Exchange correlation effects in nuclear novel phenomena

Long, Wen Hui (longwh@lzu.edu.cn)

Lanzhou University

International Symposium Commemorating the 40th Anniversary of the Halo Nuclei

October 12 - 18, 2025 | Capital Hotel, Beijing

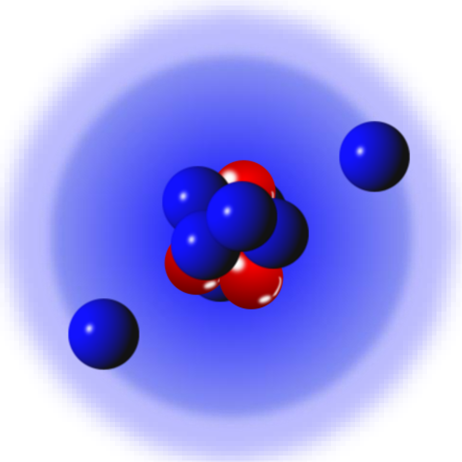
自強不息

獨樹一幟

OUTLINE

- **Introductions**
  - Nuclear novel phenomena and the challenges
  - Relativistic Hartree-Fock (RHF) theory and the extensions
- **Recent developments of the RHF theory**
  - Deformed relativistic Hartree-Fock-Bogoliubov (D-RHFB) model
  - Configuration-Interaction relativistic Hartree-Fock (CI-RHF) model
- **Exchange correlation effects**
  - Spherical halo phenomena at the drip line of Ce isotopes
  - Deformed halo structure in ^{19}C
  - Nature of the island of inversion exhibited by ^{32}Mg
- **Conclusions**

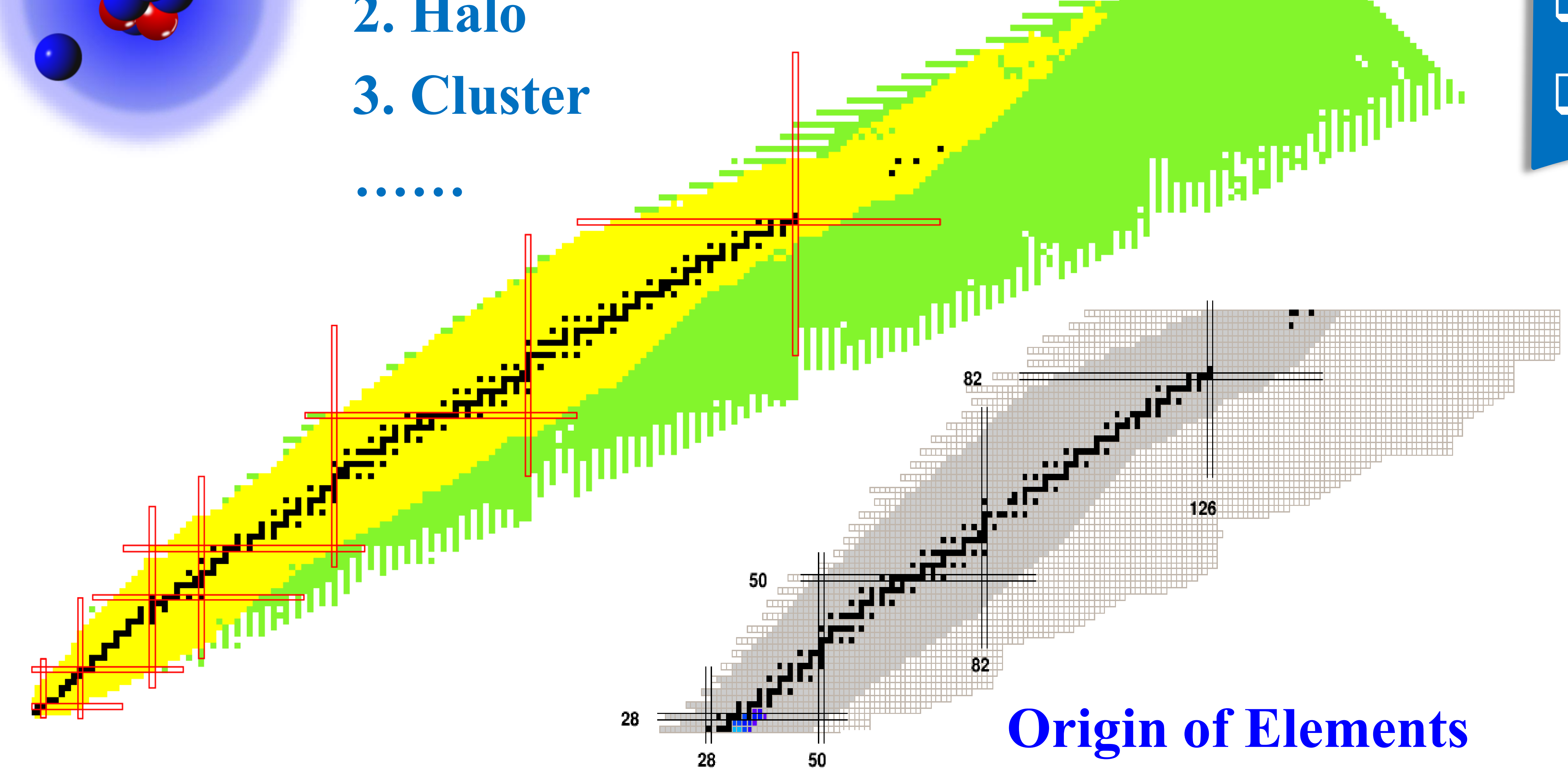
Unstable Nuclei



Novel Phenomena

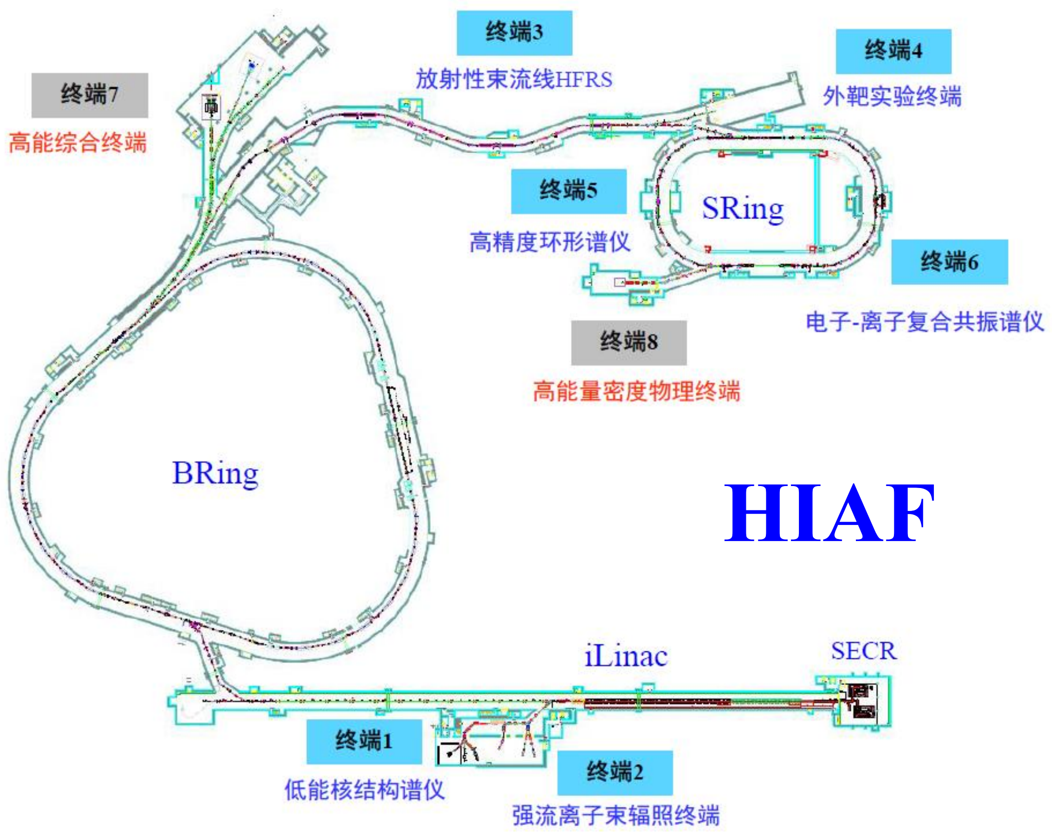
1. Island of inversion
2. Halo
3. Cluster

……



Origin of Elements

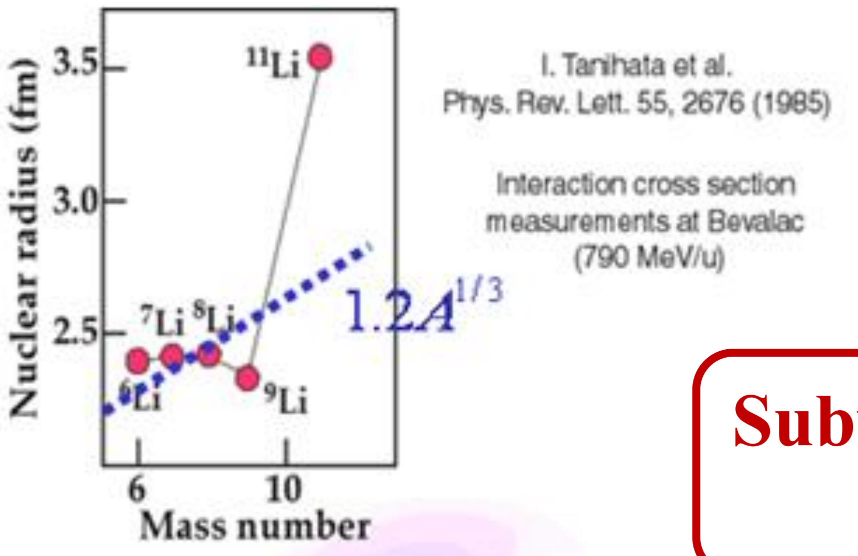
- Nucleon-Nucleon Interaction
- Origin of Heavy Elements
- Island of Superheavy nuclei

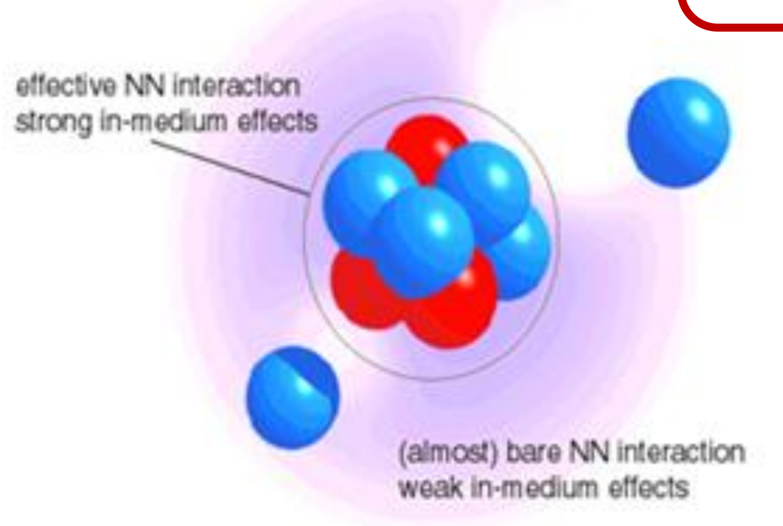


Call for comprehensive understanding on the nature of unstable nuclei

Halo and the challenges

- The first candidate of halo nuclei: ^{11}Li





I. Tanihata et al., PRL **55** (1985) 2676

Extraordinary matter radius

↓

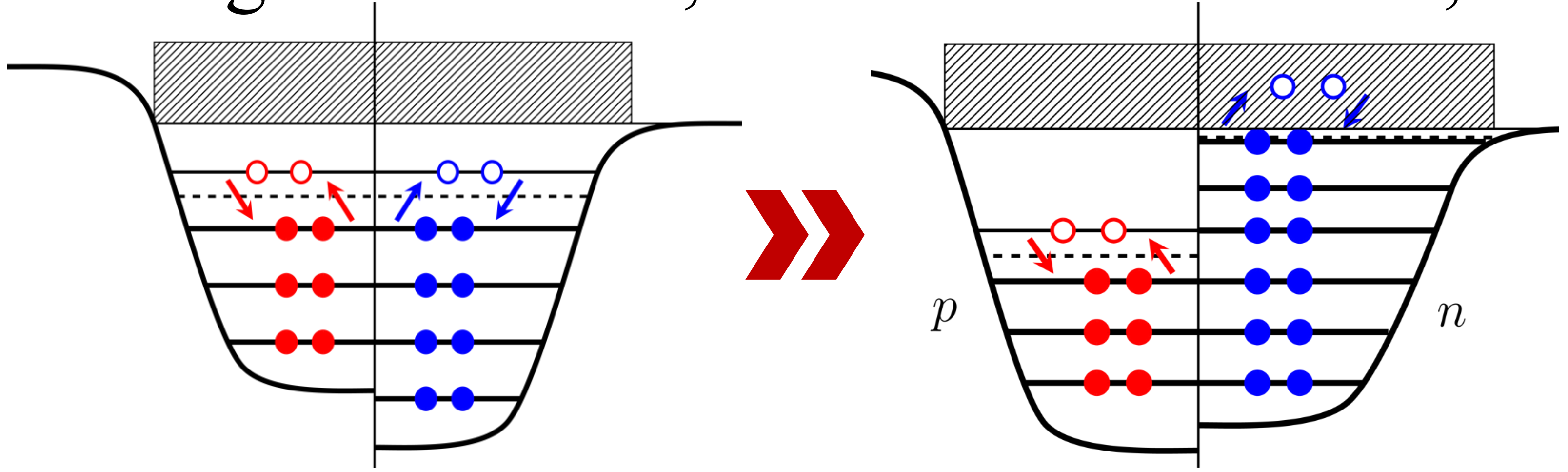
Subvert the conventional knowledge

↑

Dilute matter distribution

1. **Weak binding mechanism**

   Pairing correlations, the continuum effects, …



Meng et al., PPNP **57** (2006) 470

2. **Nuclear deformation**

   Deformed Halo, cluster, island of inversion,…

3. **Nuclear many-body state**

   Mean field approach, AMP, GCM, SM, …

How nucleons spontaneously form an atomic nucleus

Nuclear force & Nuclear many-body method

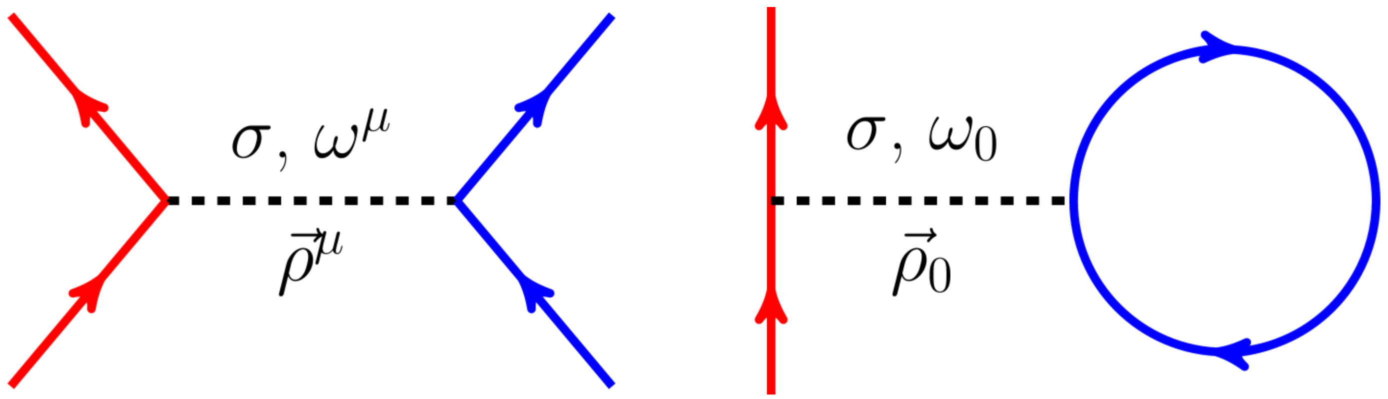
Relativistic Hartree-Fock (RHF) theory

- **Relativity**: Interplay between **strong scalar attraction** and **vector repulsion**

  Natural spin-orbit coupling, the origin of pseudo-spin symmetry and spin symmetry, …

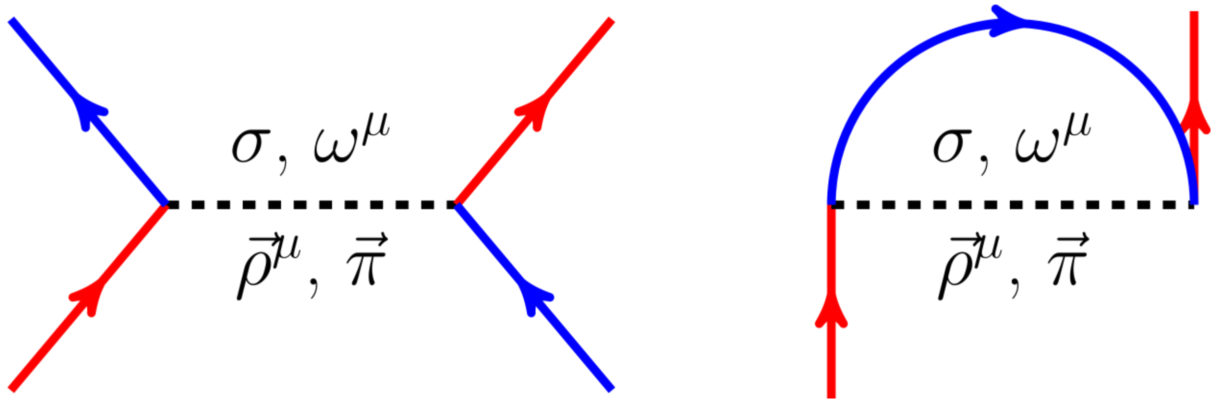
  Meng et al., PPNP **57**, 470 (2006); Ginocchio, PRL **78**, 436 (1997); Meng et al., PRC **58**, R628 (1998); Zhou et al., PRL **91**, 262501 (2003).

