Deformation and pairing in neutron-rich halo nuclei

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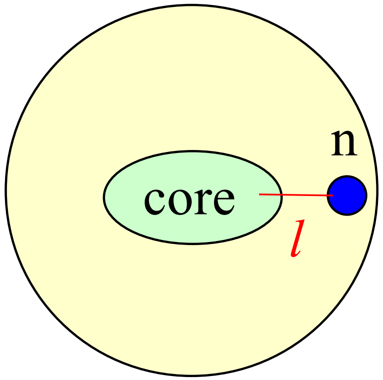
1. Introduction: halo and s.p. angular momenta
2. Deformed halo: Coulomb breakup of ^{31}Ne
3. Pairing in two-neutron halo: Coulomb breakup of ^{22}C
4. Summary

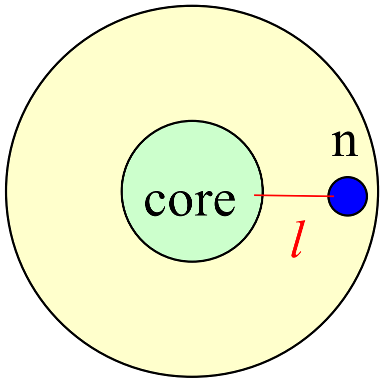
cf. a talk by Takashi Nakamura-san yesterday morning

International symposium commemorating the 40th anniversary of the halo nuclei (Halo40), Beijing, China, Oct. 12-18, 2025

I. Deformed halo nuclei



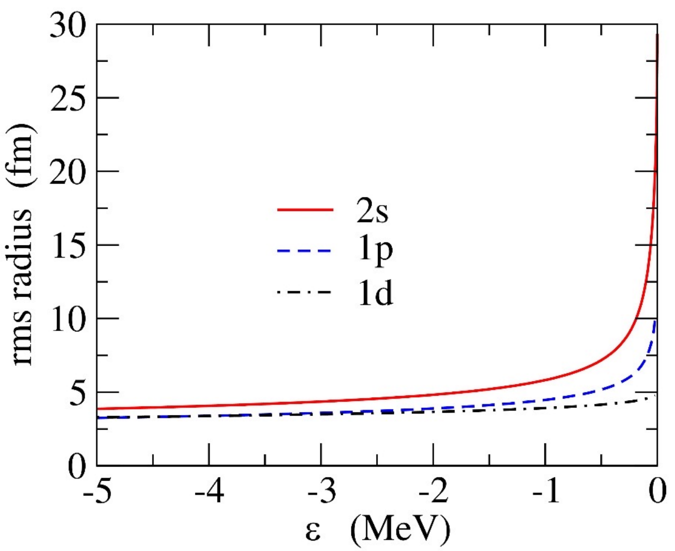
Introduction

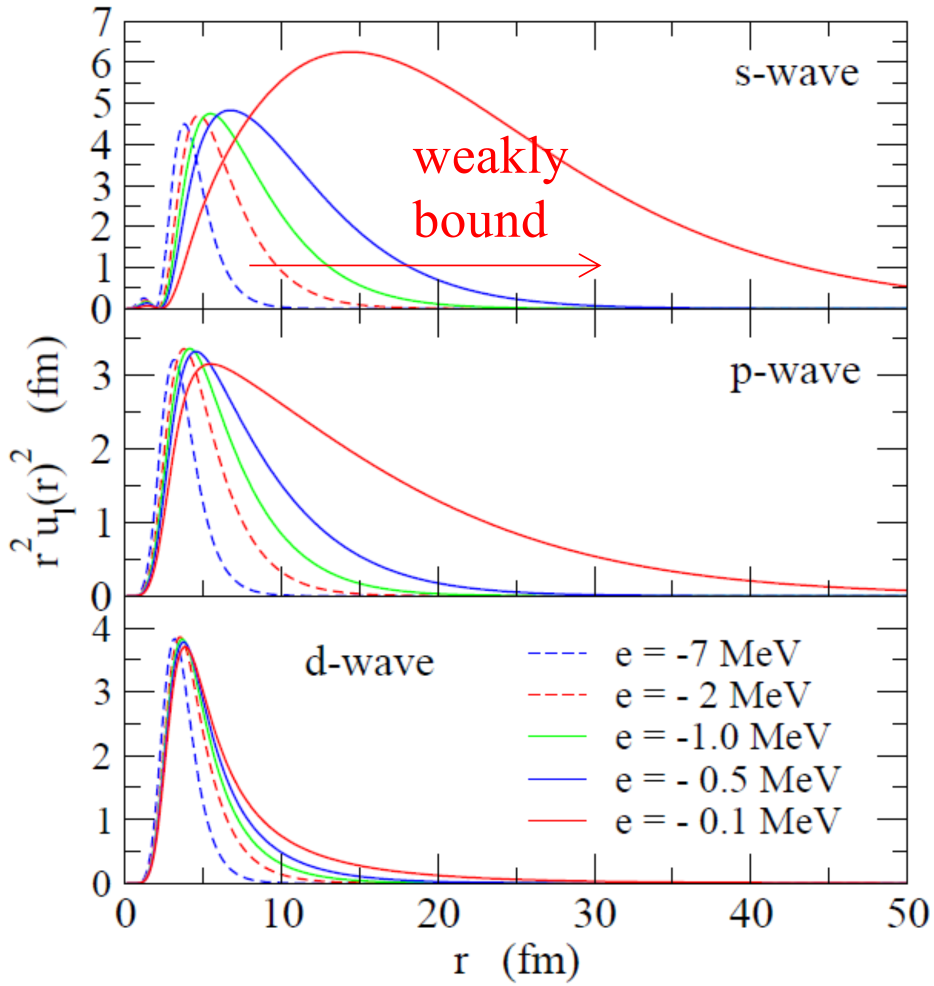


l = 0 or 1
→ halo structure

$$\langle r^2 \rangle \to \infty \quad (\epsilon \to 0)$$

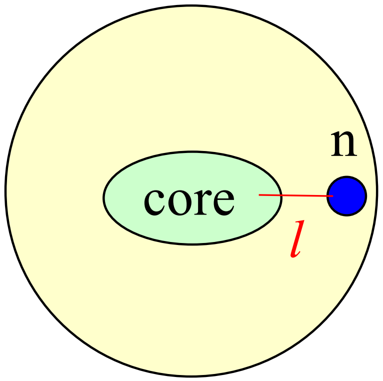
for l = 0 or 1





Deformed halo nuclei

For an axially deformed core:



