# Halo phenomena in light to medium mass nuclei with three-body models

HALO-40 INTERNATIONAL SYMPOSIUM COMMEMORATING THE 40TH ANNIVERSARY OF THE HALO NUCLEI

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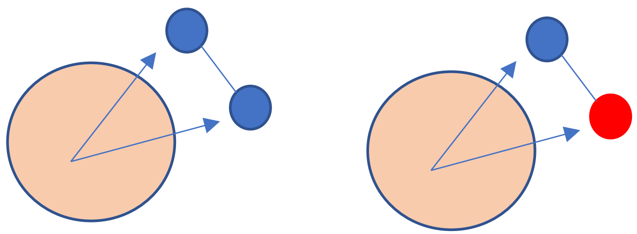

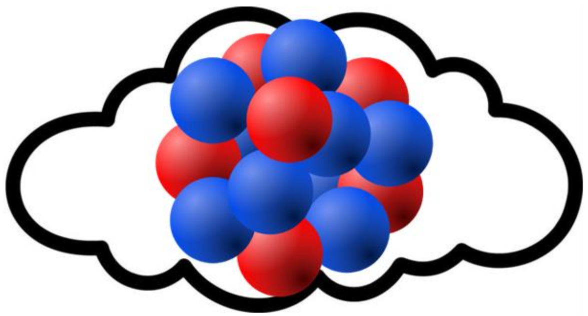

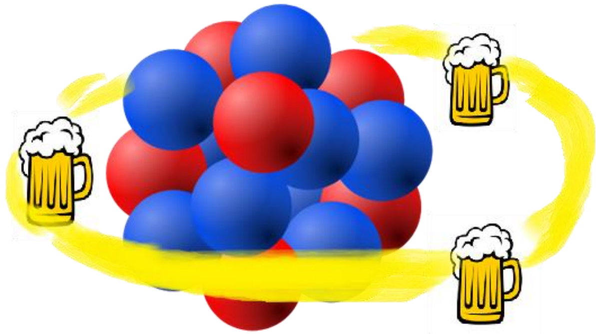

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MOdeling Nuclear STructure and REactions

# Collaborators

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A little history:

1985 11Li experiment by Isao Tanihata at the RIKEN Japan
1987 Bjørn Jonson and Hans Geissel coined the term "halo nuclei" paper published in Nuclear Physics News.

# Outline of the presentation

AIM: update you on the last 5 years of research on three-body models at the driplines

- New insights on the structure of 29F at the border of the island of inversion
- Extensions to 31F: radiii,  B(E1) and halo structure
- p-n correlations in the external orbitals of 102Sb

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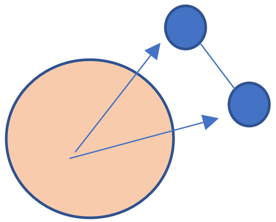
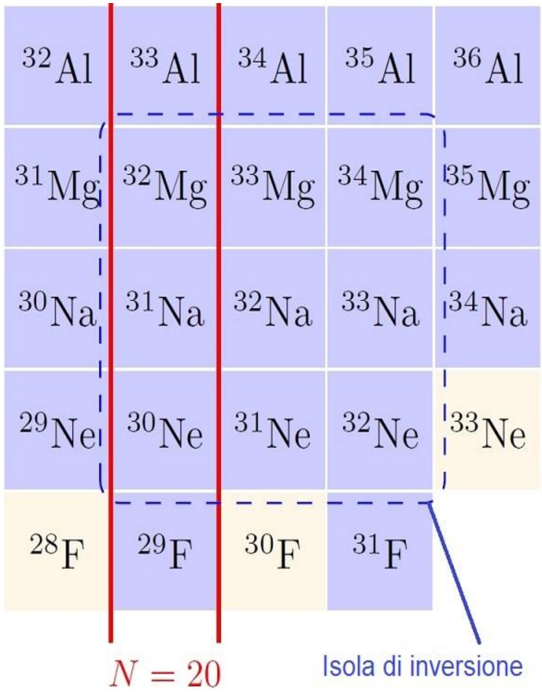
[2] J. Singh, J. Casal, W. Horiuchi, L. Fortunato and A. Vitturi, **Phys. Rev. C 101, 024310 (2020)**
[3] J .Casal, Jagjit Singh, L. Fortunato, W. Horiuchi and A. Vitturi, **Phys. Rev. C 102, 064627 (2020)**
[4] G. Singh, Jagjit Singh, J. Casal and L. Fortunato, **Phys. Rev. C 105, 014328 (2022)**
[5] T. Oishi, M. Kimura, L. Fortunato, **Phys. Rev. C 111, 034307 (2025)**

# Fluorine-29 stands on the coast of the island of inversion


COMMUNICATIONS PHYSICS






PERSPECTIVE

https://doi.org/10.1038/s42005-020-00402-5

OPEN

## The $^{29}$F nucleus as a lighthouse on the coast of the island of inversion

L. Fortunato [1,2]✉, J. Casal [1,2], W. Horiuchi [3], Jagjit Singh [4] & A. Vitturi[1,2]

# Fluorine-29 stands on the coast of the island of inversion

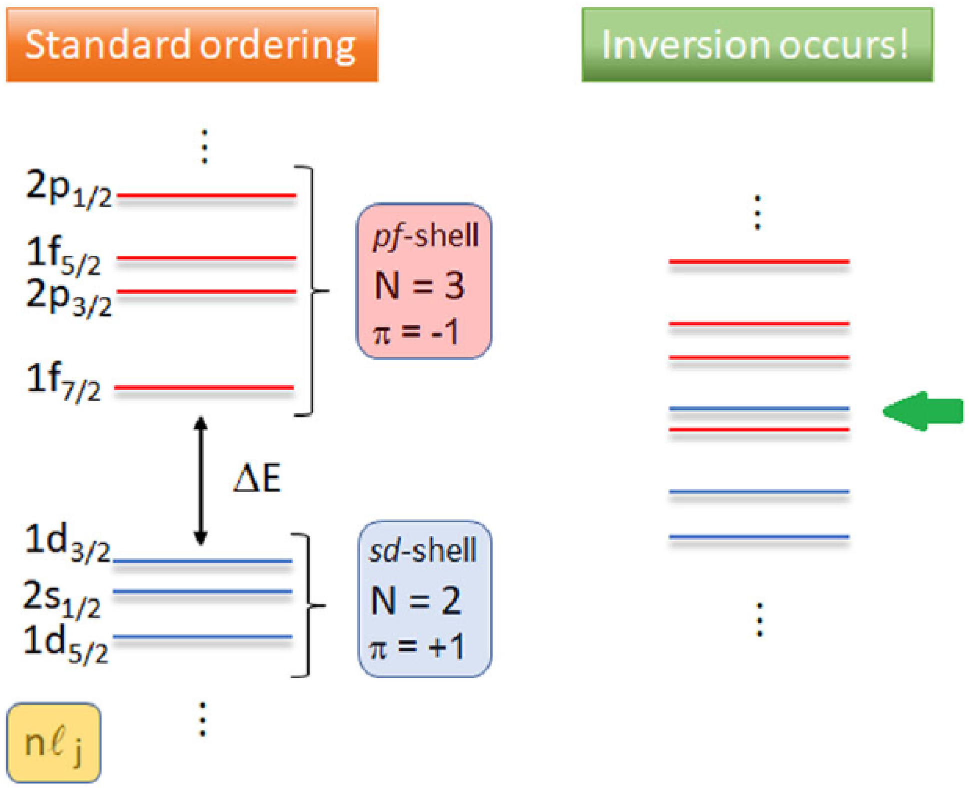



**Fig. 1 Standard ordering of shell-model energy levels and typical inversion mechanism.** The $N = 2$ *sd*-shell and the $N = 3$ *pf*-shell with positive and negative parity $\pi$, respectively, are shown on the left in the standard ordering (states are labeled by the standard set of quantum number $n\ell_j$). Inversion occur (right) when the shell gap, $\Delta E$, associated with the filling of 20 neutrons, disappears and one level (or more) of the $N = 3$ *pf*-shell gets lower than one (or more) of the levels of the $N = 2$ *sd*-shell.

