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Dipole response in deformed halo nuclei ^{42,44}Mg

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Collaborators: Z.Z. Li, X.W. Sun, and J. Meng

Exotic nuclei

- **The study of exotic nuclei is one of the frontiers of nuclear physics**

I. Tanihata et al., Phys. Rev. Lett. 55, 2676 (1985).

  - Exotic nuclei near the drip-line are loosely bound with extended density distributions
  - Proper description requires treatment of continuum, pairing, and deformation

- **The relativistic continuum Hartree-Bogoliubov (RCHB) theory successfully describes exotic nuclei ground states**

  - Self-consistently incorporates pairing correlations and continuum effects
  - Reproduces and interprets the neutron halo in ^{11}Li

J. Meng and P. Ring, Phys. Rev. Lett. 77, 3963 (1996).
J. Meng, H. Toki, S. G. Zhou, S. Q. Zhang, W. H. Long, and L. S. Geng, Prog. Part. Nucl. Phys. 57, 470 (2006).

- **Taken deformation into account, the deformed relativistic Hartree-Bogoliubov theory in continuum (DRHBc) has been successfully applied to the study of deformed exotic nuclei**

S.-G. Zhou, J. Meng, P. Ring, and E.-G. Zhao, Phys. Rev. C 82, 11301 (2010).
L. Li, J. Meng, P. Ring, E.-G. Zhao, and S.-G. Zhou, Phys. Rev. C 85, 24312 (2012).
K. Y. Zhang, C. Pan, X. H. Wu, X. Y. Qu, X. X. Lu, and G. A. Sun, AAPPS Bulletin 35, 13 (2025).

Excitations in exotic nuclei

- **In order to investigate the excitations of exotic nuclei, many-body approaches beyond the mean-field approximation should be adopted**

P. Ring and P. Schuck, *The Nuclear Many-Body Problem* (Springer Berlin, Heidelberg, 2004).

- **The widely used quasi-particle random phase approximation (QRPA) method has a very high computational cost for deformed nuclei**
- **Instead, the quasi-particle finite amplitude method (QFAM) is equivalent to QRPA and avoids the high computational cost**

P. Avogadro and T. Nakatsukasa, Phys. Rev. C 84, 14314 (2011).

  - Soft monopole mode

J. C. Pei, M. Kortelainen, Y. N. Zhang, and F. R. Xu, Phys. Rev. C 90, 51304 (2014).

  - β decay half-lives

M. T. Mustonen, T. Shafer, Z. Zenginerler, and J. Engel, Phys. Rev. C 90, 24308 (2014).

  - Collective inertia in spontaneous fission

K. Washiyama, N. Hinohara, and T. Nakatsukasa, Phys. Rev. C 103, 14306 (2021).

  - γ ray strength function

L. González-Miret Zaragoza et al., Phys. Rev. C 112, 44303 (2025).

  - …

- **Based on DRHBc theory, QFAM has been implemented using finite difference method, which can only describe the monopole modes of nuclei**

X. Sun and J. Meng, Phys. Rev. C 105, 44312 (2022).

This work

- **In halo nuclei, a soft dipole resonance is predicted to emerge from the slow oscillation of the halo against the core**

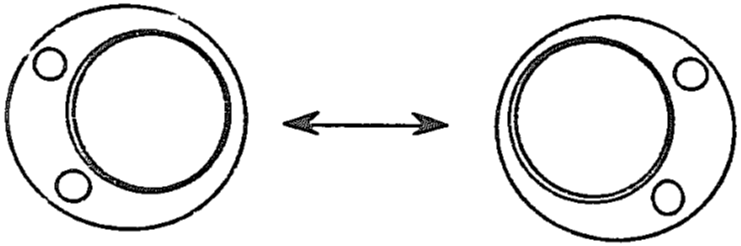


Figure from: K. Ikeda, Nucl. Phys. A 538, 355 (1992).