$$V(r) \to V(r,\theta)$$

$$\psi(\boldsymbol{r}) = R_{jl}(r)\mathcal{Y}_{jlm}(\hat{\boldsymbol{r}})$$

$$\to \sum_{j,l} R_{jl}(r)\mathcal{Y}_{jlm}(\hat{\boldsymbol{r}})$$

✓ T. Misu, W. Nazarewicz, and S. Aberg, NPA614 ('97) 44

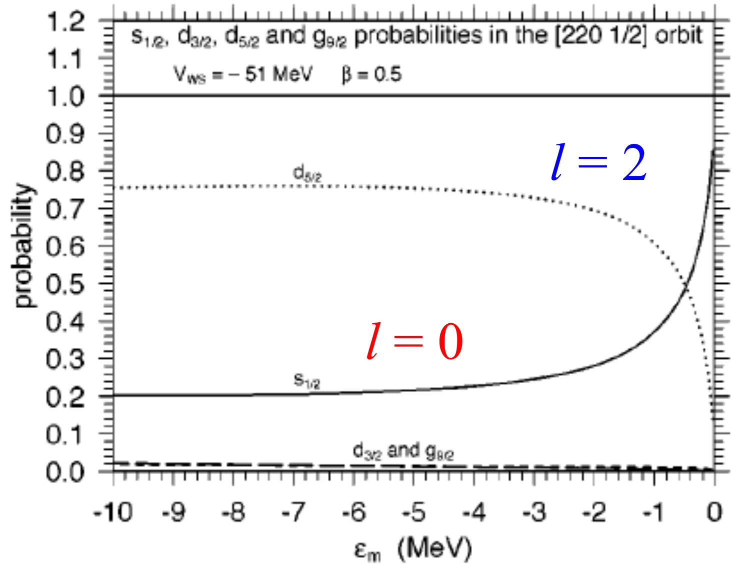
$$\psi(\boldsymbol{r}) \to R_{s_{1/2}}(r)\mathcal{Y}_{s_{1/2},m}(\hat{\boldsymbol{r}}) \quad (\epsilon \to 0)$$

s-wave dominance

$$\boxed{\begin{array}{c} d \to d+s \to s \text{ (halo)} \\ \beta \qquad (\varepsilon \to 0) \end{array}}$$

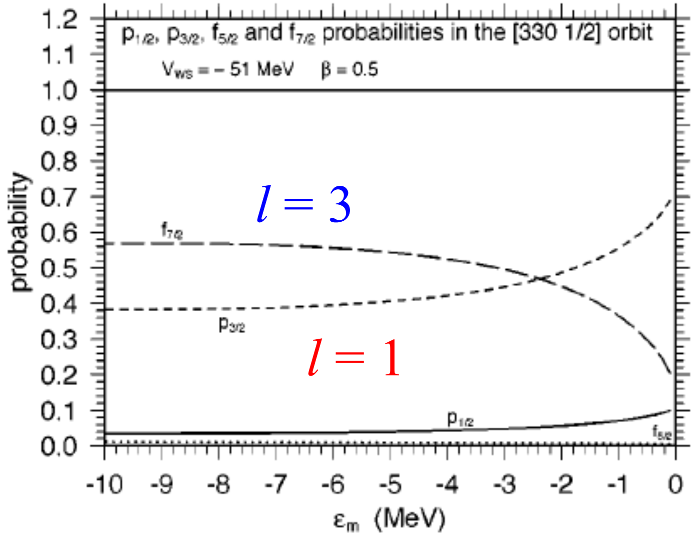
✓ I. Hamamoto, PRC69 ('04) 041306(R)

numerical demonstration with a deformed WS potential



Deformed halo nuclei

✓ I. Hamamoto, PRC69 ('04) 041306(R)



also p-wave dominance
(even though $P_{l=1} < 1$)

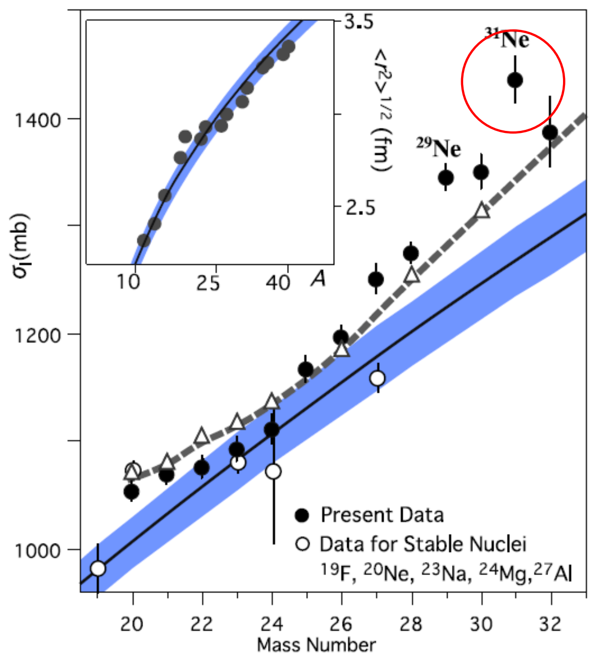
Experimental evidence: ^{31}Ne

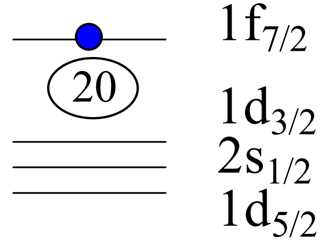
✓ large E1 cross sections

T. Nakamura et al., PRL103 ('09) 262501

✓ large reaction cross sections

M.Takechi et al., PLB 707 ('12) 357





$^{31}_{10}\mathrm{Ne}_{21}$

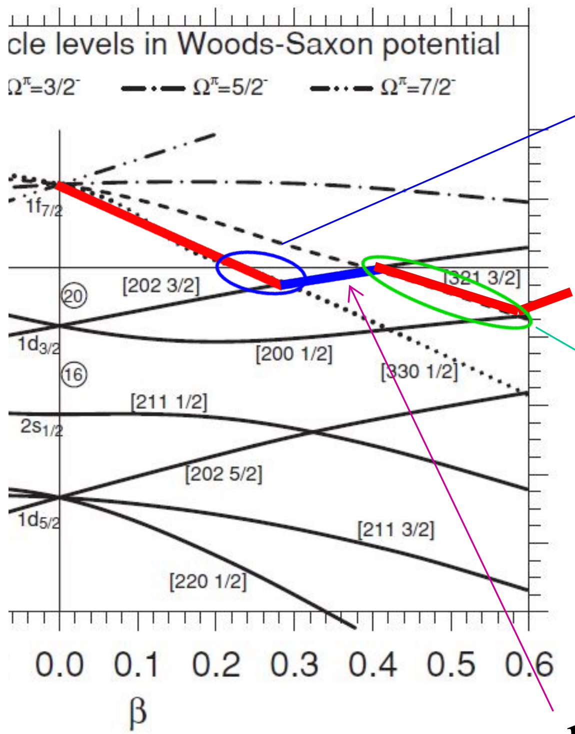
deformation:

$f \longrightarrow f+p \longrightarrow p$ (halo)