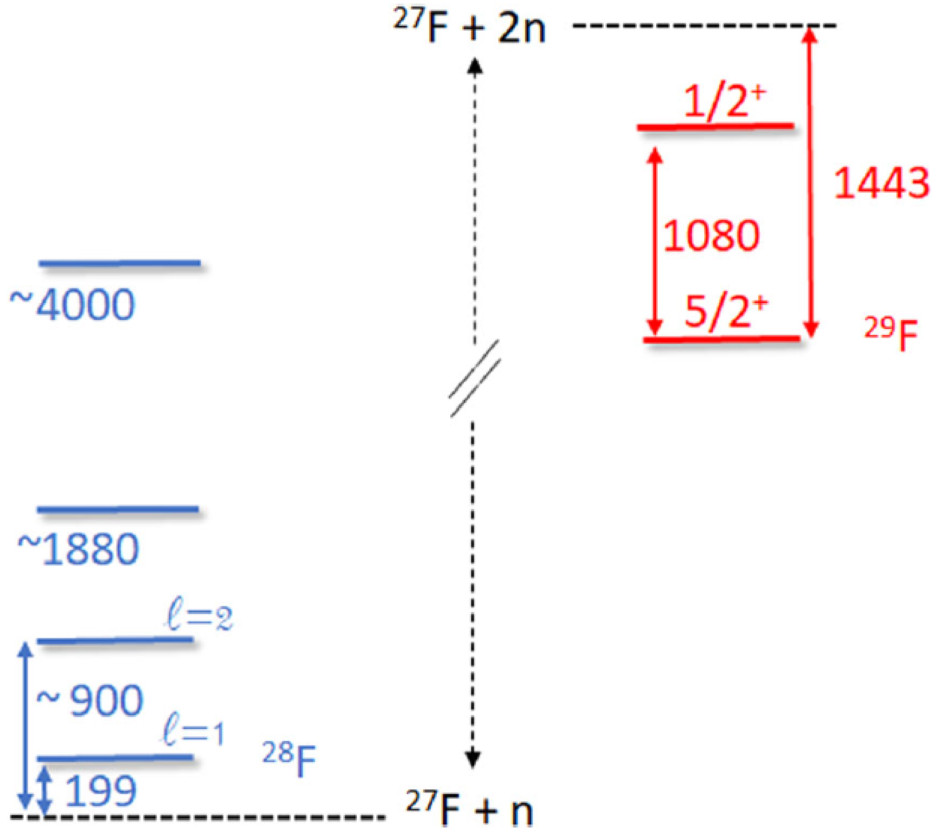



**Fig. 2 Synopsis of known experimental data on $^{28,29}$F.** All energies are in keV (not to scale) from refs. [4--6,11]. States in red are labelled by the $J^\pi$ quantum numbers and energies are referred to the $^{27}$F + 2n threshold. States in blue are inferred from the $^{29}$F(−1n) column of Fig. 2 of ref. [6], and correspond only to the states decaying to the ground state of $^{27}$F. They are labelled by the orbital angular momentum quantum number, $\ell$, when available. Energies are referred to the $^{27}$F + n threshold.

# Fluorine-29 stands on the coast of the island of inversion

We had previously PRC **101**, 024310 (2020) proposed 4 scenarios for the structure of the very exotic nucleus 29F, called A,B,C,D , based on the three-body hyperspherical formalism by J.Casal

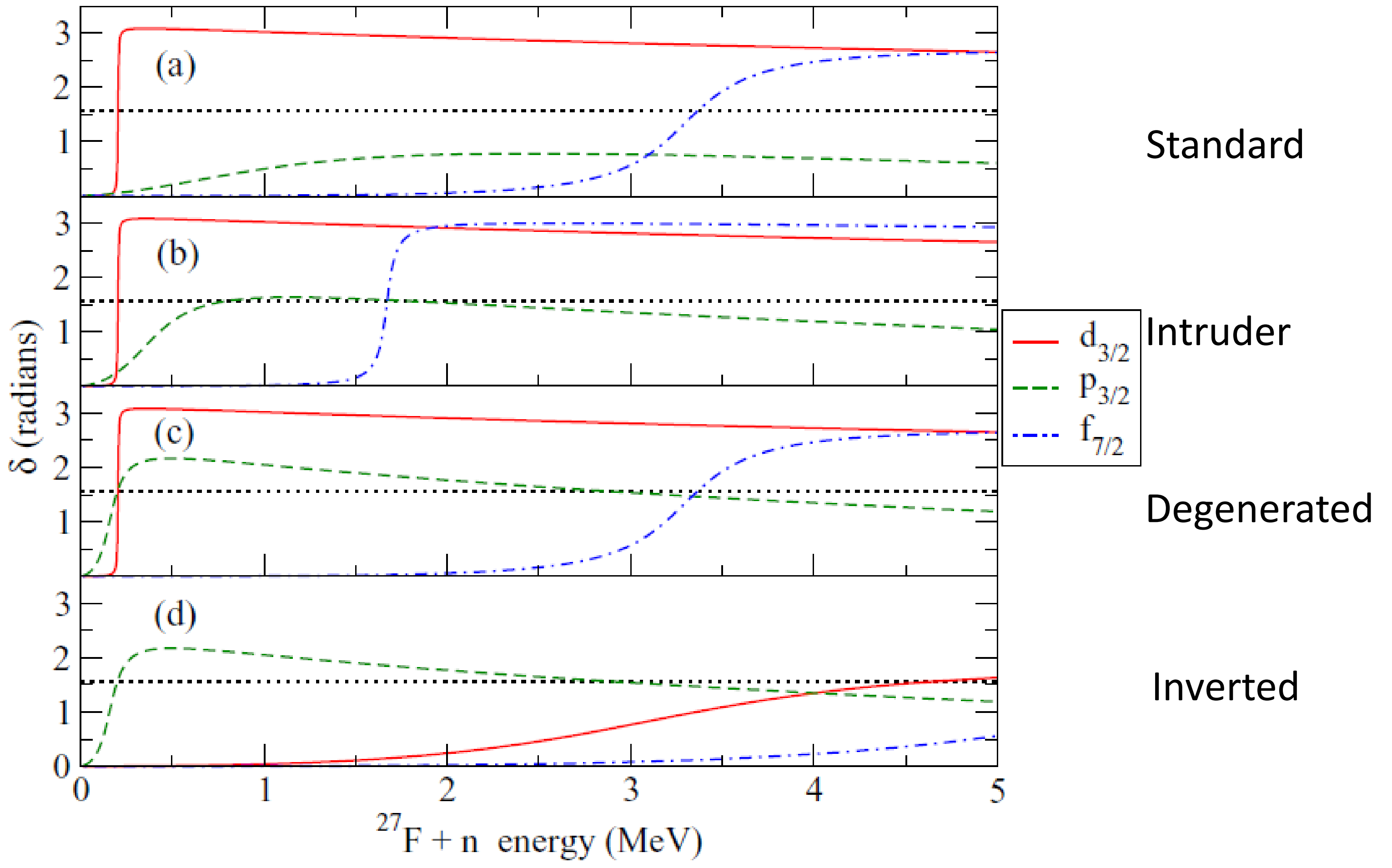
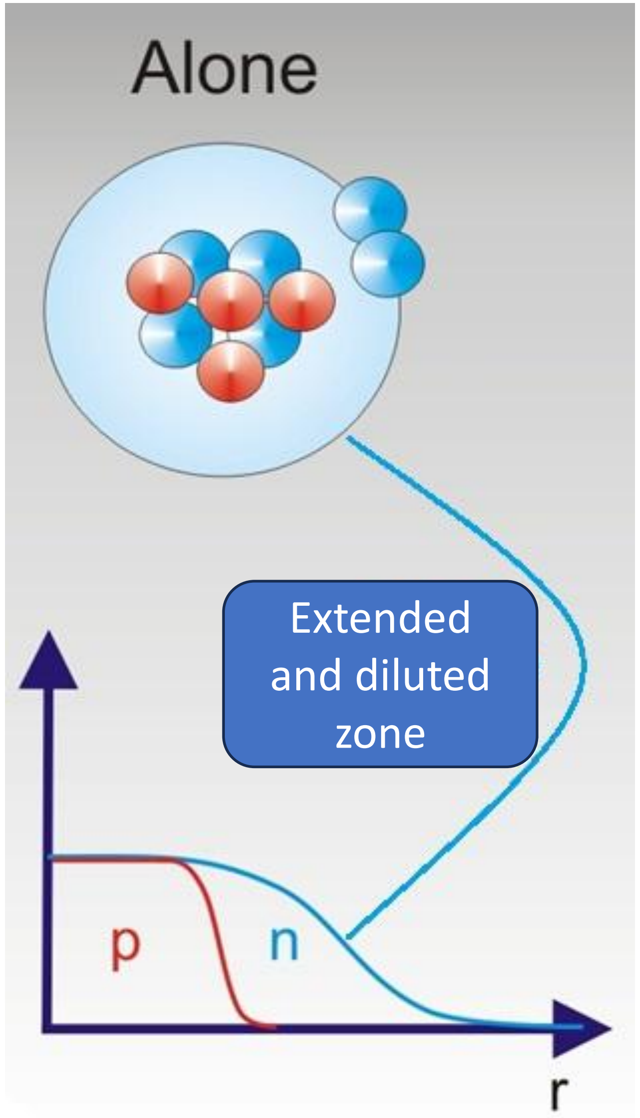



FIG. 1. $^{27}F + n$ phase shifts for $d_{3/2}$, $p_{3/2}$, and $f_{7/2}$ states, corresponding to different sets (A–D). The dotted black line corresponds to $\pi/2$.



