International Symposium Commemorating the 40th Anniversary of the Halo Nuclei
October 12 – 18, 2025 | Capital Hotel | Beijing, China

Triaxially deformed halo nuclei

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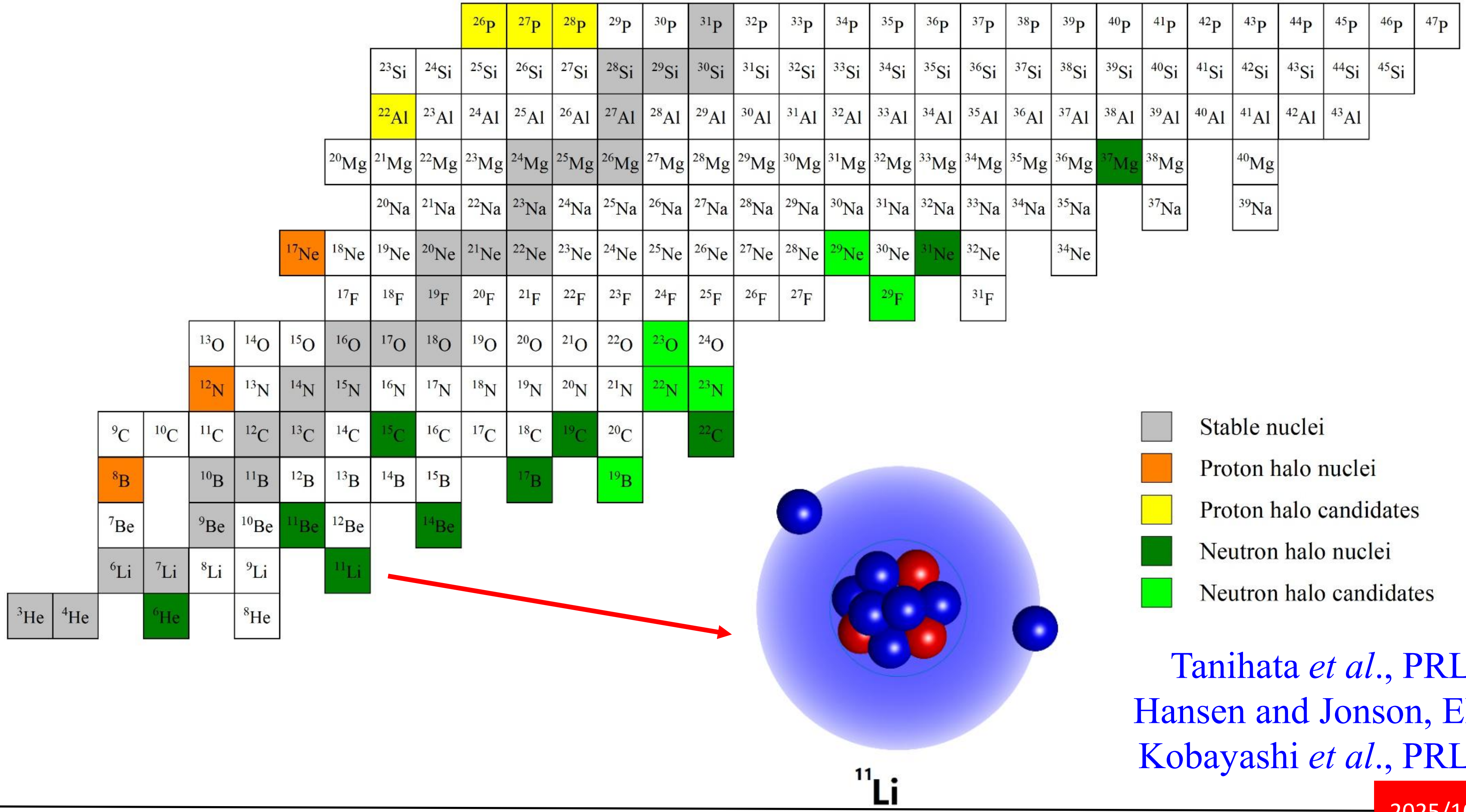
Outline

- Introduction
- Theoretical framework
- Triaxially deformed neutron halo nuclei ^{40}Al and ^{42}Al
- Triaxially deformed proton halo nucleus ^{27}P
- Summary

Exotic halo phenomena near the drip line

- ✓ To date, about 20 halo nuclei or candidates have been suggested in experiments.

Tanihata, Savajols, and Kanungo, PPNP 68, 215 (2013)
KYZ, Papakonstantinou, Mun *et al.*, PRC 107, L041303 (2023)



Tanihata *et al.*, PRL 55, 2676 (1985)
Hansen and Jonson, EPL 4, 409 (1987)
Kobayashi *et al.*, PRL 60, 2599 (1988)

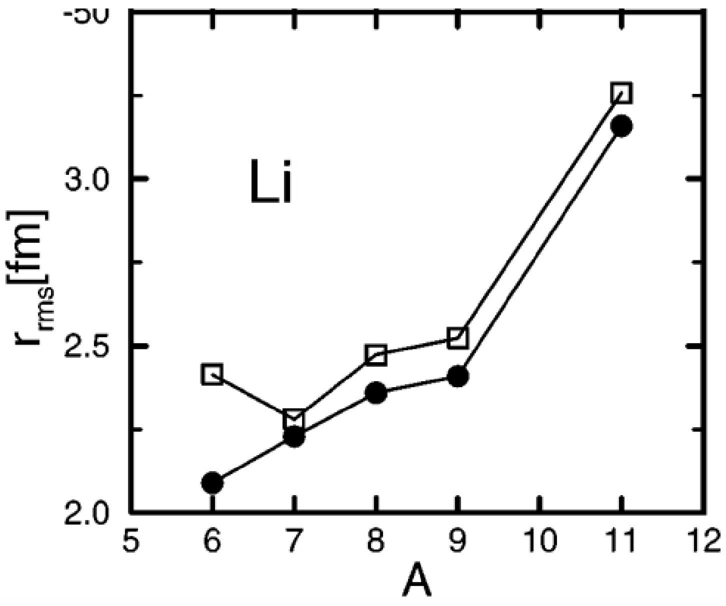
Relativistic density functional theory (RDFT)

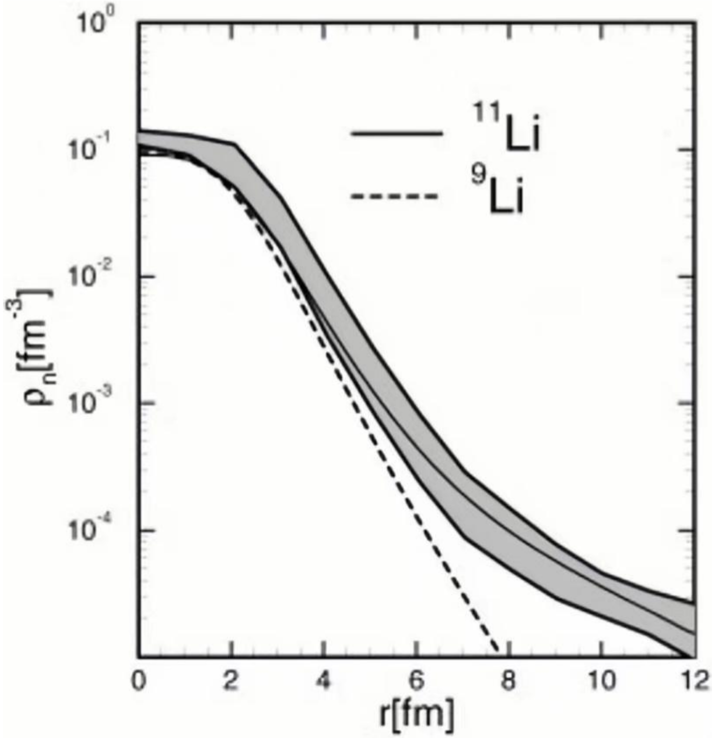
- ✓ The RDFT has been successfully applied to study various nuclear phenomena over the past decades.