β $(\varepsilon \rightarrow 0)$

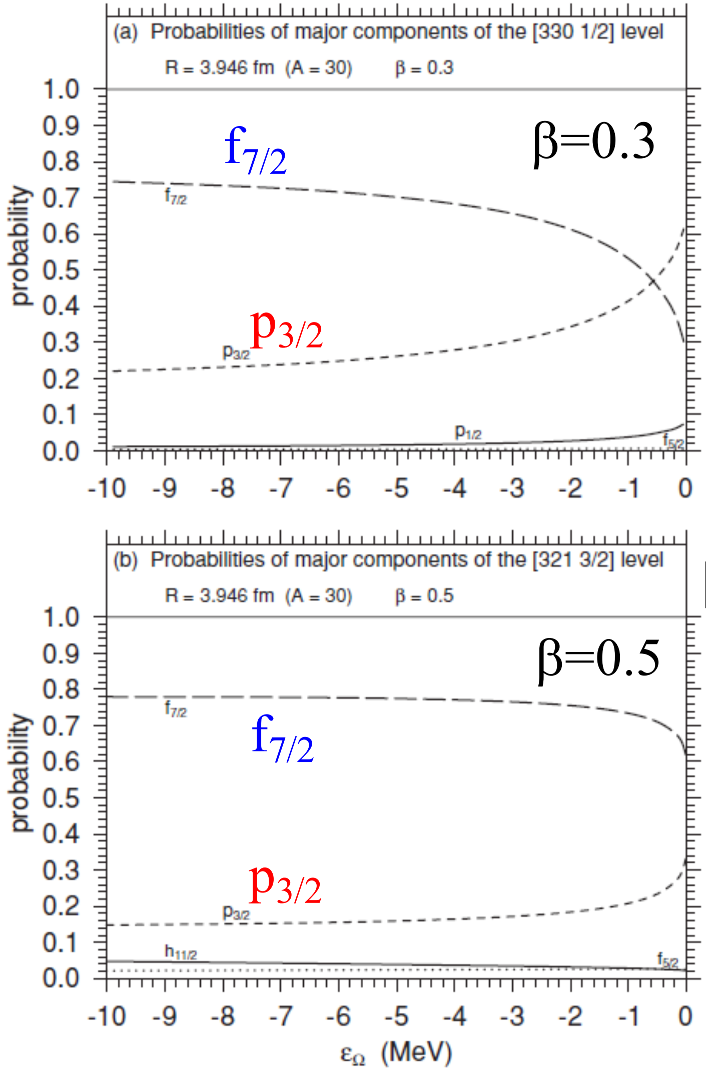
Deformed halo nuclei

An analysis with a Nilsson model
[I. Hamamoto, PRC81('10)021304(R)]



21st neutron

non-halo ($\Omega^\pi = 3/2^+$)



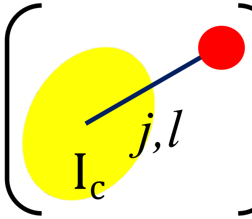
[330 1/2]
larger $p_{3/2}$

[321 3/2]
smaller $p_{3/2}$

Particle-rotor model

Y. Urata, K.Hagino, and H. Sagawa, PRC83 ('11) 041303(R)

core + neutron two-body model **with core excitations**

$$\Psi_{IM} = \sum_{I_c,j,l} \Bigg[$$


$$\Bigg]^{(IM)}$$

e.g.,

$$= |0^+ \otimes p_{3/2}\rangle + |2^+ \otimes f_{7/2}\rangle + \cdots$$

$$= \sum_{I_c,j,l} R^{(I)}_{I_cjl}(r)|[(jl)I_c]^{(IM)}\rangle$$ → coupled equations for $R(r)$

4^+ —— ε_4

2^+ —— $\varepsilon_2 = 0.801(7)$ MeV

0^+ ——

rotor

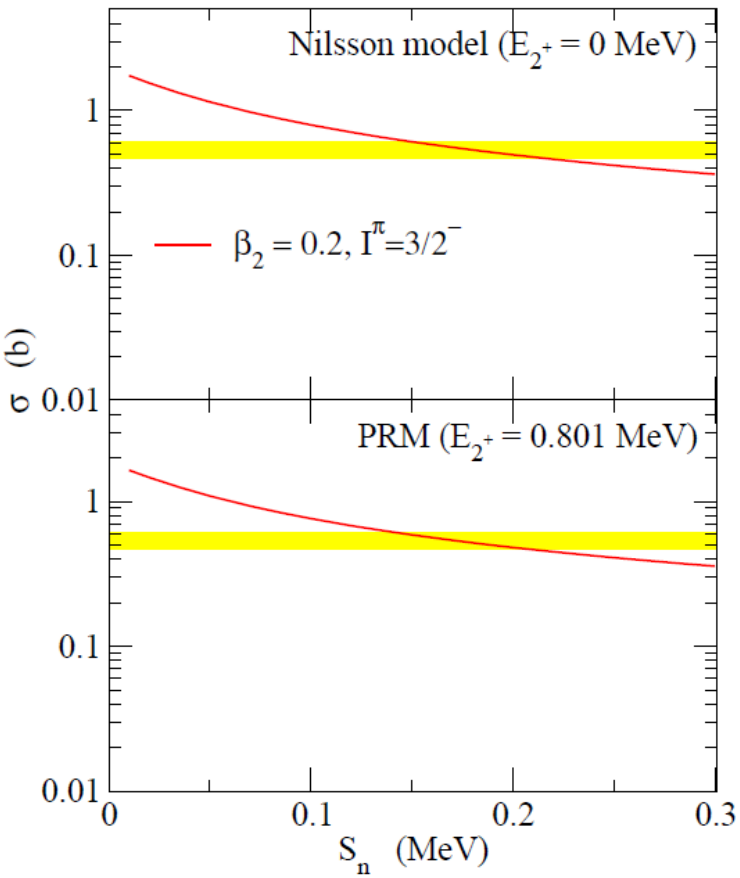
Nilsson model: $\varepsilon \to 0$
(the adiabatic approximation)