- **However, whether the soft dipole resonance exists remains an open question**

T. Nakamura, in *Handbook of Nuclear Physics*, edited by I. Tanihata, H. Toki, and T. Kajino (Springer Nature Singapore, Singapore, 2023).

- **In this work**

QFAM based on the DRHBc theory is developed and applied to investigate the electric dipole (E1) response in deformed halo nuclei ^{42,44}Mg

  - In order to study dipole response, direct variation is adopted instead of finite difference
  - We focus on the low-energy strength and the possible soft dipole resonance

Linear response theory

The TDRHB equation

$$i\hbar\frac{\partial}{\partial t}\mathcal{R}(t) = [\mathcal{H}(\mathcal{R}(t)) + \mathcal{F}(t), \mathcal{R}(t)]$$

For a weak harmonic external field, in the small amplitude limit, the density and Hamiltonian respectively oscillate around the equilibrium with the same frequency ω

$$\mathcal{F}(t) = \mathcal{F}(\omega)e^{-i\omega t} + \mathcal{F}^{\dagger}(\omega)e^{i\omega t}$$

$$\mathcal{R}(t) = \mathcal{R}_0 + \delta\mathcal{R}(\omega)e^{-i\omega t} + \delta\mathcal{R}^{\dagger}(\omega)e^{i\omega t}$$

$$\mathcal{H}(t) = \mathcal{H}_0 + \delta\mathcal{H}(\omega)e^{-i\omega t} + \delta\mathcal{H}^{\dagger}(\omega)e^{i\omega t}$$

The linear response equation in the quasi-particle basis can be obtained

$$\delta\mathcal{R}(\omega) = \begin{pmatrix} 0 & X \\ Y & 0 \end{pmatrix}$$

$$\boxed{\begin{aligned} (E_\mu + E_\nu - \hbar\omega)X_{\mu\nu} + \delta H^{20}_{\mu\nu} &= -F^{20}_{\mu\nu} \\ (E_\mu + E_\nu + \hbar\omega)Y_{\mu\nu} + \delta H^{02}_{\mu\nu} &= -F^{02}_{\mu\nu} \end{aligned}}$$

P. Avogadro and T. Nakatsukasa, Phys. Rev. C 84, 14314 (2011).
T. Nikšić, N. Kralj, T. Tutiš, D. Vretenar, and P. Ring, Phys. Rev. C 88, 44327 (2013).

- In conventional QRPA method, residual interaction matrix elements are calculated and then diagonalization is performed to solve the linear response equation

→ high computational cost for deformed heavy nuclei

Strength function

- QFAM solves the linear response equation by iteration for a given frequency ω

$$
\begin{tikzcd}
X_{\mu\nu}, Y_{\mu\nu} \arrow[d] & \delta H^{20}_{\mu\nu}, \delta H^{02}_{\mu\nu} \arrow[l] \\
\delta\rho_{kl}, \delta\kappa_{kl} \arrow[d] & \delta h_{kl}, \delta\Delta_{kl} \arrow[u] \\
\delta\rho(\boldsymbol{r}), \delta\kappa(\boldsymbol{r}) \arrow[r] & \delta h(\boldsymbol{r}), \delta\Delta(\boldsymbol{r}) \arrow[u]
\end{tikzcd}
$$

$\rightarrow$ avoids the high computational cost

With the obtained $X_{\mu\nu}(\omega), Y_{\mu\nu}(\omega)$, the strength function is given by

$$S(\omega) = -\frac{1}{\pi}\mathrm{Im}R_F(\omega) \quad , \quad R_F(\omega) = \frac{1}{2}\sum_{\mu\nu}\left\{F^{20*}_{\mu\nu}X_{\mu\nu}(\omega) + F^{02*}_{\mu\nu}Y_{\mu\nu}(\omega)\right\}$$

Based on the quasi-particle energies E_μ and wavefunctions U, V provided by the DRHBc theory, QFAM can be implemented