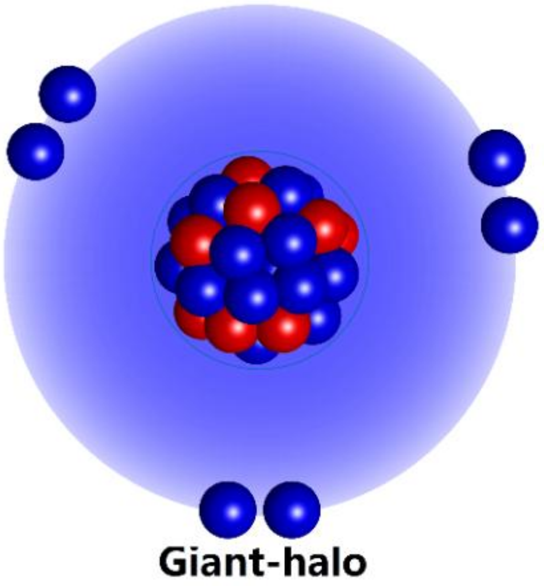
Meng (Ed.), *Relativistic Density Functional for Nuclear Structure* (2016)

- ✓ The relativistic continuum Hartree-Bogoliubov (RCHB) theory describes ^{11}Li and predicts giant halos.

Meng and Ring, PRL 77, 3963 (1996)
Meng and Ring, PRL 80, 460 (1998)







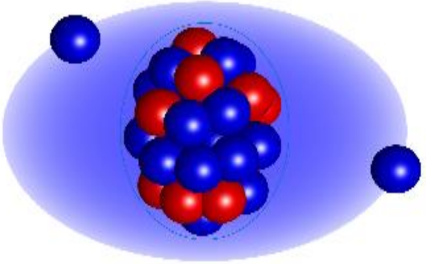
Microscopic prediction and description of deformed halo nuclei

- ✓ In 2010, the deformed relativistic Hartree-Bogoliubov theory in continuum (DRHBc) has been developed for deformed exotic nuclei.

Zhou, Meng, Ring, and Zhao, PRC 82, 011301(R) (2010)

- ✓ ^{42,44}Mg were predicted as deformed halo nuclei with novel shape decoupling.

Zhou, Meng, Ring, and Zhao, PRC 82, 011301(R) (2010)
Li, Meng, Ring, Zhao, and Zhou, PRC 85, 024312 (2012)



- ✓ Experimental evidence for deformed halos in ^{31}Ne and ^{37}Mg has been revealed.

Nakamura, Kobayashi, Kondo *et al*., PRL 112, 142501 (2014)
Kobayashi, Nakamura, Kondo *et al*., PRL 112, 242501 (2014)

T. Nakamura's talk, Oct. 13, 10:45-11:10

- ✓ Recently, the DRHBc theory has successfully described the halo phenomena in ^{31}Ne and ^{37}Mg.

Zhong, Zhang, Sun *et al*., SCPMA 65, 262011 (2022)
KYZ, Yang, An *et al*., PLB 844, 138112 (2023)
Pan, KYZ, and Zhang, PLB 855, 138792 (2024)
An, KYZ, Lu *et al*., PLB 849, 138422 (2024)

See also Qi Lu's talk, Oct. 15, 15:05-15:20

The DRHBc application to other neutron halos

- ✓ Reproduction of the *s*-wave components in the halo neutron of ^{17}B and description of the two-neutron halo nucleus ^{19}B.

Yang, Kubota, Corsi *et al.*, PRL 126, 082501 (2021)
Sun, PRC 103, 054315 (2021)

- ✓ Description of halo structures in ^{15,19,22}C and relevant reaction observables for ^{15,19}C.

Sun, Zhao, and Zhou, PLB 785, 530 (2018)
Sun, Zhao, and Zhou, NPA 1003, 122011 (2020)
Wang, KYZ, An *et al.*, EPJA 60, 251 (2024)

- ✓ Prediction of the new isotope ^{39}Na as a deformed halo nucleus with the $N = 28$ shell collapse.

KYZ, Papakonstantinou, Mun *et al.*, PRC 107, L041303 (2023)

- ✓ Exploration of rotational modes in the predicted deformed halo nuclei ^{42,44}Mg by implementing the angular momentum projection.

Sun and Zhou, Sci. Bull. 66, 2072 (2021)
Sun and Zhou, PRC 104, 064319 (2021)

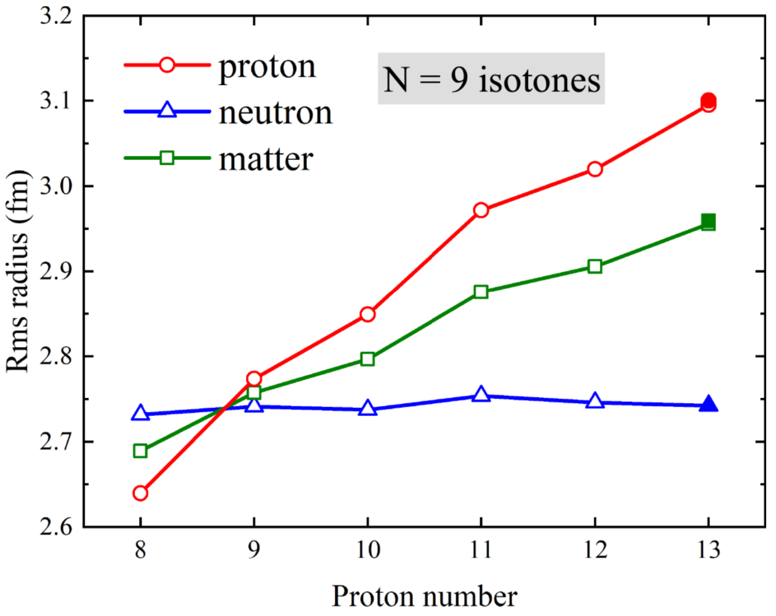
See reviews in Nucl. Phys. Rev. **41**, 75 & 191 (2024)
See speacial issue in Chin. Sci. Bull. **70(20)**, 3288 & 3326 & 3338 & 3348 (2025)
See also the DRHBc mass table webpage: https://drhbctable.jcnp.org/

The DRHBc application to proton halos

✓ Examination of the evidence for a ground-state proton halo in ^{22}Al.