P. Doornenbal et al., PRL103 ('09) 032501

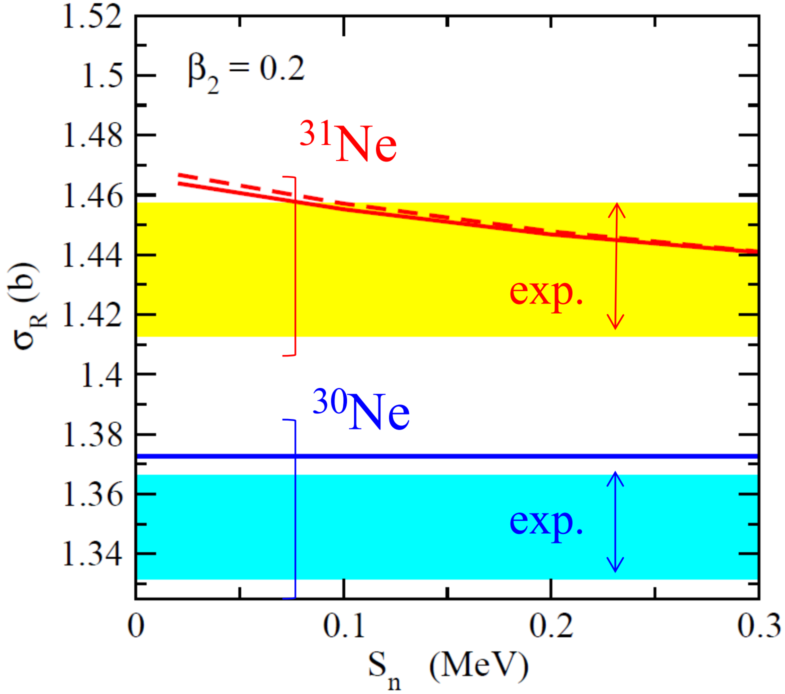
Particle-rotor model

Y. Urata, K.Hagino, and H. Sagawa, PRC83 ('11) 041303(R)
PRC86 ('12) 044613

inclusive Coulomb breakup cross sections

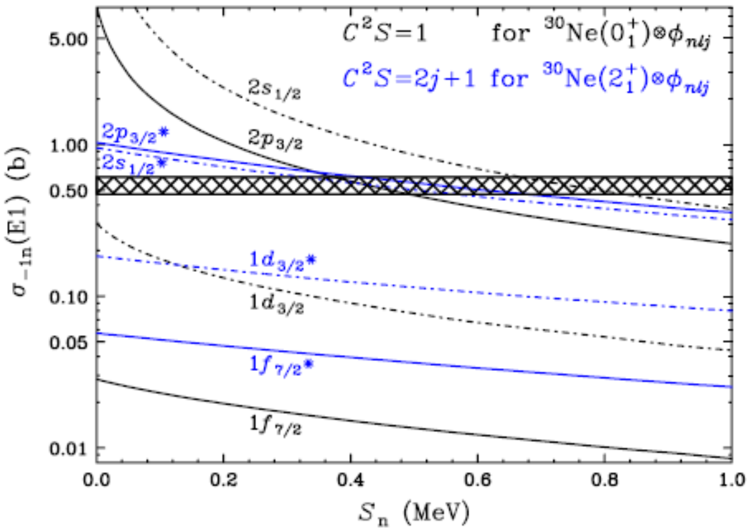


reaction cross sections



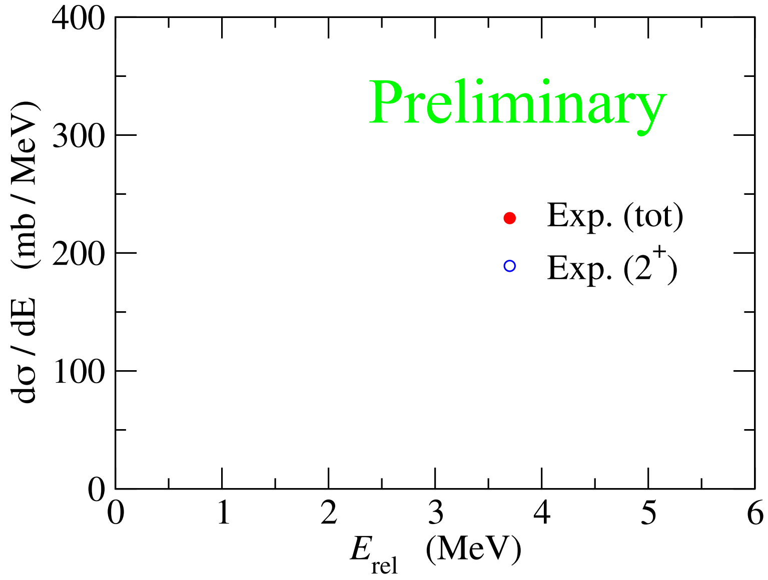
Reanalysis of ^{31}Ne

inclusive cross section



T. Nakamura et al., PRL103 ('09) 262501

exclusive cross sections (with SAMURAI)



- T. Tomai, T. Nakamura et al., in preparation
- T. Nakamura, a talk in this symposium

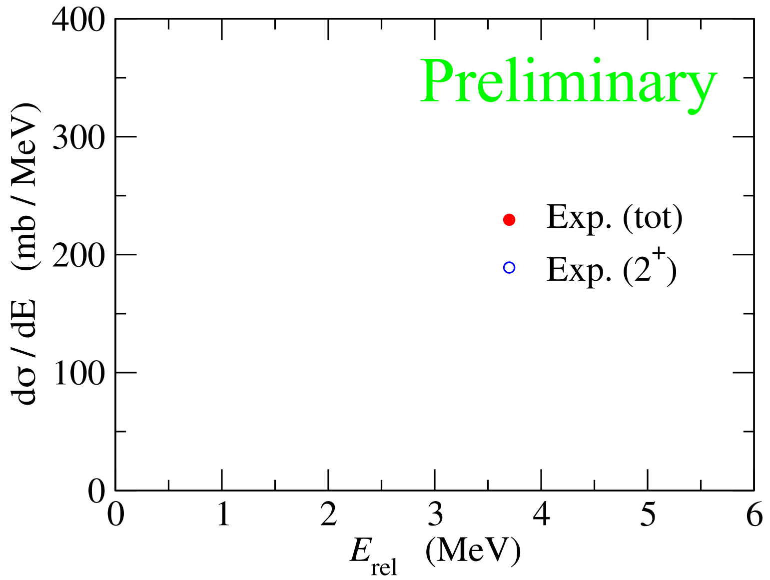
✓ a more accurate determination of S_n

S_n = 0.29 +/- 1.64 MeV → **** MeV

✓ breakup cross sections to the ^{30}Ne(2^+) state

Reanalysis of ^{31}Ne

exclusive cross sections



$$\frac{d\sigma}{dE} = \frac{16\pi^3}{9\hbar c} \cdot N_{\mathrm{E1}}(E_f - E_i) \cdot \frac{dB(E1; i \to f)}{dE}$$

$$\frac{dB(E1; i \to f)}{dE} = \frac{1}{2I_i + 1} \left| \left\langle \Psi_f \left\| \hat{D} \right\| \Psi_i \right\rangle \right|^2$$

$\leftarrow$ PRM

$$\Psi_i = \sum_{I_c, j, l} R^{(I_i)}_{I_c j l}(r) |[(jl)I_c]^{(I_i M_i)}\rangle$$

a simplified treatment: ψ_f with a spherical potential

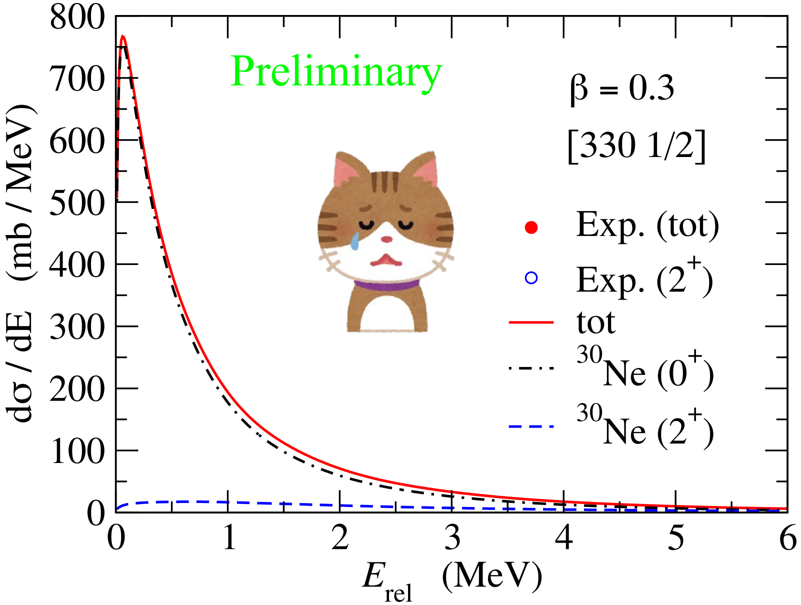
$$\Psi_f = R^{(I_f)}_{I_c j l}(r) |[(jl)I_c]^{(I_f M_f)}\rangle$$