Two ways to implement QFAM

Finite difference

For a small $\delta\rho$, calculate the h twice: $\delta h = h(\rho_0 + \delta\rho) - h(\rho_0)$

- One can directly use the subroutines from the stationary code
- Restricted by the symmetry assumptions of stationary code
  - DRHBc theory: axially deformed, spatial reflection symmetry
- Often limited to monopole excitations

DRHBc+QFAM based on the finite difference method:

X. Sun and J. Meng, Phys. Rev. C 105, 044312 (2022).

Direct variation

Directly use the variation expression: $\delta h = \delta h(\rho_0, \delta\rho)$

- One needs to derive the formulas for δh, which could be complicated
- Independent of the symmetry assumptions of stationary code
- Easily applied to multipole excitations

→ Adopted in this work

QFAM based on DRHBc

The induced Hamiltonian is $\delta h_D = \begin{pmatrix} \delta S + \delta V^0 & -\boldsymbol{\sigma} \cdot \delta \boldsymbol{V} \\ -\boldsymbol{\sigma} \cdot \delta \boldsymbol{V} & -\delta S + \delta V^0 \end{pmatrix}$

where the induced potentials are given by

$$\delta S(\boldsymbol{r}) = \left[\alpha_S + 2\beta_S \rho_S^0 + 3\gamma_S \left(\rho_S^0\right)^2\right] \delta\rho_S + \delta_S \triangle \delta\rho_S$$

$$\delta V^0(\boldsymbol{r}) = \left[\alpha_V + 3\gamma_V \left(\rho_V^0\right)^2\right] \delta\rho_V + \delta_V \triangle \delta\rho_V + \tau_3 \alpha_{TV} \delta\rho_{TV} + \tau_3 \delta_{TV} \triangle \delta\rho_{TV} + e\frac{1-\tau_3}{2} \delta A^0$$

$$\delta \boldsymbol{V}(\boldsymbol{r}) = \left[\alpha_V + \gamma_V \left(\rho_V^0\right)^2\right] \delta\boldsymbol{j}_V + \delta_V \triangle \delta\boldsymbol{j}_V + \tau_3 \alpha_{TV} \delta\boldsymbol{j}_{TV} + \tau_3 \delta_{TV} \triangle \delta\boldsymbol{j}_{TV} + e\frac{1-\tau_3}{2} \delta\boldsymbol{A}$$

The induced pairing potential is

$$\delta\Delta_{lm} = \frac{1}{2} \sum_{pq} \bar{v}_{lmpq} \delta\kappa_{pq}$$

with a density-dependent zero-range interaction

$$V(\boldsymbol{r}_1, \boldsymbol{r}_2) = V_0 \frac{1}{2} \left(1 - P^\sigma\right) \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2) \left(1 - \frac{\rho^\tau(\boldsymbol{r}_1)}{\rho_{\text{sat}}}\right)$$