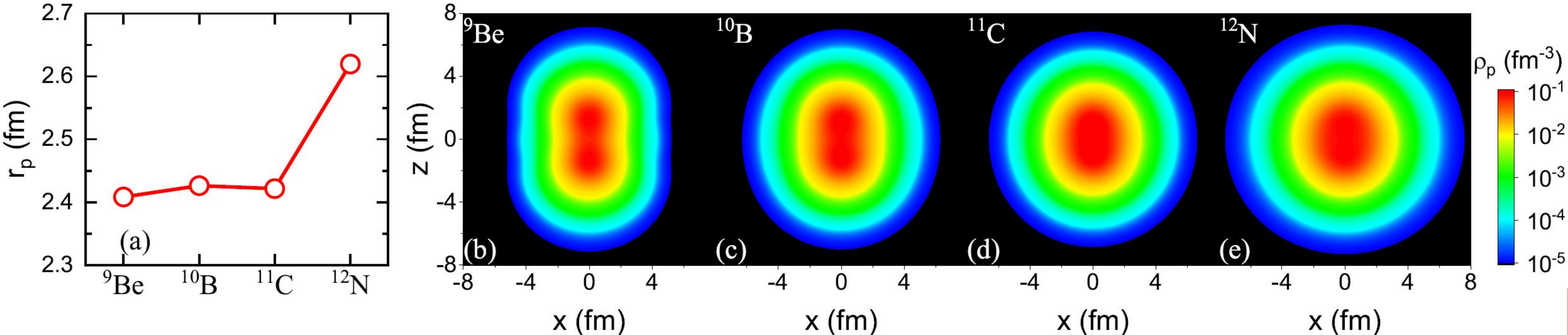
KYZ, Pan, and Wang, PRC 110, 014320 (2024)

Papakonstantinou, Mun, Pan, and KYZ, PRC 112, 044301 (2025)



✓ Microscopic description of the proton halo in ^{12}N.

KYZ and Lu, arXiv: 2510. 11038 [nucl-th]

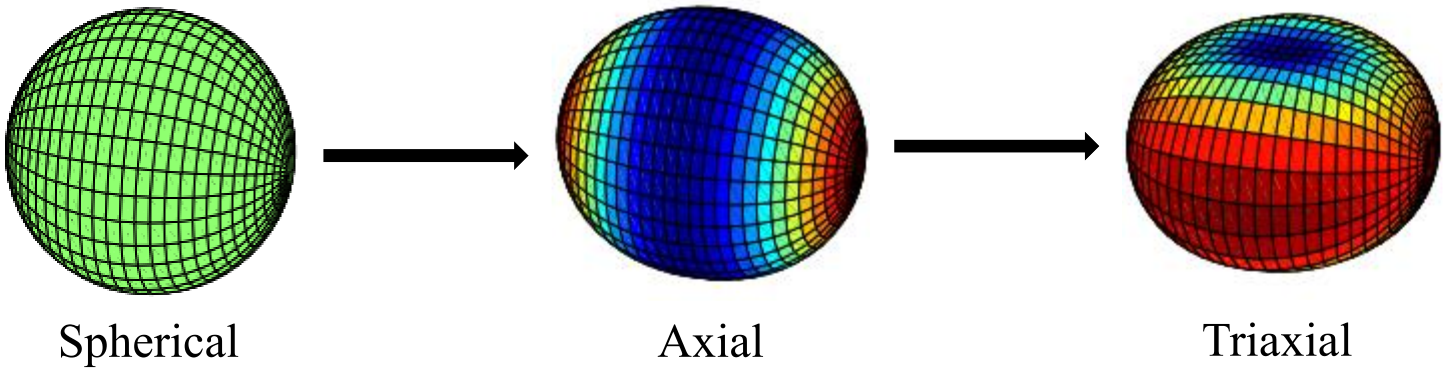


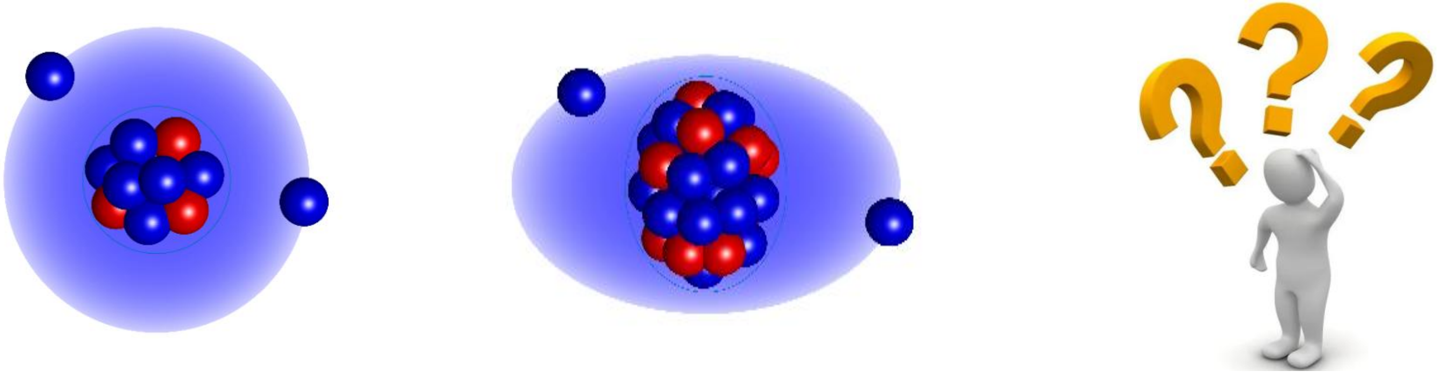
Are there triaxial halo nuclei?

✓ Possible triaxial halo nuclei have been explored based on deformed Woods-Saxon potentials.

K. Hagino's talk, Oct. 14, 14:00-14:25

Uzawa, Hagino, and Yoshida, PRC 104, L011303 (2021)
Wang, Wang, Chen, and Ren, PRC 112, 024313 (2025)





✓ Here, a triaxial relativistic Hartree-Bogoliubov theory in continuum (TRHBc) has been developed.