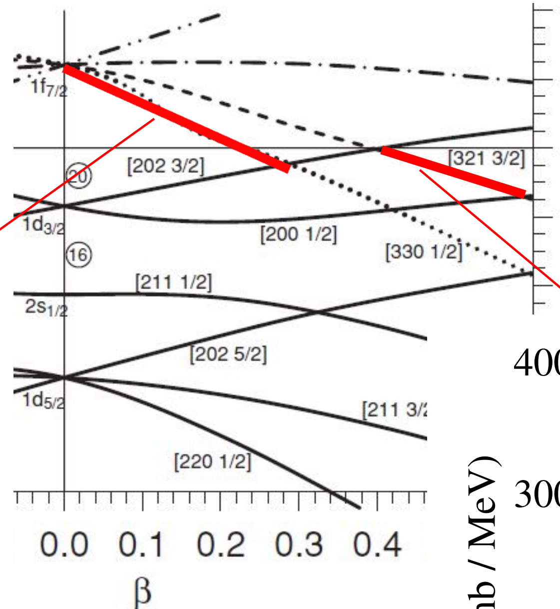
$$\langle \Psi_f | \Psi_{f'} \rangle = \delta(E_f - E_{f'})$$

Reanalysis of ^{31}Ne

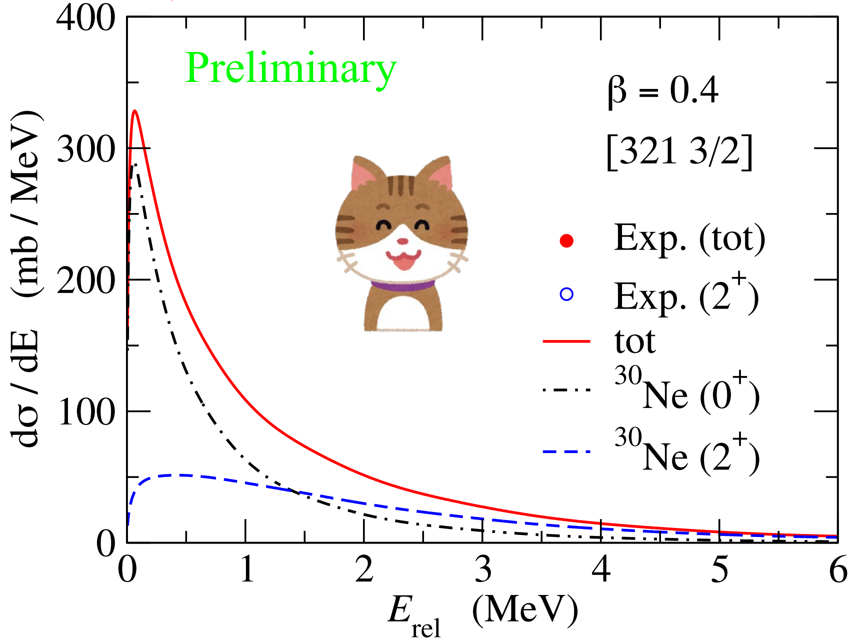
K. Hagino, T. Nakamura, and J.A. Tostevin (2025).

0^+ x $p_{3/2}$: 55.8%
2^+ x $p_{3/2}$: 11.7%
2^+ x $p_{1/2}$: 5.01%
2^+ x $f_{7/2}$: 27.0%





0^+ x $p_{3/2}$: 16.7%
2^+ x $p_{3/2}$: 29.4%
2^+ x $p_{1/2}$: 15.5%
2^+ x $f_{7/2}$: 38.3%



cf. Def. WS + a.m. projection: X. Lu, H. Sagawa, and S.-G. Zhou, PRC111, 064320 (2025) → a talk by Lu on Friday

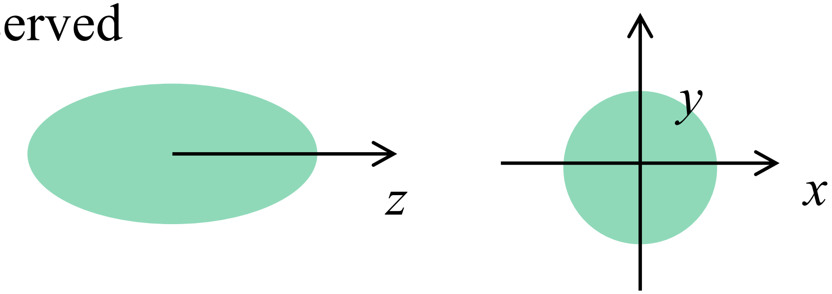
Role of tri-axial deformation in deformed halo

K. Uzawa, K. Hagino, and K. Yoshida, Phys. Rev. C104, L011303 (2021).

✓ axial symmetric deformation: $\beta \neq 0, \gamma = 0 \rightarrow j_z$: conserved

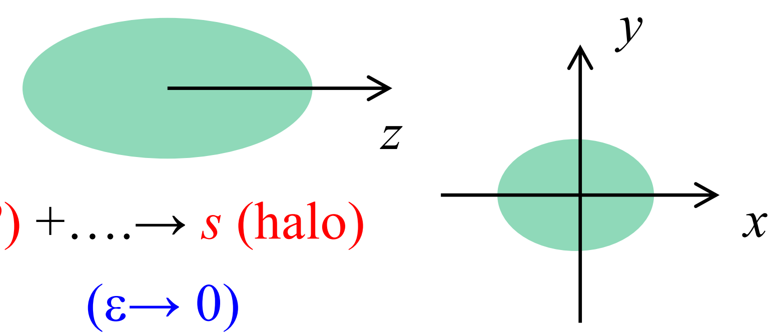
for $j_z = 1/2, \pi = +$: $d \xrightarrow[\beta]{} d + s + g \xrightarrow[(\varepsilon \rightarrow 0)]{} s$ (halo)

for $j_z = 3/2, \pi = +$: $d \xrightarrow[\beta]{} d + g \rightarrow$ *non*-halo



✓ **tri-axial deformation: $\beta \neq 0, \gamma \neq 0 \rightarrow j_z$: mixture**

$\pi = +$: $d\,(j_z=3/2) \xrightarrow[\beta,\gamma]{} d\,(j_z=3/2) + d\,(j_z=1/2) + s\,(j_z=1/2) + \ldots \xrightarrow[(\varepsilon \rightarrow 0)]{} s$ (halo)