Hartree terms



***σ*-S, *ω*-V**, $\vec{\rho}$-V, A-V

Fock terms



***σ*-S, *ω*-V**, $\vec{\rho}$-V, A-V; **$\vec{\pi}$-PV, $\vec{\rho}$-T (*ρ*-VT)**

- **Fock terms**: pure exchange degrees of freedom (DoF) such as ***π*-PV** and ***ρ*-T (*ρ*-VT)**

  Impact the modeling of nuclear binding, **nuclear tensor force**, … PRC **100**, 051301(R) (2019)

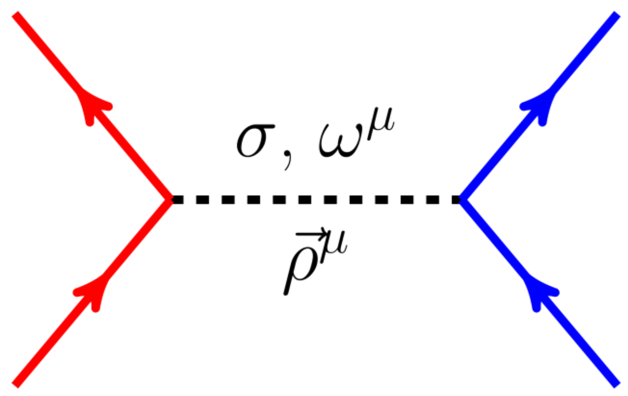
  PRC **91**, 034326 (2015); CPC **42**, 024101 (2018); PRC **98**, 034313 (2018); PRC **101**, 064306 (2020)

Exchange correlation effects in nuclear novel phenomena?

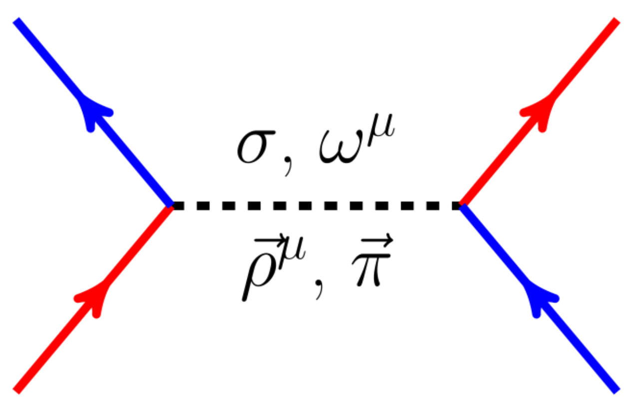
Effective RHF Lagrangians

- Nuclear in-medium effects
  1. Non-linear self-couplings of meson fields
  2. **Density-dependent coupling strengths**

  Effects of many-body correlations in nuclei



Hartree terms



Fock terms

- RHF Lagrangians PKO***i*** and PKA1

  similar quantitative accuracy as popular DFT

WHL, Giai, Meng, PLB **640**, 150 (2006); WHL, Sagawa, Giai, Meng, PRC **76**, 034314 (2007); EPL **82**, 12001 (2008).

PKO2

σ-S, ω-V
ρ-V, A-V

Conventional DoF

»

PKO1, PKO3

σ-S, ω-V, ρ-V, A-V $+\ \pi$-PV

Exchange DoF π-PV

»

PKA1

σ-S, ω-V, ρ-V, A-V $+$ π-PV, ρ-T, ρ-VT

Exchange DoF π-PV & ρ-T

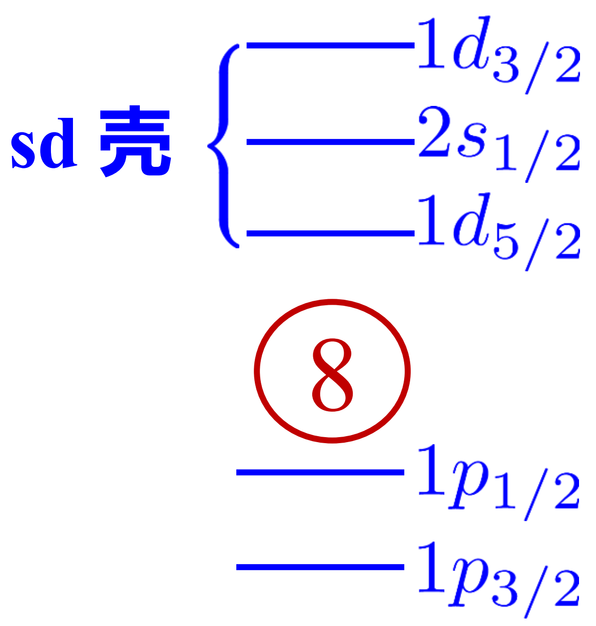
PK: **PK**U
O: **O**rsay
A: **A**izu

Pave a way to reveal the underlying effects of **the exchange correlations**

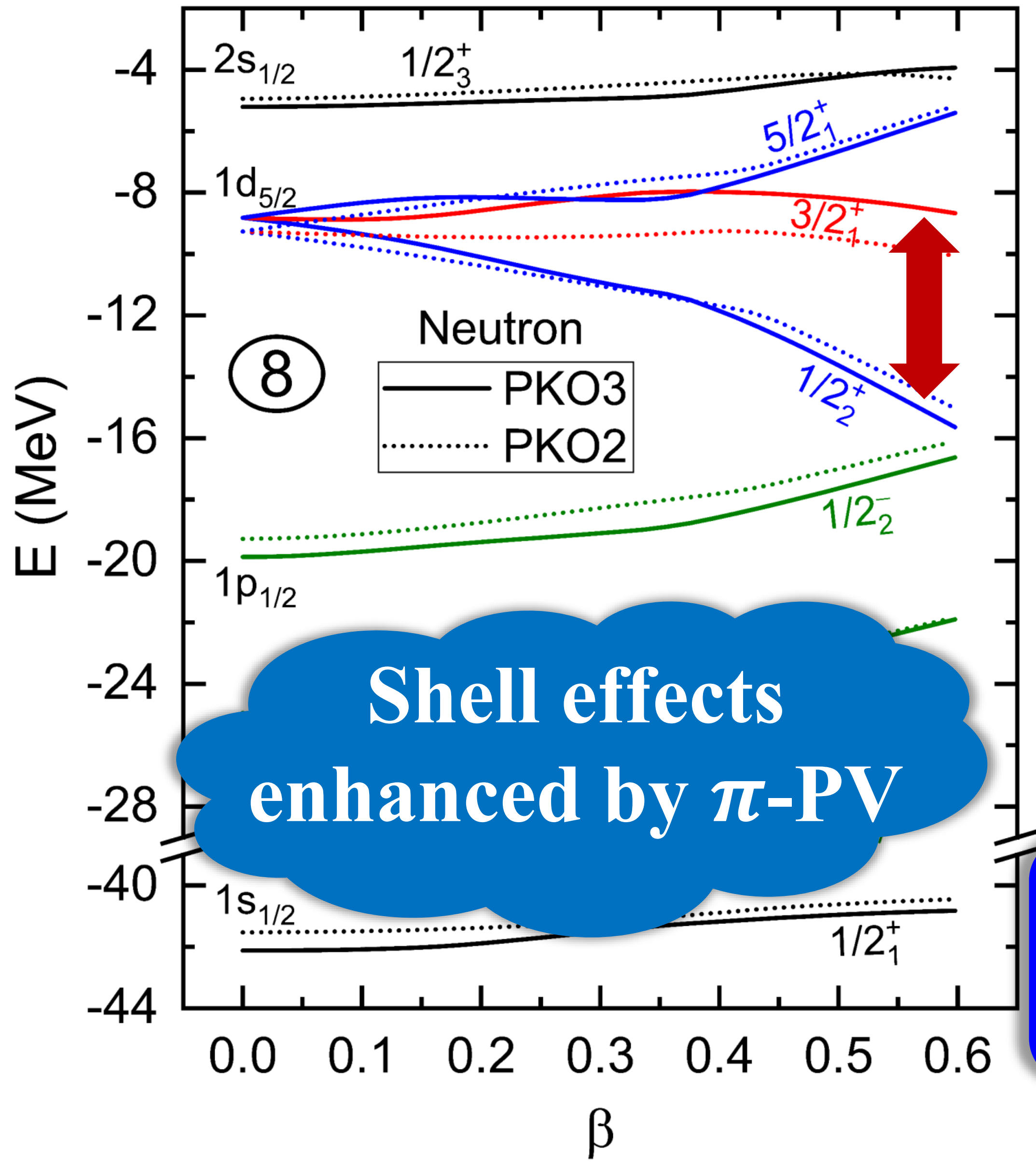
Axially deformed RHF (D-RHF) model: deformation

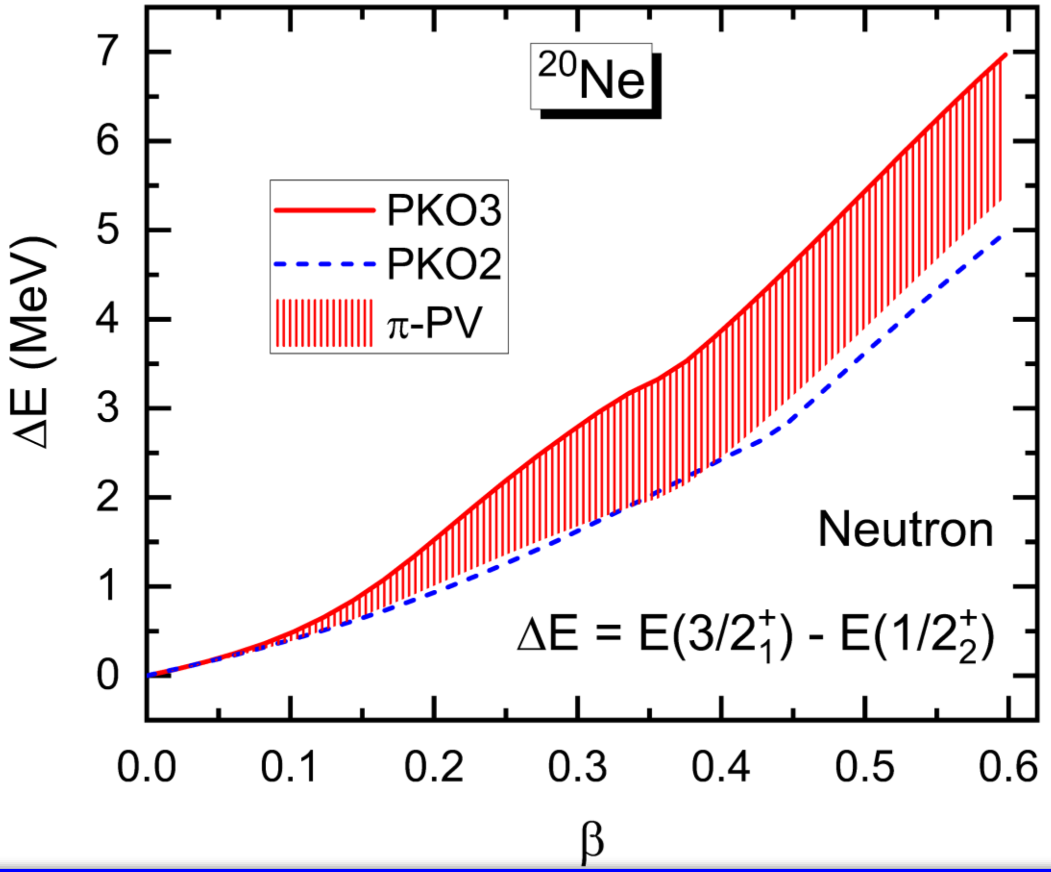
- Spherical Dirac Woods-Saxon (DWS) basis
  - **Appropriate asymptotic behaviors** of wave functions
  - **Microscopic insight** into deformed nuclei: tensor force effects

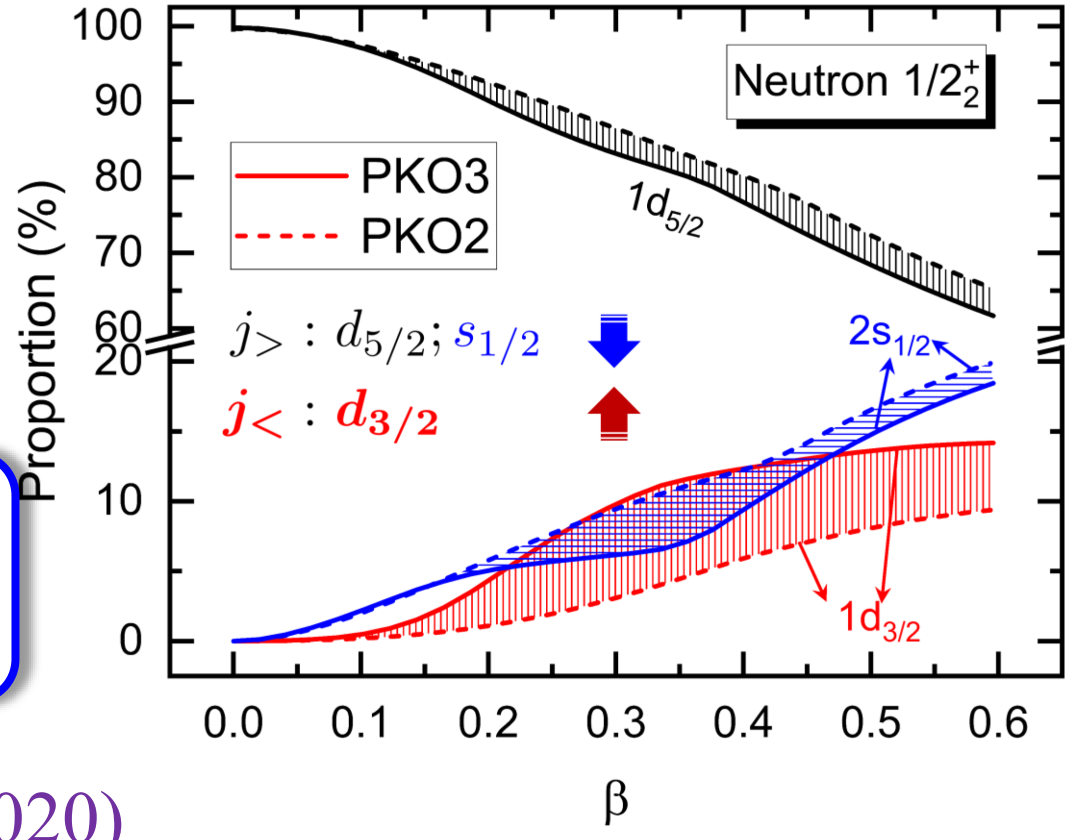
Zhou, Meng, Ring, PRC **68**, 034323 (2003)