KYZ, Zhang, and Meng, PRC 108, L041301 (2023)
KYZ, Zhang, and Meng, arXiv:2508.10420 [nucl-th], PRC in press

Relativistic Hartree-Bogoliubov equation

- ✓ The relativistic Hartree-Bogoliubov (RHB) equations read

$$\begin{pmatrix} \hat{h}_D - \lambda & \hat{\Delta} \\ -\hat{\Delta}^* & -\hat{h}_D^* + \lambda \end{pmatrix} \begin{pmatrix} U_k \\ V_k \end{pmatrix} = E_k \begin{pmatrix} U_k \\ V_k \end{pmatrix}$$

- ✓ The Dirac Hamiltonian: $h_D(\boldsymbol{r}) = \boldsymbol{\alpha} \cdot [\boldsymbol{p} - \boldsymbol{V}(\boldsymbol{r})] + V^0(\boldsymbol{r}) + \beta[M + S(\boldsymbol{r})]$

$$\begin{aligned} S(\boldsymbol{r}) =& \alpha_S \rho_S + \beta_S \rho_S^2 + \gamma_S \rho_S^3 + \delta_S \Delta \rho_S, \\ V^\mu(\boldsymbol{r}) =& \alpha_V j^\mu + \gamma_V (j_\mu j^\mu) j^\mu + \delta_V \Delta j^\mu + e A^\mu \\ & + \alpha_{TV} j_3^\mu + \delta_{TV} \Delta j_3^\mu . \end{aligned}$$

$$\begin{aligned} \rho_S(\boldsymbol{r}) &= \sum_k \bar{V}_k(\boldsymbol{r}) V_k(\boldsymbol{r}), \\ j^\mu(\boldsymbol{r}) &= \sum_k \bar{V}_k(\boldsymbol{r}) \gamma^\mu V_k(\boldsymbol{r}), \\ j_3^\mu(\boldsymbol{r}) &= \sum_k \bar{V}_k(\boldsymbol{r}) \gamma^\mu \tau_3 V_k(\boldsymbol{r}), \\ j_c^\mu(\boldsymbol{r}) &= \sum_k \bar{V}_k(\boldsymbol{r}) \gamma^\mu \frac{1-\tau_3}{2} V_k(\boldsymbol{r}). \end{aligned}$$

- ✓ The pairing potential: $\Delta(\boldsymbol{r}_1, \boldsymbol{r}_2) = V^{pp}(\boldsymbol{r}_1, \boldsymbol{r}_2) \kappa(\boldsymbol{r}_1, \boldsymbol{r}_2)$

Spherical harmonic expansion and Dirac Woods-Saxon basis

✓ In the TRHBc theory, nuclear densities and potentials are expanded in terms of spherical harmonics,

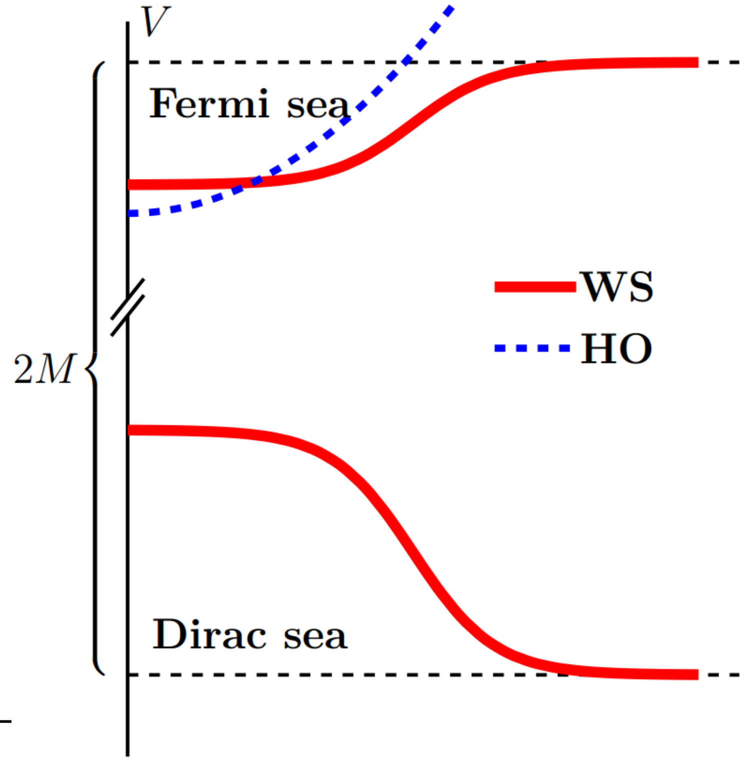
$$f(\boldsymbol{r}) = \sum_{\lambda\mu} f_{\lambda\mu}(r) Y_{\lambda\mu}(\theta, \varphi)$$

Limitations on λ and μ can be obtained by symmetry analysis.

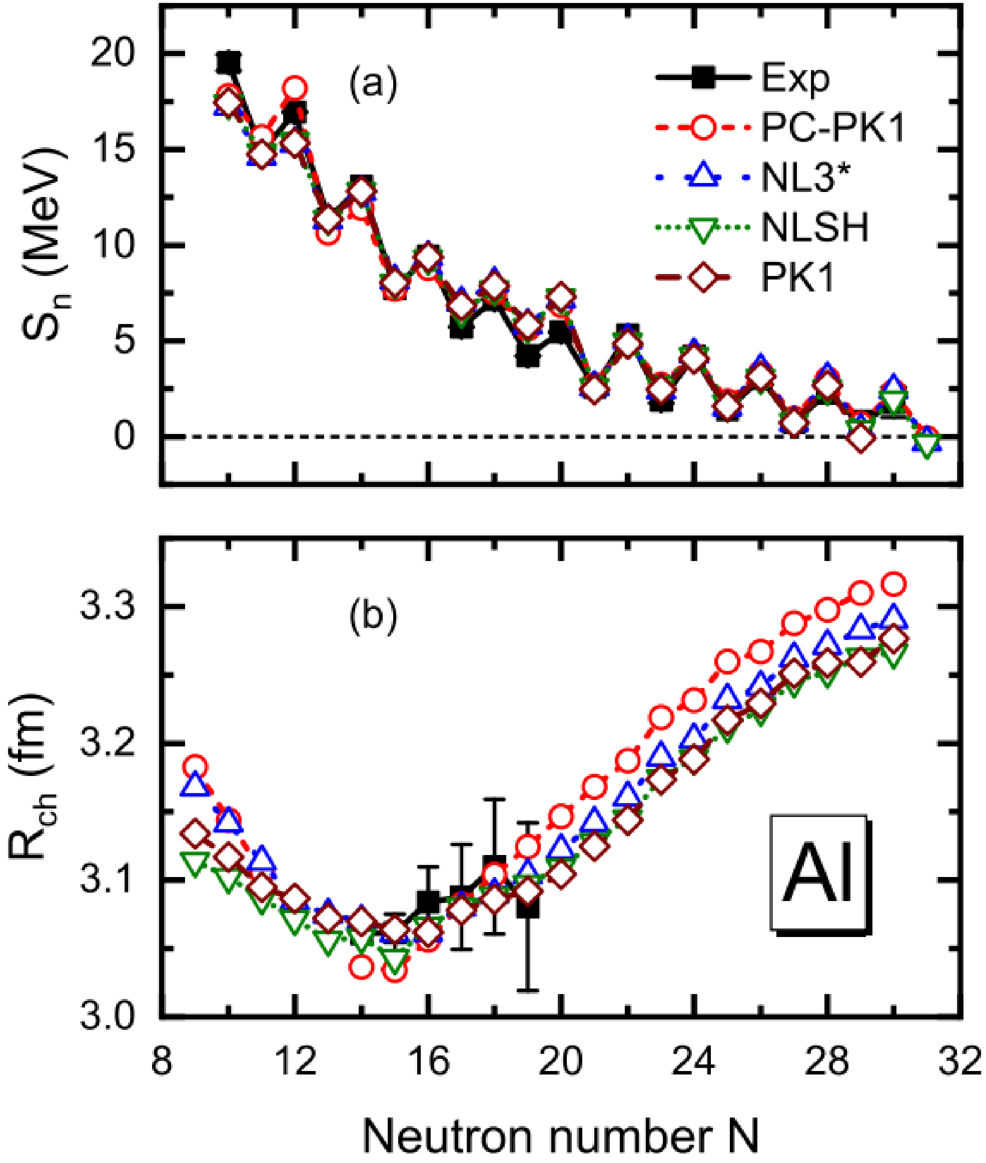
Xiang, Luo, Yang, and KYZ, Symmetry 15, 1420 (2023)