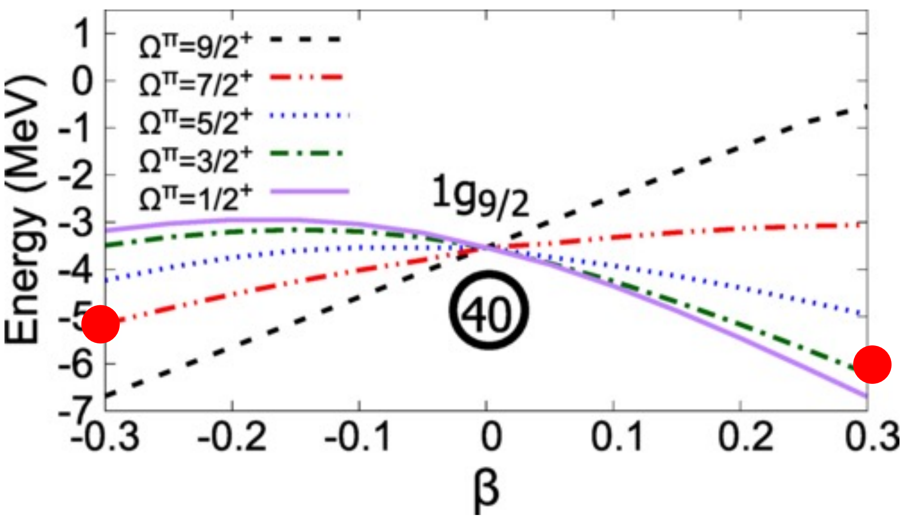


the region of halo nuclei will be extended by taking into account triaxiality

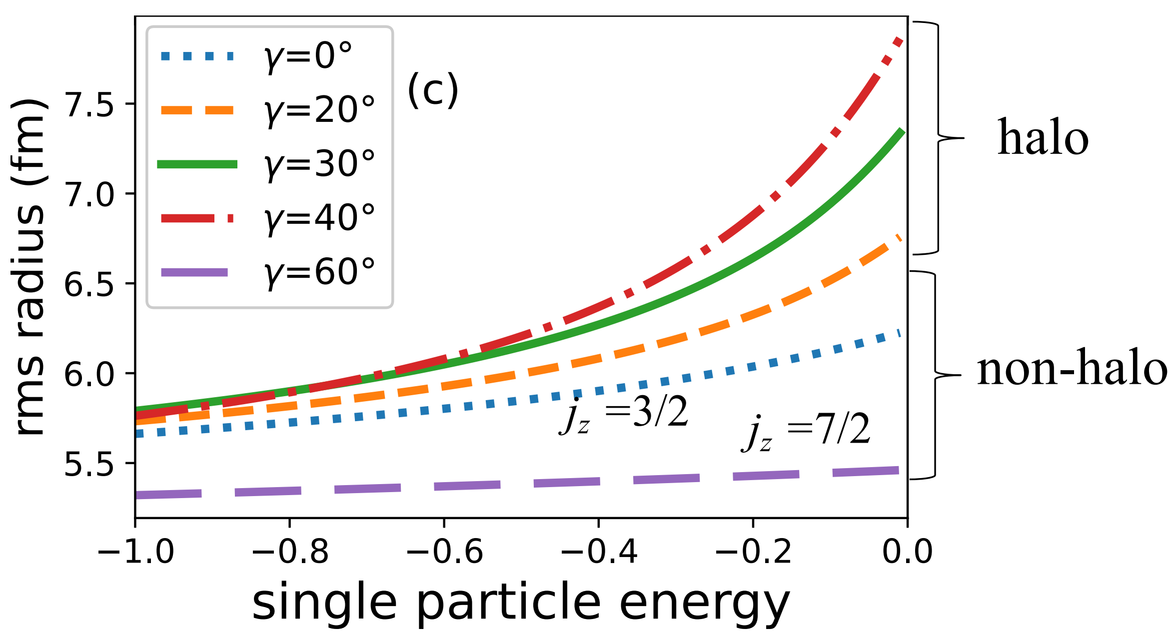
Role of tri-axial deformation in deformed halo

K. Uzawa, K. Hagino, and K. Yoshida, Phys. Rev. C104, L011303 (2021).

deformed Woods-Saxon (γ=0)



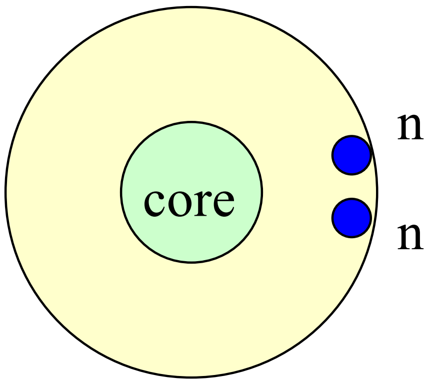
rms radius (β=0.3, N=43)



cf. a self-consistent Relativistic Hartree-Bogoliubov for ^{42}Al:
K.Y. Zhang, S.Q. Zhang, and J. Meng, Phys. Rev. C108, L041301 (2023).

→ a talk by Zhang in this session

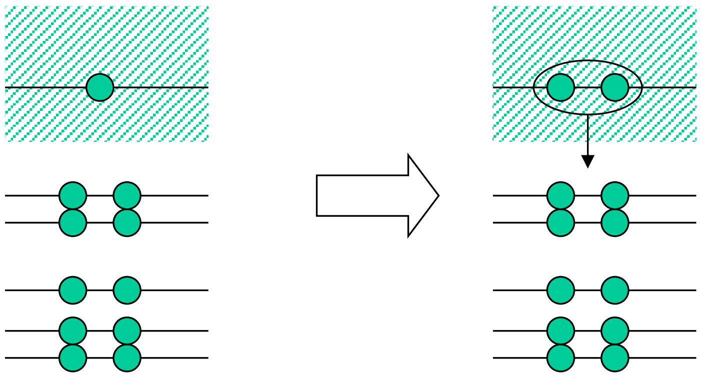
II. Two-neutron halo nuclei



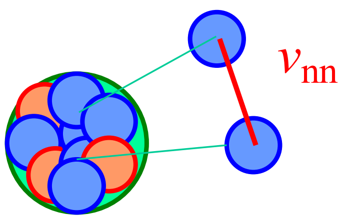
Borromean nuclei and di-neutron correlation