Numerical details

The DRHBc calculation

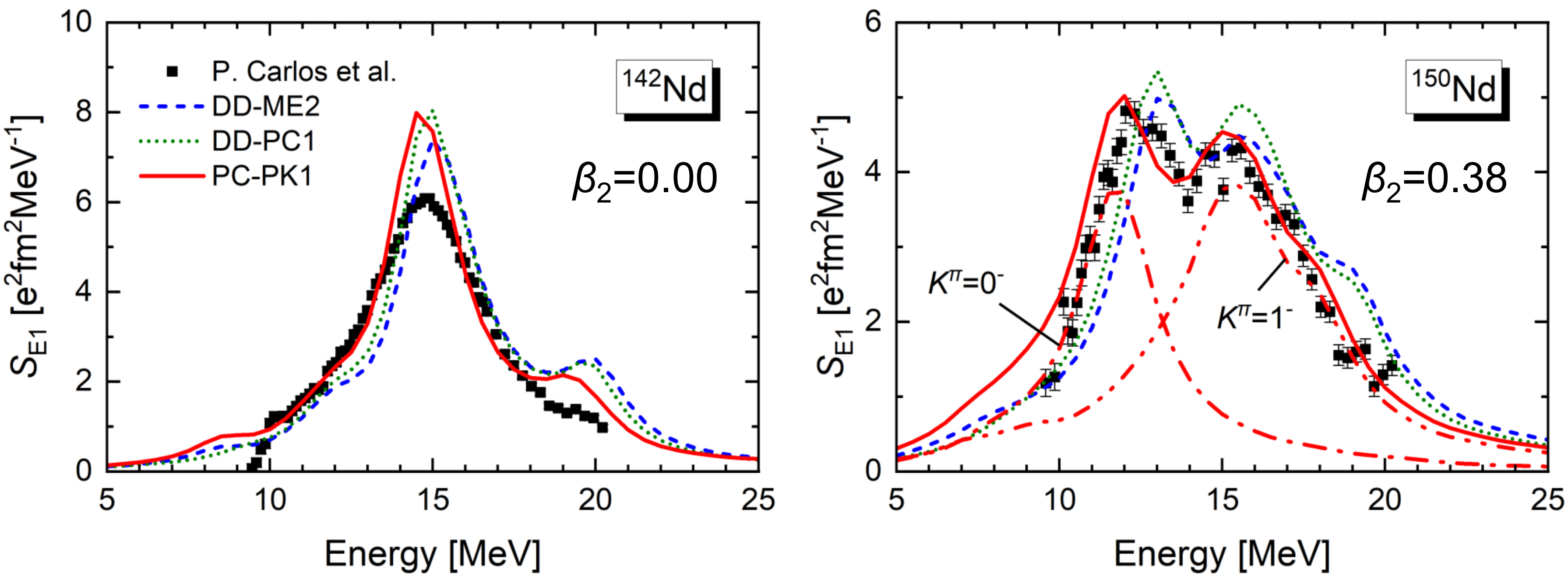
- Relativistic density functional: PC-PK1
- Pairing strength: −325.0 MeV fm^3
- Pairing window: 100 MeV
- Box size: R_{box} = 20 fm
- Mesh size: Δr = 0.1 fm
- Energy cutoff: E_{cut} = 150 MeV
- Angular momentum cutoff: J_{max} = 23/2 $\hbar$
- Legendre expansion order: λ_{max} = 6

The QFAM calculation

- Scalar/vector spherical harmonics expansion order: L_{max} = 6
- Smearing width: Γ = 2 MeV
- Quasi-particle energy cutoff: none

Results

■ E1 strength function in ^{142}Nd and ^{150}Nd



- ☐ The calculations based on DD-PC1 and DD-ME2 are performed by using HO simplex-y basis using 20 shells