# Fluorine-29 stands on the coast of the island of inversion

New experiments by

1) Revel, A. et al. “Extending the southern shore of the island of inversion to 28F” PRL 124, 152502 (2020)

and then

2) Bagchi, S. et al. “Two-neutron halo is unveiled in 29F” PRL 124, 222504 (2020)

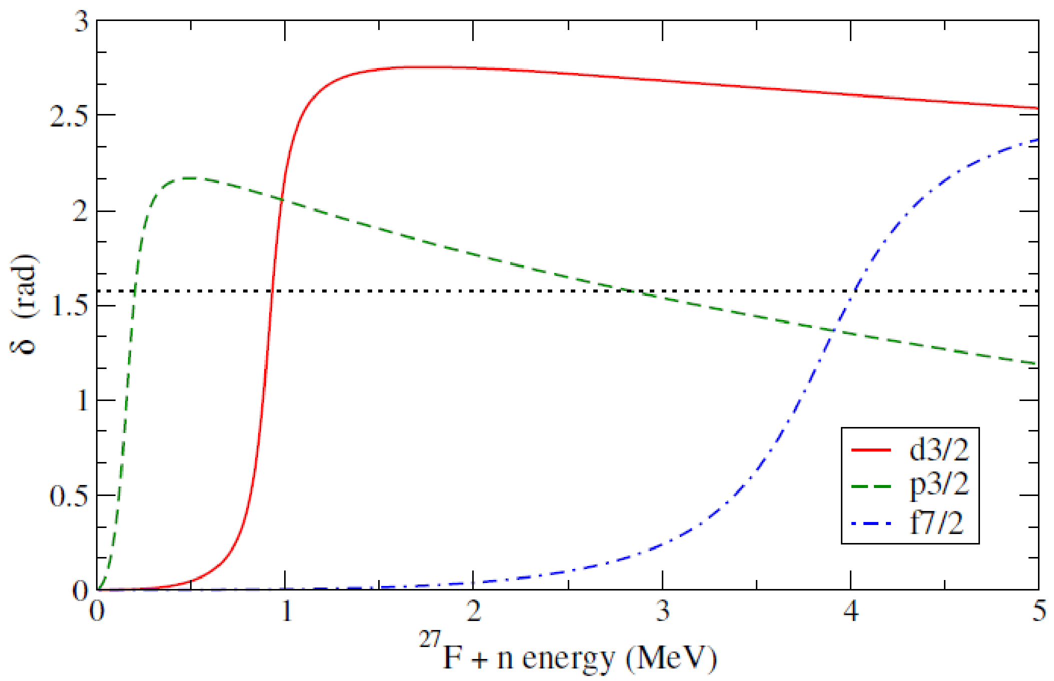
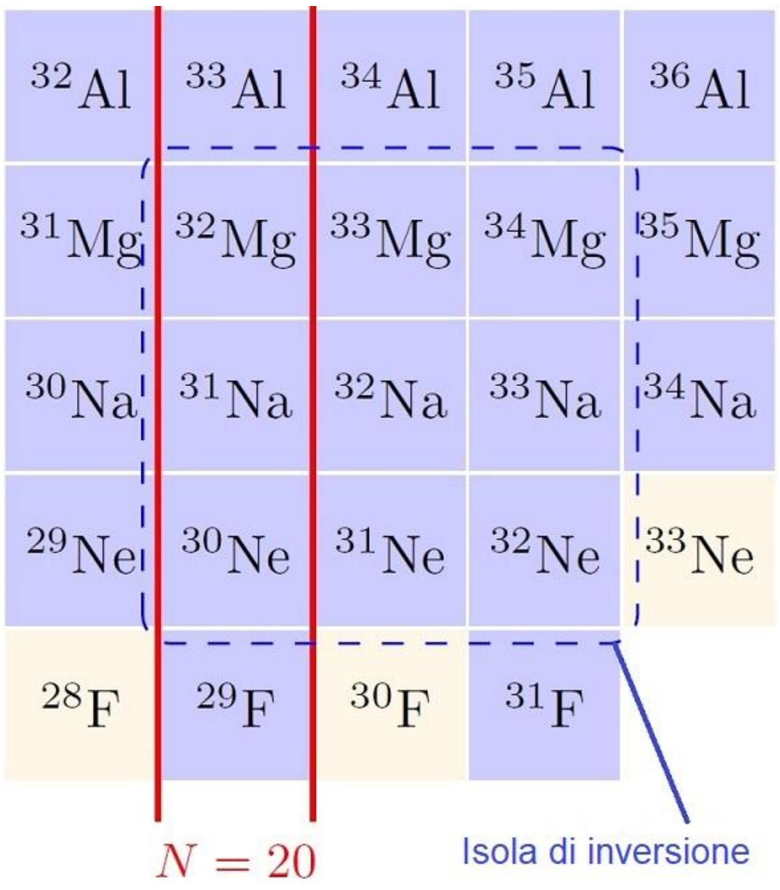


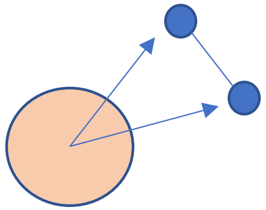

**Fig. 3 Phase-shifts, $\delta$, for the $^{27}$F + n system in the $D^\flat$ scenario as a function of the neutron-core relative energy.** Adjusting the red curve to reproduce the *d*-resonance at about 0.9 MeV, also gives the *f*-wave state (blue curve) at about 4 MeV.




Final picture … Scenario D ♭

two-neutron halo in 29F is linked to the occupancy of *pf* intruder configurations

# Fluorine-29 stands on the coast of the island of inversion



$$\frac{d\sigma}{d\varepsilon} = (\alpha Z_2)^2 \sum_{\pi\lambda\mu} \left(\frac{\Delta E}{\hbar c}\right)^{2\lambda-2} \frac{dB}{d\varepsilon}(\pi\lambda; \mathrm{gs} \rightarrow \mathrm{Cont.})|G_{\pi\lambda\mu}(\beta^{-1})|^2 g_\mu(\xi)$$