PKO2 w/o π
vs
PKO3 with π







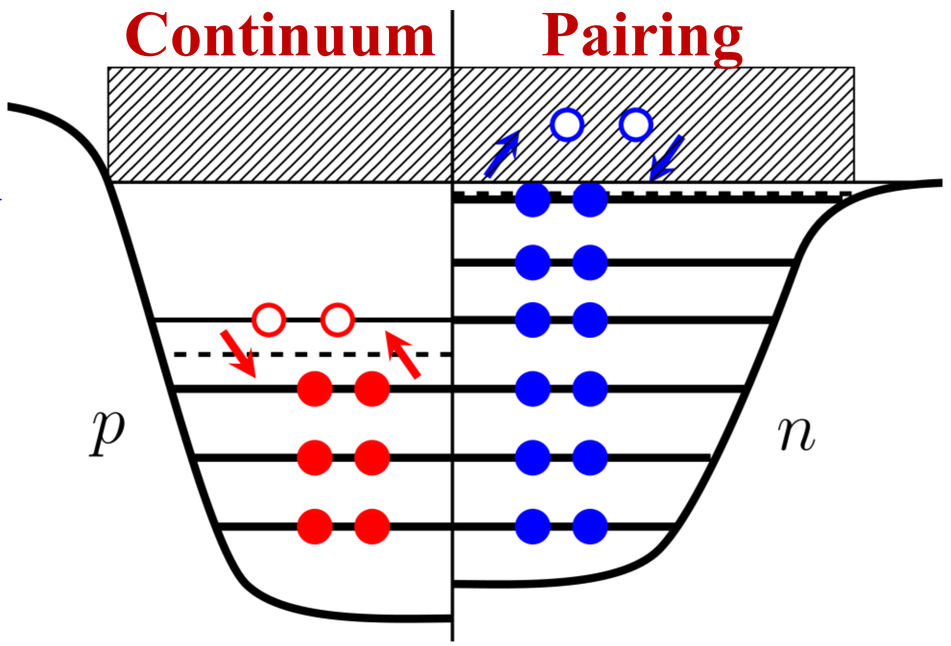
π-T: more $1d_{3/2}$ but less $1d_{5/2}$ waves mixing in $1/2_2^+$ orbit

Geng, Xiang, Sun, WHL, PRC **101**, 064302 (2020)

Axially deformed RHFB (D-RHFB) model

- **Bogoliubov transformation**: Bloch-Messiah decomposition

HF s.p. space $(c, c^\dagger) \xrightarrow{D} (a, a^\dagger) \xrightarrow{\overline{U},\ \overline{V}} (\alpha, \alpha^\dagger) \xrightarrow{C} (\beta, \beta^\dagger)$ Bogoliubov q.p. space



- **Quantize Dirac spinor in Bogoliubov quasi-particle space**

$$\psi(x) = \sum_k \left[\psi_k^U(x)\beta_k + \psi_{\tilde{k}}^V(x)\beta_k^\dagger\right] \qquad H = \hat{T} + \sum_\phi \hat{V}_\phi$$

HFB g.s.: $\beta\,|\mathrm{HFB}\rangle = 0$

$$\hat{T} = \int d\boldsymbol{x} \sum_{kk'} \bar{\psi}_{\tilde{k}}^V (-i\boldsymbol{\gamma}\cdot\boldsymbol{\nabla} + M)\psi_{\tilde{k}'}^V \beta_k \beta_{k'}^\dagger$$

Geng, WHL, PRC **105**, 034329 (2022)

$$\hat{V}_\phi = \frac{1}{2}\iint d\boldsymbol{x}_1 d\boldsymbol{x}_2 \sum_{k_1 k_1'}\sum_{k_2 k_2'} \bar{\psi}_{\tilde{k}_1}^V(x_1)\bar{\psi}_{\tilde{k}_2}^V(x_2)\Gamma_\phi D_\phi \psi_{\tilde{k}_2'}^V(x_2)\psi_{\tilde{k}_1'}^V(x_1)\beta_{k_1}\beta_{k_2}\beta_{k_2'}^\dagger\beta_{k_1'}^\dagger$$

Hartree
Mean field
Fock

$$+\frac{1}{2}\iint d\boldsymbol{x}_1 d\boldsymbol{x}_2 \sum_{k_1 k_1'}\sum_{k_2 k_2'} \bar{\psi}_{\tilde{k}_1}^V(x_1)\bar{\psi}_{k_2}^U(x_2)\Gamma_\phi D_\phi \psi_{k_2'}^U(x_2)\psi_{\tilde{k}_1'}^V(x_1)\beta_{k_1}\beta_{k_2}^\dagger\beta_{k_2'}\beta_{k_1'}^\dagger$$

Pairing correlations

Unified treatment: spin-orbit and tensor forces, deformation, pairing correlations & the continuum

Configuration-Interaction RHF (CI-RHF) model

❑ Effective Hamiltonian based on RHF model

$$H = E_{\mathrm{core}} + \sum_{ii'} \varepsilon_{ii'} :c_i^\dagger c_{i'}: + \frac{1}{4} \sum_{iji'j'} \overline{V}_{iji'j'} :c_i^\dagger c_j^\dagger c_{j'} c_{i'}:$$

EKK scheme

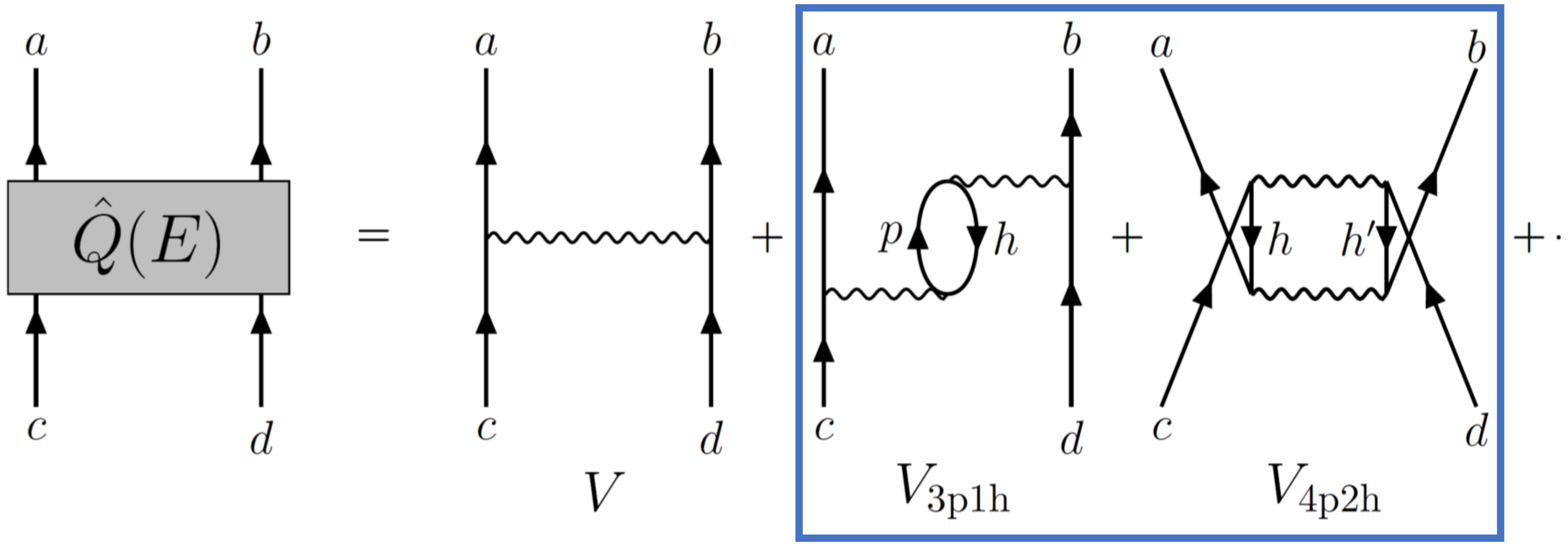
$$H_l^{\mathrm{eff}} = H^{\mathrm{BH}}(E_0) + \boxed{\sum_{k=1}^{\infty} \frac{1}{k!} \frac{d^k \widehat{Q}(E_0)}{dE_0^k} \left\{H_{l-1}^{\mathrm{eff}} - E_0\right\}^k}$$

Folded terms

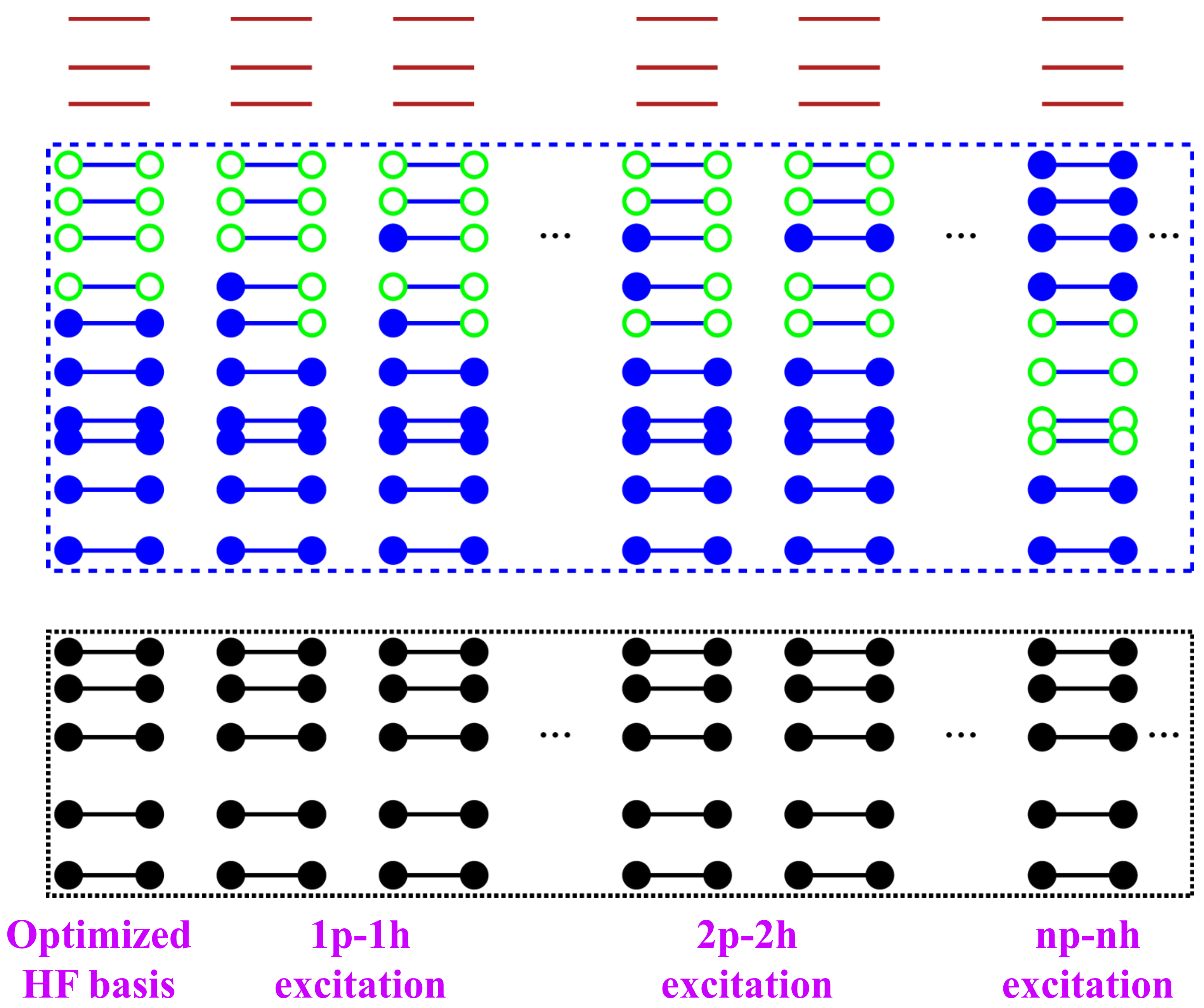
$\widehat{Q}$-box expansion

$$\hat{Q}(E) = \sum_{l=0}^{\infty} P\left[V \frac{Q}{E - H_0}\right]^l V P$$

Core polarization



J. Liu, Y.F. Niu, WHL, CPC **49**, 064104 (2025)



$$|\Psi\rangle = \sum_n C_n |\Phi_n\rangle, \qquad |\Phi_n\rangle = \prod_{i=1}^{A-N_{\mathrm{core}}} c_{n_i}^\dagger |\mathrm{core}\rangle$$

Brown et al. PLB **695** (2011) 507…

CI-RHF model: ^{18}O and Ne isotopes

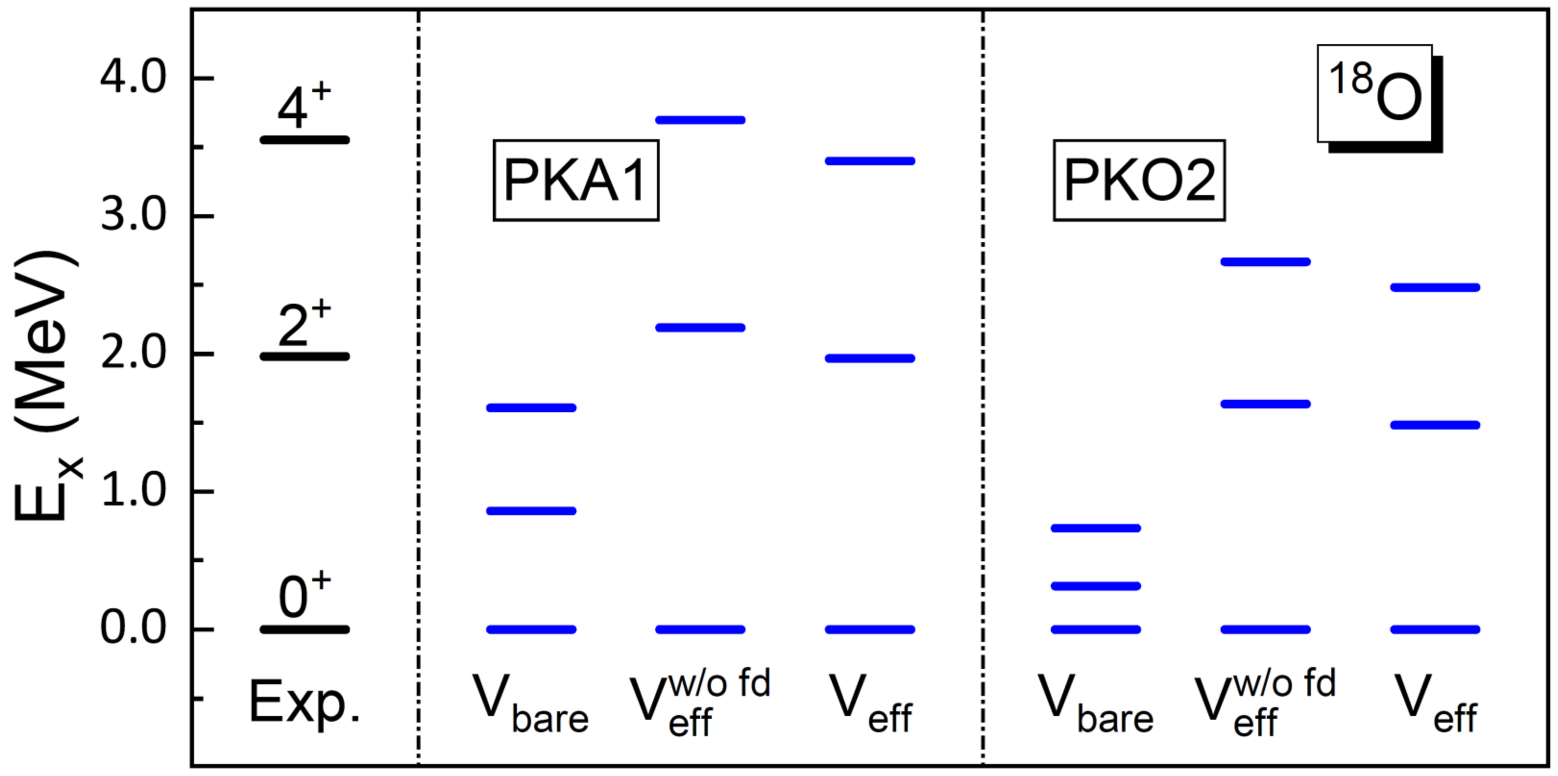
- **Without introducing any new parameters**, the effective Hamiltonian constrained by the existing RHF Lagrangians works well for describing the low-lying excitations