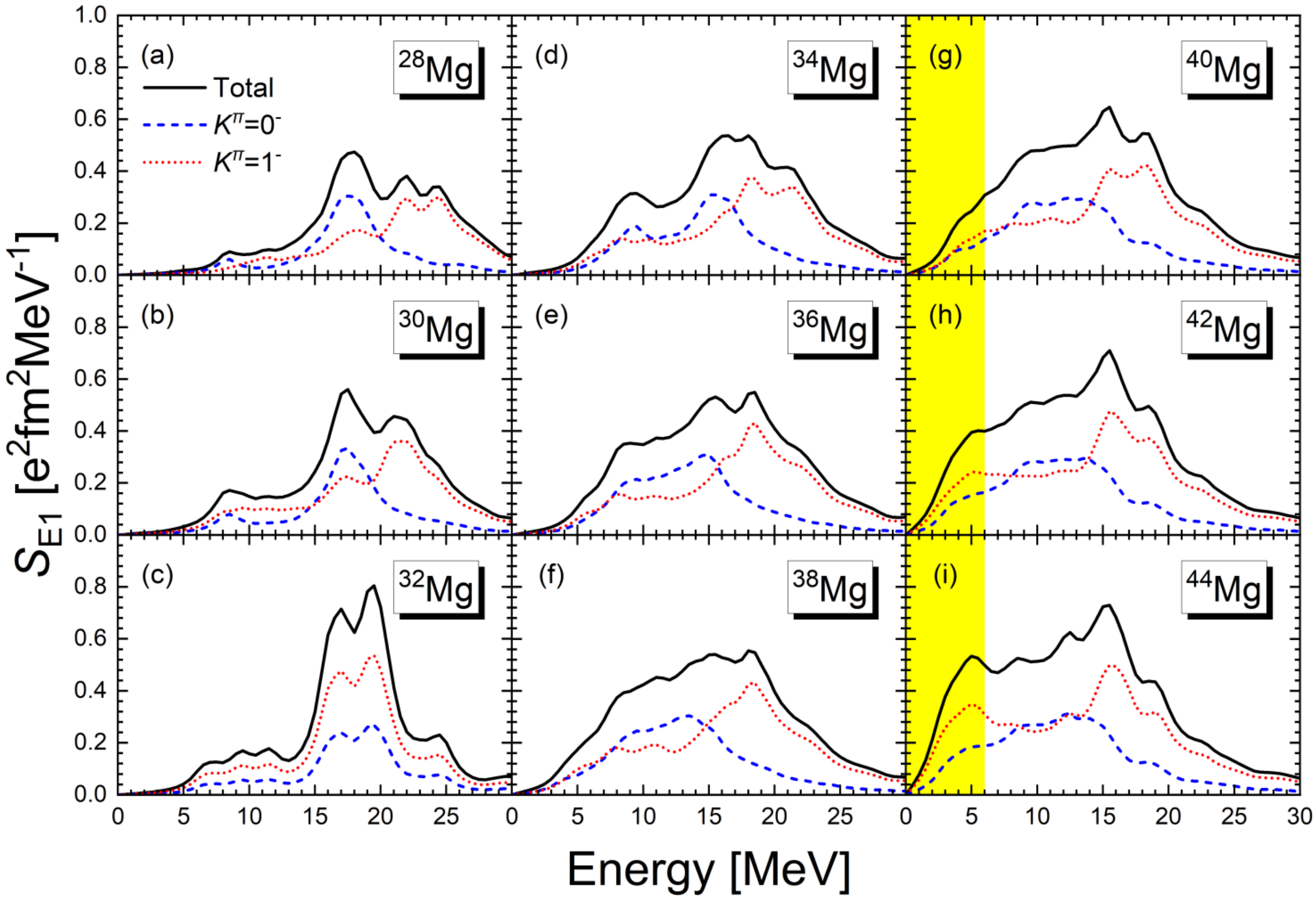
A. Bjelčić and T. Nikšić, Comput. Phys. Commun. 287, 108689 (2023).

- ☐ The PC-PK1 results in this work are close to the DD-PC1 and DD-ME2 results
- ☐ The results of this work reproduce well the experimental data