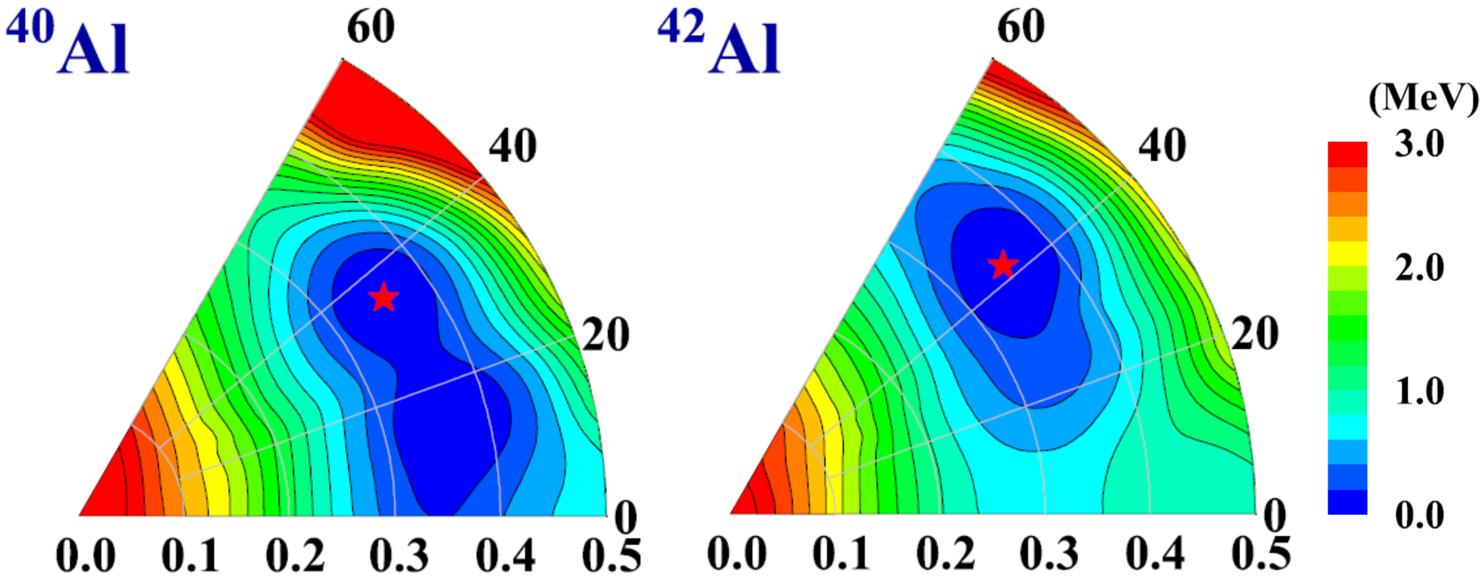
✓ The RHB equations are solved in a set of Dirac Woods-Saxon basis, the wave function of which has an appropriate asymptotic behavior at large r.

Zhou, Meng, and Ring, PRC 68, 034323 (2003)
KYZ, Pan, and Zhang, PRC 106, 024302 (2022)



Neutron separation energy and charge radii



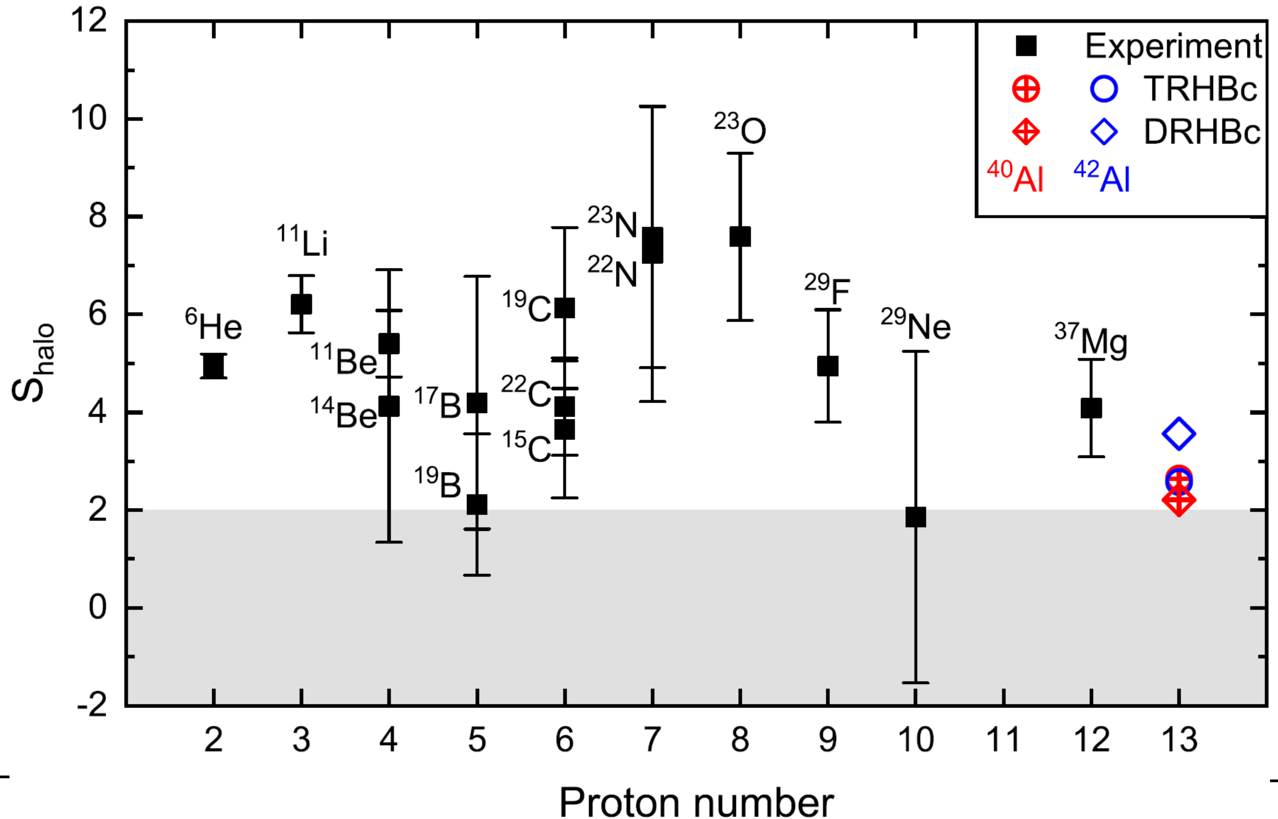


- ✓ The experimental neutron separation energies and charge radii are well reproduced.
- ✓ The neutron-rich ^{40}Al and ^{42}Al are triaxially deformed and weakly bound with S_n below 1 MeV.