J. Liu, Y.F. Niu, WHL, CPC **49**, 064104 (2025)

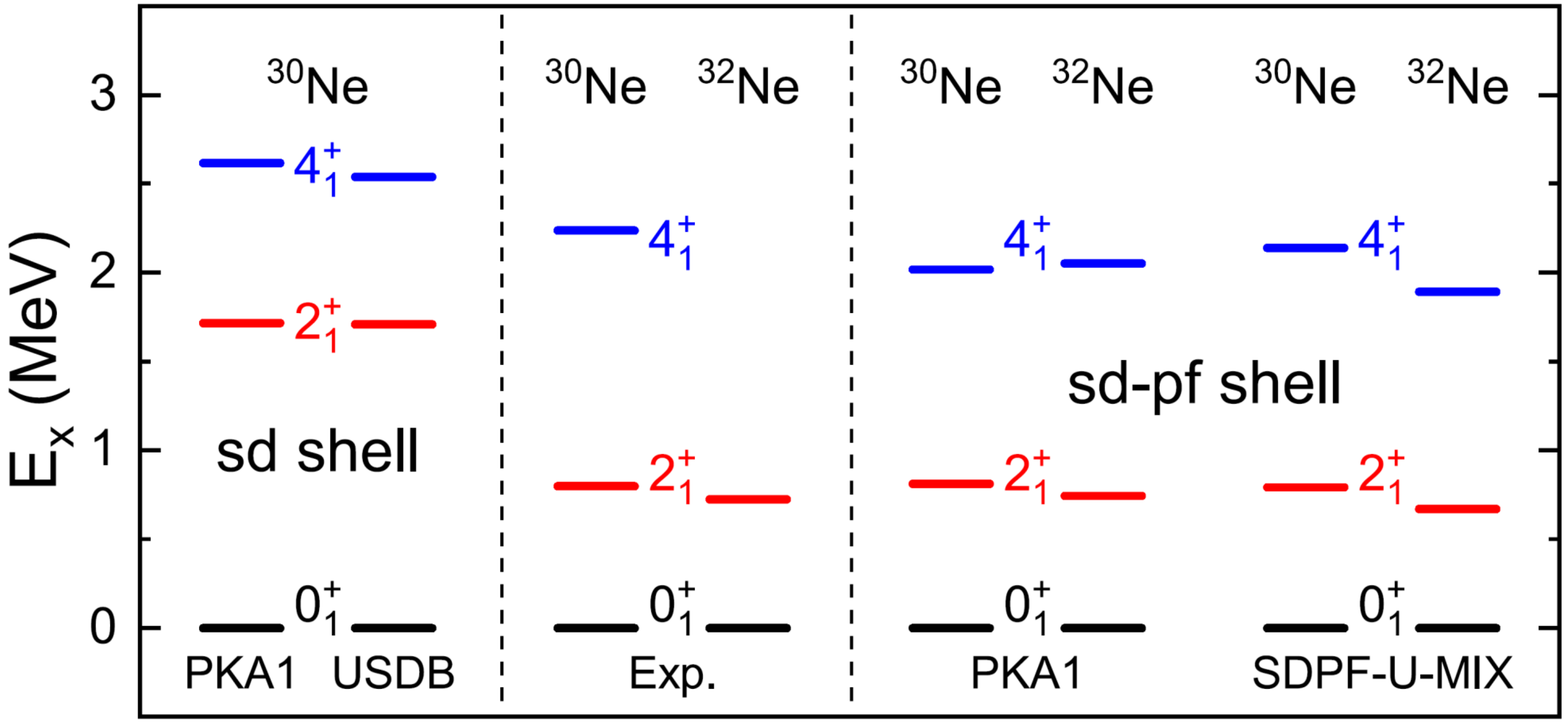
Folded terms & core polarizations

are necessitated for precise description

PKA1 seems better than PKO series



Extending from sd shell to sd-pf shell



An efficient tool to describe the g.s. and low-lying excitations of unstable nuclei

Exchange correlation effects at the RHF level

蘭州大學 LANZHOU UNIVERSITY

- Exchange degrees of freedom (π-PV and ρ-T) change the modeling of nuclear binding

RMF (DD-ME2) ❯❯ RHF (PKO3: plus π-PV) ❯❯ RHF (PKA1: plus π-PV & ρ-T)

Modeling of nuclear binding for ^{208}Pb

Models	$E_{\text{kin.+c.m.}}$	$E_{\sigma+\omega}$	$E_{\rho+\pi}$	$E_{\text{cou.}}$	Total
DD-ME2	2629.1	−5194.0	+100.3	827.2	−1637.4
PKO3	2823.2	−4609.7	−648.7	798.4	−1636.8
PKA1	2498.9	−3298.9	−1634.8	798.6	−1636.3

Fock terms, particularly the exchange degrees of freedom (DoF), impact the nuclear binding,

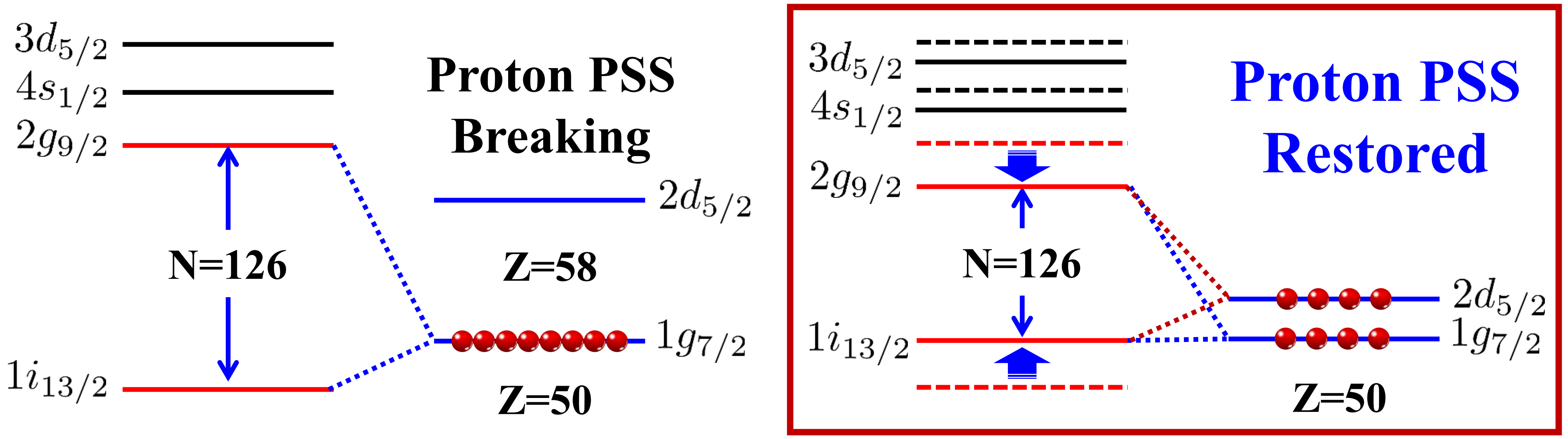
Geng, Li, WHL, Niu, Chang, PRC **100**, 051301(R) (2019)

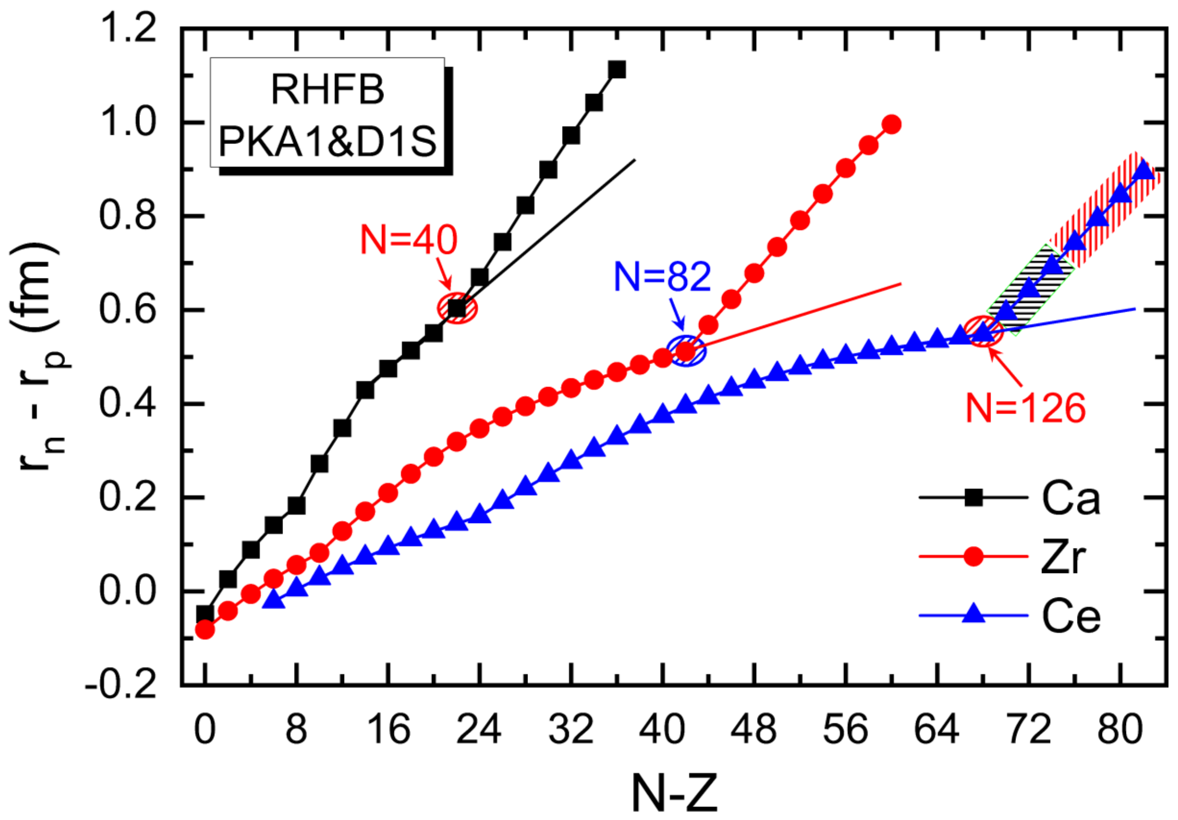
Picture of nuclear binding

σ-S **attraction**	ω-V **repulsion**	Exchange DoF **attraction**
dominant		**Sub-dominant**

Giant halo in Ce isotopes

- Exchange DoFs enhance the link between the **restored proton PSS** and the **squeezed N=126 shell**

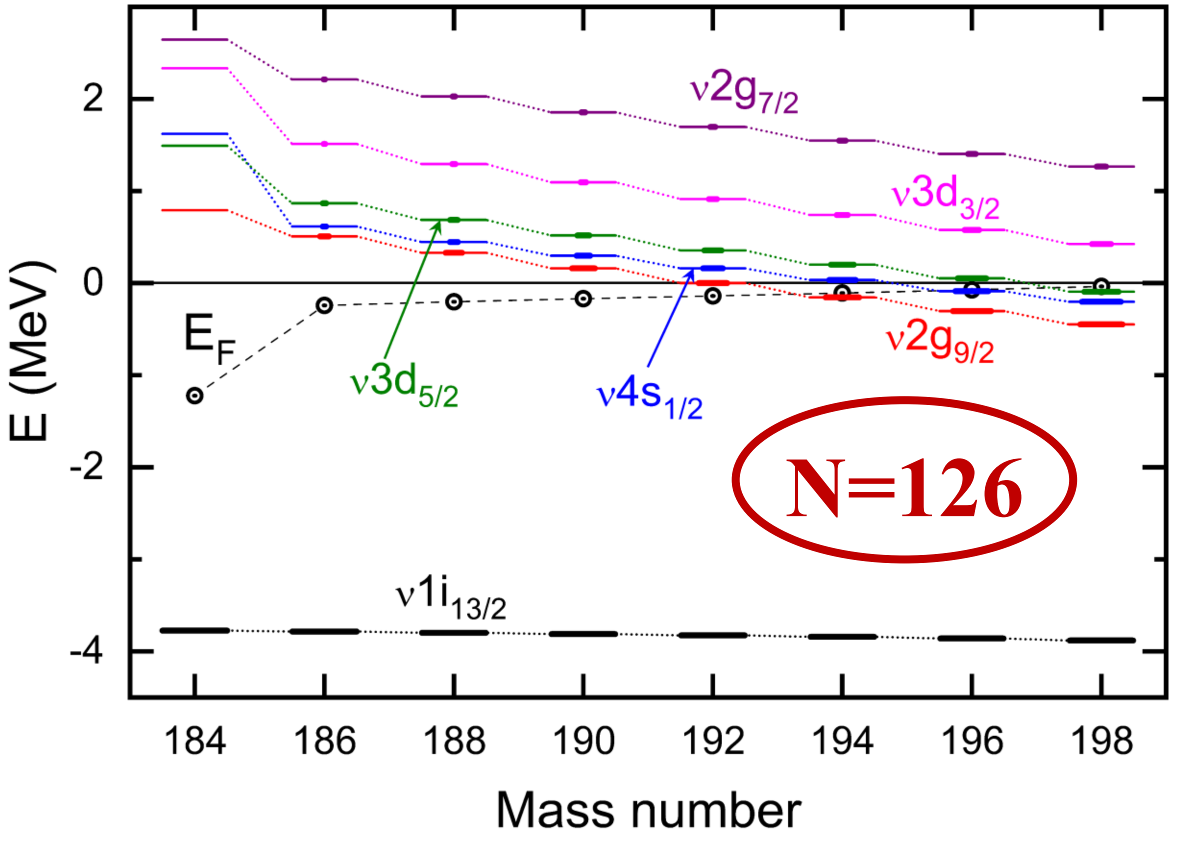




b	$a = \pi 2d_{5/2}$			$a = \pi 1g_{7/2}$		
	V_{ab}	ρ-T	π-PV	ρ-T	π-PV	V_{ab}
$\nu 1i_{13/2}$	-0.257	25.1%	2.1%	18.5%	9.8%	$\boxed{-0.386}$
$\nu 4s_{1/2}$	-0.097	17.1%	3.9%	20.2%	3.5%	-0.052
$\nu 2g_{9/2}$	-0.217	24.5%	3.0%	19.0%	8.8%	$\boxed{-0.157}$

N=126 squeezed

Nodal effects



WHL, Ring, Meng, Giai, Bertulani, PRC **81**, 031302(R) (2010)

