu_3^3 + 3k^2(4\mu_1+\pi r)}$$

  - To generate weakly bound state, we set $\mu_3 \sim M_{\rm hi}, \mu_1 \sim M_{\rm hi}$ or $M_{\rm lo}$
  - **In this way, we do not need to introduce a dimer field**

P.F. Bedaque, H.W. Hammer, U. van Kolck, PLB 569, 159–167 (2003)

Summary

- **A BPHZ-inspired subtractive renormalization method** is proposed to deal with the singular potential in few-body system
  - All divergences of the amplitude can be systematically removed
  - → The renormalizied T-matrix is obtained $(\Lambda \sim \infty)$
  - Verified by the toy-model and applied to the NN, YN interactions
  - Investigate the weakly bound P-wave halo state
  - → No need to include a dimer field to achieve the renormalization
- Next, we plan to employ the proposed **subtractive renormalization method** to chiral NN&YN interactions up to higher orders

Thank you for your attention!

Additional slides

Renormalization group

Let a physical quantity be given in some theory by

$$f(x) = \frac{x}{1 - \hbar x},$$

where x is a parameter and $\hbar$ controls the quantum corrections.

- If $|x| < 1$ we can expand $f(x)$ in Taylor series around $x = 0$, approximating the exact function by the sum of first N terms

$$f(x) \approx S_N = x + \hbar x^2 + \hbar^2 x^3 + \cdots + \hbar^{N-1} x^N .$$

- For $|x| > 1$ our expansion leads to a divergent series. In this case we can try an alternative way by rewriting the function $f(x)$ identically and expanding in a different way:

$$\begin{aligned} f(x) &= \frac{x}{1-\hbar x} \equiv \frac{x}{1 - \hbar\mu x - \hbar x(1-\mu)} \\ &= \frac{1}{1-\mu}\frac{x_\mu}{1-\hbar x_\mu} = \frac{x_\mu}{1-\mu}\left(1 + \hbar x_\mu + \hbar^2 x_\mu^2 + \cdots\right), \end{aligned}$$

where $x_\mu = x(1-\mu)/(1-\hbar\mu x)$.

- Sum of any finite number of terms depends on scale μ
- Convergence properties of the series crucially depends on the choice of μ
- If x = 2, this series converges only if μ > 3/4

Slide from J. Gegelia

Renormalization group

The advantage of using RG in perturbative calculations is based on the μ-dependence of the sum of any finite number of terms.

This example demonstrates essential features of RG applied to perturbative calculations:

Exploiting the scale dependence of finite sums of the perturbative series one chooses such values of the scale parameter which leads to optimal convergence of perturbative series.

Numerous applications ... pQCD being the best known example.

Slide from J. Gegelia