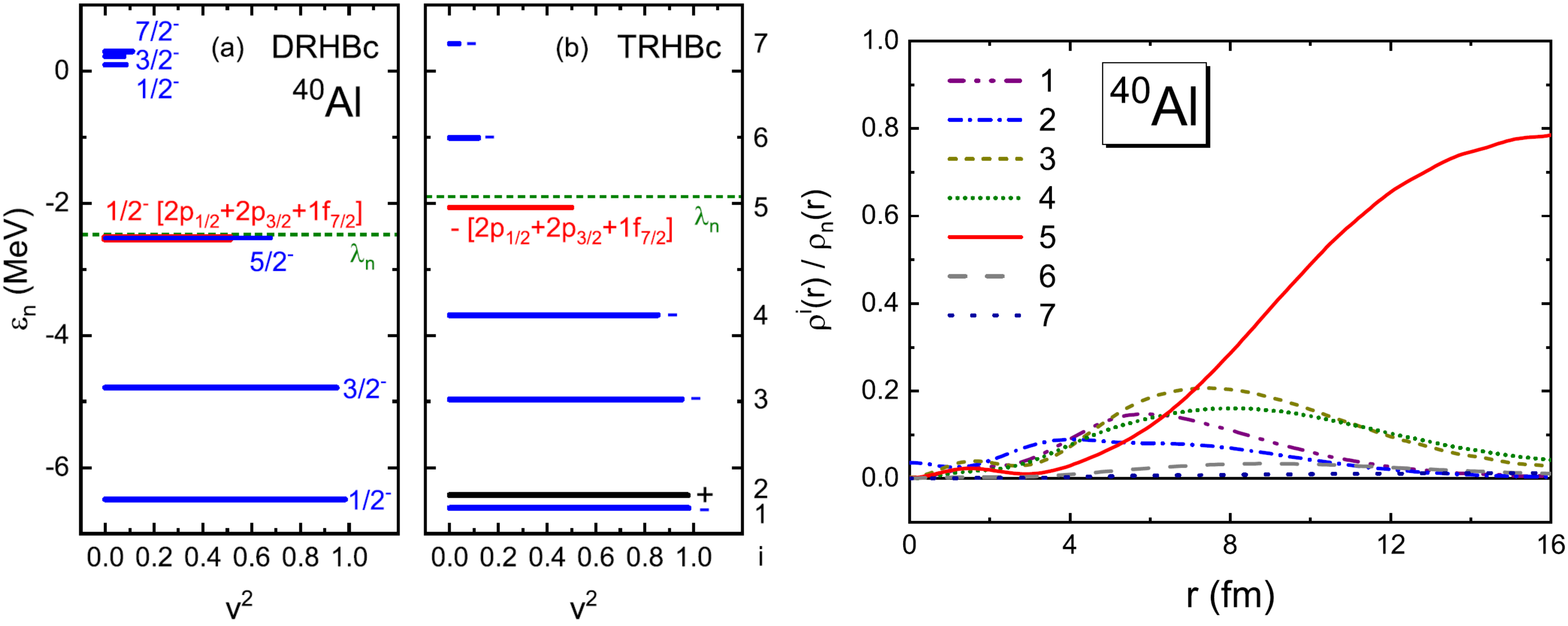
Halo scale

$$S_{\text{halo}} = \frac{\Delta R^{\text{expt(calc)}}}{\Delta R^{\text{emp}}} = \frac{R^{\text{expt(calc)}}(N+m) - R^{\text{expt(calc)}}(N)}{R^{\text{emp}}(N+m) - R^{\text{emp}}(N)}$$

The halo scale compares the contribution to the rms radius by the weakly bound neutron(s) with the conventional one (the empirical value can be estimated using $r_0A^{1/3}$)