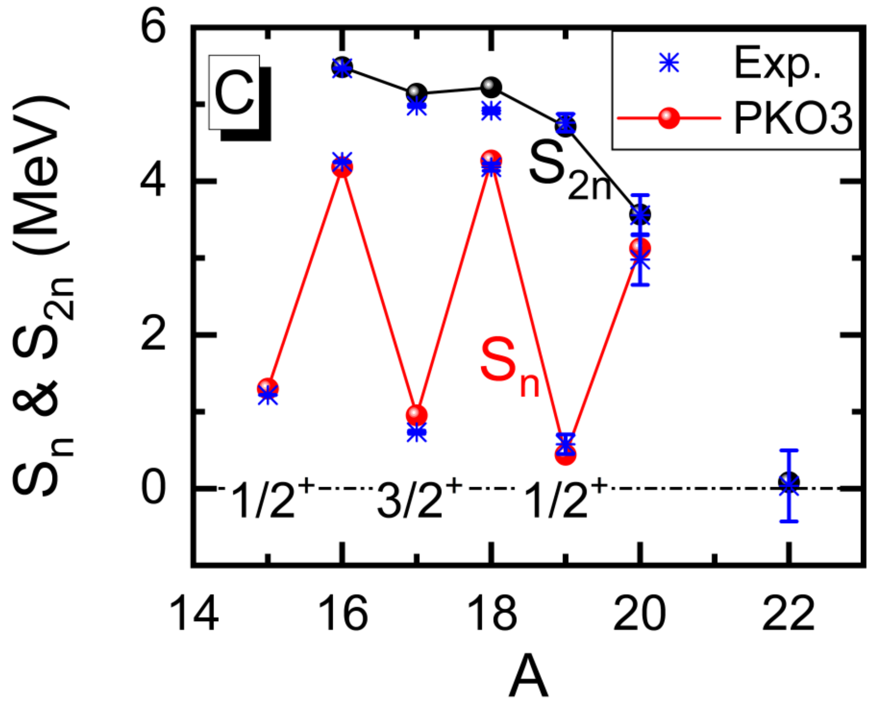
Continuum effects in the halo structure of ^{19}C

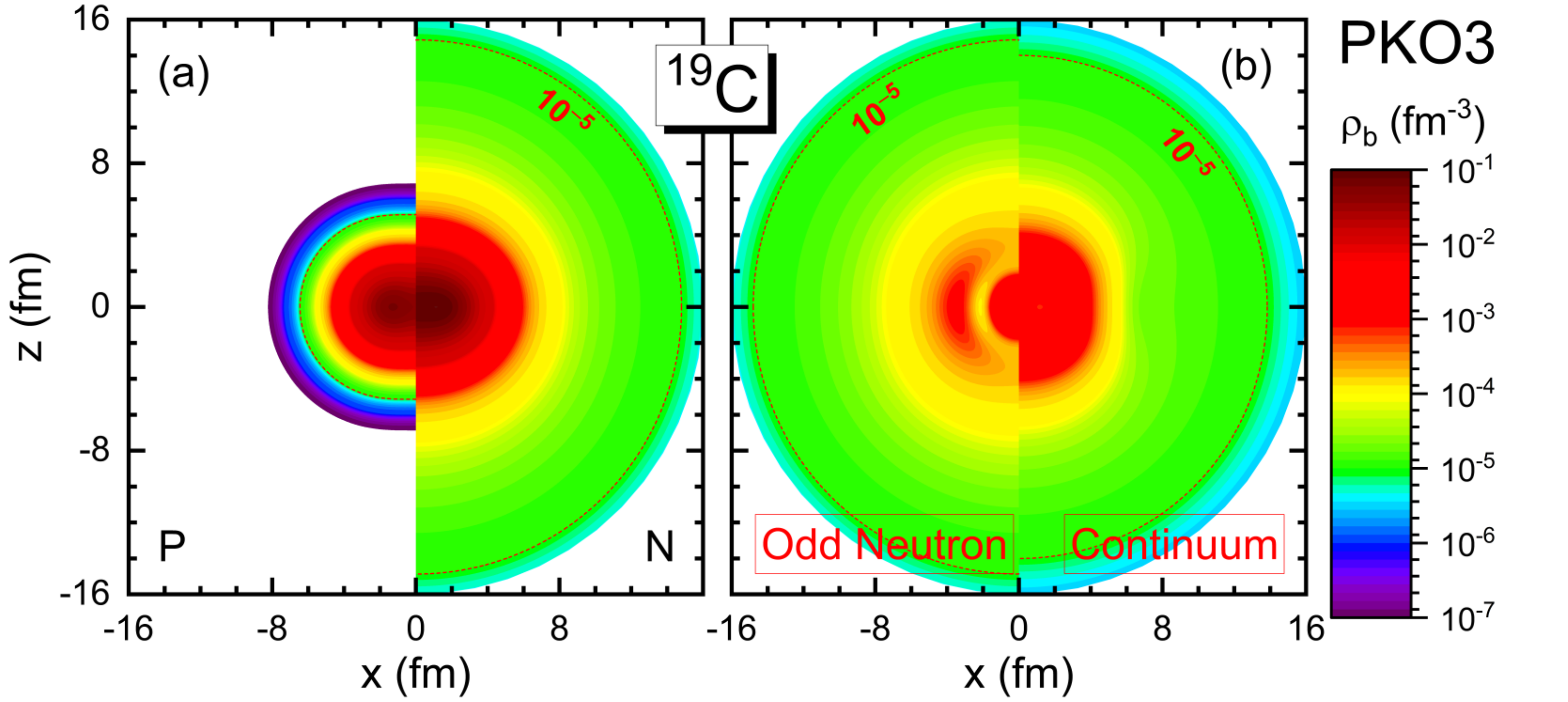
- ^{19}C: single-neutron halo, spin-parity $J^{\pi} = 1/2^{+}$

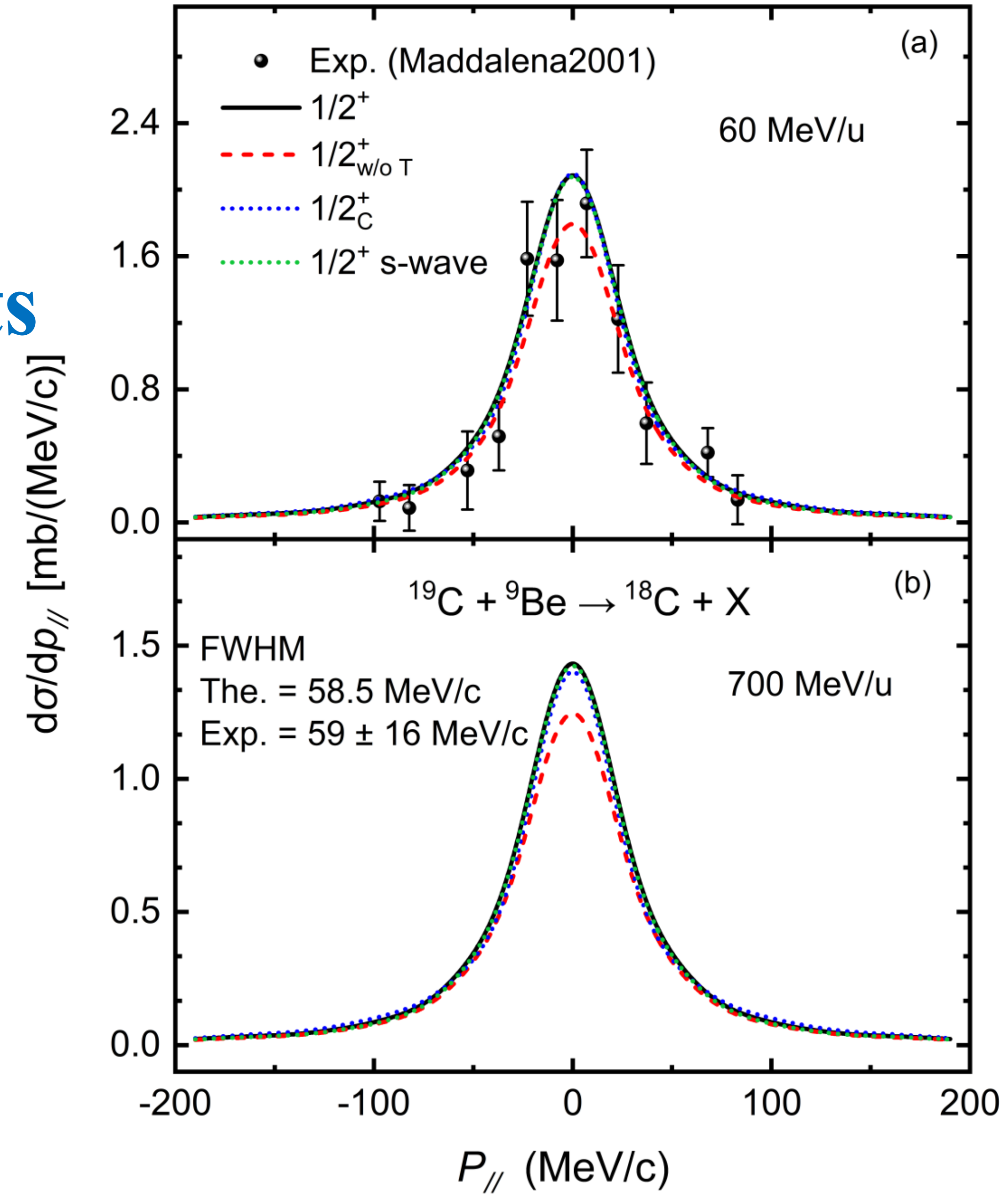
D-RHFB structure & Glauber reaction models

Structure & reaction observables: critical continuum effects

J. L. An, H. C. Li, J. Geng, WHL, S. S. Zhang, JPG **52**, 055106 (2025)







Structure: spin-parity, neutron separation energy, density

Reaction: longitude momentum

- D-RHFB model can provide reliable structure input for the reactions of unstable nuclei

The island of inversion nuclide: ^{32}Mg

LANZHOU UNIVERSITY

- Nature of the island of inversion exhibited by ^{32}Mg
  - **Neutron magic shell N = 20 vanishes,** accompanying with **notable g.s. deformation**
  - **Rotational-like g.s. band** and the **co-existence of prolate and spherical shapes**

Motobayashi, Ikedaa *et al.*, PLB **346**, 9 (1995); Pritychenko, Glasmacher *et al.*, PLB **461, 322** (1999).
Church, Campbell *et al.*, PRC **72**, 054320 (2005); Wimmer, Kroll *et al.*, PRL **105**, 252501 (2010).

- Shell model with the SDPF-M interaction

  A remarkable cross-shell excitation from sd- to pf-shells is revealed for deformed g.s. 0_1^+

Caurier, Nowacki, Poves, PRC **90**, 014302 (2014); Kitamura, Wimmer, Poves *et al.*, PLB **822**, 136682 (2021).

- Angular momentum projection based on the mean-field models: **intrusion of $\boldsymbol{pf}$-shell**

  Reproduce the deformed g.s. but fail in describing the low-lying excitation energies

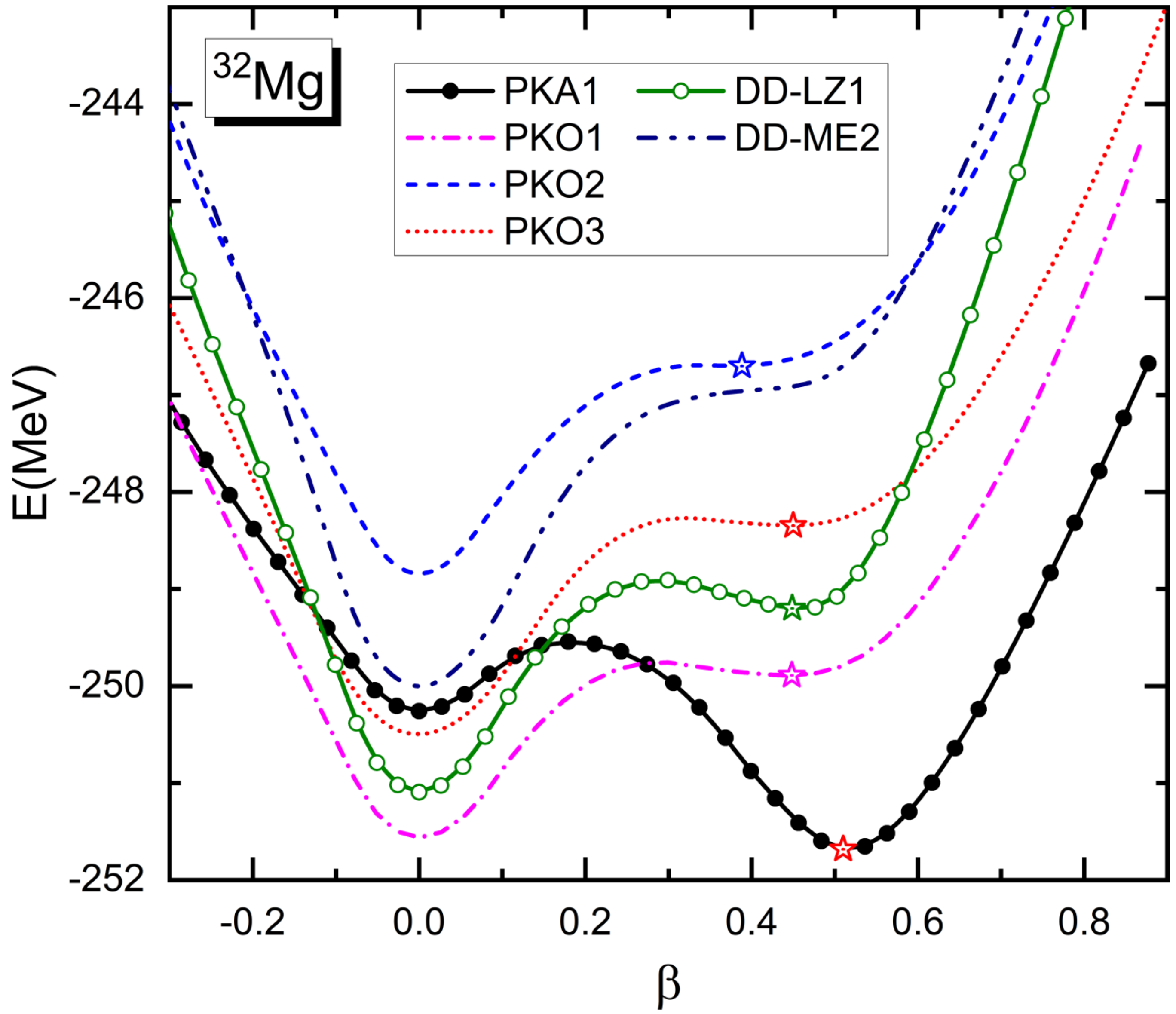
Rodríguez-Guzmán, Egido, Robledo, NPA **709**, 201 (2002); Vaquero, Egido, Rodŕguez, PRC **88**, 064311 (2013).
Yao, Meng, Ring *et al.*, PRC **79**, 044312 (2009).