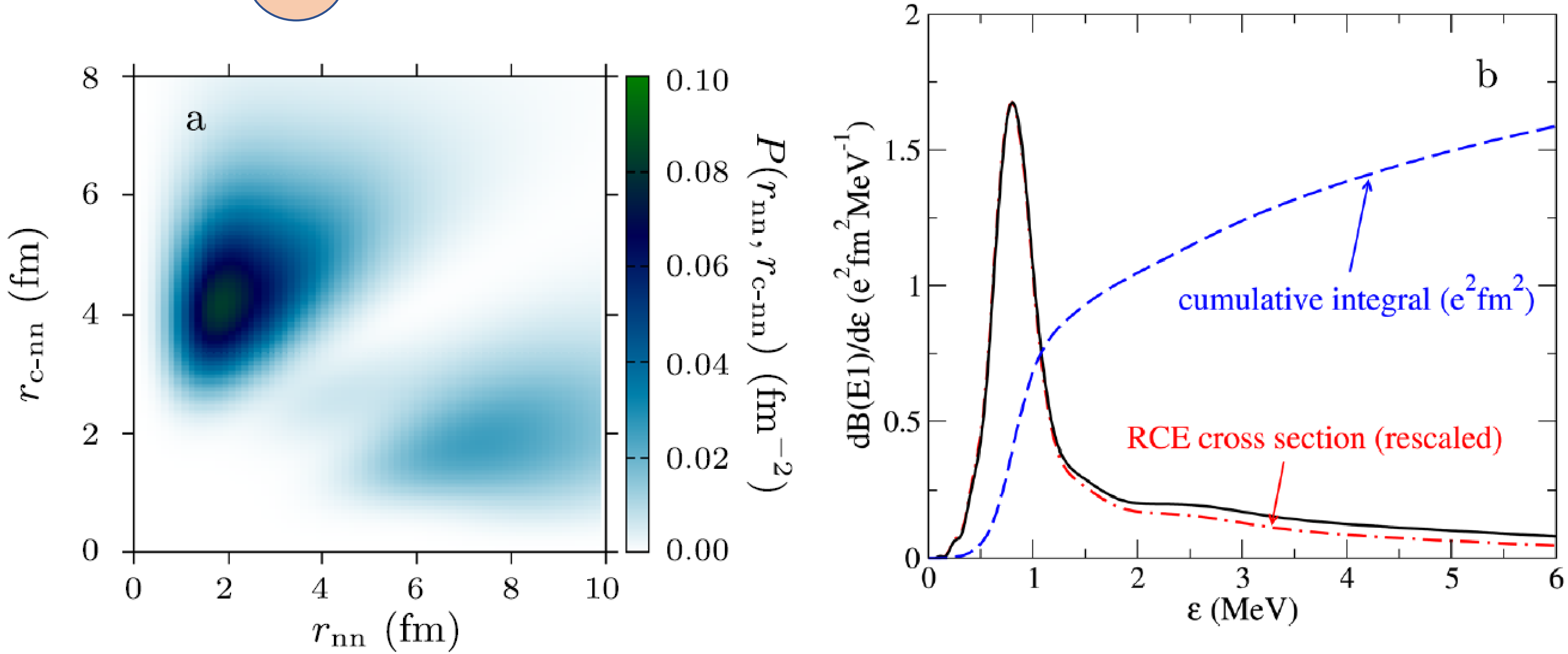



**Fig. 4 Results on $^{29}$F within the new $D^\flat$ scenario. a** Ground-state probability density of $^{29}$F as a function of the distance between the two valence neutrons ($r_{nn}$) and that between their center of mass and the core ($r_{c-nn}$). The maximum probability density corresponds to the dineutron configuration. **b** Electric dipole ($E1$) strength function from the ground state to continuum as a function of the $^{27}\mathrm{F} + \mathrm{n} + \mathrm{n}$ energy. The dashed line indicates the cumulative integral. The dash-dotted line is the corresponding Relativistic Coulomb Excitation (RCE) cross-section, scaled to the same maximum to illustrate the decreasing proportionality with the energy.

# From PRC102 : analysis of B(E1) dipole excitations

- 29F (27F+*n* + *n*) wave functions are built within the hyperspherical harmonics expansion formalism (HHE)
- total reaction cross sections are calculated using the Glauber theory for a carbon target at 240 MeV/nucleon
- continuum states and *B*(*E*1) transition probabilities are described in a pseudostate approach using THO basis
- form factors are used in continuum-discretized coupled-channels (CDCC) calculations to describe low-energy scattering.

Our predictions show the low-lying enhancement of the $E1$ response expected for halo nuclei and the relevance of dipole couplings for low-energy reactions on heavy targets

$$\langle \mathrm{g.s.}||\widehat{Q}_1||n1\rangle = \sqrt{3}Ze\sqrt{\frac{2}{A(A+2)}}\sum_{\beta,\beta'}\delta_{l_x l_x'}\delta_{S_x S_x'}(-)^{l_x+S_x}\hat{l}_y\hat{l}_y'\hat{l}\hat{l}' \times W(ll'l_y l_y';1l_x)W(01ll';1S_x)\begin{pmatrix} l_y & 1 & l_y' \\ 0 & 0 & 0 \end{pmatrix} \times \sum_{ii'} C_{\mathrm{g.s.}}^{i\beta 0} C_{\mathrm{n}}^{i'\beta' 1} I_{i\beta,i'\beta'},$$

Results nicely converge in the hyperspherical formalism ->

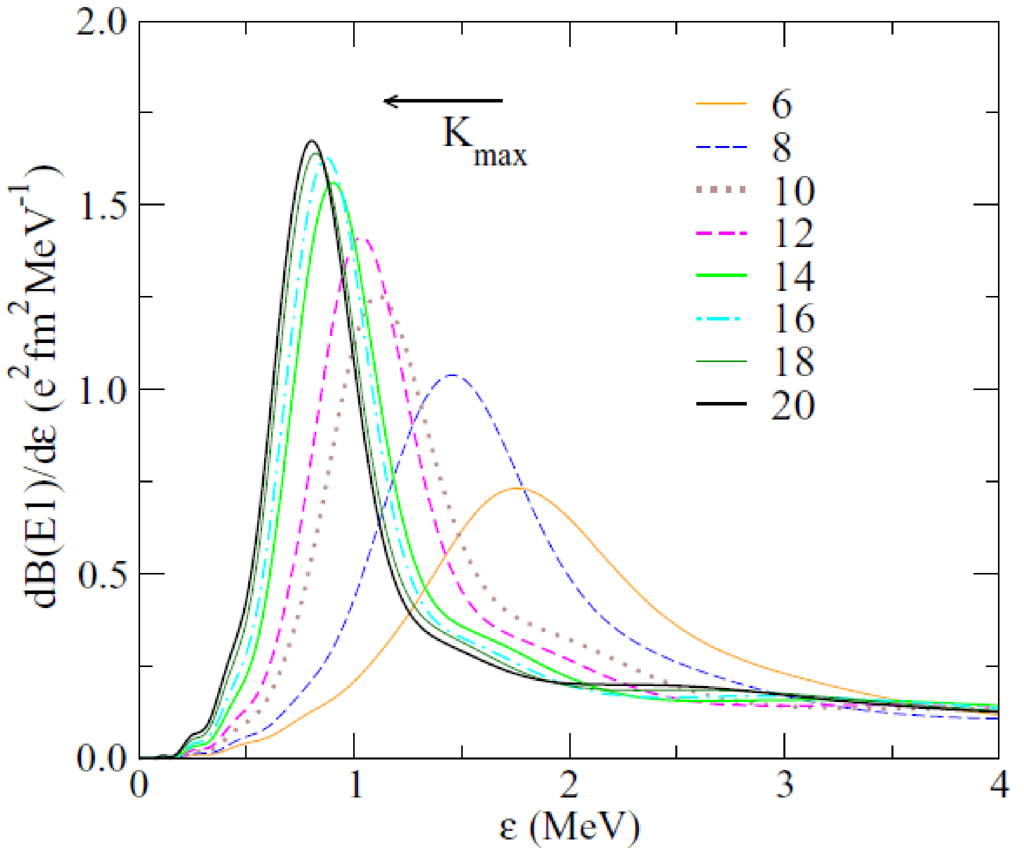
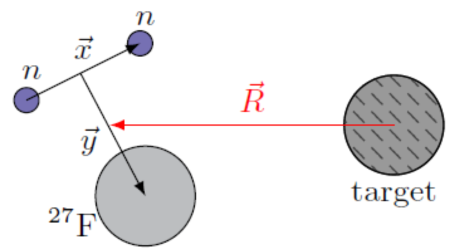



FIG. 7. Convergence of the $B(E1)$ distribution for $^{29}$F with respect to $K_{max}$.