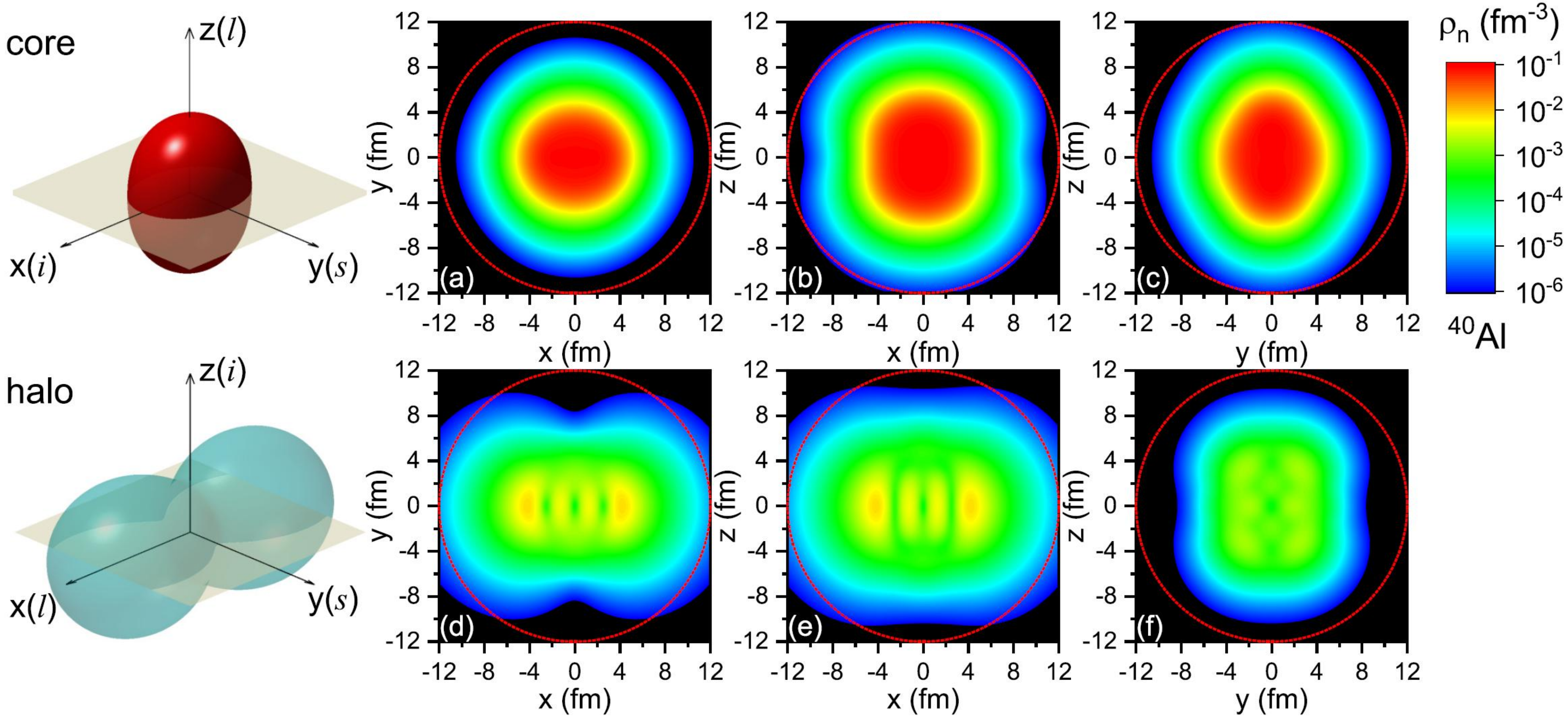


Single-neutron orbitals in ^{40}Al and their contributions



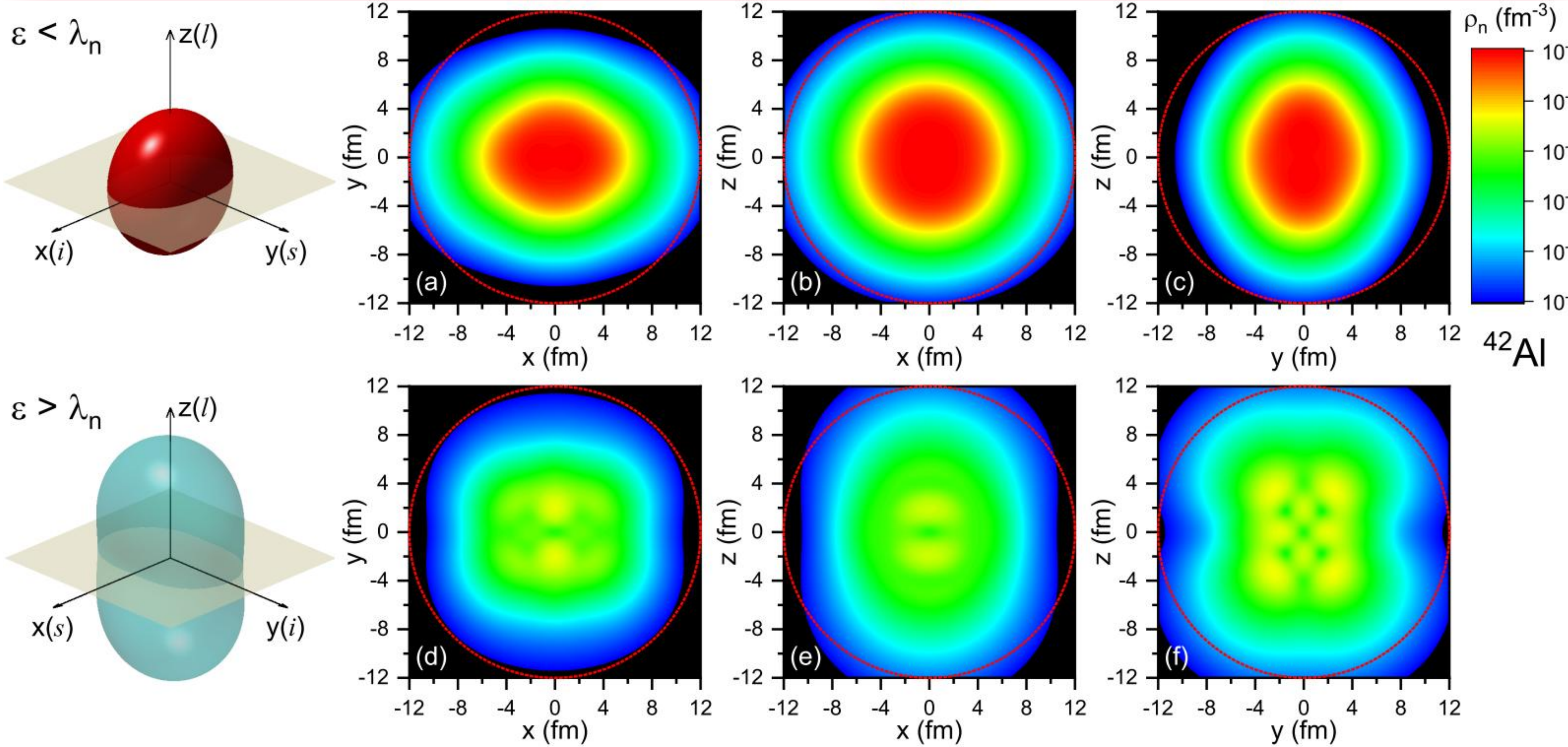
✓ The orbital of the valence neutron well decouples from those of the core only when triaxial deformation is considered.

Core and halo densities in different planes for ^{40}Al



- ✓ Core: $r = 3.73$ fm, $\beta = 0.38$, $\gamma = 25°$, $z > x > y$
- ✓ Halo: $r = 5.10$ fm, $\beta = 1.48$, $\gamma = 113°$, $x > z > y$

Core and halo densities in different planes for ^{42}Al



✓ Core: $r = 3.85$ fm, $\beta = 0.38$, $\gamma = 50°$, $z > x > y$

✓ Halo: $r = 5.26$ fm, $\beta = 0.79$, $\gamma = -23°$, $z > y > x$

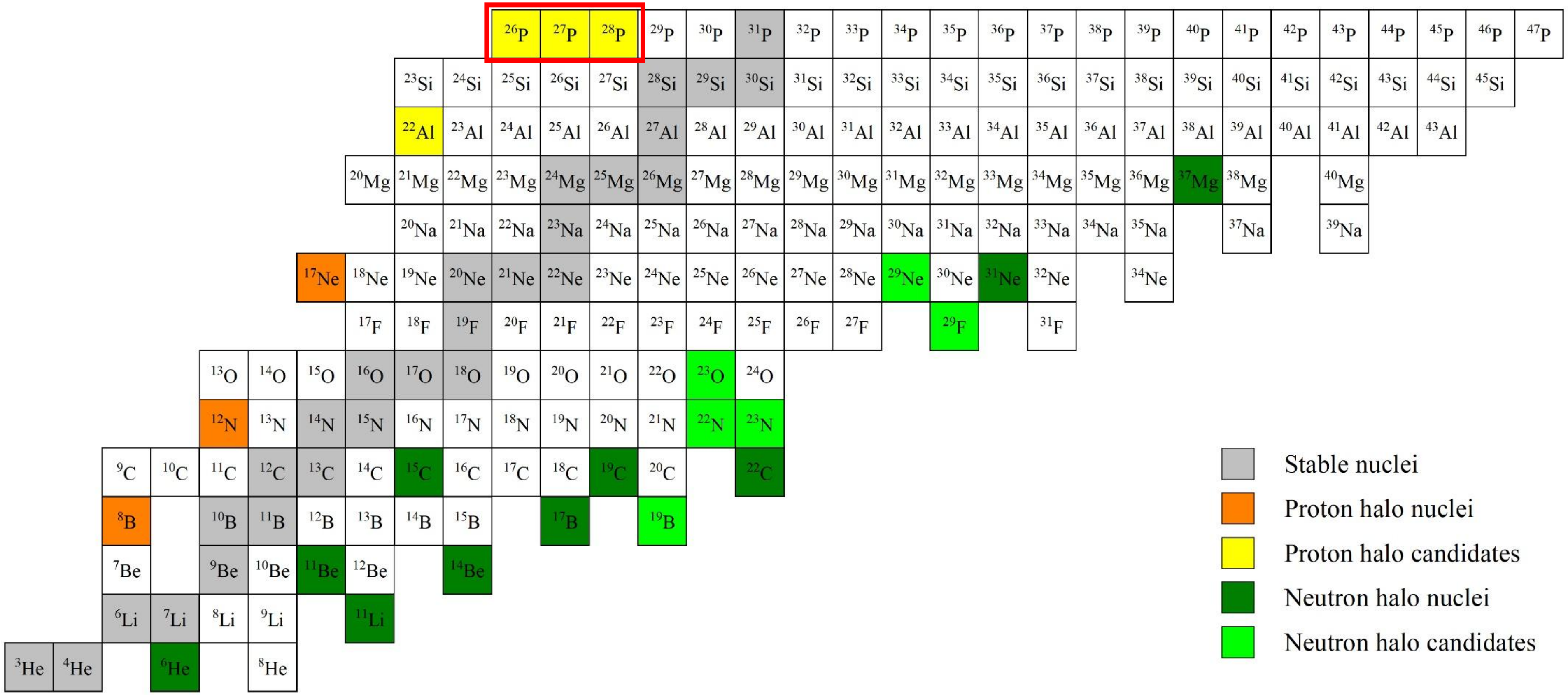
Exp hints for halos in proton-rich phosphorus isotopes