What can we learn from the nature of the island of inversion exhibited by ^{32}Mg

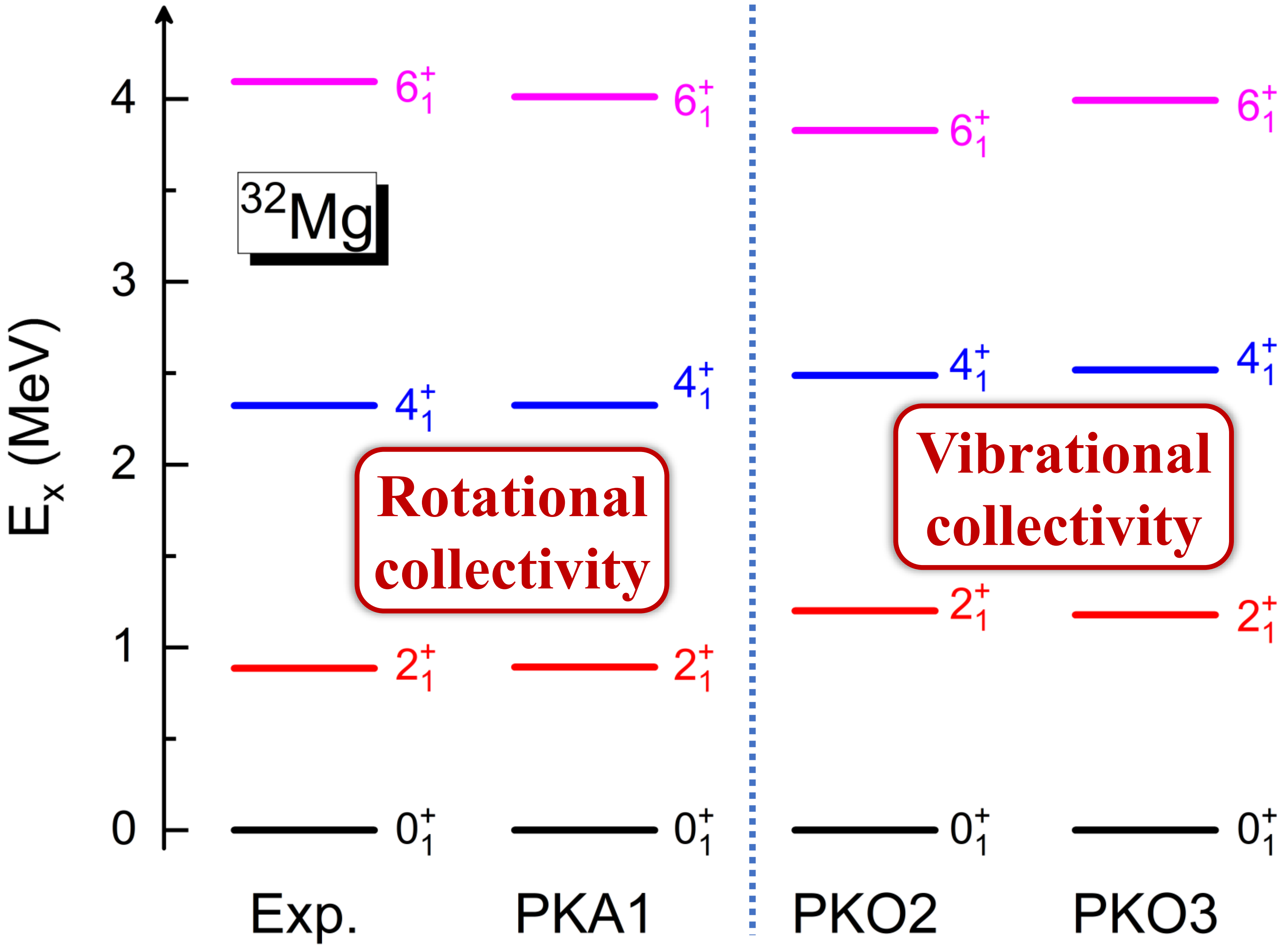
Nature of the island of inversion exhibited by ^{32}Mg

LANZHOU UNIVERSITY

- D-RHFB calculations: **deformation**



- CI-RHF calculations: **collectivity**



Link between D-RHFB and CI-RHF

Part of many-body correlations in CI-RHF model are considered by the D-RHFB model

Peng, Liu, Geng, Niu, WHL, CPC **49**, 064112 (2025)

Island of inversion: underlying mechanism

兰州大学 LANZHOU UNIVERSITY

- **Cross-shell excitation**: deformation picture in ^{32}Mg

$R = E(4_1^+)/E(2_1^+)$

	$1d_{5/2}$	$2s_{1/2}$	$1d_{3/2}$	$1f_{7/2}$	$2p_{3/2}$	R
PKA1	5.75	1.91	2.27	1.50	**0.56**	2.61
PKO2	5.73	1.92	2.38	1.81	0.16	2.07
PKO3	5.73	1.92	2.56	1.57	0.23	2.14

Rotational collectivity
stable deformation

Vibrational collectivity
soft deformation

Peng, Liu, Geng, Niu, WHL, CPC **49**, 064112 (2025)

- **Impact of exchange DoFs: binding** of ^{32}Mg

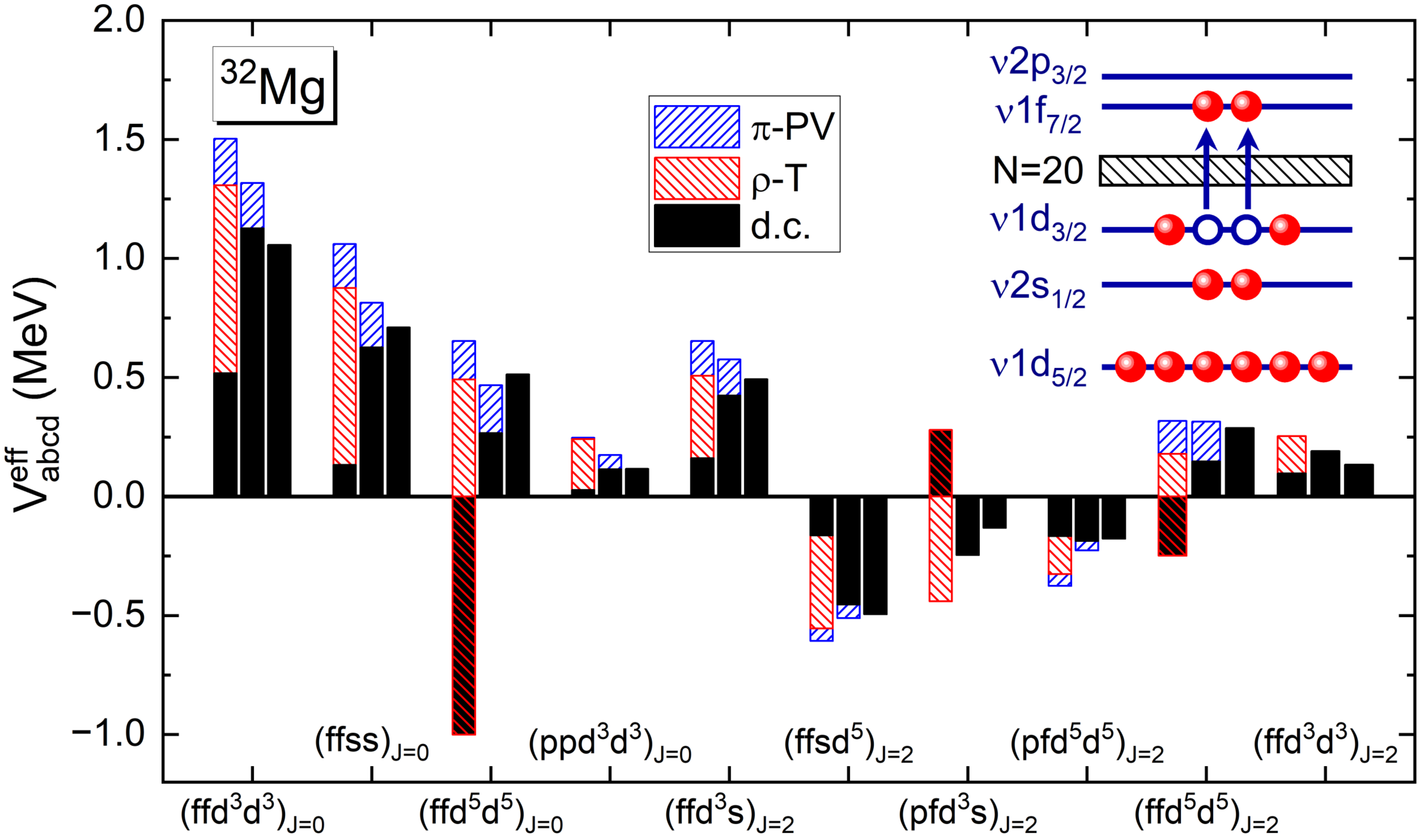
	$E_{\text{kin.}}$	$E_{\sigma+\omega}$	E_ρ	E_π	$E_{\text{oth.}}$	E
PKA1	400.8	−417.0	−217.5	−31.5	14.9	−250.2
	409.6	−434.4	−214.4	−29.9	17.4	−251.7
PKO3	438.1	−606.1	−59.7	−37.2	14.4	−250.5
	448.7	−616.3	−59.2	−36.1	14.6	−248.3
DD-LZ1	412.6	−701.3	18.3	–	19.4	−251.1
	414.1	−702.7	17.6	–	21.8	−249.2

Exchange DoFs exhibit substantial contributions to nuclear binding

Picture of nuclear binding

σ-S **attraction**	ω-V **repulsion**	Exchange DoF **attraction**
dominant		**Sub-dominant**

Configuration Interactions: ρ-T and π-PV



PKA1 (left)

PKO3 (mid.)

PKO2 (right)

Discernible π-PV &

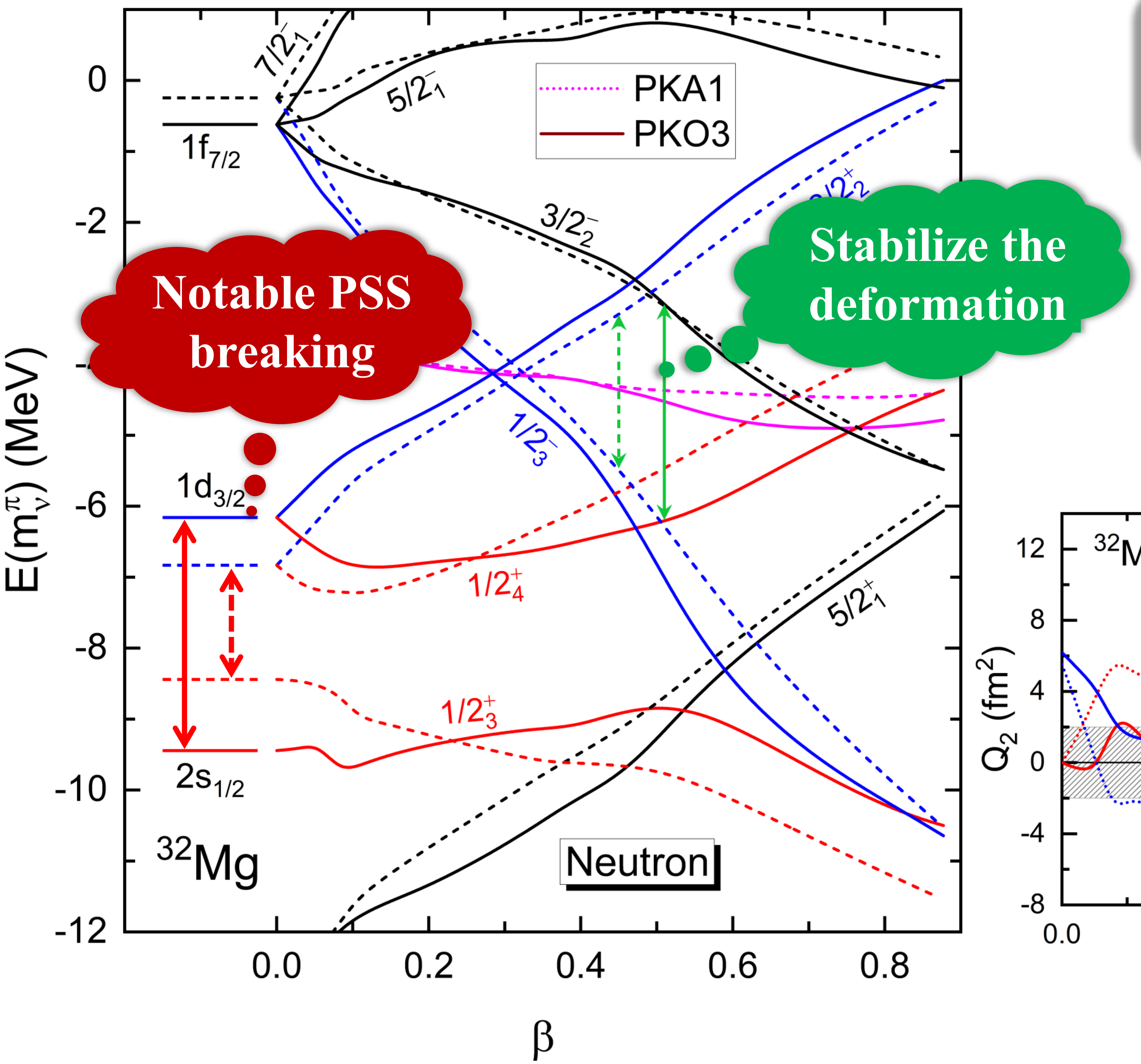
compressive ρ-T

Be determined at the Hartree-Fock level

Configuration Interactions shaped by the exchange DoFs ➔ deformation stability?

Peng, Liu, Geng, Niu, WHL, CPC **49**, 064112 (2025)

PSS breaking: an implemented insight



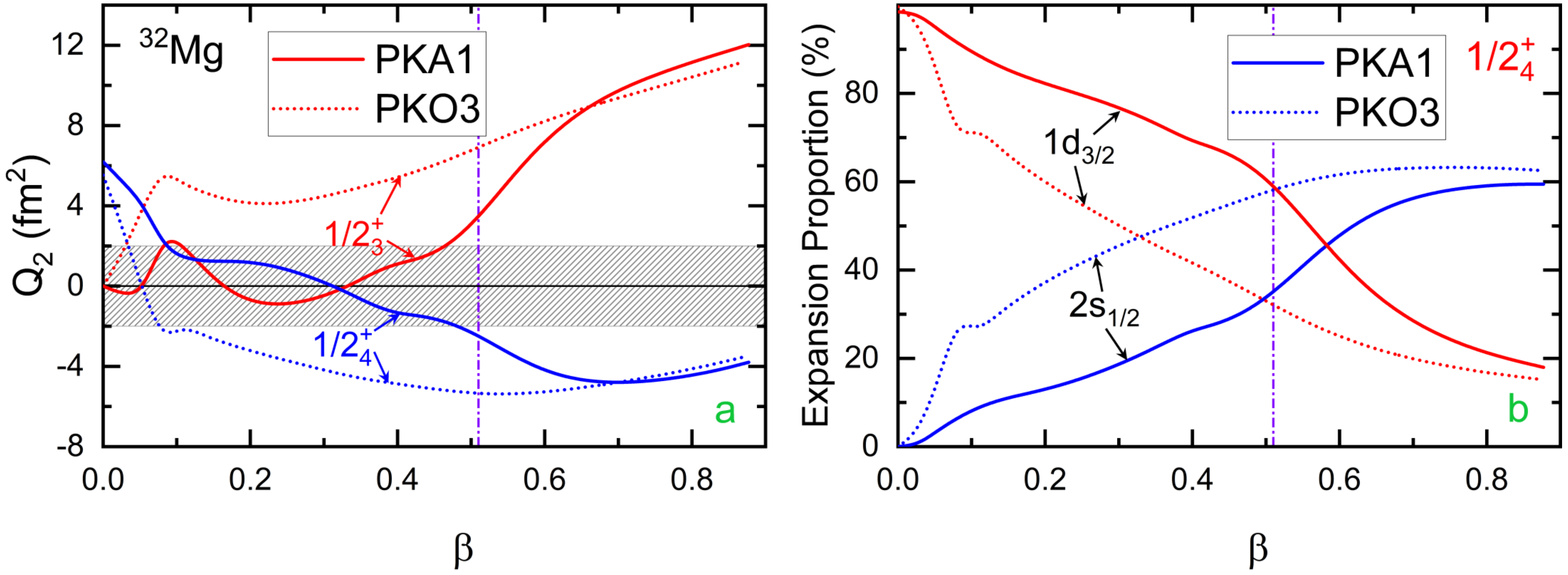
Notable deformed shell and the breaking of PSS