# Results of PRC102 on dipole excitations



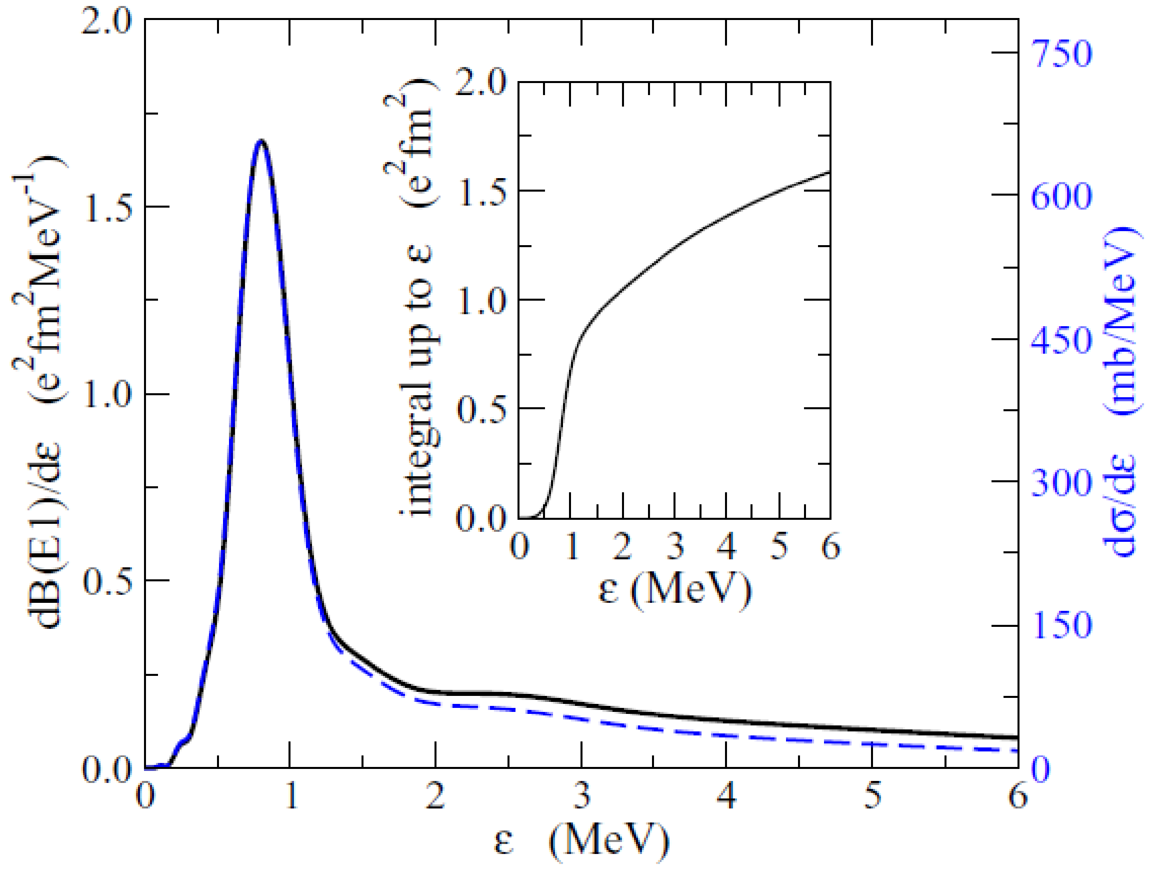




FIG. 8. $B(E1)$ distribution for $^{29}$F as a function of the continuum energy. The inset shows the cumulative integral up to $\varepsilon$. The dashed blue line corresponds to the RCE cross section of $^{29}$F at 235 MeV/u on a lead target. Note the different scales.

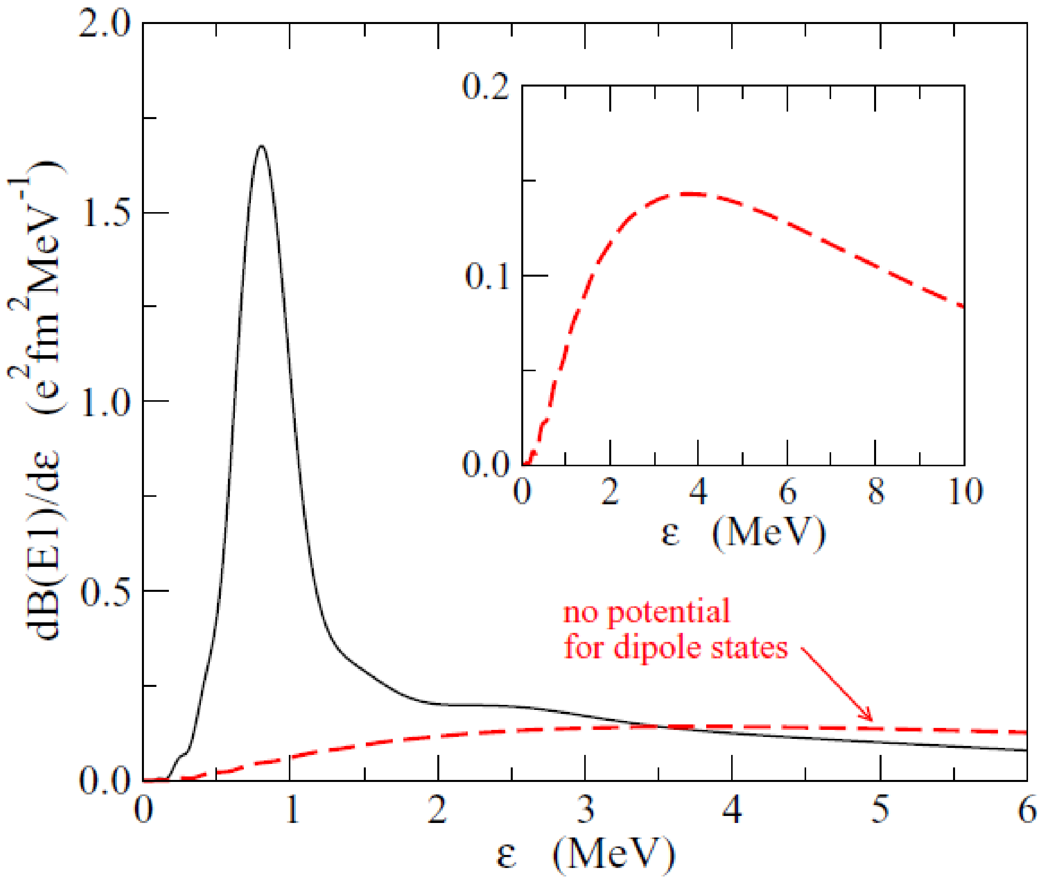
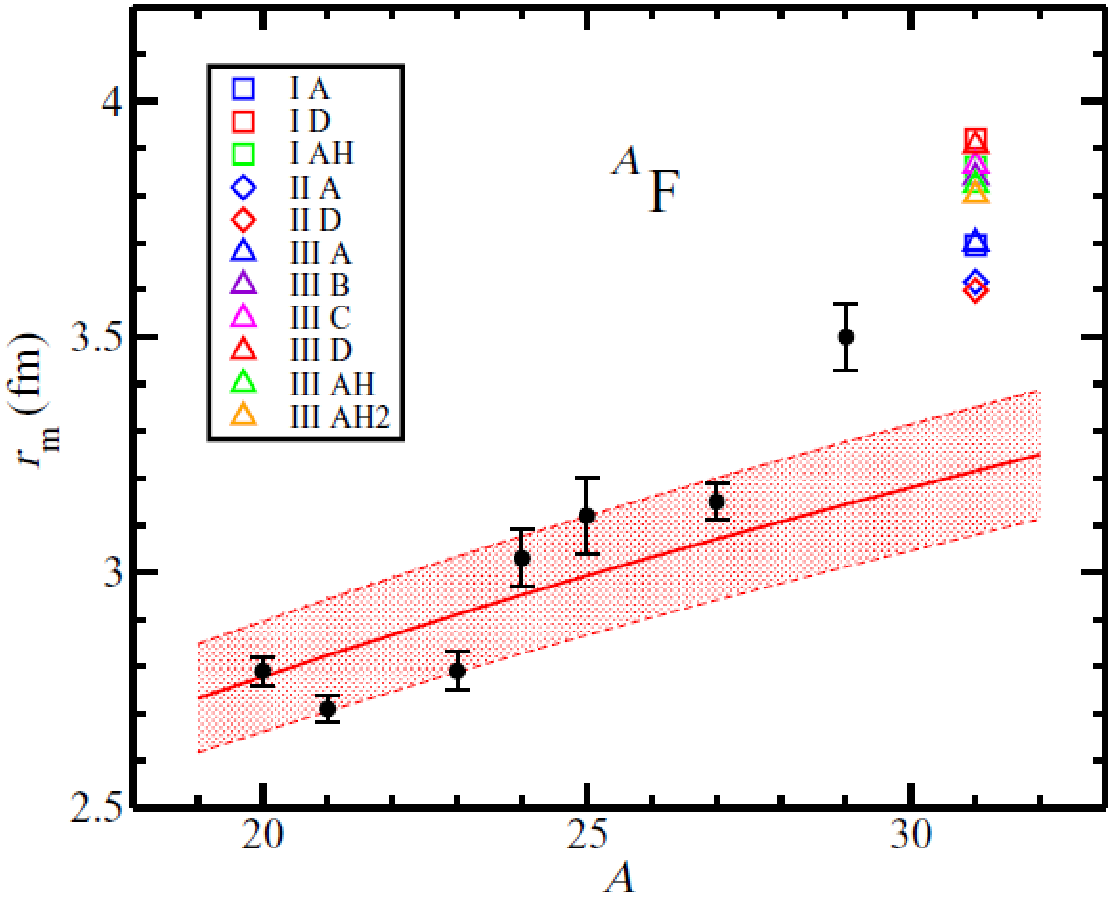



FIG. 9. $B(E1)$ distribution compared with the results setting the potential for dipole states to zero (dashed red line). The inset shows this unrealistic calculation with a more appropriate scale and up to 10 MeV in the continuum.