P. Carlos et al., Nucl. Phys. A 172, 437 (1971).

Results

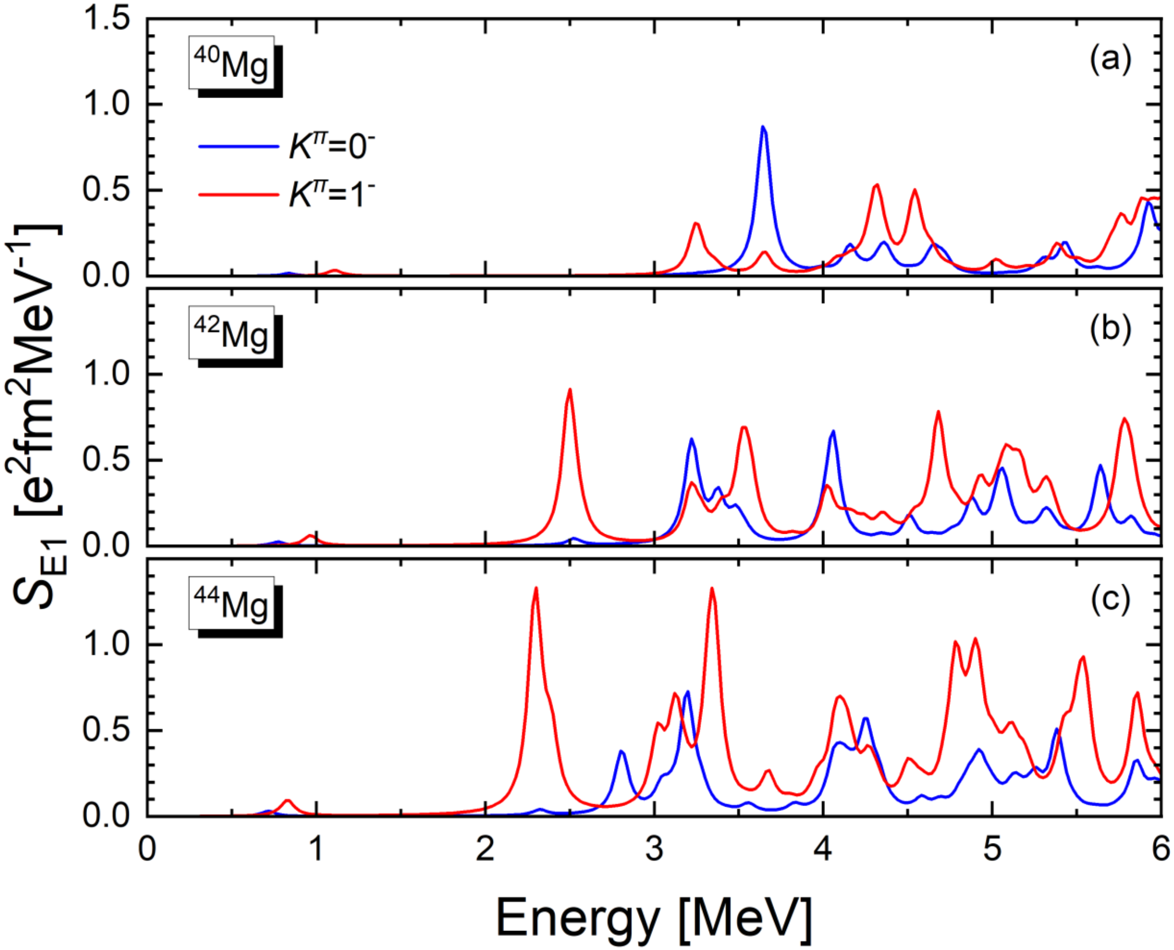
■ **E1 strength function in neutron-rich magnesium isotopes**



□ In neutron-rich Mg isotopes, an enhancement is predicted in the low-energy region and increases with neutron number

Results

■ **E1 strength function in ^{40,42,44}Mg (Γ = 0.1 MeV)**



□ A peak is found at ~2.5 MeV in ^{42,44}Mg but not in ^{40}Mg.