Underlying effects from exchange DoF

PSS breaking

Shape evolution

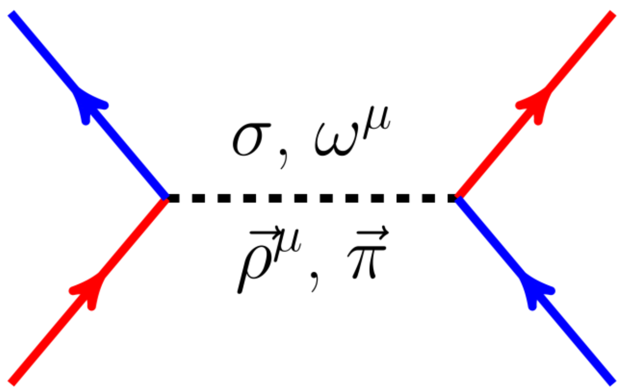
Deformation → **Spherical Shell** → **Mixture of spherical waves**



Peng, Liu, Geng, Niu, WHL, CPC **49**, 064112 (2025)

Conclusions： exchange correlation effects

- At the mean field level & beyond, exchange DoFs are more important than expected



Exchange DoFs

π-PV and ρ-T

How nucleons form an atomic nucleus

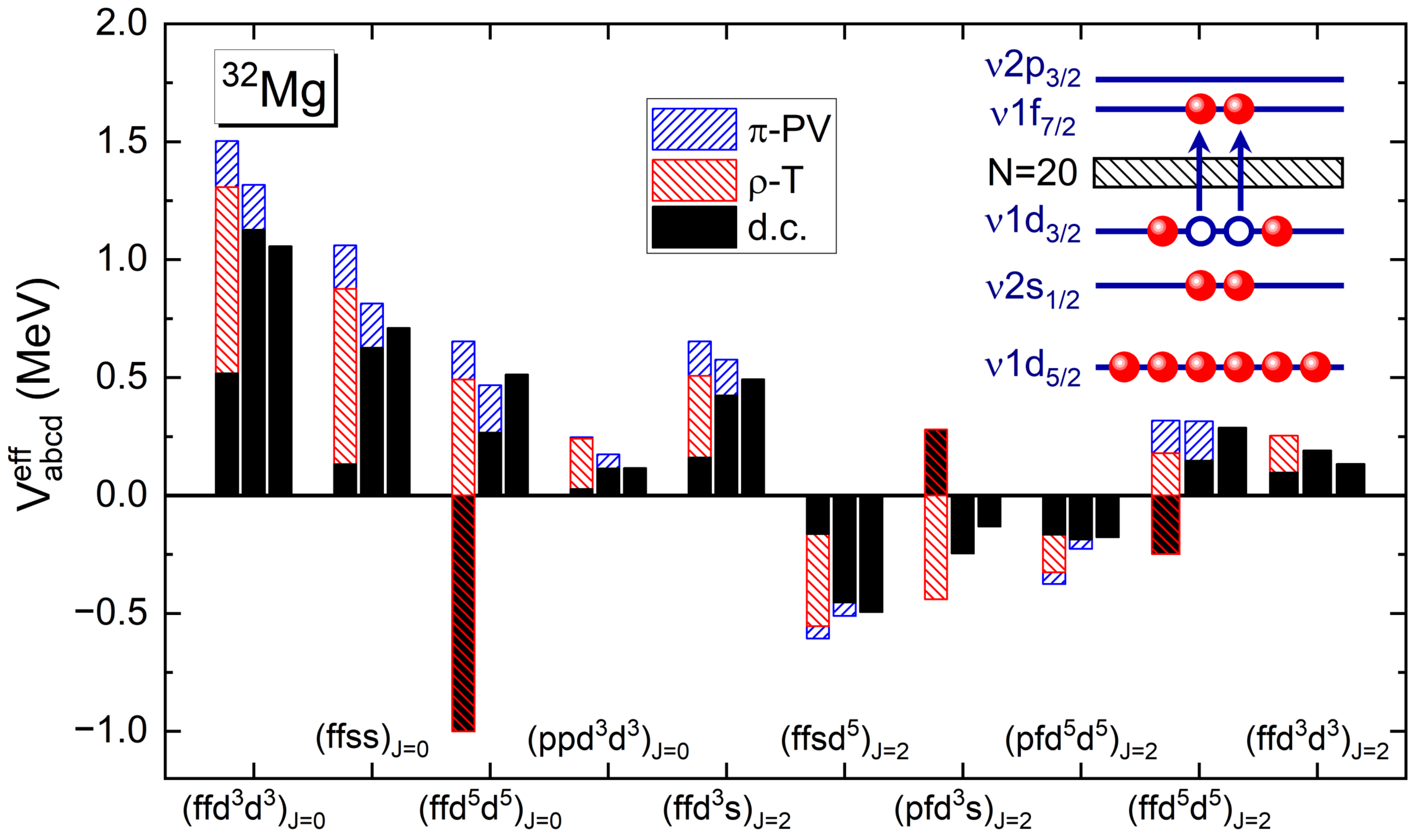
- Impact the modeling of nuclear binding
- Enhance the neutron-proton interactions

- Exchange DoFs shall be predetermined at the mean field level

Picture of nuclear binding

σ-S **attraction**	ω-V **repulsion**	Exchange DoFs **attraction**

dominant **Sub-dominant**



- Explicit Fock terms are complicated, which calls for an economical solution

Acknowledgements

Collaborators:

J. Geng, Y. Peng, Y.F. Niu @LZU

J. Liu @USC

S.S. Zhang, J.L. An, H.C. Li @BUAA

J. Meng, P.W. Zhao @PKU

J.J. Li, Z.P. Li @SWU

P. Ring @TUM

N. Van Giai @Paris-Saclay University

C.A. Bertulani @ Texas A&M University-Commerce

……

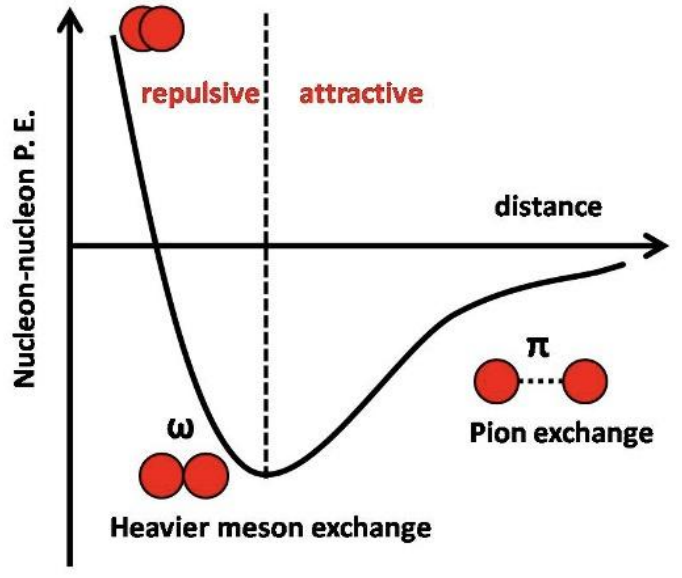
Thank you for your attentions

Long, Wen Hui (longwh@lzu.edu.cn)

Fundamental issue in nuclear physics

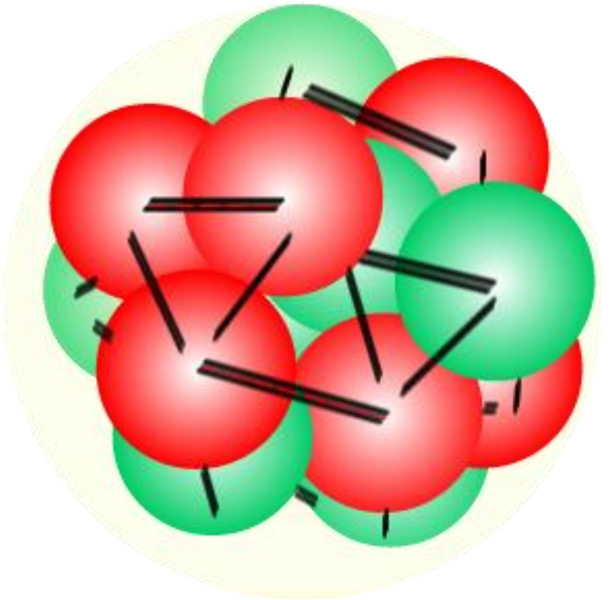
LANZHOU UNIVERSITY

How nucleons spontaneously form atomic nuclei



Nuclear force

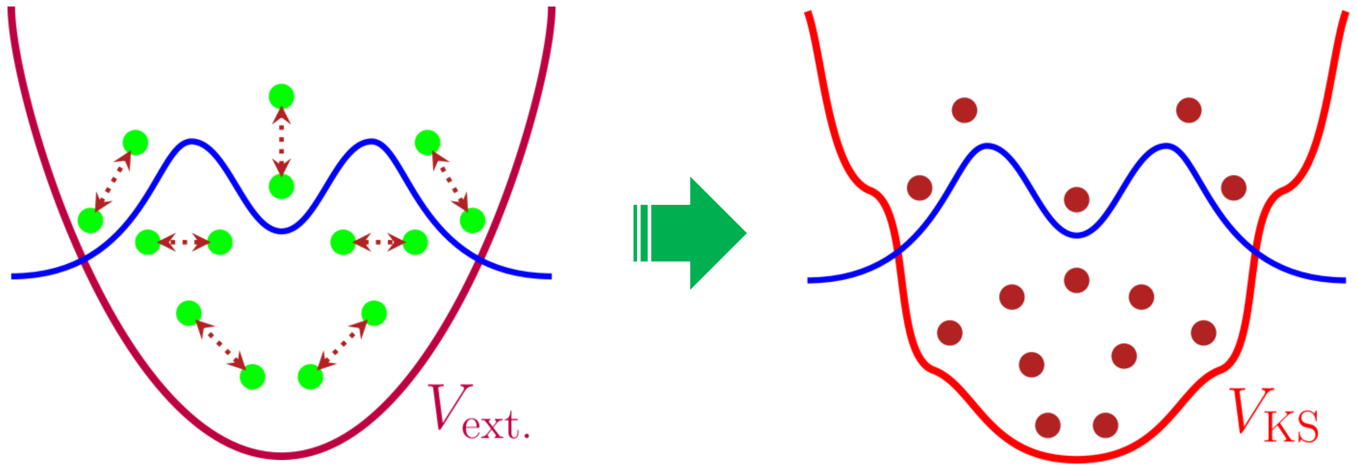
+



Many-body system

- **Density functional theory (DFT)**

many-body problem ➔ one-body problem



Drut, Furnstahl, Platter, PPNP **64** (2010) 120

- **A relativistic representation** of nuclear binding given by relativistic DFT

Interplay between **strong scalar attraction** and **vector repulsion** ➔ **bound nucleus**

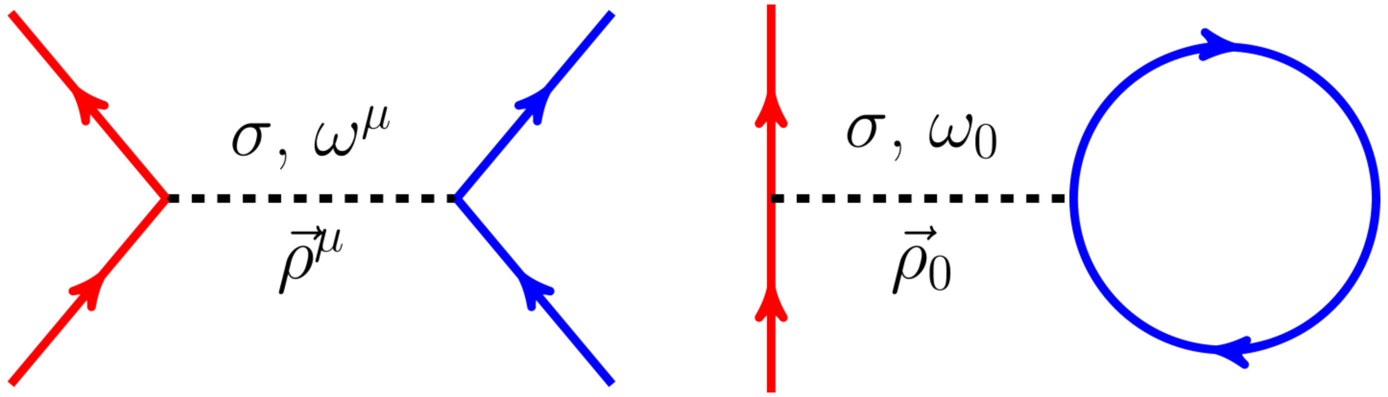
Natural spin-orbit coupling, the origin of pseudo-spin symmetry and spin symmetry, …

Meng et al., PPNP **57**, 470 (2006); Ginocchio, PRL **78**, 436 (1997); Meng et al., PRC **58**, R628 (1998); Zhou et al., PRL **91**, 262501 (2003).

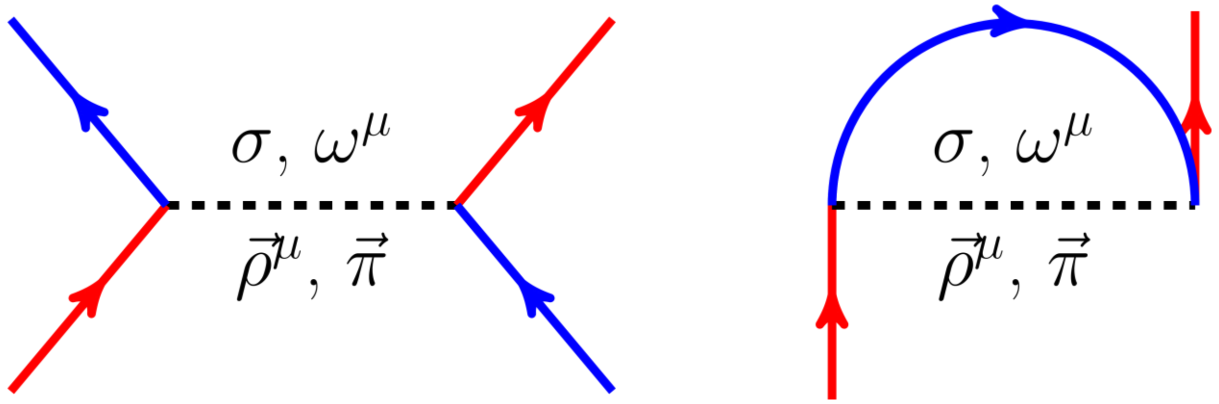
What can we learn from the novel phenomena exhibited by unstable nuclei?

Relativistic Hartree-Fock (RHF) theory

- Fock terms give rise to **pure exchange degrees of freedom**



Hartree terms: σ-S, ω-V, $\vec{\rho}$-V, A-V



Fock terms: σ-S, ω-V, $\vec{\rho}$-V, A-V; **π-PV, ρ-T (ρ-VT)**

Bouyssy (1987), Bernardos (1993), Shi (1995), Marcos (2004), Long (2004-now), Geng (2020-now), ...