# PRC105 extension to 31F, tricky because n+n+29F, but 29F=n+n+27F !

Again, we build several different scenarios and examine the consequences.
Dominance of pf-shell configurations, with various degrees of p-wave components. All point to increased radius and larger B(E1), the hallmarks of a halo formation

PHYSICAL REVIEW C **105**, 014328 (2022)




| Case | Set | $r_m$ (fm) | $r_{nn}$ (fm) | $r_{c\text{-}nn}$ (fm) | $\Delta r$ (fm) | $E1$ Sum rule ($e^2$fm$^2$) |
|---|---|---|---|---|---|---|
| I | A | 3.695 | 6.881 | 4.881 | 0.195 | 1.929 |
| | D | 3.921 | 9.640 | 6.340 | 0.421 | 3.254 |
| | AH | 3.861 | 8.963 | 5.986 | 0.361 | 2.902 |
| II | A | 3.617 | 5.728 | 4.270 | 0.117 | 1.477 |
| | D | 3.599 | 5.732 | 4.017 | 0.099 | 1.306 |
| III | A | 3.699 | 6.264 | 4.901 | 0.199 | 1.945 |
| | B | 3.839 | 8.341 | 5.995 | 0.339 | 2.910 |
| | C | 3.865 | 8.698 | 6.125 | 0.365 | 3.038 |
| | D | 3.907 | 9.488 | 6.253 | 0.407 | 3.166 |
| | AH | 3.823 | 8.400 | 5.793 | 0.323 | 2.718 |
| | AH2 | 3.801 | 7.905 | 5.742 | 0.301 | 2.670 |

# PRC105 inversion of orbitals and relative dominance of p-wave => halo

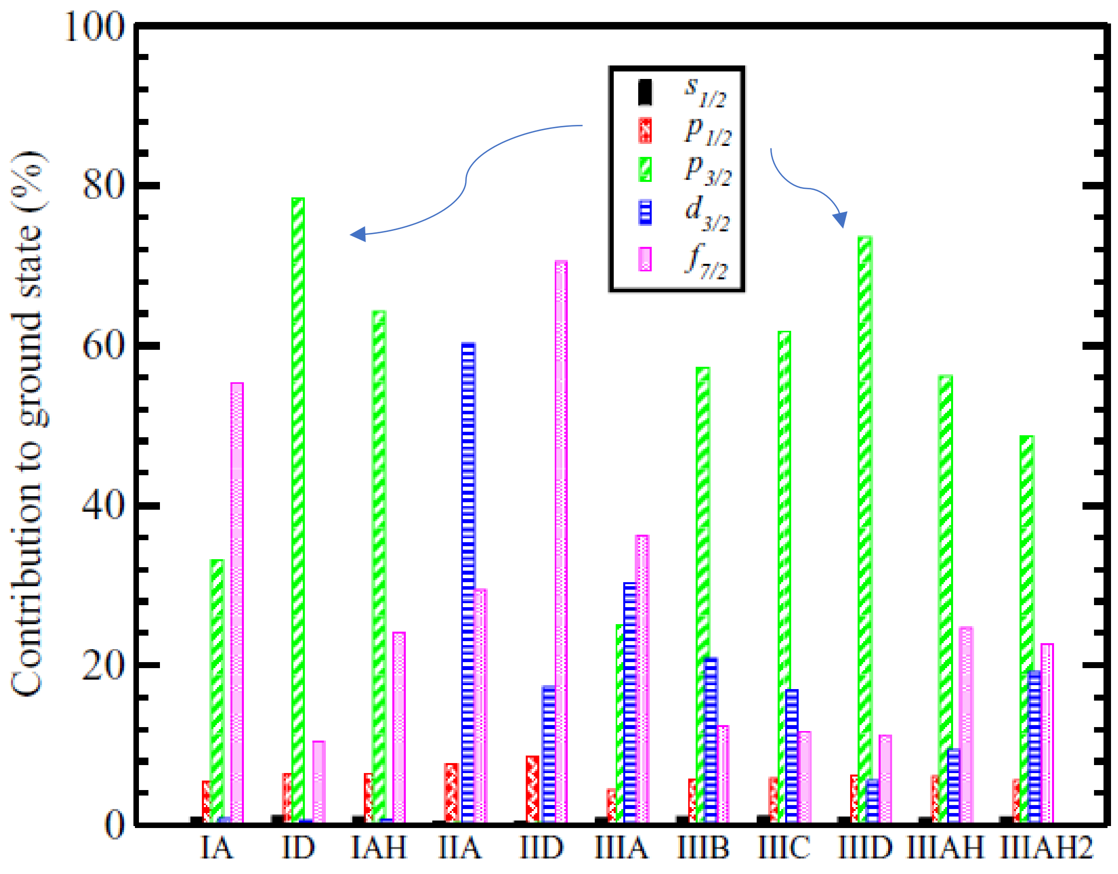



FIG. 6. The contribution by different energy orbitals in the different configurations considered for the g.s. of $^{31}$F. The dominance of $p_{3/2}$, $d_{3/2}$, and $f_{7/2}$ subshells is evident for the various schemes.