Results

■ **Transition density of ^{42,44}Mg**

^{42}Mg:

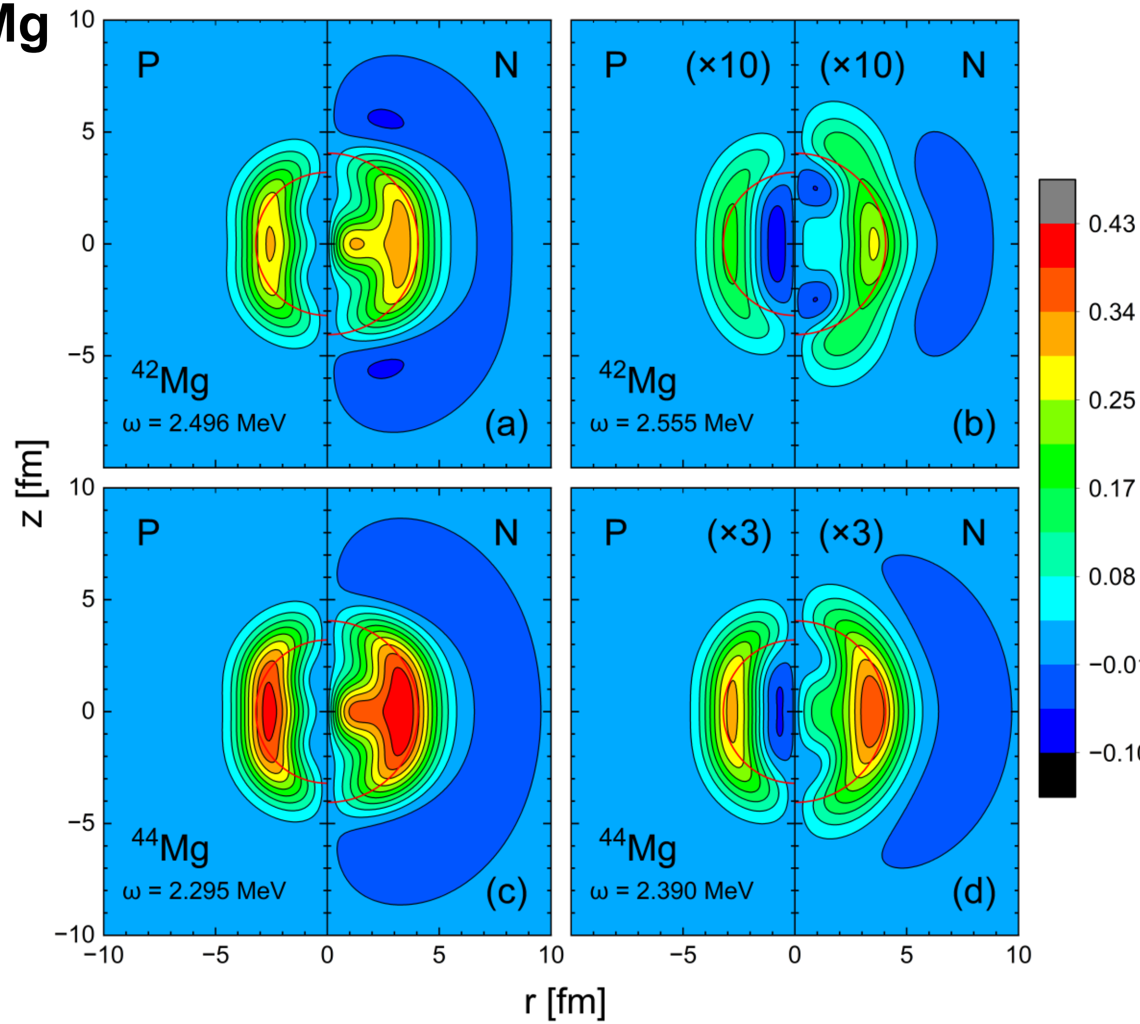
two QRPA eigenstates

2.496, 2.555 MeV

^{44}Mg:

two QRPA eigenstates

2.295, 2.390 MeV



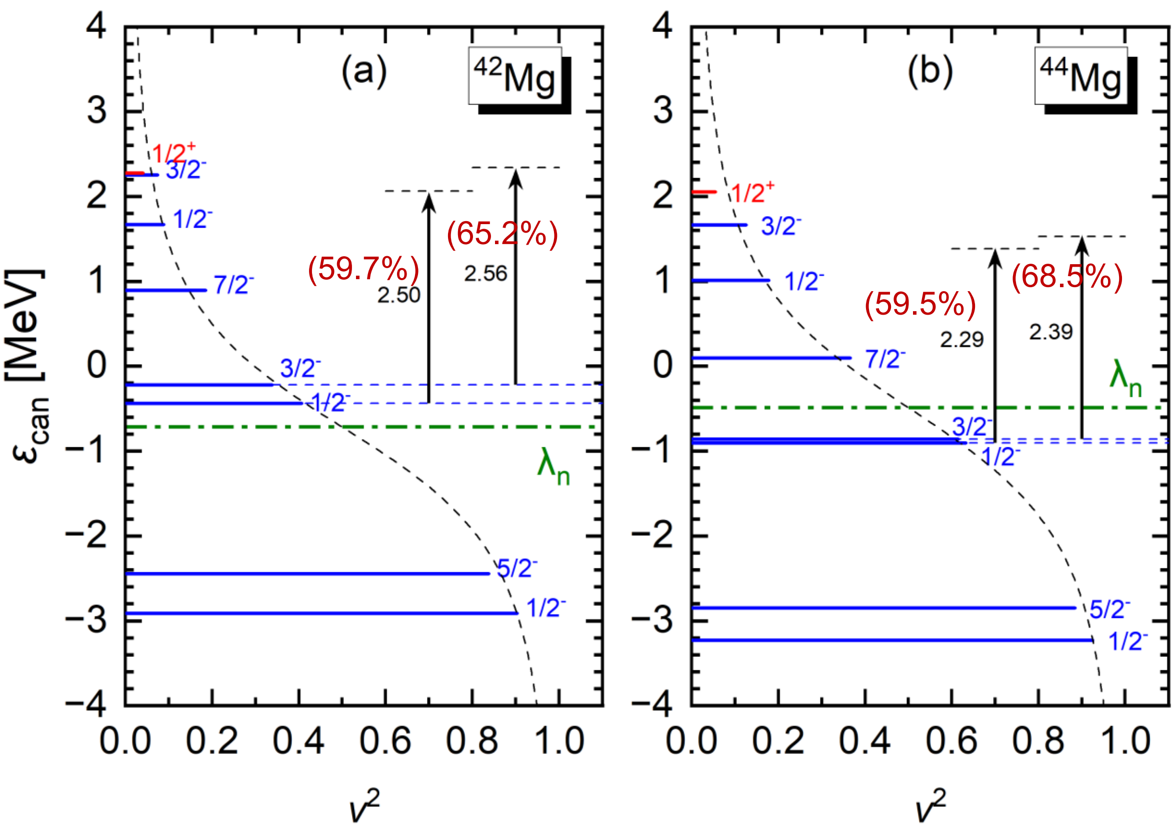
- On the surface and in the interior, neutrons and protons oscillate in phase
- Outside of the nucleus, only neutrons oscillate, and in opposite phase to the core

Results

■ The dominant transitions

- ➢ The QRPA amplitudes are extracted from the QFAM amplitudes

N. Hinohara, M. Kortelainen, and W. Nazarewicz, Phys. Rev. C 87, 64309 (2013).



- ➢ The dominant contributions come from the transition of halo neutrons

Summary and outlook

- **Summary**
  - The quasi-particle finite amplitude method (QFAM) based on the DRHBc theory is developed and applied to investigate the electric dipole (E1) response in deformed halo nuclei ^{42,44}Mg
  - A significant enhancement of E1 strength is found around 2.5 MeV
    - This enhancement predominantly arises from excitations of halo neutrons
    - The halo-core relative oscillation presents a consistent picture with the soft dipole resonance
- **Outlook**
  - Investigate the shape decoupling effect on nuclear vibrations
  - Apply method to study γ ray strength functions and *r*-process

The End

Thanks for your attention!