How are two neutrons correlated in Borromean nuclei?
→ di-neutron correlation

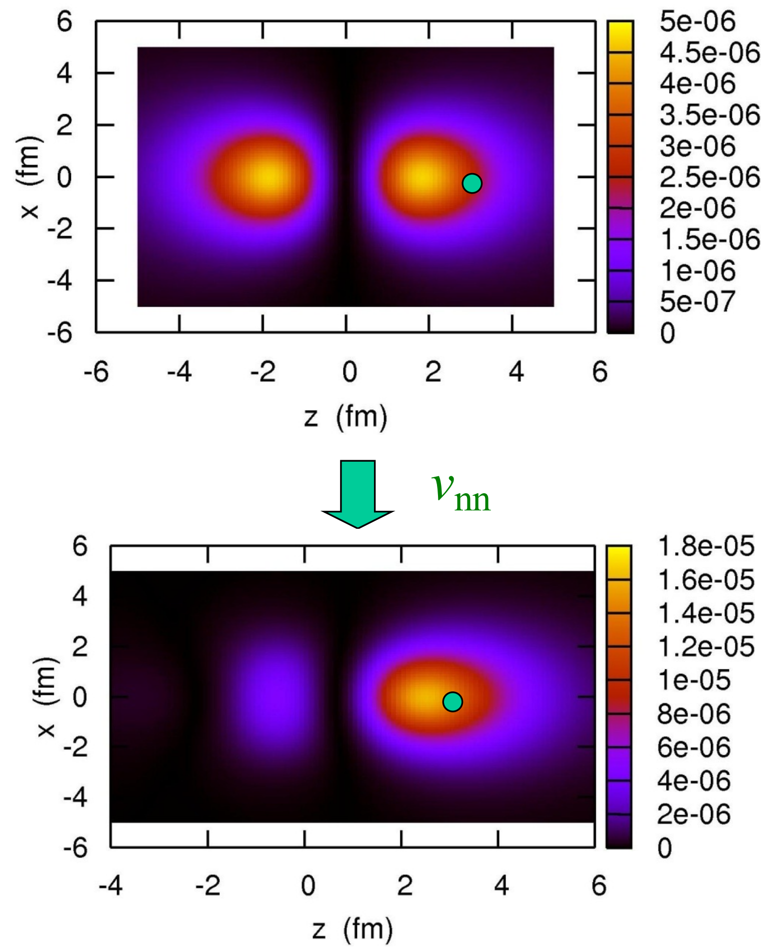
residual interaction → attractive



particle unstable   particle stable



3-body model calculation for ^{11}Li

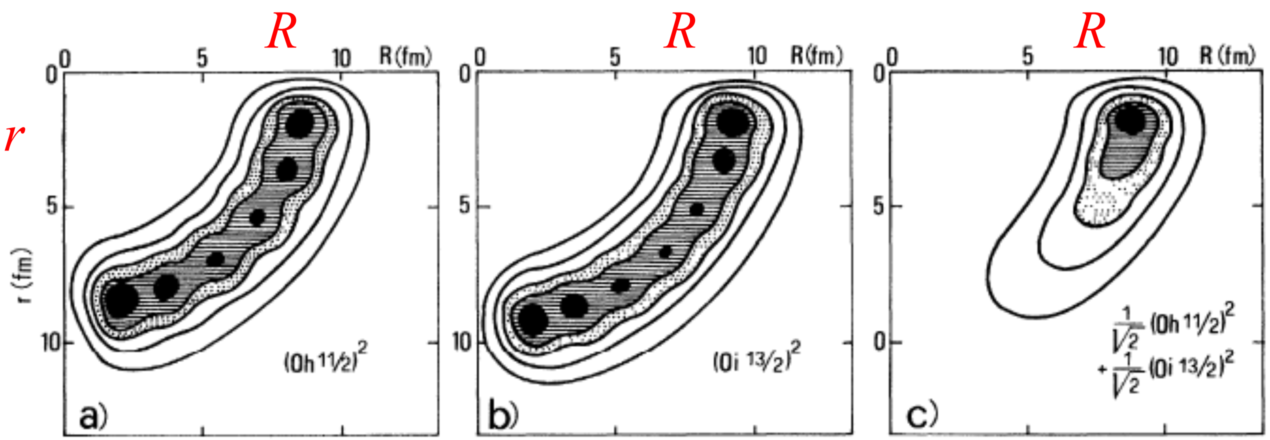


K. Hagino and H. Sagawa, PRC72 (2005) 044321

Borromean nuclei and di-neutron correlation

an essence :
admixture of configurations with
opposite parities

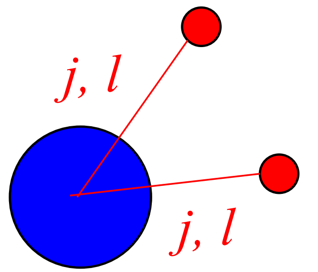
$$\Psi(\boldsymbol{r}_1, \boldsymbol{r}_2) = \sum_{j,l} C_{jl} \left[\phi_{jl}(\boldsymbol{r}_1) \phi_{jl}(\boldsymbol{r}_2) \right]^{(00)}$$

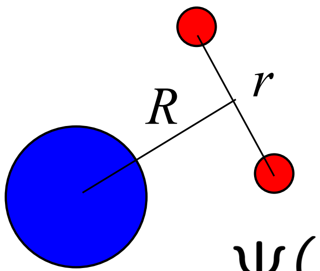


$(0h_{11/2})^2$ $\quad$ $l = 5$

$(0i_{13/2})^2$ $\quad$ $l = 6$

$\frac{1}{\sqrt{2}}(0h_{11/2})^2 + \frac{1}{\sqrt{2}}(0i_{13/2})^2$





$$\Psi(\boldsymbol{r}_1, \boldsymbol{r}_2) \to \Psi(\boldsymbol{r}, \boldsymbol{R})$$

F. Catara, A. Insolia, E. Maglione,
and A. Vitturi, PRC29 (1984) 1091

Analysis of the Coulomb breakup of ^{22}C

- ✓ Y. Togano et al., PLB761, 412 (2016)

  $\sigma_R \rightarrow$ rms radius = 3.44(8) fm
  $S_{2n} = 0.56^{+0.27}_{-0.20}$ MeV

- ✓ N. Kobayashi et al., PRC86, 054606 (2012).

  momentum distribution $\rightarrow$ a large s-d mixture
  S_{2n} = 0.40-1.20 MeV

- ✓ N. Nakatsuka, T. Nakamura et al. in preparation

  exclusive Coulomb breakup cross sections ⇐

3-body model calculation

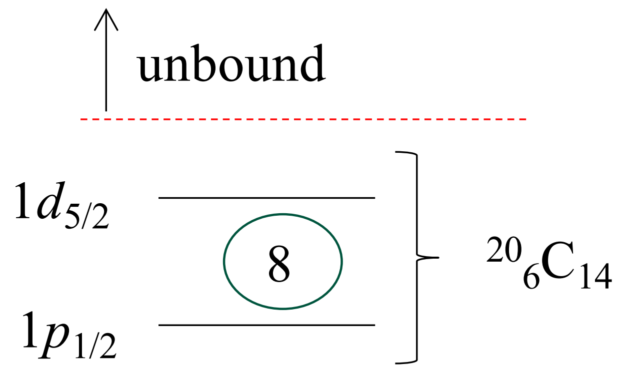
G.F. Bertsch and H. Esbensen, Ann. of Phys. 209 (1991) 327.
K. Hagino and H. Sagawa, PRC72 (2005) 044321.