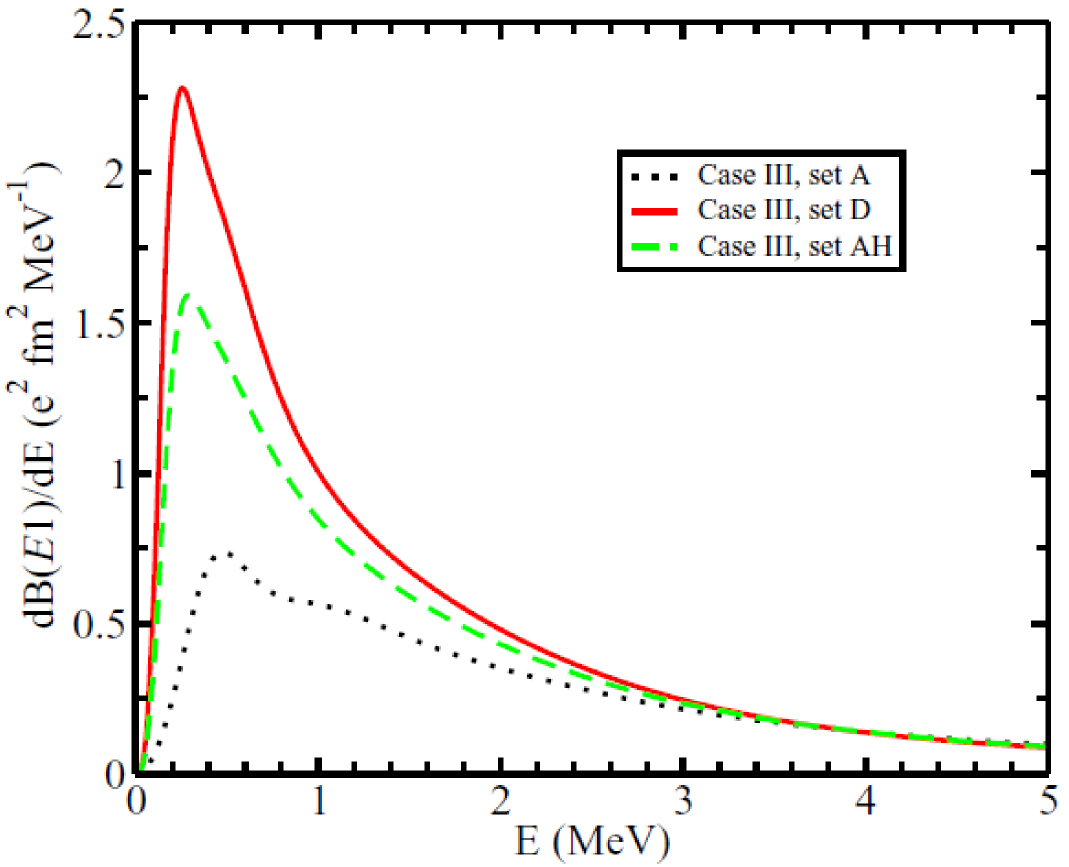


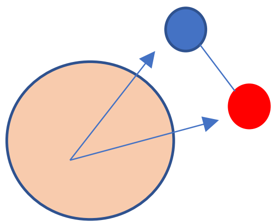

FIG. 8. The $dB(E1)/dE$ distribution for sets A (black dotted line), D (red solid line), and AH (green dashed line) of the open shell configuration (case III) considered in this study. Note the dominance of the inverted configuration, i.e., set D, in the amplitude.

Read it like this: if you see a lot of **green** and **red** (and black, but minor here), you get a halo. Even in the worst case scenario, the halo is there.

# PRC 111: proton-neutron (deuteron) correlations in proton-rich systems

## One-proton emission from $^{102}$Sb and its sensitivity to the proton-neutron interaction

Tomohiro Oishi[1,*] Masaaki Kimura,[1,†] and Lorenzo Fortunato[2,‡]

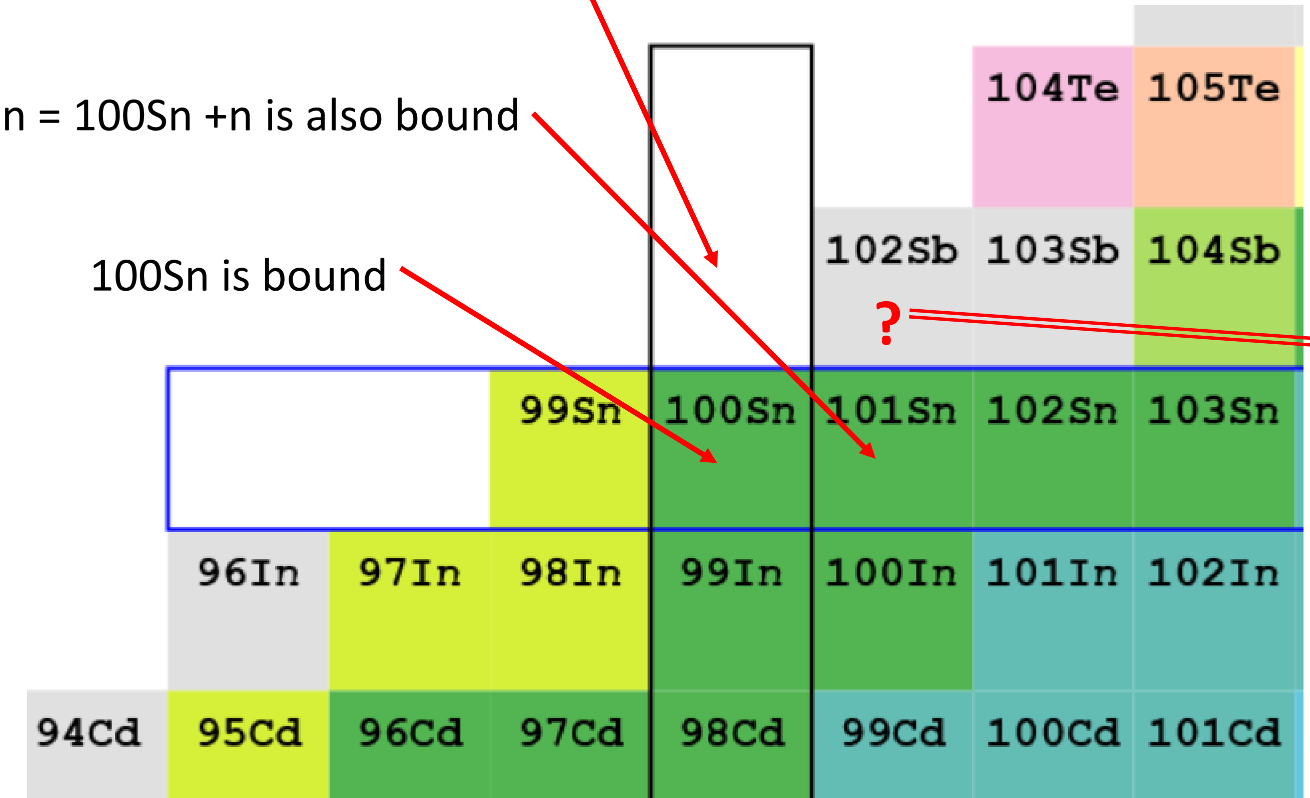


inert 100Sn core and
valence proton and neutron

101Sb = 100Sn +p is NOT bound

101Sn = 100Sn +n is also bound

100Sn is bound




the 1+ ground state of 102Sb is suggested as a possible **proton emitter**
due to a weakening effect on the proton-neutron (*pn*) interaction with respect to a bare deuteron