✓ ^{27}P has a S_p of 0.8 MeV and a narrow momentum distribution after the one-proton removal reaction.

Wang *et al.*, AME2020 CPC 45, 030003 (2021)

C. J. Lin's talk, Oct. 14, 10:50-11:10

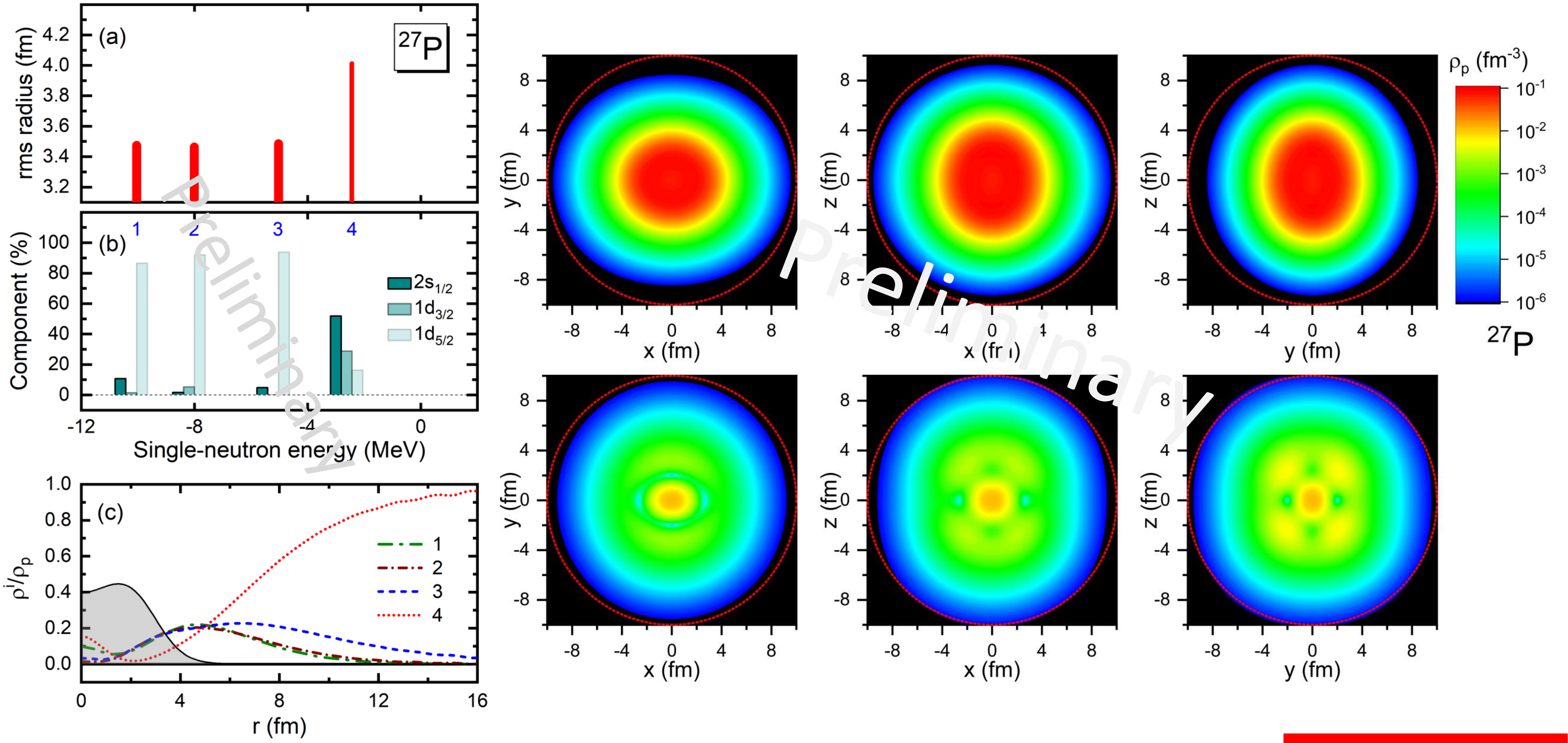
Navin, Bazin, Brown *et al.*, PRL 81, 5089 (1998)



KYZ, Papakonstantinou, Mun *et al.*, PRC 107, L041303 (2023)

Preliminary results for ^{27}P

KYZ *et al.*, in preparation



Summary

- ✓ A triaxial relativistic Hartree-Bogoliubov theory in continuum (TRHBc) has been developed to explore triaxially deformed halo nuclei.

  KYZ, Zhang, and Meng, PRC 108, L041301 (2023)
  KYZ, Zhang, and Meng, arXiv:2508.10420 [nucl-th], PRC in press

- ✓ The experimental neutron separation energies and charge radii for aluminum isotopes are well reproduced by the TRHBc theory with density functionals PC-PK1, PK1, NL3*, and NLSH.

- ✓ The weakly bound ^{40}Al and ^{42}Al are predicted as triaxially deformed halo nuclei with novel triaxial shape decoupling between the core and the halo.

- ✓ In particular, a sizable gap facilitating the separation between the halo neutron and the core in ^{40}Al is developed only when triaxial deformation is considered.

- ✓ The experimentally suggested proton halo candidate ^{27}P is investigated using the TRHBc theory. The preliminary results indicate it as a triaxially deformed halo nucleus.

Acknowledgment

This work is in collaboration with Shuangquan Zhang & Jie Meng

ACKNOWLEDGMENTS

Helpful discussions with L. S. Geng, C. Pan, P. Ring, B. H. Sun, D. Vretenar, P. W. Zhao, S.-G. Zhou, and members of the DRHBc Mass Table Collaboration are highly appreciated.

Thank you for your attention

Evolution of single-particle energy with triaxial deformation