V_{nn} : a density-dependent contact interaction

3-body model calculation for ^{22}C

$$^{22}_{6}C_{16} = {}^{20}_{6}C_{14} + n + n$$

spherical core

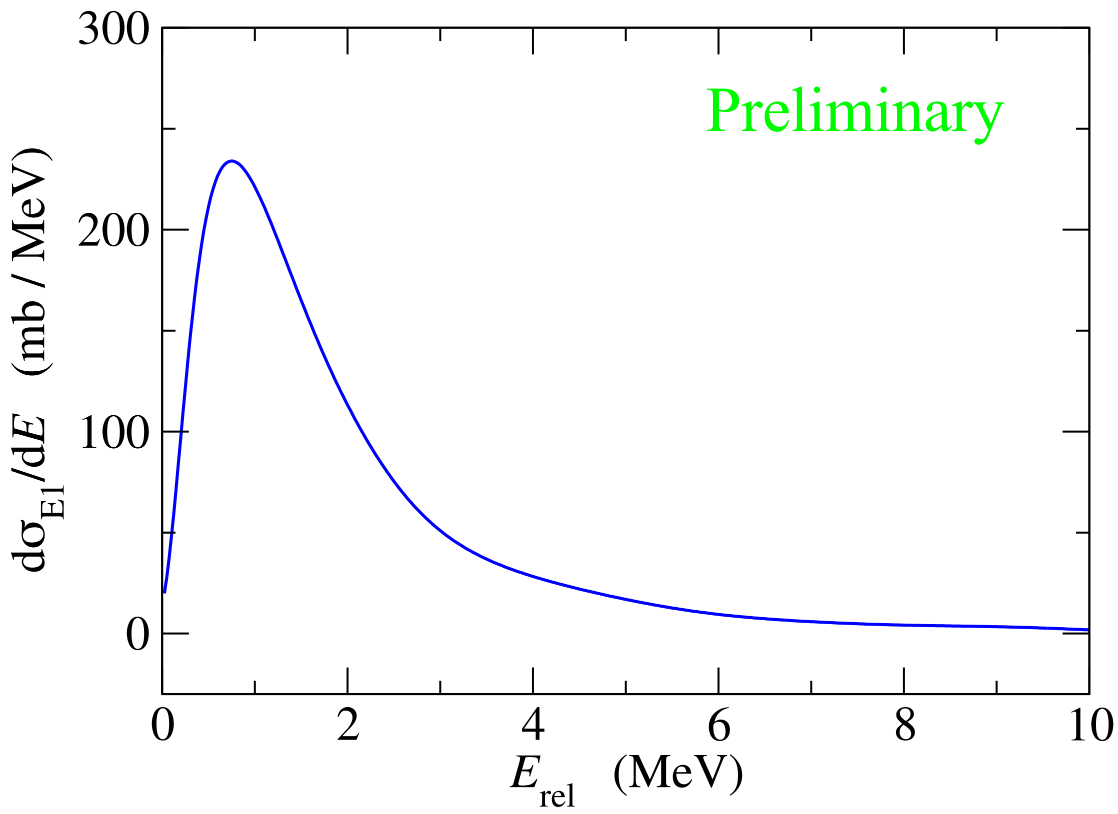


V_{nC} : Woods-Saxon

change V_0 for $l = 0$ to adjust a_{nC}

$\rightarrow$ adjustable parameters: a_{nC} and S_{2n}

Analysis of the Coulomb breakup of ^{22}C



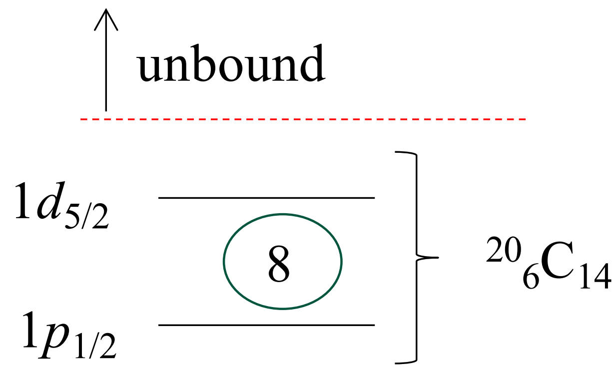
N. Nakatsuka, T. Nakamura et al. in preparation
T. Nakamura, a talk in this symposium

rms radius = 3.20 fm [Exp.: 3.44 (8) fm]

3-body model calculation for ^{22}C

$^{22}_{6}C_{16} = {}^{20}_{6}C_{14} + n + n$

spherical core



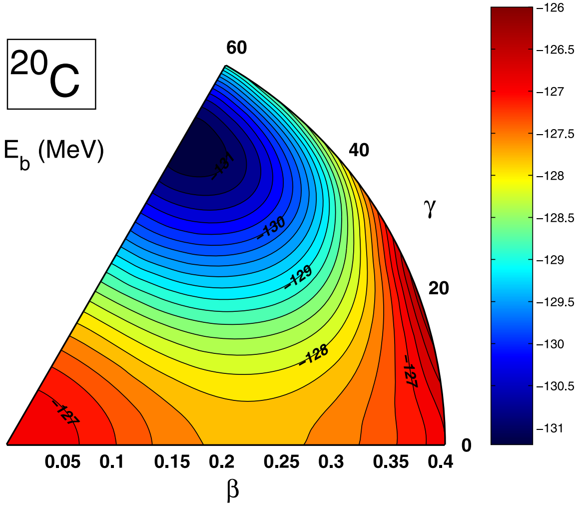
V_{nC} : Woods-Saxon

change V_0 for $l = 0$ to adjust a_{nC}

→ adjustable parameters: a_{nC} and S_{2n}

Analysis of the Coulomb breakup of ^{22}C

Open issue: is ^{20}C spherical?



Y. Zhang et al., PTP120, 129 (2008).

Skyrme HF+BCS
$\rightarrow$ oblate deformation

$\rightarrow$ open $1d_{5/2}$ orbital

Preliminary

<u>3-body model calculation for ^{22}C</u>

$${}^{22}_{6}\mathrm{C}_{16} = {}^{20}_{6}\mathrm{C}_{14} + \mathrm{n} + \mathrm{n}$$

spherical core

$\uparrow$ unbound

$1d_{5/2}$

8

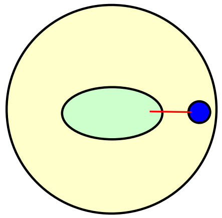
$1p_{1/2}$

${}^{20}_{6}\mathrm{C}_{14}$

N. Nakatsuka, T. Nakamura et al.
in preparation

Skyrme HF+RPA (but no pairing)
T. Inakura et al., PRC89 ('14) 064316.

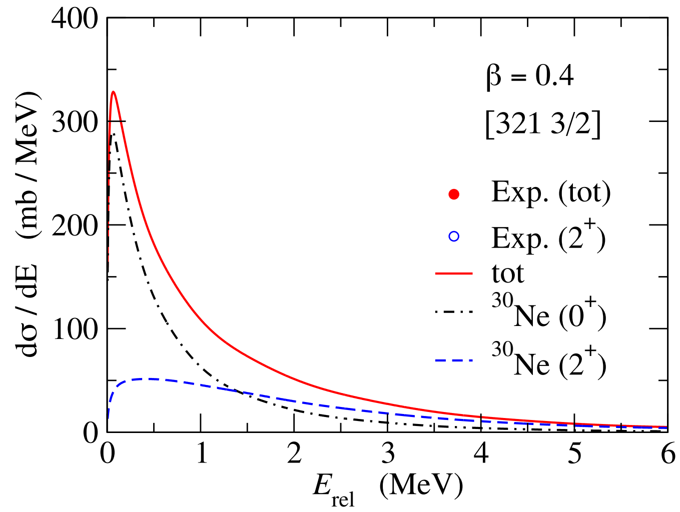
Summary



I. deformed halo nuclei

- a particle-rotor model for the Coul-b.u. of ^{31}Ne
  → configuration of the valence neutron
- role of triaxial deformation
  → an extension of the range of halo nuclei

cf. K. Uzawa et al., Phys. Rev. C104, L011303 (2021).
K.Y. Zhang et al., Phys. Rev. C108, L041301 (2023).



II. Coulomb breakup of ^{22}C

- 3-body model calculation
- an open issue (theory): a small $(s_{1/2})^2$ component

Future perspective

2n-halo nuclei with a deformed core

→ a rotor + 2 neutrons?

maybe important in ^{22}C (a deformed ^{20}C)