# Three-body model and time evolution calculations H(t)

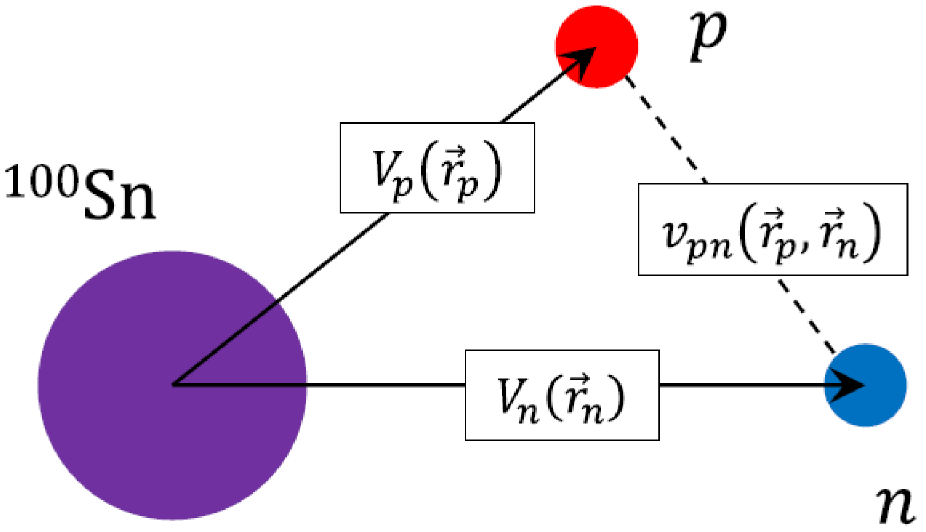


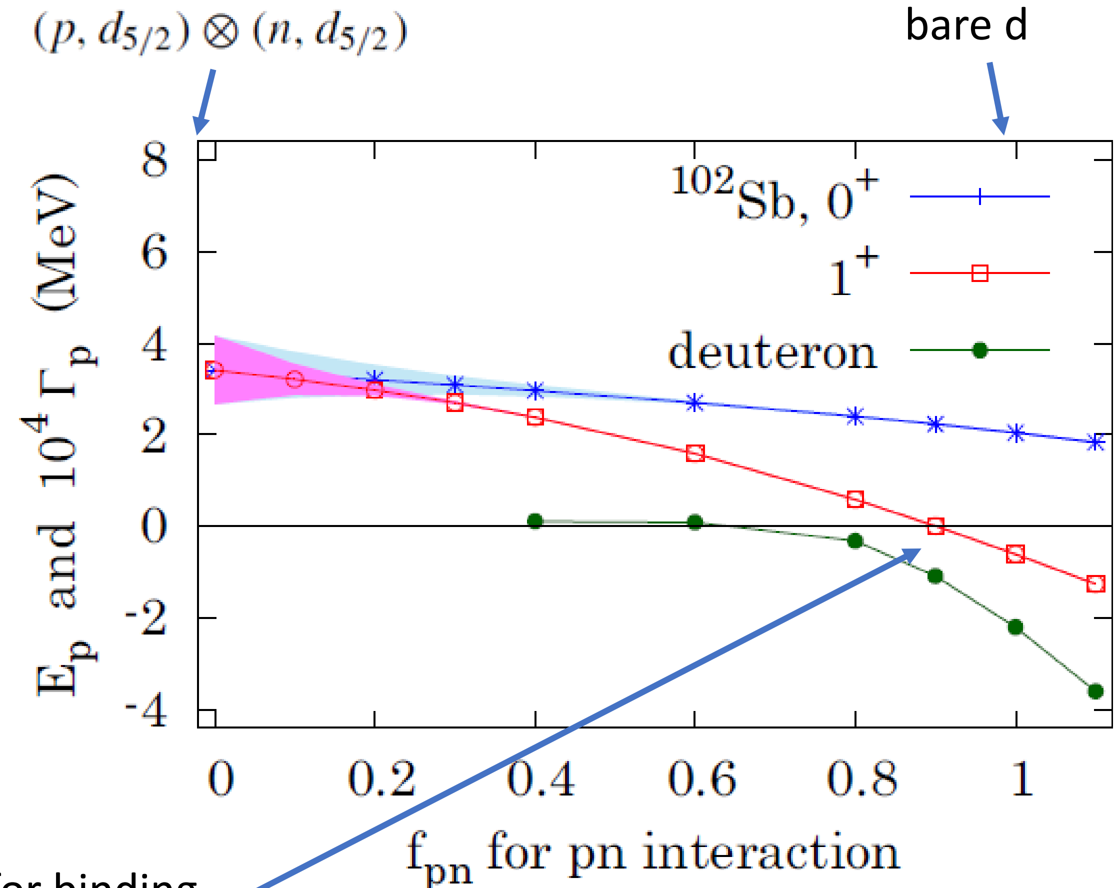

FIG. 1. Three-body model of $^{102}$Sb.

$$|\Psi(t)\rangle = \exp\left[-it\frac{H_{3B}}{\hbar}\right]|\Psi(0)\rangle$$

the tuning factor is employed to modulate the strength of the $pn$ interaction (deuteron-correlation energy):

$$v_{pn}(\boldsymbol{r}_p, \boldsymbol{r}_n) \longrightarrow f_{pn} v_{pn}(\boldsymbol{r}_p, \boldsymbol{r}_n).$$

Thus, $f_{pn} = 1$ indicates the bare-deuteron energy.




f ≈ 0.92 is the threshold for binding

# PRC111: similar trends to other crucial isotopes with p-n correlations

TABLE IV. The $pn$-separation energies, $S_{pn,\text{calc.}} = -\langle H_{3B} \rangle$, calculated in this work. Experimental data of $^{42}$Sc and $^{18}$F are presented for comparison. Note that $^{42}$Sc ($1^+$) is not the ground but the first-excited state.

| | $f_{pn}$ | $S_{pn,\text{calc.}}$ [MeV] | $S_{pn,\text{expt.}}$ [6] [MeV] | $p$-emitter ? |
|---|---|---|---|---|
| $^{102}$Sb ($1^+$) | 1.00 | 11.43 | – | No |
| | 0.60 | 9.23 | – | Yes |
| | 0.40 | 8.44 | – | Yes |
| $^{42}$Sc ($1^+$) | 0.38 | 12.06 | 12.023 | No |
| $^{18}$F ($1^+$) | 0.58 | 9.75 | 9.750 | No |

The $^{102}$Sb nucleus is suggested as a proton emitter. We reached this conclusion by considering the weakening of $pn$ interaction. This weakening effect is introduced from the $pn$-separation energies of other core-$pn$ systems, $^{42}$Sc and $^{18}$F. In this proton-emitting scenario, the lower limit of lifetime is evaluated as $\tau \gtrsim 4.4 \times 10^{-18}$ s in both the $1^+$ and $0^+$ cases. This limit is determined by assuming no interactions between the valence proton and neutron. However, since the finite $pn$ interaction actually exists, the longer lifetime is expected.

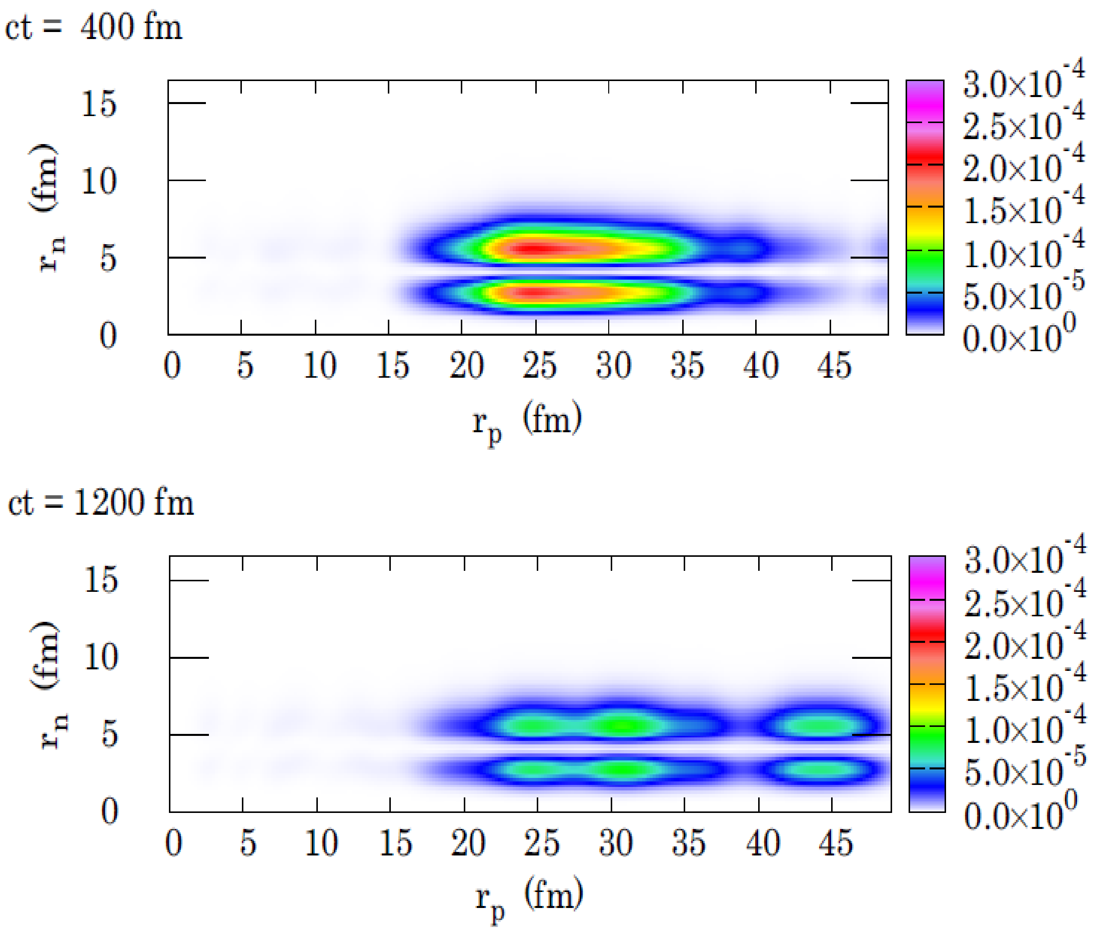



FIG. 6. Time-dependent density distribution of the decaying state, $\rho_{dcy}(t, r_p, r_n)$, with $f_{pn} = 0.4$.

# Summary

- ✓ **Comm.Physics 3 (2020)** and also **PRC 101, 024310 (2020)** We have succesfully interpreted new experimental results on the structure of 29F indicating it lies at the border of the island of inversion.

- ✓ In **PRC 102, 064627 (2020)** we have studied in quite some detail the electric dipole response of the 29F system highlighting futher aspects of its halo nature.

- ✓ In **PRC 105, 014328 (2022)** we have extended our calculations to 31F, which is very neutron-rich, finding that the formation of a halo is very probable.

- ✓ In **PRC 111, 034307 (2025)** , instead, we have investigated an unstable systems 102Sb, with core plus p-n structure, finding that it is a probable one-proton emitter due to the weakening of the p-n correlations

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