- **Exchange degrees of freedom (DoF) impact the modelling of nuclear binding**
  π-PV and ρ-T (ρ-VT) PRC **100**, 051301(R) (2019)
- **Nuclear tensor force** can be naturally introduced: e.g., via the π-PV and ρ-T couplings
  PRC **91**, 034326 (2015); CPC **42**, 024101 (2018); PRC **98**, 034313 (2018); PRC **101**, 064306 (2020)

- **Not exactly a DFT**: **explicit** Fock terms (meson exchange) are **not a functional of local density**

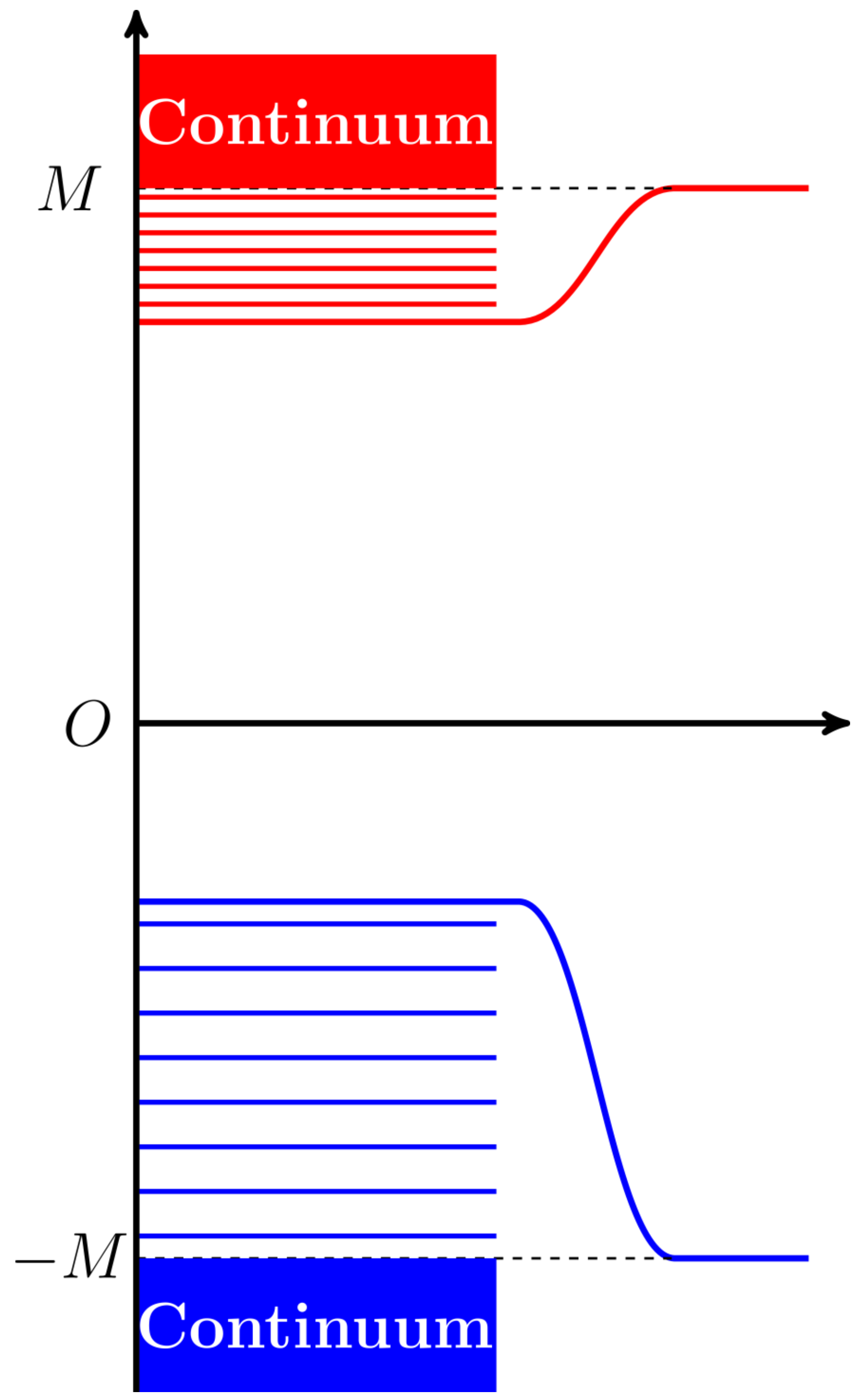
Energy function of DFT $$E(\widehat{\rho}) = \langle\Psi|\,\widehat{H}\,|\Psi\rangle \approx \langle\Phi|\,H_{\text{eff.}}\,|\Phi\rangle = T + E_H(\rho) + \boxed{E_{\text{xc}}(\rho)}$$ **Exchange correlations**

Axially deformed RHF (D-RHF) model

- Spherical Dirac Woods-Saxon (DWS) basis

Zhou, Meng, Ring, PRC **68**, 034323 (2003)

  - **Appropriate asymptotic behaviors** of wave functions
  - **Microscopic insight** into deformed nuclei: effects of tensor force



- Yukawa propagator: the decomposition in spherical coordinate

$$\frac{1}{4\pi}\frac{e^{-m|\boldsymbol{x}-\boldsymbol{x}'|}}{|\boldsymbol{x}-\boldsymbol{x}'|} = \frac{1}{rr'}\sum_{\lambda=0}^{\infty} I_{\lambda+\frac{1}{2}}(mr_<)K_{\lambda+\frac{1}{2}}(mr_>)\sum_{\mu=-\lambda}^{\lambda} Y_{\lambda\mu}(\boldsymbol{\Omega})Y^*_{\lambda\mu}(\boldsymbol{\Omega})$$

Singularity appears when $\boldsymbol{x}=\boldsymbol{x}'$

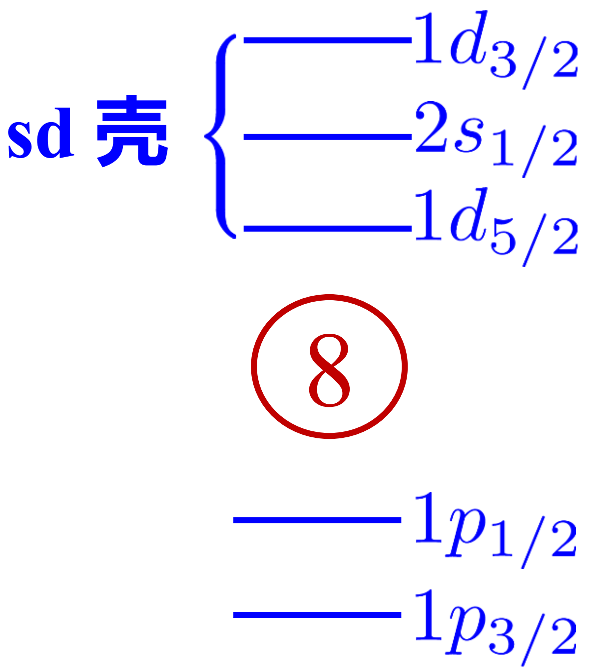
$$V_\phi = \frac{1}{2}\int d\boldsymbol{x}d\boldsymbol{x}' \sum_{\alpha\beta;\alpha'\beta'} \left[\bar{\psi}_\alpha\Gamma_\phi\psi_{\alpha'}\right]_{\boldsymbol{x}} \frac{1}{4\pi}\frac{e^{-m_\phi|\boldsymbol{x}-\boldsymbol{x}'|}}{|\boldsymbol{x}-\boldsymbol{x}'|}\left[\bar{\psi}_\beta\Gamma^\phi\psi_{\beta'}\right]_{\boldsymbol{x}'} c^\dagger_\alpha c^\dagger_\beta c_{\beta'} c_{\alpha'}$$

($\lambda\mu$)-terms are truncated by the DWS basis to avoid singularity

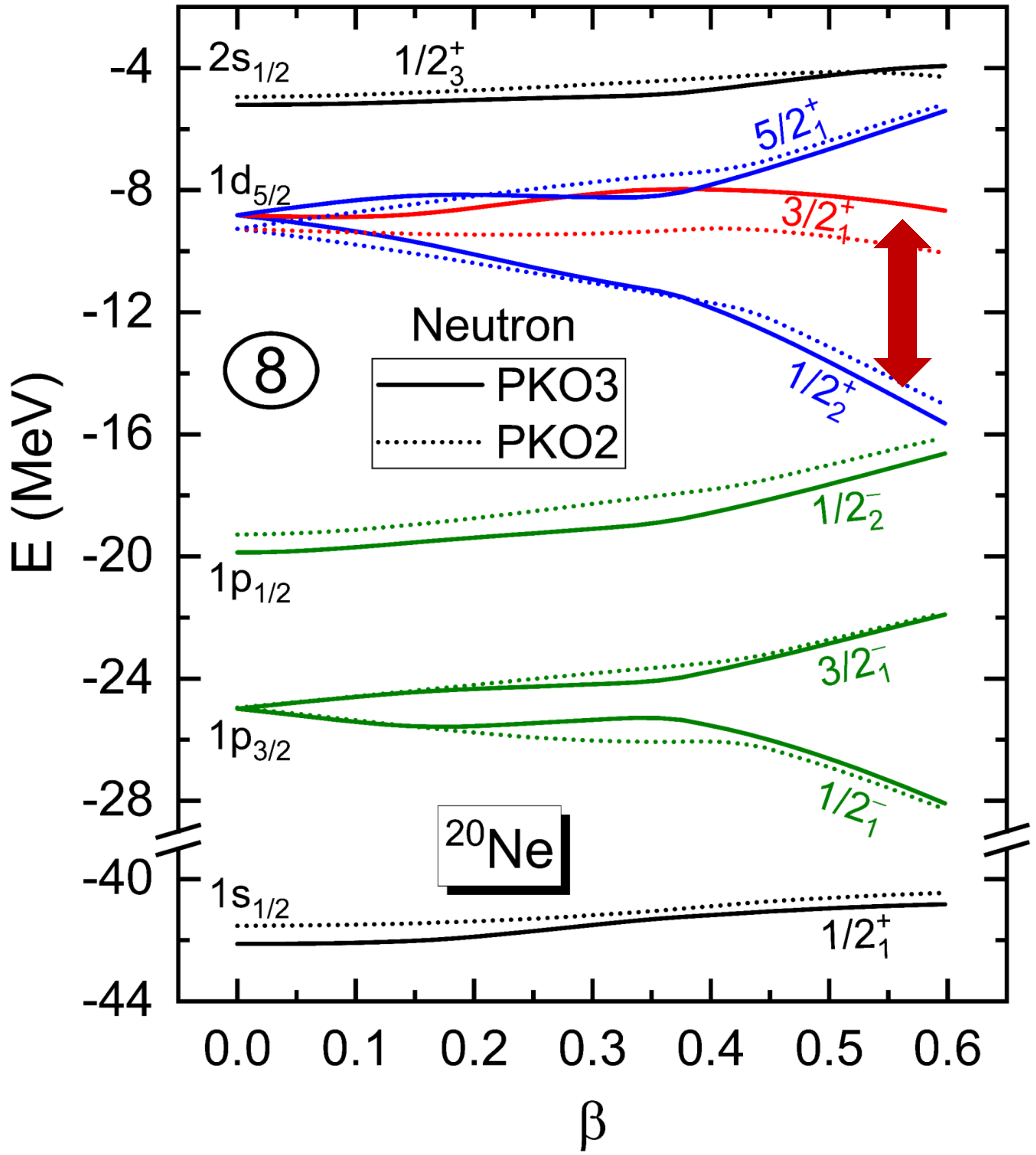
Geng, Xiang, Sun, WHL, PRC **101**, 064302 (2020)

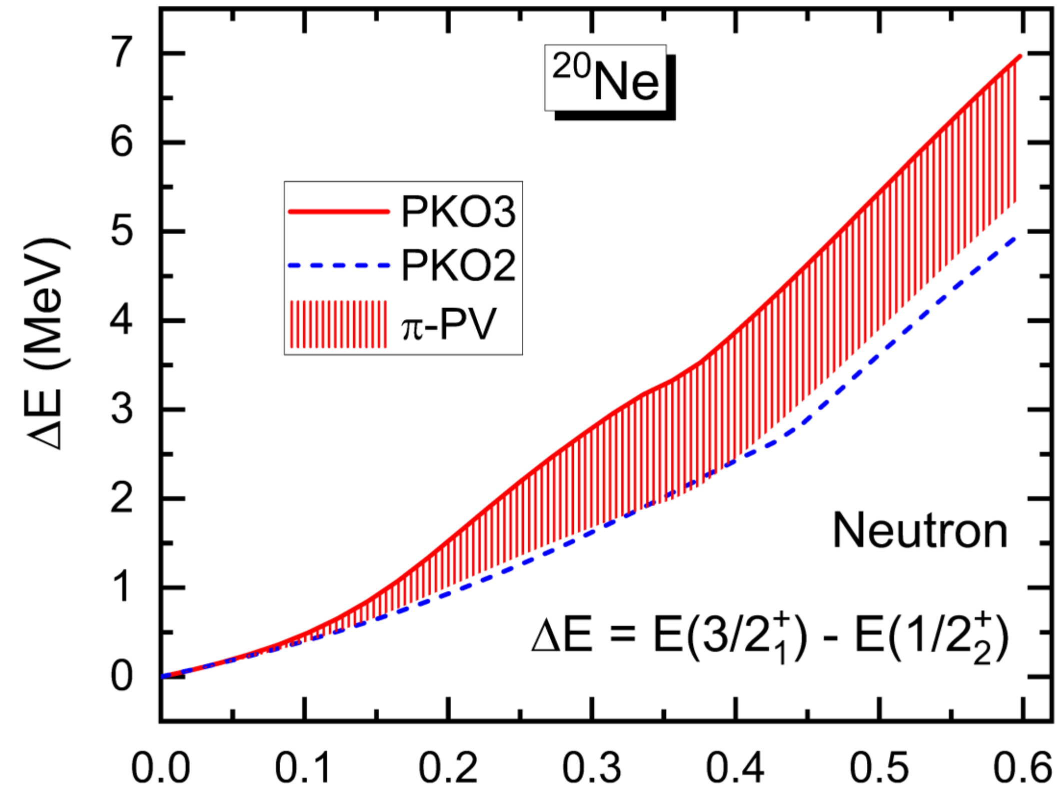
Tensor force effects in ^{20}Ne

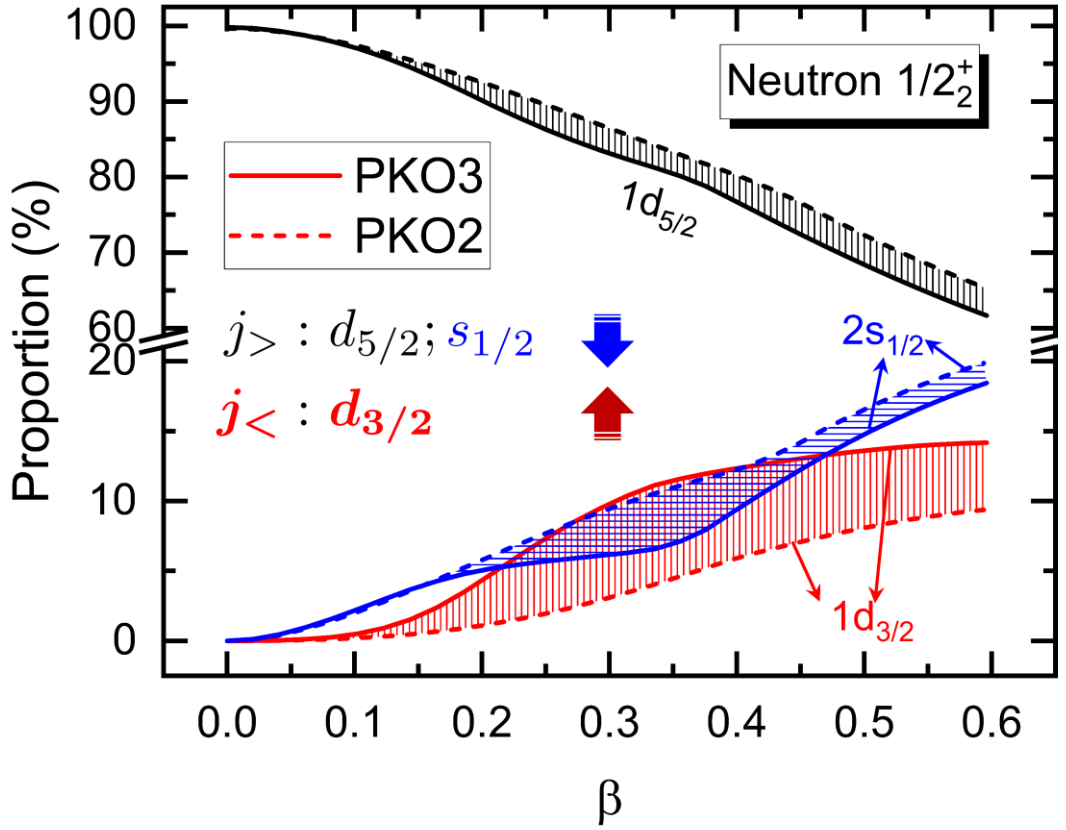
- Tensor force changes the mixing of spherical waves



PKO2
w/o π
vs
PKO3
with π







***π*-T** enhances the intrude **of $1d_{3/2}$ waves**

Shell effects enhanced by *π*-PV

PRC **101**, 064302 (2020)